Equalizer

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Definition

An equalizer adjusts the response frequency using audio filters to change the spectral characteristics of the audio input. Frequency ranges can be emphasized or attenuated using low-pass, high-pass, band-pass or band-stop (notch) filters[1]. Equalization is mostly used to improve the tonal quality of a recording or instrument, although filtering some undesirable frequencies is also highly employed on stage and recording sessions.

Three-band equalizer

The model presented here is a parametric three-band equalizer, based on first-order filters, with a low-pass, followed by a band-pass and high-pass passive filters. Equalizers with four or more bands are possible by adding band-pass filters in between, as well as other types of filters, like Butterworth, second-order, active, etc. The three-band equalizer can be mathematically modeled with resistors and capacitors, with all filters in parallel. The equalizer can also have all filters in series, with different formulation. The band-pass filter is easily constructed joining a low-pass filter and a high-pass filter in series, which results in a second-order filter. Therefore, there are 4 filters in the three-band equalizer: two low-pass and two high-pass filters. Figure 1 shows the electric circuits of a low-pass (C_l, R_l) and high-pass (C_h, R_h) filters.

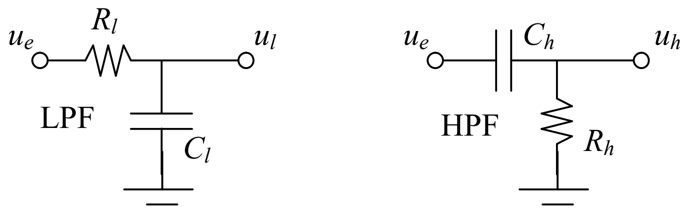

Fig. 1 – Low-pass filter (LPF) and high-pass filter (HPF) circuits.

The Z transform of a resistor R and capacitor C are, respectively

$$U(z) = R\, I(z)$$
$$U(z) = \frac{T}{2C}\frac{1+z^{-1}}{1-z^{-1}} I(z),$$

where T is the audio sampling period, and u and i are the voltage and current on the resistor or capacitor, respectively.

Low-pass filter

The equations for the low-pass filter (LPF) is expressed using the Z transform formulas for the resistor and capacitor, resulting in:

$$U_l = \frac{T}{2C_l}\frac{1+z^{-1}}{1-z^{-1}} I\,,$$
$$U_e = U_l + R_l\, I\,.$$

The transfer function for the low-pass filter can now be obtained, resulting in

$$U_l = \frac{T\,z^{-1} + T}{(T - 2R_l\,C_l)\,z^{-1} + T + 2R_l\,C_l} U_e\,,$$

or, when presented in time domain:

$$u_l(k) = b_{l0}\,u_e(k) + b_{l1}\,u_e(k-1) - a_{l1}\,u_l(k-1)\,,$$

and whose diagram is presented in Figure 2, in which

$$a_{l0} = T + 2R_l\,C_l$$
$$a_{l1} = \frac{T - 2R_l\,C_l}{a_{l0}}$$
$$b_{l0} = \frac{T}{a_{l0}}$$
$$b_{l1} = \frac{T}{a_{l0}}$$

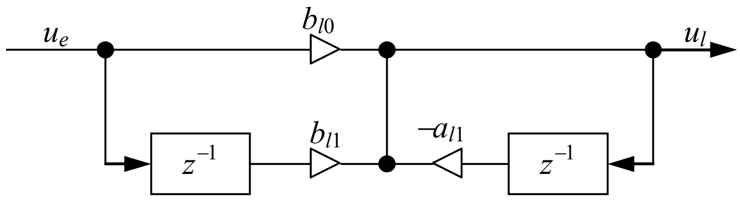

Fig. 2 – Z transform block diagram of a low-pass filter.

High-pass filter

The elementary equations in Z transform of the high-pass filter (HPF) are:

$$U_e = U_h + \frac{T\,(1+z^{-1})}{2C_h\,(1-z^{-1})} I\,,$$
$$U_h = R_h\, I\,,$$

which results in the following transfer function

$$U_h = \frac{2R_h\,C_h\,z^{-1} - 2R_h\,C_h}{(2R_h\,C_h - T)\,z^{-1} - 2R_h\,C_h - T} U_e\,,$$

that can be transformed in time domain resulting in

$$u_h(k) = b_{h0}\, u_e(k) + b_{h1}\, u_e(k-1) - a_{h1}\, u_h(k-1)$$

where the coefficients are given by

$$a_{h0} = T + 2\,R_h\,C_h\,,$$
$$a_{h1} = \frac{T - 2\,R_h\,C_h}{a_{h0}}\,,$$
$$b_{h0} = \frac{2\,R_h\,C_h}{a_{h0}}\,,$$
$$b_{h1} = -\frac{2\,R_h\,C_h}{a_{h0}}\,.$$

The block diagram of the transfer function of the high-pass filter can be seen in Figure 3.

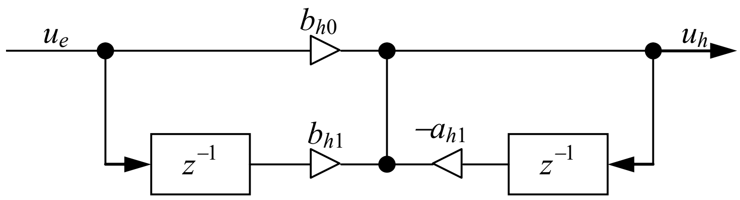

Fig. 3 – Block diagram in Z transform of a high-pass filter.

Band-pass filter

The band-pass filter (BPF) is composed by high-pass and low-pass circuits in series as shown in Figure 4. Although both circuits are connected by u_i, it is assumed that the current flowing through this interface can be neglected, which significantly simplifies the equations without compromising the filter effectiveness.

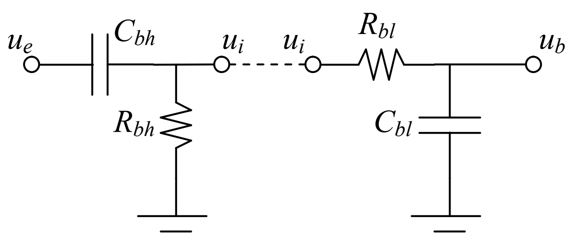

Fig. 4 – Band-pass filter circuit.

As usual, the elementary equations for this filter in Z transform are:

$$U_e = \frac{T\,(1+z^{-1})}{2\,C_{bh}\,(1-z^{-1})}\, I_h + U_i\,,$$
$$U_i = R_{bh}\, I_h\,,$$
$$U_i = R_{bl}\, I_l + U_b\,,$$
$$U_b = \frac{T\,(1+z^{-1})}{2\,C_{bl}\,(1-z^{-1})}\, I_l\,,$$

which were solved by eliminating both currents and the voltage U_i, resulting in the second order transfer function:

$$U_b = \frac{B_{b2}\, z^{-2} + B_{b0}}{A_{b2}\, z^{-2} + A_{b1}\, z^{-1} + A_{b0}} U_e \,,$$

where the coefficients are given by

$$A_{b0} = -2T\, R_{bl}\, C_{bl} - 2T\, R_{bh}\, C_{bh} - T^2 - 4\, R_{bl}\, C_{bl}\, R_{bh}\, C_{bh} \,,$$
$$A_{b1} = 8\, R_{bl}\, C_{bl}\, R_{bh}\, C_{bh} - 2T^2 \,,$$
$$A_{b2} = 2T\, R_{bl}\, C_{bl} + 2T\, R_{bh}\, C_{bh} - T^2 - 4\, R_{bl}\, C_{bl}\, R_{bh}\, C_{bh} \,,$$
$$B_{b2} = 2T\, R_{bh}\, C_{bh} \,,$$
$$B_{b1} = 0 \,,$$
$$B_{b0} = -2T\, R_{bh}\, C_{bh} \,.$$

The transfer function can now be expressed in time domain, resulting

$$u_b(k) = b_{b0}\, u_e(k) + b_{b1}\, u_e(k-1) + b_{b2}\, u_e(k-2) - a_{b1}\, u_b(k-1) - a_{b2}\, u_b(k-2) \,,$$

and such that

$$a_{b0} = A_{b0} \,,$$
$$a_{b1} = A_{b1} \,/\, A_{b0} \,,$$
$$a_{b2} = A_{b2} \,/\, A_{b0} \,,$$
$$b_{b0} = B_{b0} \,/\, A_{b0} \,,$$
$$b_{b1} = B_{b1} \,/\, A_{b0} \,,$$
$$b_{b2} = B_{b2} \,/\, A_{b0} \,.$$

The block diagram of the transfer function can be seen in Figure 5.

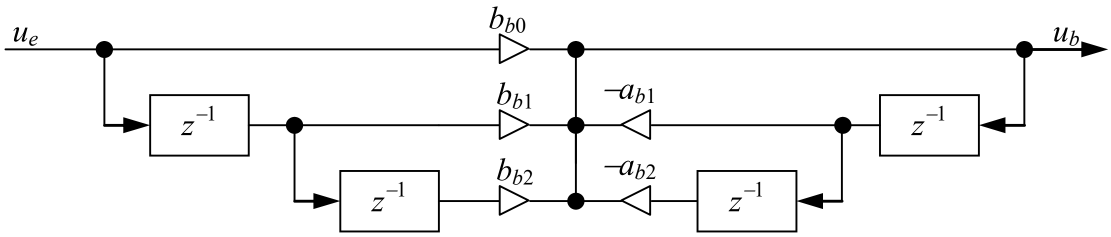

Fig. 5 – Block diagram of the band-pass filter.

Parametric Equalizer

In the three-band equalizer it is assumed that the output response is given by a linear combination from each filter such that

$$u_s = \alpha_l\, u_l + \alpha_b\, u_b + \alpha_h\, u_h \,,$$

where each α_i ($i = l$, b or h) can be understood as a filter regulator ($0 \leq \alpha_i \leq 1$), similar to a volume potentiometer. The block diagram of the Equalizer is shown in the Figure 6.

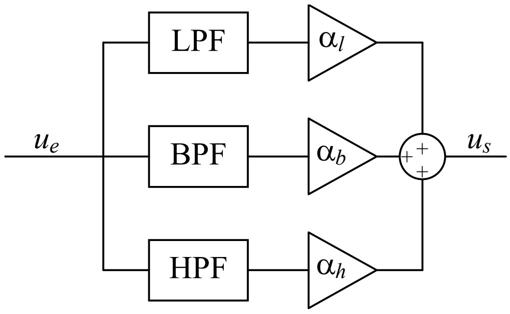

Fig. 6 – Block diagram of the three-band equalizer.

The cutoff frequency of a filter is given by:

$$f_c = \frac{1}{2\pi RC},$$

which can be solved to express the *RC* product as a function of the cutoff frequency. So, instead specifying each resistor or capacitor, it is preferable to choose the cutoff frequency for each filter. Moreover, the cutoff frequency of the LPF shall be equal to the cutoff frequency of the high-pass circuit of the BPF. In other words, $R_l\, C_l = R_{bh}\, C_{bh}$. Similarly, the cutoff frequency of the HPF shall be identical to the cutoff frequency of the low-pass circuit of the BPF ($R_h\, C_h = R_{bl}\, C_{bl}$). If f_l and f_h are the cutoff frequencies for the LPF and HPF of the 3-band Equalizer, respectively, then

$$R_l\, C_l = R_{bh}\, C_{bh} = \frac{1}{2\pi f_l}$$
$$R_h\, C_h = R_{bl}\, C_{bl} = \frac{1}{2\pi f_h}$$

The Bode diagram of the transfer function can be seen in the Figure 7, for cutoff frequencies of $f_l = 100$ Hz and $f_h = 1000$ Hz, and for regulators α of 0.1 (blue), 0.5 (green) and 1 (red). Although the filters change the phase of the audio input, it is considered that this effect is hardly perceived in normal auditions.

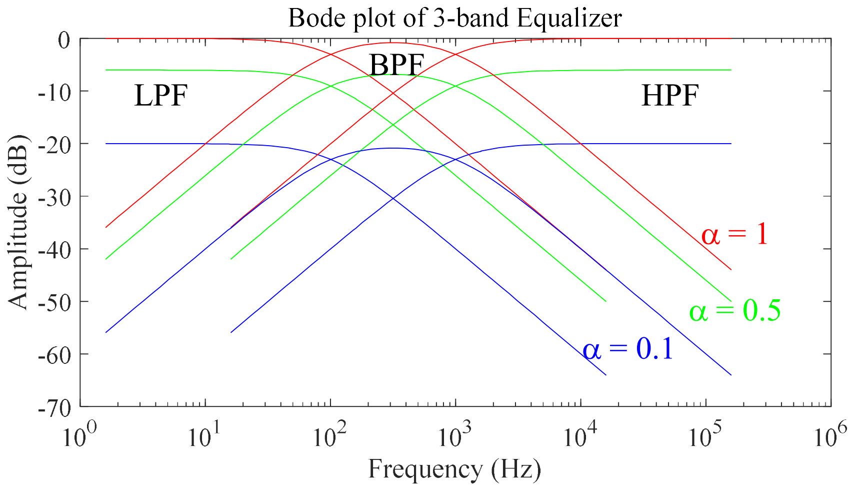

Fig. 7 – Bode diagram (amplitude only) of the three-band equalizer.

References

1 Wikipedia. Equalizer (Audio). Available at: <https://en.wikipedia.org/wiki/Sound_effect>, 2023.