Report for mechanics of soft bodies questions

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Q1 - Problem statement

A soft robot moves inside a smooth rigid tube. The robot body consists of a cylindrical soft tube (length L, outer radius R, inner radius r) and thin rigid plates attached to the both ends of the tube. Young's modulus of the tube material is given by E. Air pressure P is applied inside the tube through its one end. Assume that the robot extends along its central axis alone and radial deformation is negligible. Let $L = 100mm$, $R = 10mm$, $r = 6mm$, $E = 1.0MPa$, and $P = 0.10MPa$, estimate the extensional deformation of the robot.

Solution

Determine the cross-sectional area A of the tube: Since the tube has an inner radius r and an outer radius R , the cross-sectional area is the area of the annular section:

$$A = \pi(R^2 - r^2) \tag{1}$$

Plugging in the values:

$$A = \pi\left(10^2 - 6^2\right)\,\mathrm{mm}^2 = \pi(100 - 36)\,\mathrm{mm}^2 = 64\pi\,\mathrm{mm}^2 \tag{2}$$

Calculate the force F exerted by the internal pressure: The force due to the internal pressure P is the product of P and the cross-sectional area A:

$$F = P \cdot A \tag{3}$$

Using $P = 0.10\,\mathrm{MPa}$ and $A = 64\pi\,\mathrm{mm}^2$:

$$F = 0.10 \times 64\pi\,\mathrm{N} = 6.4\pi\,\mathrm{N} \tag{4}$$

Compute the extensional deformation ΔL using Hooke's Law: In an elastic material, deformation can be calculated by:

$$\Delta L = \frac{F \cdot L}{E \cdot A} \tag{5}$$

Substituting the values:

$$\Delta L = \frac{6.4\pi \times 100}{1.0 \times 64\pi}\,\mathrm{mm} \tag{6}$$

Simplifying, we find:

$$\Delta L = \frac{640\pi}{64\pi}\,\mathrm{mm} = 10\,\mathrm{mm} \tag{7}$$

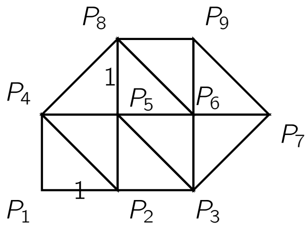
Final Answer

The extensional deformation ΔL of the robot is estimated to be:

$$\boxed{\Delta L = 10\,\mathrm{mm}} \tag{8}$$

Q2 - Problem statement

Show inertia matrix M and connection matrices J_λ, J_μ of the two-dimensional body below. Length of orthogonal sides of all isosceles right triangles is 1. Thickness of the two-dimensional body is $h = 2$ and its density is $\rho = 12$.



Solution

We can first get $M_{i,j,k}$ for each triangle.

$$M_{1,2,4} = M_{2,3,5} = M_{5,4,2} = M_{6,5,3} = M_{6,3,7} = M_{5,8,4} = M_{5,6,8} = M_{6,7,9} = M_{9,8,6} = \frac{\rho h \Delta}{12} \begin{bmatrix} 2I_{2\times2} & I_{2\times2} & I_{2\times2} \\ I_{2\times2} & 2I_{2\times2} & I_{2\times2} \\ I_{2\times2} & I_{2\times2} & 2I_{2\times2} \end{bmatrix} \tag{9}$$

$$\frac{\rho h \triangle}{12} = \frac{12 \times 2}{12} = 2$$

and

$$\begin{aligned} M &= M_{1,2,4} \oplus M_{2,3,5} \oplus M_{5,4,2} \oplus M_{6,5,3} \oplus M_{6,3,7} \oplus M_{5,8,4} \oplus M_{5,6,8} \oplus M_{6,7,9} \oplus M_{9,8,6} \\ &= 2 \begin{bmatrix} 2I_{2\times2} & I_{2\times2} & & I_{2\times2} & & & & & \\ I_{2\times2} & 6I_{2\times2} & I_{2\times2} & 2I_{2\times2} & 2I_{2\times2} & & & & \\ & I_{2\times2} & 6I_{2\times2} & & 2I_{2\times2} & 2I_{2\times2} & I_{2\times2} & & \\ I_{2\times2} & 2I_{2\times2} & & 6I_{2\times2} & 2I_{2\times2} & & & I_{2\times2} & \\ & 2I_{2\times2} & 2I_{2\times2} & I_{2\times2} & 10I_{2\times2} & 2I_{2\times2} & 2I_{2\times2} & & \\ & & I_{2\times2} & & I_{2\times2} & 10I_{2\times2} & 2I_{2\times2} & 2I_{2\times2} & 2I_{2\times2} \\ & & & & & 2I_{2\times2} & 4I_{2\times2} & & I_{2\times2} \\ & & & I_{2\times2} & 2I_{2\times2} & 2I_{2\times2} & & 6I_{2\times2} & I_{2\times2} \\ & & & & & 2I_{2\times2} & I_{2\times2} & I_{2\times2} & 4I_{2\times2} \end{bmatrix}. \end{aligned} \tag{10}$$

For J_λ, we need to calculate $J_\lambda^{1,2,4}$, $J_\lambda^{2,3,5}$, $J_\lambda^{5,4,2}$, $J_\lambda^{6,5,3}$, $J_\lambda^{6,3,7}$, $J_\lambda^{5,8,4}$, $J_\lambda^{5,6,8}$, $J_\lambda^{6,7,9}$, and $J_\lambda^{9,8,6}$.
For $J_\lambda^{1,2,4}$, we have:

$$\boldsymbol{a} = \frac{1}{2\triangle} \begin{bmatrix} y_j - y_k \\ y_k - y_i \\ y_i - y_j \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \boldsymbol{b} = -\frac{1}{2\triangle} \begin{bmatrix} x_j - x_k \\ x_k - x_i \\ x_i - x_j \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \tag{11}$$

then we can get:

$$\begin{aligned} H_\lambda &= \begin{bmatrix} \boldsymbol{a}\boldsymbol{a}^{\mathrm{T}} & \boldsymbol{a}\boldsymbol{b}^{\mathrm{T}} \\ \boldsymbol{b}\boldsymbol{a}^{\mathrm{T}} & \boldsymbol{b}^{\mathrm{T}} \end{bmatrix} h\triangle \\ H_\lambda &= \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & -1 \\ -1 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 1 & -1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & -1 & 0 & 1 \end{array}\right] \end{aligned} \tag{12}$$

Because 1,4, 2, 5, 3, 6 rows and columns of H_λ are 1, 2, 3, 4, 5, 6 rows and columns of $J_\lambda^{1,2,4}$.

$$J_\lambda^{1,2,4} = \left[\begin{array}{rr|rr|rr} 1 & 1 & -1 & 0 & 0 & -1 \\ 1 & 1 & -1 & 0 & 0 & -1 \\ \hline -1 & -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 1 & 0 & 0 & 1 \end{array}\right] \tag{13}$$

We can do the same thing for $J_\lambda^{2,3,5}$, $J_\lambda^{5,4,2}$, $J_\lambda^{6,5,3}$, $J_\lambda^{6,3,7}$, $J_\lambda^{5,8,4}$, $J_\lambda^{5,6,8}$, $J_\lambda^{6,7,9}$, and $J_\lambda^{9,8,6}$.
And finally, we can get J_λ by:

$$\begin{aligned} J_\lambda &= J_\lambda^{1,2,4} \oplus J_\lambda^{2,3,5} \oplus J_\lambda^{5,4,2} \oplus J_\lambda^{6,5,3} \oplus J_\lambda^{6,3,7} \oplus J_\lambda^{5,8,4} \oplus J_\lambda^{5,6,8} \oplus J_\lambda^{6,7,9} \oplus J_\lambda^{9,8,6} \\ &= \begin{bmatrix} 1 & 1 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 2 & 1 & -1 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & -1 & 0 & 1 & 0 & -1 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & -2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ -1 & -1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & -2 & 0 & 4 & 1 & -2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -2 & 1 & 0 & 0 & 0 & 1 & 4 & -1 & 0 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & -1 & 4 & 1 & -2 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & -1 & 0 & 1 & 4 & 0 & 0 & 1 & 0 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -2 & 0 & 2 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -2 & 1 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 1 & 1 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -2 & 1 & 0 & -1 & 0 & 1 & 2 \end{bmatrix} \end{aligned} \tag{14}$$

Similarly, we can get J_μ by:

$$\begin{aligned} J_\mu &= J_\mu^{1,2,4} \oplus J_\mu^{2,3,5} \oplus J_\mu^{5,4,2} \oplus J_\mu^{6,5,3} \oplus J_\mu^{6,3,7} \oplus J_\mu^{5,8,4} \oplus J_\mu^{5,6,8} \oplus J_\mu^{6,7,9} \oplus J_\mu^{9,8,6} \\ &= \begin{bmatrix} 3 & 1 & -2 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 3 & 0 & -1 & 0 & 0 & -1 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2 & 0 & 6 & 1 & -2 & -1 & 0 & 1 & -2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 1 & 6 & 0 & -1 & 1 & 0 & -1 & -4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 4 & 0 & 0 & 0 & 0 & 1 & -2 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & 5 & 0 & 0 & 1 & 0 & 0 & -4 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 1 & 0 & 0 & 5 & 0 & -4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 & 0 & 0 & 0 & 4 & 0 & -2 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -2 & -1 & 0 & 1 & -4 & 0 & 12 & 1 & -4 & -1 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & -1 & -4 & 1 & 0 & 0 & -2 & 1 & 12 & -1 & -2 & 0 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 & -4 & -1 & 12 & 1 & -4 & 0 & 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 & 0 & -4 & 0 & 0 & -1 & -2 & 1 & 12 & 0 & -2 & 1 & 0 & -1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -2 & 0 & 0 & 1 & 0 & 0 & 4 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4 & 1 & 0 & 0 & 0 & 0 & 5 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & -1 & 0 & 1 & -2 & -1 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -4 & 0 & 0 & 0 & -1 & 1 & 5 \end{bmatrix} \end{aligned} \tag{15}$$