Report #8

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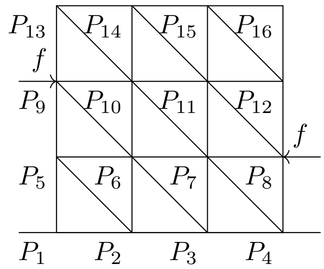
Deformation simulation of a viscoelastic object

Simulate the deformation of a rectangular viscoelastic object shown in the figure. The bottom surface is fixed to the ground. A pair of forces with the same magnitude f are applied to the both sides for a while, then the forces are released. Action lines of the forces do not coincide. Use appropriate values of geometrical and physical parameters of the object.



Preset

We first define the rectangular object as a 2D grid with 4×4 points, and the height and the width of the object are 3 and 3 respectively. The thickness of the object is 1. The material properties of the viscoelastic object are defined as follows: Young's modulus $E = 1.0 \times 10^6$ Pa, damping coefficient $c = 0.04 \times 10^3$ Ns/m, Poisson's ratio $\nu = 0.48$, and density $\rho = 0.020$ kg/m^3.
According to the problem, we need to define the following constraints:

- The bottom surface is fixed to the ground.
- A pair of forces with the same magnitude f are applied to the both sides for a while, then the forces are released.

According to this two constraints, we can define the following constraint matrix A:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ & & & & \vdots & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

And the external force matrix F:

$$F = \left[0, \ldots, \text{f,-f}, \ldots, 0\right]^\top$$

Here f is the magnitude of the force applied to the object, I will set $f = 0.8 \times 10^6$ N.
The time length with the force applied is 0.1 s, and the time length with the force released is 0.9 s. The total time length of the simulation is 1 s.
With the above parameters, we can simulate the deformation of the viscoelastic object.

Simulation Results

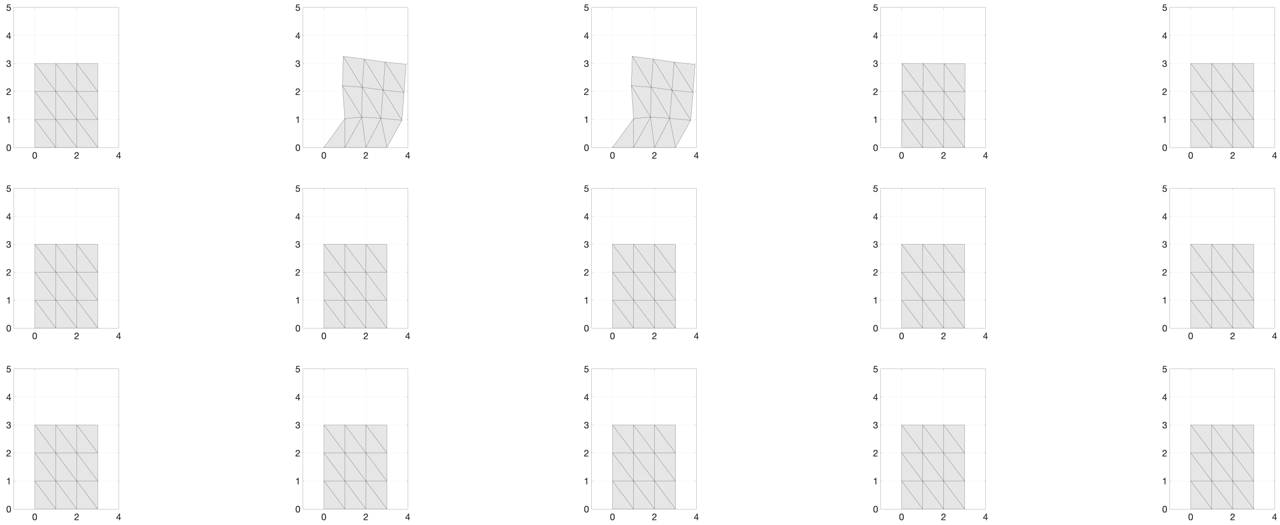


Figure 1: Deformation simulation results at different time steps

Appendix

```
  m = 4; n = 4;
thickness = 1;
[ points, triangles ] = rectangular_object( m, n, 3, 3 );

% E = 0.1 MPa; c = 0.004 kPa s; rho = 0.020 g/cm^3
Young = 1.0*1e+6; c = 0.04*1e+3; nu = 0.48; density = 0.020;

[ lambda, mu ] = Lame_constants( Young, nu );
[lambda_vis, mu_vis] = Lame_constants(c, nu);

npoints = size(points,2);
ntriangles = size(triangles,1);
elastic = Body(npoints, points, ntriangles, triangles, thickness);
elastic = elastic.mechanical_parameters(density, lambda, mu);
elastic = elastic.viscous_parameters(lambda_vis, mu_vis);
elastic = elastic.calculate_stiffness_matrix;
elastic = elastic.calculate_damping_matrix;
elastic = elastic.calculate_inertia_matrix;

alpha = 1e+6;
A = elastic.constraint_matrix([1,2,3,4]);
b0 = zeros(2*4,1);
b1 = zeros(2*4,1);

tp = 0.1;
tf = 0.9;
fpush = -0.8*1e+6;

% external force
interval = [0,tp];
qinit = zeros(4*npoints,1);
external = zeros(2*npoints,1);
external(8) = fpush;
external(9) = -fpush;
beam_bending_external_force = @(t,q) beam_bending_external_force_param_Cauchy_strain(t,q, elastic, A,b0,b1, exter
[time_push, q_push] = ode15s(beam_bending_external_force, interval, qinit);
%[time_push, q_push] = ode45(beam_bending_external_force, interval, qinit);
```