K: ~~L1:~~ LS $\frac{1-4}{9}$ Kolokwium 28 listopada

3.7) a) $\sqrt[n]{3^n+2^n} \xrightarrow[n\to\infty]{?}$

$$\sqrt[n]{3^n} \le \sqrt[n]{3^n+2^n} \le \sqrt[n]{2\cdot 3^n}$$

$\downarrow_{n\to\infty}$ $\qquad\qquad$ $\downarrow_{n\to\infty}$

$$3 \le \qquad \le 3\sqrt[n]{2}$$

To 3C

$$\lim_{n\to\infty} \sqrt[n]{3^n+2^n} = 3$$

$$\sqrt[n]{3^n\left(1+\left(\frac{2}{3}\right)^n\right)} = 3\left(1+\left(\frac{2}{3}\right)^n\right)^{\frac{1}{n}}$$

$\downarrow$ $\qquad\qquad$ $\downarrow$

$3\cdot 1^{\infty}$ $\qquad\qquad$ $3\cdot 1^{0} = 3$

3.8) b) $\left(1-\frac{1}{n^2}\right)^n =$

$$\left(1+\frac{1}{-n^2}\right)^n = \left(\left(1+\frac{1}{-n^2}\right)^{-n^2}\right)^{-\frac{1}{n}} \xrightarrow[n\to\infty]{} (e)^{0} = 1$$

$\lim_{x\to\cancel{\infty}}(1+x)^{\frac{1}{x}} = e$

e) $$\left(\frac{2^n+1}{2^n-1}\right)^{2^{n+1}} = \left(\frac{2^n-1+2}{2^n-1}\right)^{2^{n+1}} = \left(1+\frac{2}{2^n-1}\right)^{2^{n+1}} = e^{2^{n+1}} =$$

$2^n-1 = x\,2^{n+1}$

$\frac{2^n-1}{2^{n+1}} = x$

$\lim (1+x)^{\frac{1}{x}} = e$

$[1^{\infty}] \xrightarrow{\text{wbw}}$ wzór „e"

3.9) $a_n = n - n^2 = n^2\left(\frac{1}{n} - 1\right) \xrightarrow{n\to\infty} -\infty$

$[+\infty \cdot (-1)]$

Tw $a_n \to \infty$

$b_n \to b < 0$

$a_n b_n \to -\infty$

$a_n = \frac{\text{arctg}\, n}{\text{arcctg}\, n} \xrightarrow{n\to\infty} \left[\frac{\frac{\pi}{2}}{0^+}\right] = \infty$

g) $\left(\frac{2n-1}{n+2}\right)^n = \left[2^{+\infty}\right] = +\infty$

3.10) d $\underbrace{\frac{2}{n+3}}_{\substack{\downarrow n\to\infty \\ 0}} \underbrace{\sin\frac{n\pi}{2}}_{\text{ogr.}} \xrightarrow{n\to\infty} 0$

a) $a_n = n \cdot \sin\frac{n\pi}{3}$ rozbieżny

$\exists (a_{m_k}), (a_{n_k})$

$a_{m_k} \to a_1$

$\neq \iff$ rozbieżny

$a_{n_k} \to a_2$

$\frac{n\pi}{3} = k\pi \iff n = 3k$

$a_{3k} = 3k \cdot \sin k\pi = 0 \xrightarrow{k\to\infty} 0$

$\frac{n\pi}{3} = \frac{\pi}{3} + 2k\pi \iff n = 6k+1$

$a_{6k+1} = (6k+1)\sin\frac{\pi}{3} \to \left[+\infty \cdot \frac{\sqrt{3}}{2}\right] = +\infty$

16) Tw: $(a_n)\nearrow$ + ogr z góry $\Rightarrow \exists \lim_n a_n = \sup_n a_n$

... być zmiją ... okrętności

$(a_n)\searrow$ + ogr z dołu $\Rightarrow \exists \lim_n a_n = \inf_n a_n$

$$a_n = \frac{2^n}{n!}$$

$$\frac{2^n}{n!} \qquad \frac{2^n \cdot 2}{(n+1)n!} \qquad \frac{2^n \cdot 2 \cdot 2}{(n+1)(n+2)n!}$$

$$\left.\begin{array}{l} 1 > \frac{2}{n+1} \\ \forall_{n \geq 1}\ 0 \leq a \end{array}\right\} \Rightarrow \exists_{g \in \mathbb{R}} \lim_{n \to \infty} a_n = g$$

$$a_n \left\{ \sqrt{2},\ \sqrt{2+\sqrt{2}},\ \sqrt{2+\sqrt{2+\sqrt{2}}} \ \dots \right\}$$

$\lim a_n = g$

$$a_{n+1} = \sqrt{2+a_n}$$

$$g = \sqrt{2+g}$$

$$2 = \sqrt{2+2}$$

$$g = 2$$

Tw ZC $\quad a_n \leq b_n$

(1) $a_n \to +\infty \Rightarrow b_n \to +\infty$

(2) $b_n \to -\infty \Rightarrow a_n \to -\infty$

$\lim (1+x)^{\frac{1}{x}} = e$

$[1^{\infty}] \xrightarrow{\text{wtedy}}$ użyć „e"

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…być ruuja …okrotności

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