Midterm - Guiyang Han

Lecture 2:

- Define a function in Python:

Ex:

```
def gaussian(x, mu, sig):
    return exp(-power(x-mu, 2.0) / (2.0 * power(sig, 2.0)))
```

function name (you set by yourself. ! Don't set to exist names)

indicate you defining a functn

2.0 instead of 2 because float gives float

Build-in function (python's ~~does~~ function).

This indicate what after this is the output when function is called.

- note that if you run the code, there is nothing shown in Shell. unless you 'use' the function, either in Shell or Script.

Ex:

```
y = gaussian(2, 10, 5)
print y
```

set value y to the return value.

let's assume $\mu=10, \sigma=5$

- To plot a gaussian curve:

Ex:

```
x = range(0,10)
y=[]
for i in range(0,10):
    y = y+[gaussian(i, 10, 5)]
```

set range of x

for-loop to calculate each y-value for x.

```
plot(x, y)
```

~~import matplotlib~~.

from matplotlib import plot

this plot points and connect them

Lecture 3:

- List; list is a python 'thing'. we could define it by "[ ]", note that 1st element of the list index as "0".

Ex:

```
A = [1, 2, 3, 4, 5]
print A[0]
```

→ this gives 1.

```
A.append(24)
```

→ this add one more to end of list.

```
print A[0:2]
```

→ this gives a list: [1,2]

- Array; This is only defined in numpy. OR "array is list in numpy"

The reason is to do this it enables many array operators in numpy.

Ex:

```
A=array([1,2,3,4,5])
```

→ we define an array using a list.

Note: Be extererely careful when doing "b=a" where a is a list. You probably means

lecture 4:

- for-loop: As in word, you are trying to look each element in a list. You do not need to predefine the label (usually i)

Ex:

```
for i in range(5):
    print i.
```

range(5) is a list: [0, 1, 2, 3, 4]

You do not to define i, system knows i=1, then i=2 ....

Ex:

```
p=0
for i in range(5):
    p = p+i
```

p is like empty slot that does not reset for each loop.

so, p keeps grow

- while-loop: while loop run(execute) the codes below it iff and only if the condition meet, then, if condition not meet, it moves on.

Ex:

```
x = 6
while x > 0:
    x = x+1
    print x
```

This code will run forever

Because x=6, then, while x>0 is true, the code below makes x=7. then, it goes back check if x>0 again.

lecture 5:

- Data:

Ex:

```
import numpy as np
thedata = np.loadtxt('filename.txt')
```

in order to use command

This product a list called thedata.

thedata: variable name

'filename.txt': name of file for the data.

- mean (average), usually denote as ~~$\mu(x)$~~ μ or $\bar{x}$,

$$\bar{x} := \frac{1}{N}\sum_{i=1}^{N} x_i \quad , \quad x_i \text{ is the value for one instance.}$$

Ex:

```
mean = sum(thedata) / len(thedata)
```

sum over all data

divide by length of data a.k.a N.

- variance, denote by σ^2 (pronounce 'sigma')

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N} (x_i - \bar{x})^2$$

$\bar{x}$: average (mean)

- standard deviation is square root of variance.

$$\sigma = \sqrt{\sigma^2}.$$

Lecture 3 (cont4):

- Gaussian (Normal):
  - formula: $p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\bar{x})^2}{2\sigma^2}}$; $\bar{x}, \sigma$ are defined by the given data. are not unknowns.

↑ ~~this is NOT the probability given x !!!~~ Second thought, this could mean probability at exactly that point...

This is called probability amplitude.

(Think about it, when you say $p(x=4)$, do you mean x is exactly 4? what about 4.00001? So, probability is defined as a range, like $4.0 < x < 4.1$)

Refer to Lecture 2 review.

Ex:

```
def gaussian(x, mu, sig):
    return exp(-power(x-mu, 2.0)/(2.0*power(sig, 2.0)))
```

- Probability: The probability is the area we define the x.

Probability of $a<x<b$.

so, $\text{prob}(a<x<b) = \int_a^b dx \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\bar{x})^2}{2\sigma^2}}$ ← number, not a variable

↑ integrate over $x=a$ to b.

$$= -\frac{1}{2}\text{erf}\left(\frac{a}{\sigma}\right) + \frac{1}{2}\text{erf}\left(\frac{b}{\sigma}\right) \Leftrightarrow \frac{1}{2}(\text{erf}(-a) + \text{erf}(b))$$

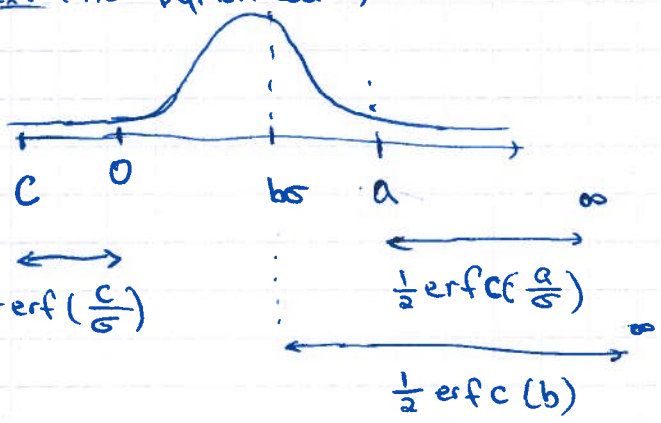
Probability of get a between 0 to a. prob of $0<x<b$.

- $\text{erf}\left(\frac{a}{\sigma}\right)$: probability get $-a<x<a$.
- $\text{erfc}\left(\frac{a}{\sigma}\right) = 1 - \text{erf}\left(\frac{a}{\sigma}\right)$: prob get $-\infty \leq x \leq -a$ plus $\infty > x > a$
- $\frac{1}{2}\text{erf}\left(\frac{a}{\sigma}\right)$: prob getting $0<x<a$
- $\frac{1}{2}\text{erfc}\left(\frac{a}{\sigma}\right)$: prob getting $\infty > x > a$
- $\frac{1}{2}\text{erfc}(a)$: prob get $\infty > x > a\sigma$ ← sigma.
- $\text{erfc}(a)$: prob get $\infty > x > a\sigma$, plus $-\infty < x < -a\sigma$

This is called a-significant.

lecture 5 (cont) - (cont):

Ex: (not python code)



or b-significance.

- Model fitting: we when we have a distribution, we use "chi-squared" to calculate the error:

$$\chi^2 := \sum_i \frac{(y_i - m_i)^2}{2\sigma_i^2}$$; y_i the actual value, m_i the predict value.

σ_i^2 ↖ variance

$$\chi^2 = \frac{\text{different between predict and real}}{\text{~~over~~ rescale the number}}$$

because large diff in large variance model is not a big deal.

- So, overall, we minimize the χ^2 value to better fit the data.

Lecture 7:

- $\binom{n}{k}$: This means n choose k. This gives number of ways to draw k items from total of n items.

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$; $3! = 3 \times 2 \times 1$.

- Binomial Distribution:

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$; → set this either from a given data or knowledge.

p is the probability event A happens.

n is the total events number.

Ex: (not a python)

$$p(k) = \binom{10}{k} (0.4)^k (1-0.4)^{10-k}$$ → let's say from 10 coin flips what are probs get exactly k heads, when this coin has 40% head chance.

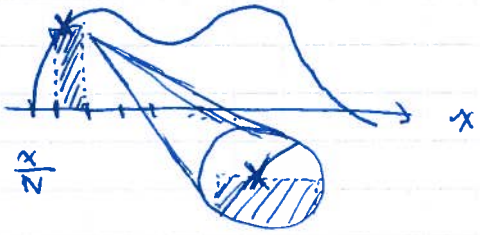
↙ how many combinations of exactly k heads

↘ for 1 combination, the probability.

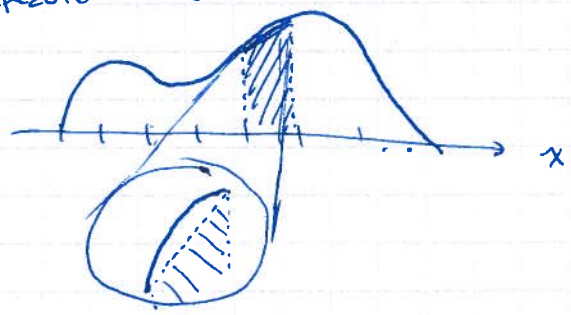
Lecture 9

- Riemann Sum: use midpoint to construct a rectangle.
  Trapezoid Rule: use slopes to construct a Tpzoid.

Riemann Sum



Trapezoid Rule:



Ex:

```
x = range(0, 20, 0.01)
sum = 0
for a in ~~range to~~ x :
    midpoint = a + 0.05
    y = f(midpoint)
    sum += 0.01 * y
print sum
```

Set $\Delta x = 0.01$, x is x-axis.

Rie Sum use the midpoint so, we + half a interval.

height of the function

Area calculate. next a: (a+1)

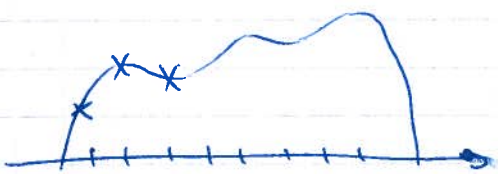
use $A = 0.01 * \frac{f(a) + f(a+0.01)}{2}$, for Trapezoid Rule.

area is this

number for integrate f over range 0 to 20.
aka $\int_0^{20} f(x)\,dx$

- Simpson's Rule: use $ax^2 + bx + c$ to fit the data to calculate area

Ex: (not codes)



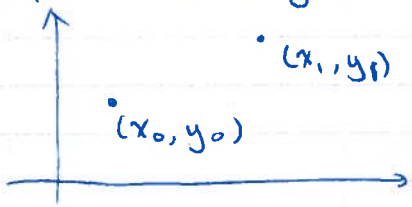
① pick 3 consecutive points

② fit $ax^2 + bx + c$

③ use a, b, c to find the area for this part

Lecture 10

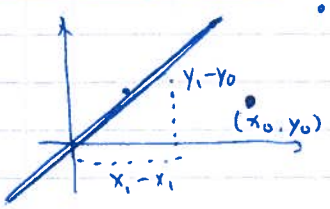
- Interpolation: a technique to fit datas.
  - 2 points is easy:



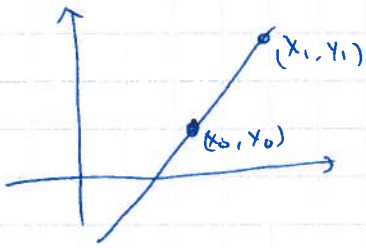
To fit ① we need to match the slope.

② put the line through (x_0, y_0).

Lecture 10 (cont'd)



① Step 1: find slope $\frac{y_0 - y_1}{x_0 - x_1}$



② $y = \left(\frac{y_0 - y_1}{x_0 - x_1}\right) x + b$ ← unknown

step 2: put the line on point (x_0, y_0)

$$y_0 = \left(\frac{y_0 - y_1}{x_0 - x_1}\right) x_0 + b$$

$$\Rightarrow b = y_0 - \frac{y_0 - y_1}{x_0 - x_1} x_0$$

so, to fit 2 point, we use functn

$$y = \left(\frac{y_0 - y_1}{x_0 - x_1}\right) x + \left[y_0 - \frac{y_0 - y_1}{x_0 - x_1} x_0\right]$$ same as pg 7 in lecture 10

- more than 2 point:

$$f(x) = \sum_i \omega_i y_i \quad ; \quad \omega_i = \prod_{j \neq i} \frac{x - x_j}{x_i - x_j}$$

This fitting is 'stupid'. observe if $x = x_i$, the $\omega_i = 1$, and $\omega_j \neq \omega_i = 0$ so, this always garentee to predict correct y_i.

Ex: given data $x = x_1 \quad x_2 \quad x_3 \quad x_4$ } gives x_1, outcome is y_1.
$y = y_1 \quad y_2 \quad y_3 \quad y_4$

$$f(x) = \omega_1 y_1 + \omega_2 y_2 + \omega_3 y_3 + \omega_4 y_4 \; ; \quad \omega_1 = \left(\frac{x - x_2}{x_1 - x_2}\right)\left(\frac{x - x_3}{x_1 - x_3}\right)\left(\frac{x - x_4}{x_1 - x_4}\right)$$

$$\vdots$$

Then $f(x_3) = 0 \cdot y_1 + 0 \cdot y_2 + 1 \cdot y_3 + 0 \cdot y_4 = y_3$.

~~Lecture~~ Error Rate:

$$\int_a^b f(x)\,dx = \sum_{k=1}^{N} \int_{x_{k-1}}^{x_k} f(x)\,dx$$

divide $\int$ into N pieces.

$$= \frac{1}{2} h \sum_{k=1}^{N} \left[f(x_{k-1}) - f(x_k)\right] + \frac{1}{4} h^2 \left[f'(a) - f'(b)\right] + \frac{1}{12} h^3 \sum_{k=1}^{N} \left[f''(x_{k-1}) + f''(x_k)\right] + O(h^4)$$

This is ~~Simpson~~ Trapozoid predictin.

So, error is about $\sim \frac{1}{4} h^2$ we say $O(h^2)$

This is error from Simpson. $O(h^4)$