


Przykład 1 Obliczyć wyznacznik macierzy

a)  $\det [-2] = -2$

b)  $\det \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = +1 \cdot 4 - 2 \cdot 3 = -2$



c)

$$\det \begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 2 \\ 0 & 2 & 3 \end{bmatrix} = +1 \cdot 1 \cdot 3 + (-1) \cdot 2 \cdot 0 + 0 \cdot 2 \cdot 2 - 0 \cdot 1 \cdot 0 - 2 \cdot 2 \cdot 1 - 3 \cdot 2 \cdot (-1) =$$

$$= 3 + 0 + 0 - 0 - 4 + 6 = 5$$

Diagram showing the expansion of the determinant using the first row. The signs for the elements are indicated by green circles with minus signs:  $\ominus$  for the first element (1),  $\ominus$  for the second element (2), and  $\ominus$  for the third element (0). The signs for the elements in the second and third rows are indicated by red circles with plus signs:  $\oplus$  for the first element (1),  $\oplus$  for the second element (2), and  $\oplus$  for the third element (3).

$$\det \begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 2 \\ 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & 1 \\ 0 & 2 \end{bmatrix} = +1 \cdot 1 \cdot 3 + 2 \cdot 2 \cdot 0 + 0 \cdot (-1) \cdot 2 - 0 \cdot 1 \cdot 0 - 1 \cdot 2 \cdot 2 - 2 \cdot (-1) \cdot 3 =$$

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d)

$$\det \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = 1 \cdot 5 \cdot 9 + 4 \cdot 8 \cdot 3 + 7 \cdot 2 \cdot 6 - 3 \cdot 5 \cdot 7 - 6 \cdot 8 \cdot 1 - 9 \cdot 2 \cdot 4 = 0$$

The matrix is annotated with green and red lines and signs to show the expansion by minors:  
 - Green lines connect (1,1) to (2,2) to (3,3), (1,2) to (2,3), and (1,3) to (2,1).  
 - Red lines connect (1,1) to (2,3), (1,2) to (2,1), and (1,3) to (2,2).  
 - Green circles with minus signs are placed to the left of the first column.  
 - Red circles with plus signs are placed to the right of the first row.

Przykład: Aby użyć wzoru Laplace'a obliczyć wyznacznik macierzy

$$a) \det \begin{bmatrix} 2 & -1 & 3 \\ 0 & 1 & -4 \\ 3 & 1 & -1 \end{bmatrix} = 0 \cdot (-1)^{2+1} \det \begin{bmatrix} -1 & 3 \\ 1 & -1 \end{bmatrix} + 1 \cdot (-1)^{2+2} \det \begin{bmatrix} 2 & 3 \\ 3 & -1 \end{bmatrix} + (-4) \cdot (-1)^{2+3} \det \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix} =$$

$$= 0 + (-2 - 9) + 4(2 - (-3)) = -11 + 4 \cdot 5 = 9$$

$$b) \det \begin{bmatrix} 1 & 0 & 3 & 0 \\ 2 & 1 & 0 & 2 \\ 0 & -1 & 1 & 0 \\ 1 & 0 & 1 & 3 \end{bmatrix} = 0 + 1 \cdot (-1)^{2+2} \det \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix} + (-1)(-1)^{3+2} \det \begin{bmatrix} 1 & 3 & 0 \\ 2 & 0 & 2 \\ 1 & 1 & 3 \end{bmatrix} + 0 = \textcircled{*}$$

$$\det I = \det \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix} = 1(-1)^{2+2} \det \begin{bmatrix} 1 & 0 \\ 1 & 3 \end{bmatrix} = 3$$

$\textcircled{-}$ 
 $\textcircled{+}$

$$\det II = \det \begin{bmatrix} 1 & 3 & 0 \\ 2 & 0 & 2 \\ 1 & 1 & 3 \end{bmatrix} = 0 + 0 + 6 - 0 - 2 - 18 = -14$$

$\begin{matrix} 1 & 3 & 0 \\ 2 & 0 & 2 \end{matrix}$

$$\textcircled{*} = 3 + (-14) = -11$$

$$c) \det \begin{bmatrix} 1 & 3 & 0 & 2 \\ 2 & 0 & 1 & 4 \\ 0 & -2 & 1 & -1 \\ -1 & 1 & 5 & 0 \end{bmatrix} \begin{matrix} \overline{I}_w(-2) + \overline{II}_w \\ \\ \overline{I}_w + \overline{IV}_w \end{matrix} = \det \begin{bmatrix} 1(-2)+2 & 3(-2)+0 & 0(-2)+1 & 2(-2)+4 \\ 0 & -2 & 1 & -1 \\ 1+(-1) & 3+1 & 0+5 & 2+0 \end{bmatrix} = \det \begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & -6 & 1 & 0 \\ 0 & -2 & 1 & -1 \\ 0 & 4 & 5 & 2 \end{bmatrix} =$$

$$= 1 \cdot (-1)^{1+1} \det \begin{bmatrix} -6 & 1 & 0 \\ -2 & 1 & -1 \\ 4 & 5 & 2 \end{bmatrix} = -12 - 4 - 30 + 4 = -42$$

↓  
methode Sarruse

$$\begin{matrix} -6 & 1 & 0 \\ -2 & 1 & -1 \end{matrix}$$

alternativweise

$$= \det \begin{bmatrix} -6 & 1 & 0 \\ -2 & 1 & -1 \\ 4 & 5 & 2 \end{bmatrix} \begin{matrix} \overline{II}_k(6) + \overline{I}_k \\ \\ \end{matrix} = \det \begin{bmatrix} 1(6)+(-6) & 1 & 0 \\ 1(6)+(-2) & 1 & -1 \\ 5 \cdot 6 + 4 & 5 & 2 \end{bmatrix} = \det \begin{bmatrix} 0 & 1 & 0 \\ 4 & 1 & -1 \\ 34 & 5 & 2 \end{bmatrix} =$$

$$= 1(-1)^{1+2} \det \begin{bmatrix} 4 & -1 \\ 34 & 2 \end{bmatrix} = (-1)(8 - (-34)) = -42$$

$$d) \det \begin{bmatrix} -2 & 0 & 3 & 1 \\ 0 & -1 & 3 & 2 \\ 3 & 4 & 2 & 0 \\ 1 & 2 & 1 & -5 \end{bmatrix} \begin{matrix} \overline{I}_w(-2) + \overline{II}_w \\ = \\ \overline{I}_w(5) + \overline{IV}_w \end{matrix} = \det \begin{bmatrix} -2 & 0 & 3 & 1 \\ (-2)(-2)+0 & 0(-2)+(-1) & 3(-2)+3 & 1(-2)+2 \\ 3 & 4 & 2 & 0 \\ (-2) \cdot 5 + 1 & 0 \cdot 5 + 2 & 3 \cdot 5 + 1 & 1 \cdot 5 + (-5) \end{bmatrix} =$$

$$= \det \begin{bmatrix} -2 & 0 & 3 & 1 \\ 4 & -1 & -3 & 0 \\ 3 & 4 & 2 & 0 \\ -9 & 2 & 16 & 0 \end{bmatrix} = 1 \cdot (-1)^{1+4} \det \begin{bmatrix} 4 & -1 & -3 \\ 3 & 4 & 2 \\ -9 & 2 & 16 \end{bmatrix} \begin{matrix} \overline{II}_w(4) + \overline{I}_w \\ = \\ \overline{II}_w(-3) + \overline{III}_w \end{matrix} \det \begin{bmatrix} (-1) \cdot 4 + 4 & -1 & (-1)(-3)+3 \\ 4 \cdot 4 + 3 & 4 & 4(-3)+2 \\ 2 \cdot 4 + (-9) & 2 & 2(-3)+16 \end{bmatrix} =$$

$$= (-1) \det \begin{bmatrix} 0 & -1 & 0 \\ 18 & 4 & -10 \\ -1 & 2 & 10 \end{bmatrix} = (-1)(-1)(-1)^{1+2} \det \begin{bmatrix} 18 & -10 \\ -1 & 10 \end{bmatrix} = (-1) \cdot (180 - 10) =$$

$\ominus$   $\oplus$

= -180