Report #2

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Problem Statement

Assume that a system is described by four coordinates q_1 through q_4. Two constraints R1 and R2 are imposed on the system. Let $\mathbf{q} = [\mathbf{q_1}, \mathbf{q_2}, \mathbf{q_3}, \mathbf{q_4}]^\top$ and $\mathbf{R} = [\mathbf{R_1}, \mathbf{R_2}]^\top$. Let $\mathbf{g_1}$ and $\mathbf{H_1}$ be gradient vector and Hessian matrix related to R_1 while g_2 and H_2 be gradient vector and Hessian matrix related to R_2. Let J be Jacobian given by

$$J = \begin{bmatrix} \partial R_1/\partial q_1 & \partial R_1/\partial q_2 & \partial R_1/\partial q_3 & \partial R_1/\partial q_4 \\ \partial R_2/\partial q_1 & \partial R_2/\partial q_2 & \partial R_2/\partial q_3 & \partial R_2/\partial q_4 \end{bmatrix} \tag{1}$$

Show the following equations:

$$\begin{aligned} \dot{\boldsymbol{R}} &= J\dot{\boldsymbol{q}} \\ \ddot{\boldsymbol{R}} &= J\ddot{\boldsymbol{q}} + \begin{bmatrix} \dot{\boldsymbol{q}}^\top H_1 \dot{\boldsymbol{q}} \\ \dot{\boldsymbol{q}}^\top H_2 \dot{\boldsymbol{q}} \end{bmatrix} \end{aligned} \tag{2}$$

Solution

1. $\dot{\mathbf{R}} = J\dot{q}$

$$\dot{R} = \frac{\partial R}{\partial q}\dot{q} = J\dot{q} \tag{3}$$

Here, we have

$$J = \frac{\partial R}{\partial q} \tag{4}$$

So, the equation $\dot{\mathbf{R}} = J\dot{q}$ is proved.

2. $\ddot{\mathbf{R}} = J\ddot{q} + \begin{bmatrix} \dot{q}^T H_1 \dot{q} \\ \dot{q}^T H_2 \dot{q} \end{bmatrix}$

Differentiating the equation $\dot{\mathbf{R}} = J\dot{q}$ with respect to time, we have

$$\ddot{\mathbf{R}} = \frac{d}{dt}(J\dot{q}) = \frac{dJ}{dt}\dot{q} + J\ddot{q} \tag{5}$$

The time derivative of J can be expanded using the gradient and Hessian matrices:

$$\ddot{\mathbf{R}} = \frac{d}{dt}(J\dot{q}) = \frac{dJ}{dt}\dot{q} + J\ddot{q} \tag{6}$$

Finally, we have

$$\ddot{\mathbf{R}} = J\ddot{q} + \begin{bmatrix} \dot{q}^T H_1 \dot{q} \\ \dot{q}^T H_2 \dot{q} \end{bmatrix} \tag{7}$$