In [1]:

```python
"""
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AMATH 301 B
"""

import numpy as np
import scipy.linalg
import scipy.optimize
import matplotlib.pyplot as plt
import scipy.interpolate
import pandas as pd
```

In [2]:
```
# Problem 1

data = np.genfromtxt('CO2_data.csv', delimiter=',')
t, co2 = data[0, :], data[1, :]
n = len(t)

# a)

coeffs = np.polyfit(t, co2, 1)

yhat = np.polyval(coeffs, t)
yhat2 = lambda arb : arb[0] * np.exp(arb[1]*t) + arb[2]

RMS_Error = lambda coeffs : np.sqrt((1 / n) * np.sum((yhat2(coeffs) - co2) **
2))
coeff_min = scipy.optimize.minimize(RMS_Error, np.array([30, 0.03, 300]), meth
od='Nelder-Mead')

yhat3 = lambda ARB : ARB[0] * np.exp(ARB[1]*t) + ARB[2] + ARB[3] * np.sin(ARB[
4]*(t - ARB[5]))
RMS_Error2 = lambda coeffs : np.sqrt((1 / n) * np.sum((yhat3(coeffs) - co2) **
2))
coeff_min2 = scipy.optimize.minimize(RMS_Error2, np.array([coeff_min.x[0], coe
ff_min.x[1], coeff_min.x[2], -5, 4, 0]),
                                     method='Nelder-Mead', options={'maxiter':
1000})
plt.figure(figsize=(10, 8))
plt.title('Atmospheric CO2 at Mauna Loa')
plt.xlabel('time (in years since Jan 1958)')
plt.ylabel('CO2 (ppm)')
plt.grid()
plt.plot(t, co2, 'k', t, yhat, 'r', t, yhat2(coeff_min.x), 'b', t, yhat3(coeff
_min2.x), 'g')
plt.legend(['CO2 over time', 'Linear fit', 'Exponential fit', 'Exponential and
Sin fit'])

# b)

exp_sin_error = RMS_Error2(coeff_min2.x)
print("Min Error:", exp_sin_error)
"""
The lowest RMS error came from the 'Exponential and Sin fit' function (f(t) =a
e^rt+b+Asin(B(t-C))). Although the exponential fit by
itself modeled the shape of the data quite well, the added trig function here
 significantly increased its accuracy, reducing the
error to a low of 0.9497282235549688. This is due to the sine wave oscillation
s in addition to the exponential curve. I think
this model captures patterns in CO2 very well due to the sine wave, and it doe
sn't seem overfitted due to the similarity of the
real data.
"""
```

```
Min Error: 0.9497282235549688
```

Out[2]:

```
"\nThe lowest RMS error came from the 'Exponential and Sin fit' function (f
(t) =ae^rt+b+Asin(B(t-C))). Although the exponential fit by\nitself modeled t
he shape of the data quite well, the added trig function here significantly i
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is is due to the sine wave oscillations in addition to the exponential curve.
I think \nthis model captures patterns in CO2 very well due to the sine wave,
and it doesn't seem overfitted due to the similarity of the\nreal data.\n"
```

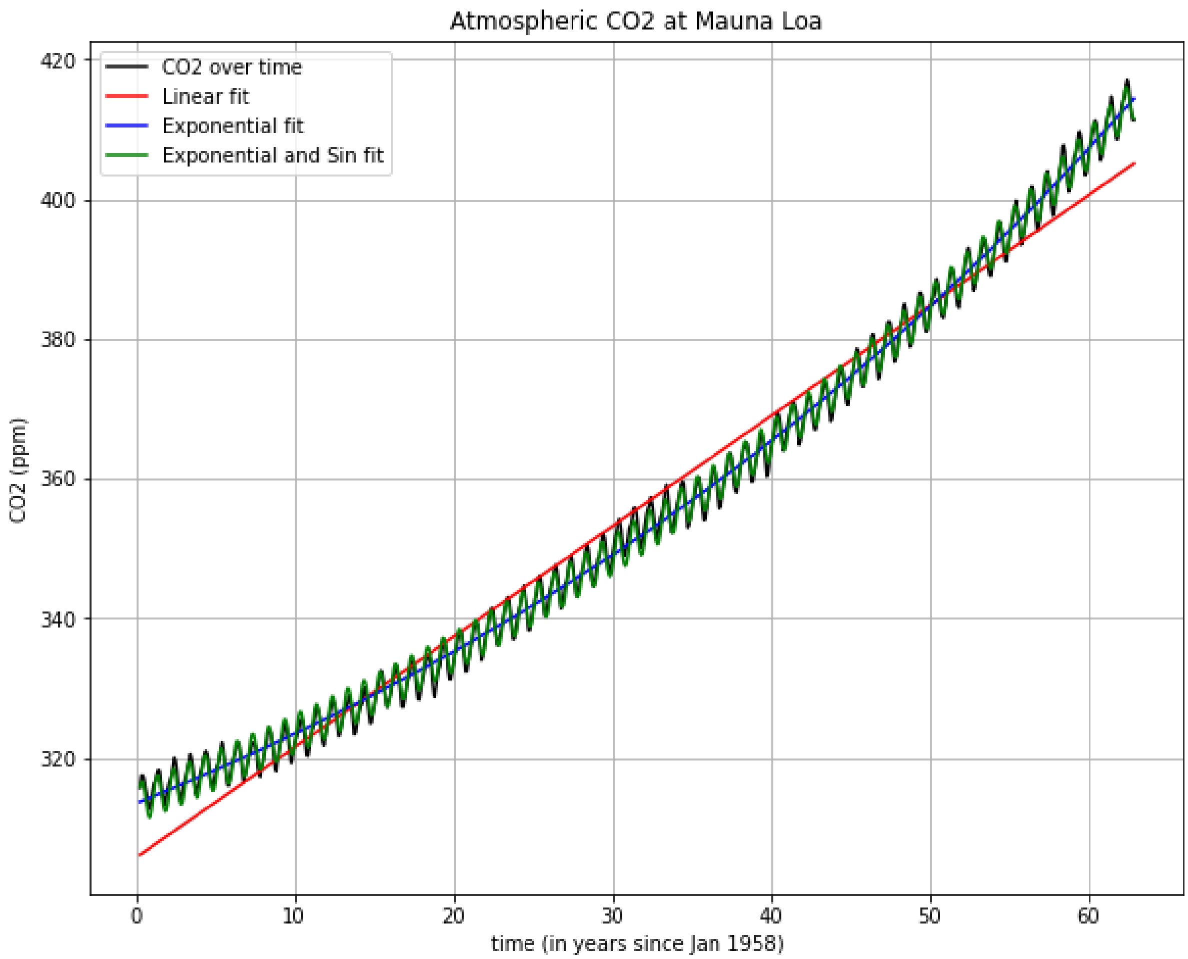

In [3]:
```
# c)

interp_func_linear = scipy.interpolate.interp1d(t, co2, fill_value='extrapolat
e')
interp_func_cubic = scipy.interpolate.interp1d(t, co2, kind='cubic', fill_valu
e='extrapolate')
t = 62 + 11/12
actual = 412.88

yhat_prediction = coeffs[0] * t + coeffs[1]
yhat2_prediction = yhat2(coeff_min.x)
yhat3_prediction = yhat3(coeff_min2.x)
cubic_prediction = interp_func_cubic(t)
linear_prediction = interp_func_linear(t)

data = {'model': ['Linear', 'Exponential', 'Exp/Sin', 'Cubic Spline', 'Linear
 Interpolation'],
        'prediction': [yhat_prediction, yhat2_prediction, yhat3_prediction, cu
bic_prediction, linear_prediction],
        'absolute error': [np.abs(yhat_prediction - actual), np.abs(yhat2_pred
iction - actual),
                           np.abs(yhat3_prediction - actual), np.abs(cubic_pre
diction - actual), np.abs(linear_prediction - actual)]}
table = pd.DataFrame(data)
display(table)

"""
As shown in the dataframe below, the Cubic Spline method yielded the most accu
rate prediction for the CO2 concentration in
November of 2020 (least absolute error of 0.494935). The Exp/Sin model is clos
e behind.
"""

# d)

"""
If I wanted to predict concentration levels as far out as 2040, I would rely o
n the Exp/Sin method, as it had the second lowest
absolute error (0.765773) and seems to fit the real data model very well. Look
ing at the graph, it seems to follow the
exponentially increasing/sine oscillating curve of the true data almost perfec
tly, and I would not worry about overfitting since
I am not using some high-order polynomial. Even though the Cubic Spline interp
olation model ostensibly worked well due to its
low error (0.494935) for the November 2020 prediction, extrapolating would lik
ely be disastrous for predictions as far out as
2040. The reason it worked well for the November 2020 prediction is because we
are predicting very close to the original data
set. November is one month away from October, which is where our data ends. Si
nce it is within the normal spacing of our data
points, it seems to fit the data very well. Once we go beyond 2020 though, the
model would likely fall apart completely. To sum
up, I would use Exp/Sin to predict 2040 concentration levels due to its reliab
ility and excellent fit, and I believe the Cubic
Spline would perform the worst in 2040 due to it being based on extrapolation.
"""
```

	model	prediction	absolute error
0	Linear	405.173	7.706653
1	Exponential	414.581	1.700963
2	Exp/Sin	412.114	0.765773
3	Cubic Spline	413.3749348097231	0.494935
4	Linear Interpolation	411.2699999999999	1.610000

Out[3]:
```
'\nIf I wanted to predict concentration levels as far out as 2040, I would re
ly on the Exp/Sin method, as it had the second lowest\nabsolute error (0.7657
73) and seems to fit the real data model very well. Looking at the graph, it
seems to follow the \nexponentially increasing/sine oscillating curve of the
true data almost perfectly, and I would not worry about overfitting since\nI
am not using some high-order polynomial. Even though the Cubic Spline interpo
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would perform the worst in 2040 due to it being based on extrapolation.\n'
```
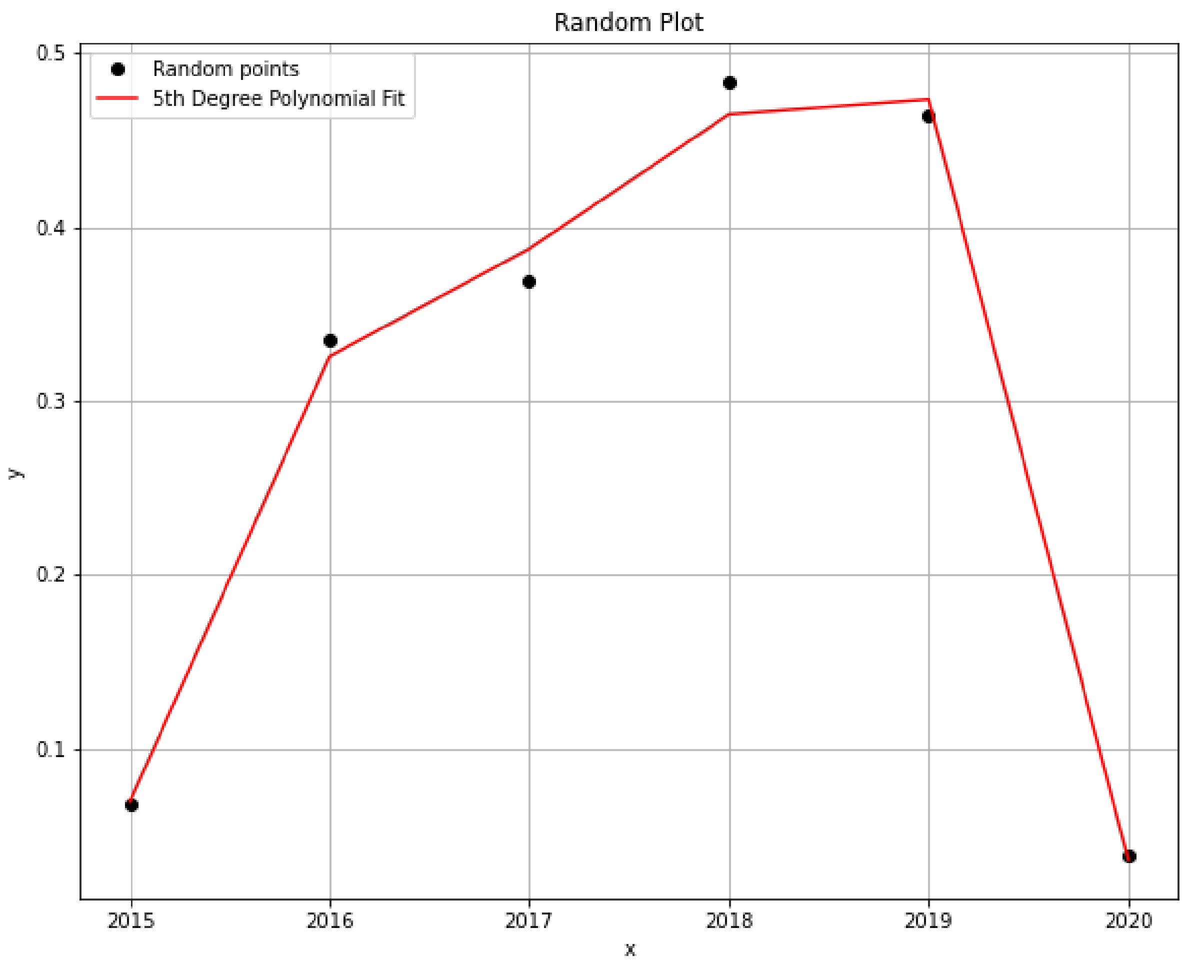

In [14]:
```
# Problem 2

np.random.seed(1222)  # Set seed so my results don't change every time

# a)

x = np.arange(2015, 2021, 1)
y = np.random.rand(x.size)

plt.figure(figsize=(10, 8))
plt.plot(x, y, 'ko')
plt.title('Random Plot')
plt.xlabel('x')
plt.ylabel('y')
plt.grid()

# b)

coeffs = np.polyfit(x, y, 5)
print(coeffs)
"""
The coefficients are [-5.49123829e-06  3.32359766e-02 -4.46515153e+01 -9.04984
470e+04
                      2.73560303e+08 -1.83970699e+11]
"""

# c)

y5 = np.polyval(coeffs, x)
plt.plot(x, y5, 'r')
plt.legend(('Random points', '5th Degree Polynomial Fit'))

p2020 = y5[-1:]
abs_error = np.abs(p2020 - y[-1:])
print("error:", abs_error)
"""
The absolute error is 0.00188466. It is clearly not very far off from the true
value, but in this context with only five values,
I believe the error is significant enough to warrant concern.
"""

# d)

print('\nPrediction values:')
print(coeffs[5] * 2020**5)
print(coeffs[4] * 2020**5)
print(coeffs[3] * 2020**5)
print(coeffs[2] * 2020**5)
print(coeffs[1] * 2020**5)

"""
Since these values are all massive, we are likely experiencing catastrophic ca
ncellation in this problem, which can lead to
immense rounding error.
"""
```

```
# e)

x = np.arange(0, 7, 1)
y = np.random.rand(x.size)

coeffs = np.polyfit(x, y, 5)

y5 = np.polyval(coeffs, x)

p5 = y5[-1:]
abs_error = np.abs(p5 - y[-1:])
print('new error:', abs_error)

"""
The new error between p5 and the actual value is very small ([1.98452366e-1
3]). By using smaller values instead of 2020 - 2025,
we avoid catastrophic cancellation and make much more accurate calculations in
Python.
"""
```

```
[-3.02927926e-06  1.83307113e-02 -2.46418687e+01 -4.97650733e+04
  1.50522543e+08 -1.01204289e+11]
error: [0.00188466]

Prediction values:
-3.4037352034265084e+27
5.0624225713529e+24
-1.67371494926022e+21
-8.287632522449402e+17
616504378591431.4
new error: [0.01067647]

C:\Users\arjun\anaconda3\lib\site-packages\IPython\core\interactiveshell.py:3
343: RankWarning: Polyfit may be poorly conditioned
  exec(code_obj, self.user_global_ns, self.user_ns)
```

Out[14]:
```
'\nThe new error between p5 and the actual value is very small ([1.98452366e-
13]). By using smaller values instead of 2020 - 2025,\nwe avoid catastrophic
cancellation and make much more accurate calculations in Python.\n'
```



In [ ]: