7.1 c) $[3,1] \longmapsto [5,7,5]$, $[4,3] \longmapsto [0,1,5]$ $\qquad \forall_{v,w \in V} \forall_{a,b \in K} \ \varphi(av+bw) \overset{*}{=} a\varphi(v) + b\varphi(w)$ $\qquad \varphi: V \to V'$

(R) $\varphi: \mathbb{R}^2 \to \mathbb{R}^3$

Wsk do 7.1, 7.2

Przedstawiamy dowolny wektor $[x_1, x_2] \in \mathbb{R}^2$ jako kombinację $[3,1]$ i $[4,3]$.

$[x_1, x_2] = a[3,1] + b[4,3]$ dla pewnych $a, b \in \mathbb{R}$

$$\begin{cases} 3a + 4b = x_1 \\ a + 3b = x_2 \end{cases} \Rightarrow \begin{cases} a = \frac{3}{5}x_1 - \frac{4}{5}x_2 \\ b = -\frac{1}{5}x_1 + \frac{3}{5}x_2 \end{cases}$$

$$\varphi([x_1, x_2]) = \varphi(a[3,1] + b[4,3]) \overset{*}{=} a\varphi([3,1]) + b\varphi([4,3]) = a[5,7,5] + b[0,1,5] =$$

$$= \left(\frac{3}{5}x_1 - \frac{4}{5}x_2\right)[5,7,5] + \left(-\frac{1}{5}x_1 + \frac{3}{5}x_2\right)[0,1,5] = \left[3x_1 - 4x_2,\ \frac{21}{5}x_1 - \frac{28}{5}x_2 - \frac{1}{5}x_1 + \frac{3}{5}x_2,\ 3x_1 - 4x_2 - x_1 + 3x_2\right] =$$

$$= [3x_1 - 4x_2,\ 4x_1 - 5x_2,\ 2x_1 - x_2]$$

odp.

Przykład 1 Przekształcenie liniowe $\varphi: \mathbb{R}^2 \to \mathbb{R}^2$ określone jest wzorem $\varphi([x_1, x_2]) = [4x_1 + 5x_2, 3x_1 + 4x_2]$. Wyznaczyć macierz $M_{\mathcal{B}\mathcal{C}}(\varphi)$, gdy $\mathcal{B} = ([-4,4],[9,-5])$, $\mathcal{C} = ([3,1],[1,1])$.

Wsk. do ✓. 3

R) $\varphi([\overset{x_1}{-4}, \overset{x_2}{4}]) = [4(-4) + 5 \cdot 4, 3(-4) + 4 \cdot 4] = [4, 4]$

$\varphi([\overset{x_1}{9}, \overset{x_2}{-5}]) = [4 \cdot 9 + 5(-5), 3 \cdot 9 + 4(-5)] = [11, 7]$

$[4,4] = a[3,1] + b[1,1]$

$[11,7] = c[3,1] + d[1,1]$

$\begin{cases} 3a + b = 4 \\ a + b = 4 \end{cases}$ $\begin{cases} 3c + d = 11 \\ c + d = 7 \end{cases}$

$\begin{cases} a = 0 \\ b = 4 \end{cases}$ $\begin{cases} c = 2 \\ d = 5 \end{cases}$

odp. $M_{\mathcal{B}\mathcal{C}}(\varphi) = \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 4 & 5 \end{bmatrix}$