Simulation of the dynamic motion of a pendulum under viscous friction with Cartesian coordinates

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Problem Statement

Simulate the dynamic motion of a pendulum under viscous friction described with Cartesian coordinates x and y. Apply constraint stabilization method to convert the constraint into its almost equivalent ODE, then apply any ODE solver to solve a set of ODEs (equations of motion and equations for constraint stabilization) numerically.

Dynamic analysis

According to previous content about viscous friction, the equation of the viscous friction is given by:

$$\tau = -b\dot{\theta} \tag{1}$$

where τ is the torque, b is the viscous friction coefficient, and $\dot{\theta}$ is the angular velocity.
We can easily get the equation of motion under Cartesian coordinates:

$$f_v = -\frac{b}{l}v \tag{2}$$

where f_v is the force, b is the viscous friction coefficient, l is the length of the pendulum, and v is the velocity.
Then we can get the equations for x and y:

$$\begin{bmatrix} f_x \\ f_y \end{bmatrix} = -\frac{b}{l}\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \tag{3}$$

where f_x and f_y are the viscous forces in x and y directions, respectively.
Treat the viscous forces as external forces, and use the above equations in the system that described in the sides. we can get the equations of motion:

$$\begin{gathered} \dot{x} = v_x \\ \dot{y} = v_y \\ \begin{bmatrix} m & & -R_x \\ & m & -R_y \\ -R_x & -R_y & \end{bmatrix}\begin{bmatrix} \dot{v}_x \\ \dot{v}_y \\ \lambda \end{bmatrix} = \begin{bmatrix} f_x \\ -mg + f_y \\ C\left(x, y, v_x, v_y\right) \end{bmatrix} \end{gathered} \tag{4}$$

where m is the mass of the pendulum, g is the acceleration of gravity, λ is the Lagrange multiplier, R_x and R_y are the constraint forces in x and y directions, respectively.
$C\left(x, y, v_x, v_y\right)$ is the constraint function, which is given by:

$$\begin{aligned} C\left(x, y, v_x, v_y\right) = \begin{bmatrix} v_x & v_y \end{bmatrix}\begin{bmatrix} R_{xx} & R_{xy} \\ R_{yx} & R_{yy} \end{bmatrix}\begin{bmatrix} v_x \\ v_y \end{bmatrix} \\ + 2\alpha\left(R_x v_x + R_y v_y\right) + \alpha^2 R \end{aligned} \tag{5}$$

where α is a constant, and R is the distance between the pendulum and the constraint.
R_x, R_y, R_{xx}, R_{xy}, R_{yx}, and R_{yy} are the derivatives of R with respect to x and y.
With Equation 4, we can start our simulation in MATLAB.

MATLAB code

The code of modified part is shown below:

Listing 1: MATLAB code for pendulum simulation

```
function dotq = pendulum_cartesian (t,q)
  global mass; global length; global grav; global alpha; global viscous_coeiff;
  x = q(1); y = q(2); vx = q(3); vy = q(4);

  dotx = vx; doty = vy;
  R = sqrt(x^2+(y-length)^2) - length;
  P = 1/sqrt(x^2+(y-length)^2); Rx = x*P; Ry = (y-length)*P;
  Rxx = P - x^2*P^3; Ryy = P - (y-length)^2*P^3;
  Rxy = -x*(y-length)*P^3;
  C = [vx,vy]*[Rxx, Rxy; Rxy, Ryy]*[vx;vy] ...
    + 2*alpha*(Rx*vx + Ry*vy) ...
    + alpha^2*R;
  A = [mass, 0, -Rx; 0, mass, -Ry; -Rx, -Ry, 0];
  \textcolor{red}{b = [ -viscous_coeiff*dotx; -mass*grav - viscous_coeiff*doty; C ];}

  s = A \ b;
  dotvx = s(1); dotvy = s(2);
  dotq = [dotx; doty; dotvx; dotvy];
end
```

Using ode45 function in MATLAB, we can solve the ODEs numerically.

Results

Using the following parameters: mass of 0.01 kg, length of 2.0 m, gravitational acceleration of 9.8 m/s^2, alpha of 1000, viscous coefficient of 0.01, initial angle of $\frac{\pi}{3}$ radians, and a simulation time of 10 seconds, we can obtain the simulation results as shown below:

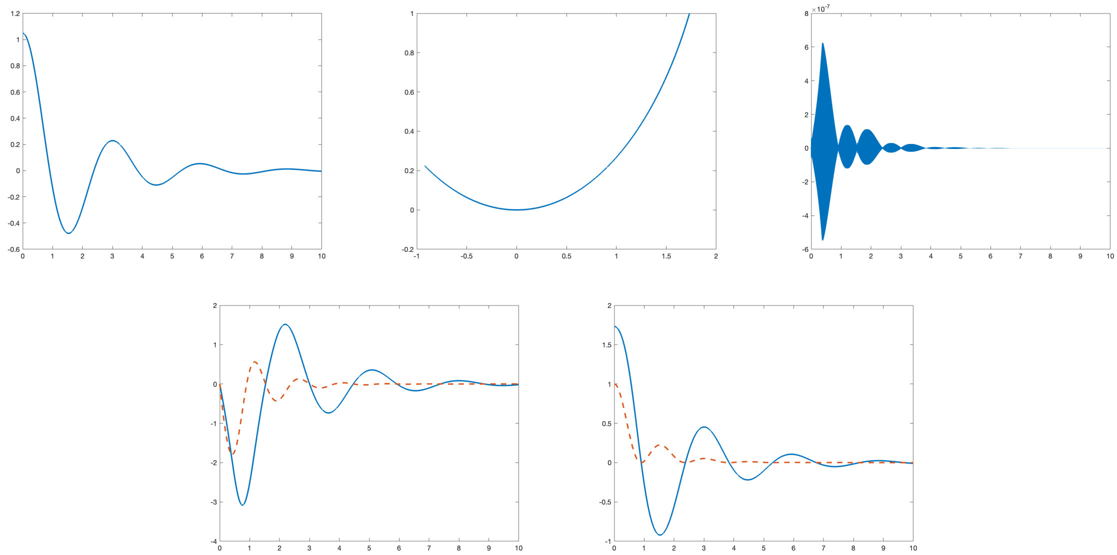

Figure 1: (a) Computed angle of the pendulum, (b) Path of the pendulum, (c) Distance R between the pendulum and the constraint, (d) Velocities v_x and v_y, (e) Positions x and y of the pendulum.(a) - (e) are ordered from left to right and top to bottom.