Overdrive (Distortion)

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Definition

Distortion, sometimes also called Overdrive or Fuzz, can produce distorted sounds with increasing loudness. Basically a distortion effect amplifies the input audio, following by clipping the signal up to a preselected level.

The clipping process in digital sound processing is usually made with hard clipping, i.e., just saturating the absolute input to a given maximum value. However, this easy procedure produces high and undesirable 3rd order harmonics on the original sound. Other clipping functions, like logarithm and arc tangent are largely employed but they also have some drawbacks similar to the hard clipping. These problems drove the solution to choose a distortion algorithm that mimics some existing analog distortion. It was choose the Big Muff (BM) which gives smooth transition between soft and hard distortion.

Big Muff Distortion

The Big Muff has three stages: a driver or pre-amplifier, 2 steps of distortion and a filter (tone) section. This pedal has also three control buttons: Sustain, Volume and Tone. All these controls were also included in the model.

Clipping Stages

Figure 1 shows the first two stages of Big Muff: the driver in red and the first step of distortion in green. The Sustain potentiometer can be seen at driver's output. The distortion stage presents two anti parallel diodes for audio clipping. A simulation of this circuit has shown that the 470 uF capacitor can be removed from the circuit without loss. However even considering this simplification, the distortion stage is still too complex to be simulated in real time in a microcontroller.

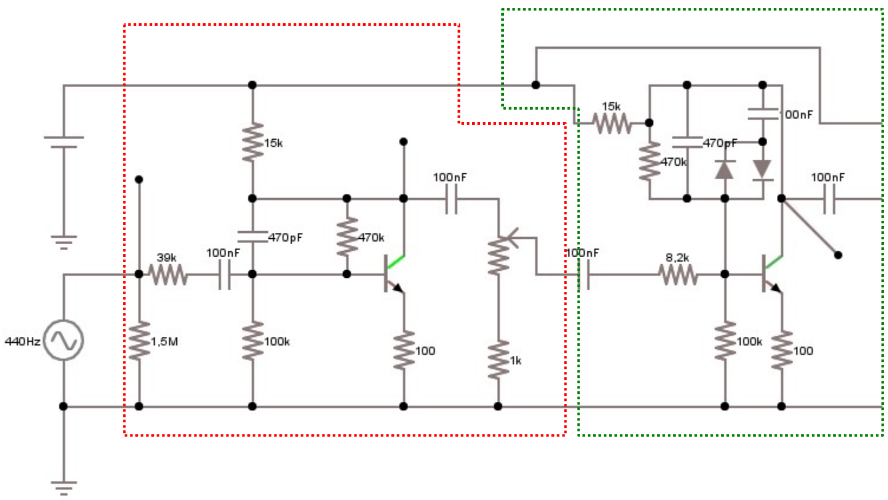

Fig. 1 – Big Muff's input driver (red) and first clipping stage (green)

The simplest circuit that can be built with a pair of clipping diodes is the one shown in Figure 2, based on the Big Muff pedal. The diodes are placed in series with a resistor and capacitor, which act as a low-pass filter. It is evident that the diodes cannot be modeled as being ideal, because ideal diodes act like hard limiter, so having the same problems of hard clipping, and, therefore, loses important characteristics of dynamics. The main effect that is desired is the non-linear characteristic presented by the diodes.

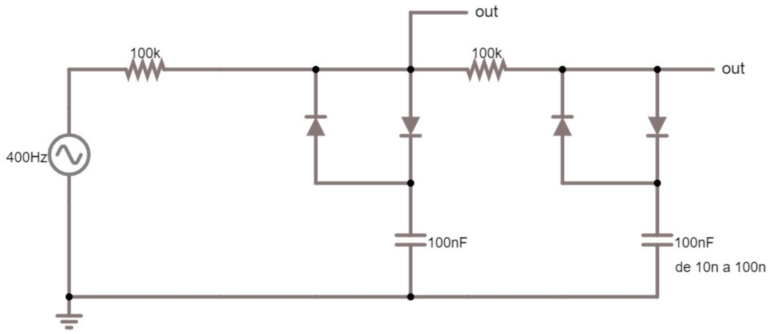

Fig. 2 – A simple circuit of two-stage diode clipping.

BM uses 1N914 diode, which can be modeled using the *I* x *V* curve from the datasheet, giving

$$i_d = 10^{(10v_d - 9)},$$

where v_d is the voltage (V) applied on the diode and i_d is the current in Amperes. It is assumed that the reverse current is negligible. From this model it can be easily seem that a non-zero current flows through the diode even when the applied voltage is null. To correct this anomaly, this model is slightly changed to

$$i_d = 10^{(10v_d - 9)} - 10^{-9}$$

which makes $i_d = 0$ when $v_d = 0$. Since both distortion steps are identical, only one will be modeled. The clipping circuit is shown in Figure 3, comprising the diodes in series with a resistor and a capacitor. The elementary Z-transform for the resistor and capacitor are:

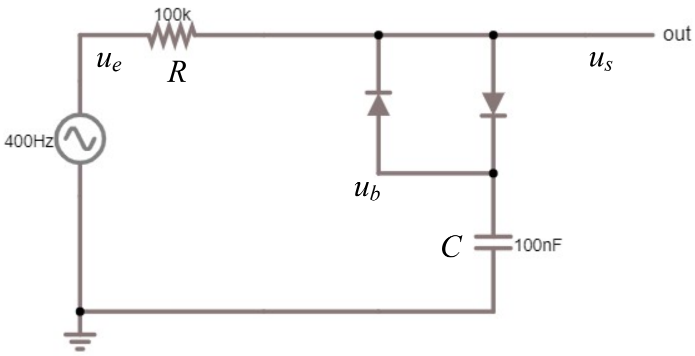

Fig. 3 – Simplified Big Muff's clipping circuit.

$$U_e = R\,I + U_s$$

$$U_b = \frac{T}{2C}\frac{1+z^{-1}}{1-z^{-1}} I$$

where T is the period of the sampling rate. The non linear diode function can only be derived in the inverse Z-transform as function of the applied voltage v_d, which takes to

$$U_s = V_d + U_b$$

The above equations can now be solved for the output voltage U_s as function of the input voltage U_e and the diode voltage V_d, resulting in

$$U_s = T\frac{z^{-1}+1}{(T-2CR)\, z^{-1}+2CR+T}U_e - \frac{2CR\,(z^{-1}-1)}{(T-2CR)\, z^{-1}+2CR+T}V_d\,,$$

and since the diode voltage needs to be computed, the solution for U_b needs also be obtained which results

$$U_b = T\frac{z^{-1}+1}{(T-2CR)z^{-1}+2CR+T}U_e - \frac{T\,(z^{-1}+1)}{(T-2CR)z^{-1}+2CR+T}V_d\,.$$

In time domain, these expressions can be formulated by

$$u_s(k) = a_{1s}\, u_s(k-1) + b_{1s}\, u_e(k-1) + b_{0s}\, u_e(k) + c_{1s}\, v_d(k-1) + c_{0s}\, v_d(k)\,,$$

in which the coefficients are

$$a_{1s} = \frac{2CR-T}{2CR+T}\,,$$
$$b_{1s} = \frac{T}{2CR+T}\,,$$
$$b_{0s} = \frac{T}{2CR+T}\,,$$
$$c_{1s} = -\frac{2CR}{2CR+T}\,,$$
$$c_{0s} = \frac{2CR}{2CR+T}\,,$$

and

$$u_b(k) = a_{1b}\, u_b(k-1) + b_{1b}\, u_e(k-1) + b_{0b}\, u_e(k) + c_{1b}\, v_d(k-1) + c_{0b}\, v_d(k)\,,$$

where

$$a_{1b} = \frac{2CR-T}{2CR+T}\,,$$

$$b_{1b} = b_{0b} = \frac{T}{2CR + T},$$

$$c_{1b} = c_{0b} = -\frac{T}{2CR + T},$$

and such that R = 100 kΩ, C = 100 10^{-9} F, and the sampling period T is 1/44100 sec. In order to include the diode model, from Kirchoff law in time domain the resistor current can be expressed by

$$i_r = \frac{u_e - v_d - u_b}{R},$$

but this current is also equal to the diode current ($i_r = i_d$), which leads to the transcendental equation

$$u_e = v_d + u_b + [10^{(10 v_d - 9)} - 10^{-9}]\, R\,.$$

which is valid only if $u_e > u_b$ (for v_d positive). An example of curves of i_r (blue) and i_d (red) can be seen in Figure 4. By substituting the expression of u_b in the above equation, it yields

$$\begin{aligned}(1 - b_{0b})\, u_e(k) = (1 + c_{0b})\, v_d(k) + [10^{(10 v_d - 9)} - 10^{-9}]\, R + \\ + a_{1b}\, u_b(k-1) + b_{1b}\, u_e(k-1) + c_{1b}\, v_d(k-1)\end{aligned},$$

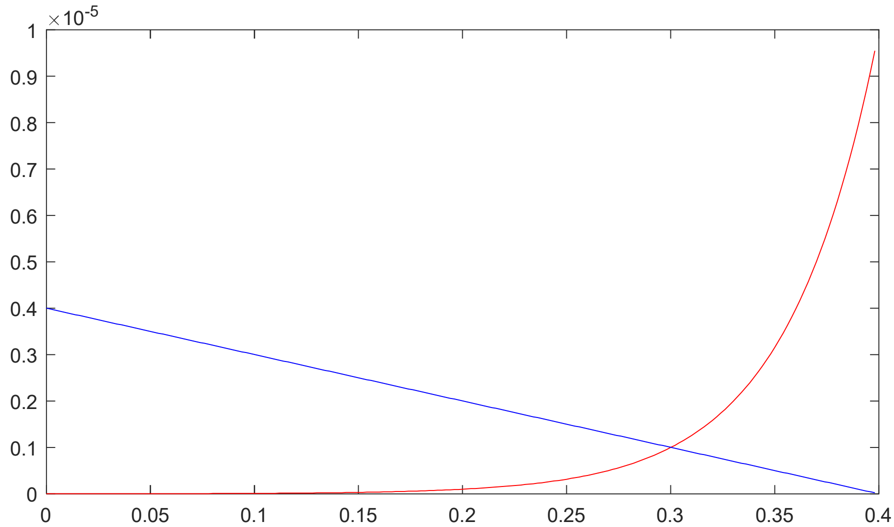

Fig. 4 – Current through resistor (blue) and diode (red).

This expression can be rearranged as function of v_d, resulting

$$f(v_d) = (1 + c_{0b})\, v_d(k) + [10^{[10 v_d(k) - 9]} - 10^{-9}]\, R + q(k) - (1 - b_{0b})\, u_e(k) = 0,$$

where

$$q(k) = a_{1b}\, u_b(k-1) + b_{1b}\, u_e(k-1) + c_{1b}\, v_d(k-1),$$

that can only be solved numerically. By applying a Newton-Raphson method to $f(v_d)$ an iterative solution is obtained:

$$\frac{df}{dv_d} = 1 + c_{0b} + 10\,R\,10^{[10 v_d(k) - 9]}\ln(10)$$

$$v_d^j(k) = v_d^{j-1}(k) - \frac{(1+c_{0b})\,v_d^{j-1}(k) + [10^{[10 v_d^{j-1}(k)-9]} - 10^{-9}]\,R + q(k) - (1-b_{0b})\,u_e(k)}{1 + c_{0b} + 10\,R\,10^{[10 v_d^{j-1}(k)-9]}\ln(10)}$$

where superscript j indicates the number of iterations (j = 0, 1, 2, …). With only three iterations a good approximation of the true value is achieved. However, a single iteration was adopted, to increase algorithm speed. This does not jeopardize the solution, because from an input u_e at time k to the next input at k+1 only small differences occurs in the audio signal, which means that the true value is reached after few audio samples. By adopting $v_d^0(k) \approx v_d^1(k-1)$ the iteration converges quickly, providing changes in signal level remain bounded. The exponential term in the above equation can be provided by a suitable lookup table.

Each clipping stage results in three equations for $u_s(u_e, v_d)$, $v_d(u_e, u_b)$ and $u_b(u_e, v_d)$. However, both u_s and u_b depend on v_d, and v_d is also function of u_b, because u_b is required to know whether v_d is positive or negative. The exact solution for this system is to iterate u_b and v_d, and then using v_d to obtain u_s. In other words, the sequence of computation is to assume some previous value to u_b and v_d (null value, for instance), and then to compute iteratively $v_d^0(k) \rightarrow u_b^0(k) \rightarrow v_d^1(k) \rightarrow u_b^1(k) \rightarrow \ldots$ till the solution reaches a required error. The proposed solution consists of considering $u_b(k) \approx u_b(k-1)$ to define the sign of v_d (positive or negative), and then computing v_d in a single iteration. The following algorithm is then proposed:

1. Initialize variables with

   $$u_s(0) = u_b(0) = v_d(0) = 0\,.$$

2. Starting with $k = 1$, compute

   If $u_e(k) > u_b(k-1)$, make $v_d^0(k) = |v_d(k-1)|$, and compute

   $$q(k) = a_{1b}\,u_b(k-1) + b_{1b}\,u_e(k-1) + c_{1b}\,v_d(k-1)\,,$$

   $$v_d^1(k) = v_d^0(k) - \frac{(1+c_{0b})\,v_d^0(k) + [10^{[10 v_d^0(k)-9]} - 10^{-9}]\,R + q(k) - (1-b_{0b})\,u_e(k)}{1 + c_{0b} + 10\,R\,10^{[10 v_d^0(k)-9]}\ln(10)}\,,$$

   and

   $$v_d(k) = v_d^1(k)\,.$$

Otherwise, if $u_e(k) < u_b(k-1)$, then make $v_d^0(k) = |v_d(k-1)|$, and

$$q(k) = a_{1b}\, u_b(k-1) + b_{1b}\, u_e(k-1) + c_{1b}\, v_d(k-1),$$

$$v_d^1(k) = v_d^0(k) - \frac{(1+c_{0b})\, v_d^0(k) + [10^{[10 v_d^0(k)-9]} - 10^{-9}]\, R - q(k) + (1-b_{0b})\, u_e(k)}{1 + c_{0b} + 10\, R\, 10^{[10 v_d^0(k)-9]} \ln(10)},$$

and

$$v_d(k) = -v_d^1(k).$$

Otherwise, if $u_e(k) = u_b(k-1)$, then $v_d(k) = 0$

3. Now compute the output u_s and update u_b:

$$u_s(k) = a_{1s}\, u_s(k-1) + b_{1s}\, u_e(k-1) + b_{0s}\, u_e(k) + c_{1s}\, v_d^1(k-1) + s_{ub}\, c_{0s}\, v_d^1(k),$$

$$u_b(k) = a_{1b}\, u_b(k-1) + b_{1b}\, u_e(k-1) + b_{0b}\, u_e(k) + c_{1b}\, v_d(k-1) + c_{0b}\, v_d(k)$$

4. For the second clipping stage the variables are initialized with

$u_s(0) = u_b(0) = v_d(0) = 0$, and $u_e(k) \leftarrow u_s(k)$. The above algorithm can be repeated, except that new variables have to be created for u_s, u_b and v_d.

5. Increment k and repeat steps 2, 3 and 4.

This algorithm has been tested and it worked correctly. A comparison with the exact solution (several iterations) showed that the single iteration differs from the exact solution by only 2% of the peak value. This error is most pronounced during changes in the v_d sign. Figure 5 shows the output for a sinusoidal input signal with a frequency of 440 Hz and amplitude of 1 V, with only one stage of distortion. The same signal is presented in Figure 6, but with two identical stages. A gain of 2.5 was applied in the output of the first stage. It can be noted that the second stage makes the waveform even sharper, that is, the second stage increases the distortion level, yet keeping the desired smoothness to decrease the predominance of the odd harmonics.

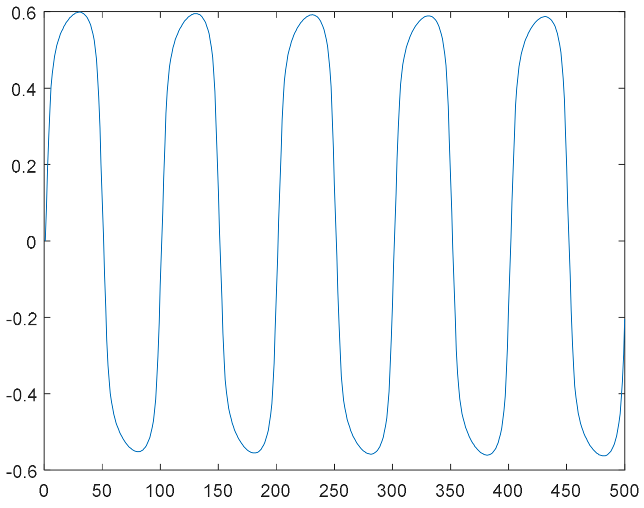

Fig. 5 – First stage clipping of a sinusoidal signal.

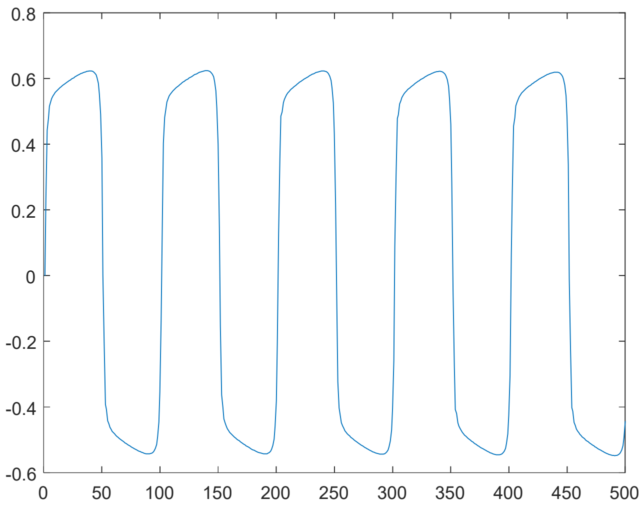

Fig. 6 – First and second clipping stages applied to a sinusoidal signal.

Sustain Level

Despite the promising results, it was later noted that a simple change in the input level by applying a selected gain does not mimic the effect of BM's Sustain potentiometer. The best result was obtained with an input gain given by $g_1 = 3\ (0.95\ s + 0.05)$, a fixed gain of $g_2 = 2$ after the first clipping stage, and a gain of $g_3 = 0.6\ (4 - 2.5\ s)$ after the second clipping stage (before the filter section), where s stands for the Sustain level ($0 \leq s \leq 1$). Some of these adjustments were made by comparing the wave shapes of a sine audio signal on the BM simulator and the real pedal wave on oscilloscope. Figure 7 indicates the gain level at each stage of the digital BM.

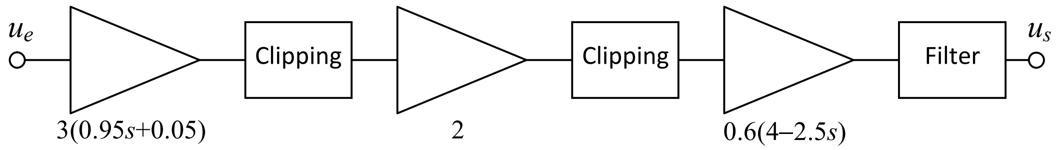

Fig. 7 – Inter stage gains of the Big Muff model.

Filter Stage

The filter stage was based on the circuit of the Big Muff's tone filter. This filter is composed of a low-pass filter (LPF) and a high-pass filter (HPF), whose outputs are combined by a potentiometer, as can be seen in the Figure 8. Each individual filter has one resistor and one capacitor. The input to the filter is the u_e voltage, while the output is the u_s voltage.

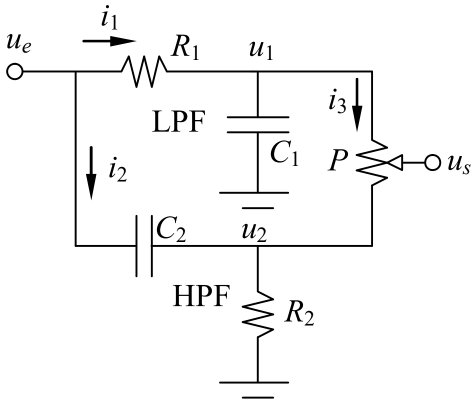

Fig. 8 – Filter circuit of Big Muff.

By applying the Z transform to this circuit, the elementary equations are given by

$$
\begin{aligned}
&U_e - U_1 = R_1\, I_1 \\
&U_e - U_2 = \frac{T}{2C_2}\frac{1+z^{-1}}{1-z^{-1}}\, I_2 \\
&U_1 - U_2 = P\, I_3 \\
&U_2 = R_2\,(I_2 + I_3) \\
&U_1 = \frac{T}{2C_1}\frac{1+z^{-1}}{1-z^{-1}}\,(I_1 - I_3) \\
&U_s = U_1 - \alpha\, P\, I_3
\end{aligned}
\quad ,
$$

in which α is the potentiometer gain ($0 \le \alpha \le 1$), from LPF up to HPF. This set of equations can be solved for U_s as function of U_e, resulting in

$$
U_s = \frac{B_0 + B_1\, z^{-1} + B_2\, z^{-2}}{A_0 + A_1\, z^{-1} + A_2\, z^{-2}} U_e
$$

where

$$B_2 = P + R_2 - (P + R_1) X_2 R_2 + \alpha P (X_1 X_2 R_1 R_2 - 1)$$
$$B_1 = 2P + 2R_2 - 2\alpha P (X_1 X_2 R_1 R_2 + 1)$$
$$B_0 = P + R_2 + (P + R_1) X_2 R_2 + \alpha P (X_1 X_2 R_1 R_2 - 1)$$
$$A_2 = P + R_1 + R_2 - P(X_1 R_1 + X_2 R_2) - (X_1 + X_2) R_1 R_2 + P X_1 X_2 R_1 R_2$$
$$A_1 = 2P + 2R_1 + 2R_2 - 2P X_1 X_2 R_1 R_2$$
$$A_0 = P + R_1 + R_2 + P(X_1 R_1 + X_2 R_2) + (X_1 + X_2) R_1 R_2 + P X_1 X_2 R_1 R_2$$
$$X_1 = \frac{2C_1}{T}$$
$$X_2 = \frac{2C_2}{T}$$

The inverse Z transform now can be performed and yields:

$$u_s(k) = -a_1 u_s(k-1) - a_2 u_s(k-2) + b_0 u_e(k) + b_1 u_e(k-1) + b_2 u_e(k-2),$$

in which $a_i = A_i/A_1$, and $b_i = B_i/A_1$. The resistors and capacitors values adopted here differ a bit from the original BM pedal: $C_1 = 10$ nF, $C_2 = 3.9$ nF, $R_1 = 39$ kΩ, $R_2 = 100$ kΩ, $P = 100$ kΩ, such that the coefficients result in

$\beta_0 = 0.038252490268891$
$\beta_1 = 0.003071673766032$
$\beta_2 = -0.035180816502860$

$\Delta\beta_0 = 0.907850385995433$
$\Delta\beta_1 = -1.818772445756898$
$\Delta\beta_2 = 0.907850385995433$

$a_1 = -1.813565958723474$
$a_2 = 0.820907259024290$

with $b_i = \beta_i + \alpha\, \Delta\beta_i$.

BM Output

Figures 9 and 10 show the output from the BM model and from the analog pedal, respectively. The first graphic was obtained from computation on personal computer. The next figure was captured from an oscilloscope. The input signal was a sine wave for both, simulated and the real pedal. The Sustain control was adjusted to maximum.

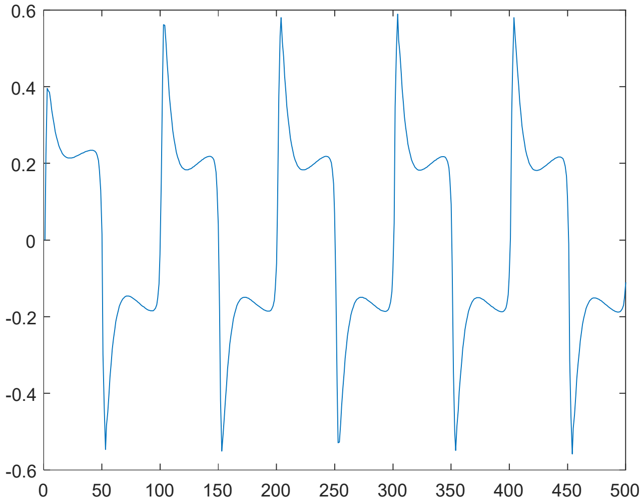

Fig. 9 – Simulated Big Muff output with maximum Sustain level.

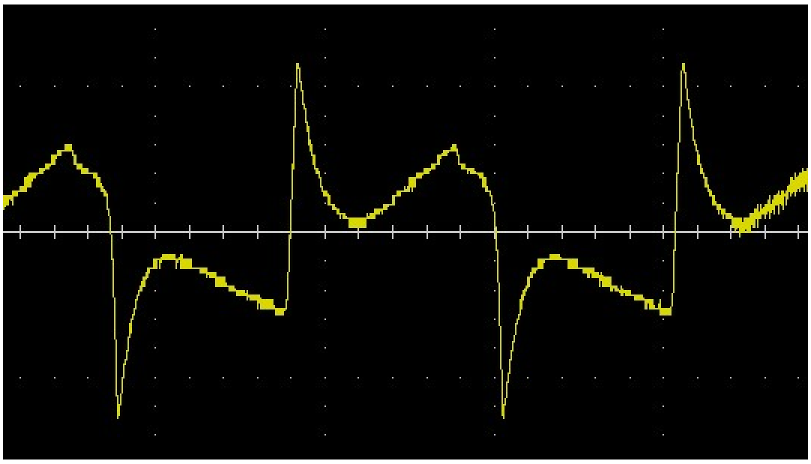

Fig. 10 – Big Muff output of a sine wave input with maximum Sustain level.

Figures 11 and 12 are similar to the previous ones, except that the Sustain level was adjusted to minimum. Again the similarity between the BM model and the pedal is astonishing.

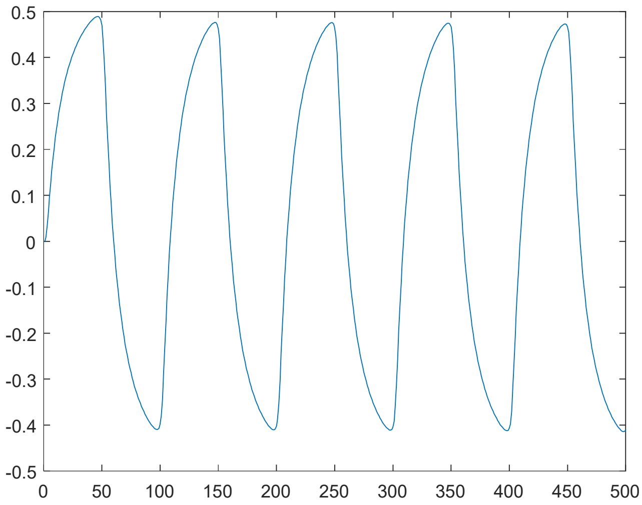

Fig. 11 – Simulated Big Muff output with minimum Sustain level.

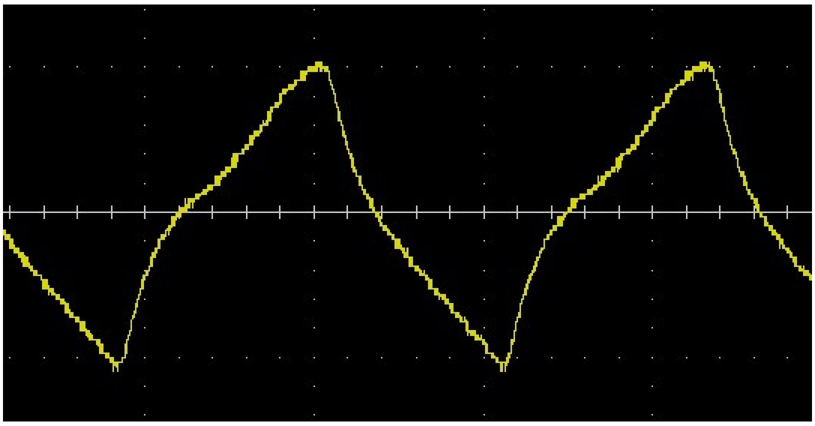

Fig. 11 – Big Muff output of a sine wave input with minimum Sustain level.

The configurable parameters of the Distortion effect are, therefore: the sustain *s*, tone or tonality α, a mixer *m* that balances the output between the dry ($m = 0$) and the wet ($m = 1$) signals, and the output gain *g*.