Wah-wah

VCarrara

Definition

The Wah-wah effect changes the tone of the input signal in response to a rocking potentiometer. The circuit is based on a second order resonant filter (RCL – Resistor, Capacitor, Inductor), in low-pass filter configuration. A variable capacitor should adjust the resonant and cutoff frequency, but it is expensive and also large. To overcome this problem, normally the variable capacitor is replaced by a Miller effect capacitor circuit, which feeds the signal back through the capacitor by means of two transistors. Increasing the current in the capacitor has the same effect as an increase in its capacitance. The current is modulated by an action potentiometer (pedal), and therefore changes the resonant frequency. Although the Wah-wah circuit is rather complex, to simulate it a simple RCL resonant circuit is necessary, with a variable capacitor, as can be seem in Figure 1.

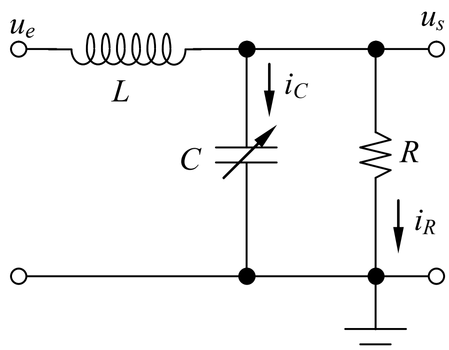

Fig. 1 – RCL resonant circuit of a wah-wah pedal.

The elementary equations in the Z transform domain are:

$$U_s = R\, I_R\,,$$

$$U_s = \frac{T}{2C}\frac{1+z^{-1}}{1-z^{-1}} I_C\,,$$

$$I_C + I_R = \frac{T}{2L}\frac{1+z^{-1}}{1-z^{-1}}(U_e - U_s)\,,$$

for the resistor, capacitor and inductor, respectively. This set of 3 equations can be solved for the input and output signals, resulting in

$$U_s = \frac{B_2 z^{-2} + B_1 z^{-1} + B_0}{A_2 z^{-2} + A_1 z^{-1} + A_0} U_e\,.$$

in which

$$A_2 = R T^2 - 2 L T + 4 C L R$$

$$A_1 = 2RT^2 - 8CLR$$
$$A_0 = RT^2 + 2LT + 4CLR$$
$$B_2 = RT^2$$
$$B_1 = 2RT^2$$
$$B_0 = RT^2$$

where T is the sampling rate period ($T = 1/f_s$). In time domain the transfer function becomes

$$u_s(k) = b_0\, u_e(k) + b_1\, u_e(k-1) + b_2\, u_e(k-2) - a_2\, u_s(k-2) - a_1\, u_s(k-1)\,.$$

where the coefficients are obtained by

$$a_2 = \frac{A_2}{A_0} = \frac{RT^2 - 2LT + 4CLR}{RT^2 + 2LT + 4CLR}$$
$$a_1 = \frac{A_1}{A_0} = \frac{2RT^2 - 8CLR}{RT^2 + 2LT + 4CLR}$$
$$b_2 = \frac{B_2}{A_0} = \frac{RT^2}{RT^2 + 2LT + 4CLR}$$
$$b_1 = \frac{B_1}{A_0} = \frac{2RT^2}{RT^2 + 2LT + 4CLR}$$
$$b_0 = \frac{B_0}{A_0} = \frac{RT^2}{RT^2 + 2LT + 4CLR}$$

The capacitance can be linearly adjusted in software by means of

$$C = C_0 + C_1\, f_{lfo}(t)$$

where C_0 and C_1 are constants and f_{lfo} ($0 \le f_{lfo} \le 1$) can be obtained by a low frequency oscillator (LFO), by reading a potentiometer attached on the pedal, or by a function generator. Simulations performed on Circuit Simulator from Falstad[1] had shown that the bias current in the capacitor changes its capacitance from 15 nF minimum, up to something close to 110 nF, therefore resulting in C_0 = 15 nF, and C_1 = 95 nF, approximately. The RCL circuit of the Wah-wah Vox pedal uses a 33 kOhms resistor, a 10 nF capacitor, and a 600 mH inductor. The Bode diagram of the response shows a peak at the circuit's resonant frequency, which changes with the action of the potentiometer attached to the Miller circuit. A higher capacitor causes a reduction in the resonant frequencies. According to Electrosmash's Vox page[2], the resonant frequencies of the Vox V847 wah-wah range from 450 to 1.6 kHz, as seen in the Bode diagram of Figure 2.

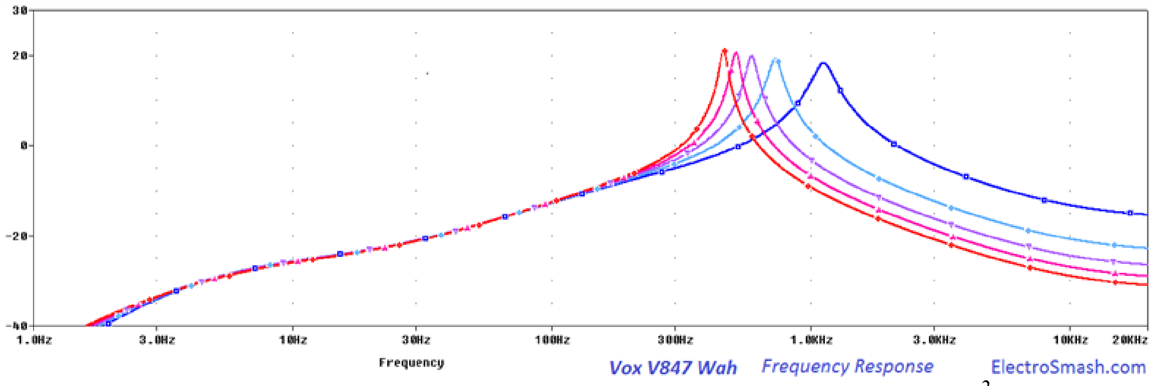

Fig. 2 – Bode diagram of the Vox V847 Wah-wah pedal (Source: [2]*)*

However, the resonant peaks in the bode diagram of the RCL transfer function show a different interval, ranging from 3.8 kHz up to 10 kHz. The resonant frequencies can be adjusted by changing the RCL values, but the best values based on the output audio are the original ones from the Vox pedal with doubled inductor (L = 1.2 H), whose Bode diagram is presented in Figure 3, for f_{lfo} ranging from 0 (red) to 1 (blue), with 0.25 increments.

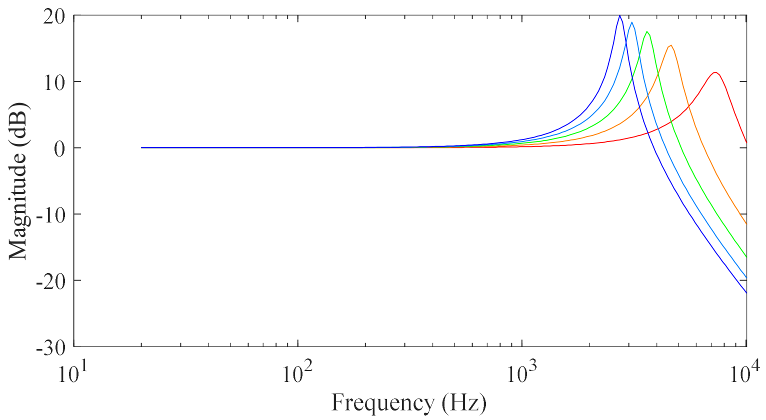

Fig. 3 – Bode diagram of Wah-wah simulator.

Higher values of the capacitor reduce the resonant frequencies and increase the height of peaks. On the other hand, by raising the inductor the frequencies also decrease, together with the peaks. In other words, an increasing of both the inductor and capacitor can lower the resonant frequencies while keeping the resonant peaks almost unchanged. As consequence, wah-wah tone becomes lower. The resistor R affects the peak height, by increasing the peaks for higher values of the resistance, still leaving the resonant frequency constant. This makes the wah-wah output louder, with larger frequency range. Based on these results, a new approach to compute the coefficients of the transfer function is proposed:

$$C_1 = f_{CL}\, C_v\,,$$
$$L = f_{CL}\, L_v\,,$$
$$R = f_R\, R_v\,,$$

where f_{CL} and f_R are gains to increase or decrease the wah-wah tone and effect depth, respectively. Low values of f_{CL} increase high frequencies on output, and high values of f_R increase the depth. It was noted that changing C_0 produces only small variations on the wah-wah output, so it can remain static. With the before mentioned values of the resistor, capacitor and inductor, f_{CL} can be adjusted between 0 (no effect) up to 4. On the other hand f_R shall be kept in the range between 0.2 to 4, although high values tend to promote distortion due to saturation on audio signal.

References

[1] Falstad. Circuit Simulator. Available at: <https://www.falstad.com/circuit/>, 2023.

[2] Electrosmach. Vox V847 Wah-Wah Analysis. Available at: <https://www.electrosmash.com/vox-v847-analysis>, 2023.