ZESTAW: C **I Kolokwium Algebra** GRUPA: Piątek 17.00

2 listopada 2022

Zadanie 1

Znaleźć rozwiązania poniższego równania w zbiorze liczb zespolonych

$z \neq 0$

$$\left(\frac{z+i}{z}\right)^4 = (2+i)^4 .$$

$\frac{z+i}{z} = \sqrt[4]{(2+i)^4} = \{2+i, -1+2i, -2-i, 1-2i\}$

(1) $\frac{z+i}{z} = 2+i$

(5 pkt)

(2) $\frac{z+i}{z} = -1+2i$, (3) $\frac{z+i}{z} = -2-i$, (4) $\frac{z+i}{z} = 1-2i$

Zadanie 2

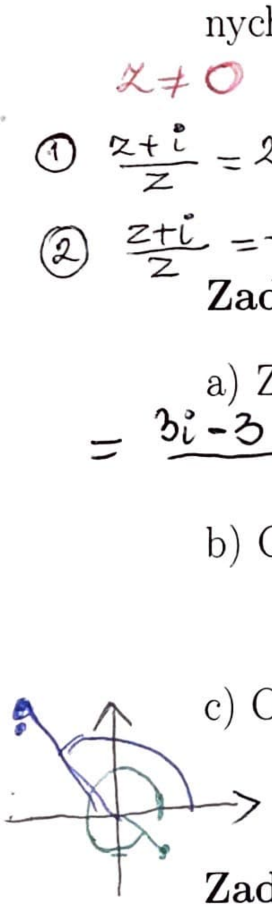
a) Znaleźć część rzeczywistą i urojoną liczby zespolonej $z = \frac{3i-\sqrt{27}}{1-i} \cdot \frac{1+i}{1+i} =$

$= \frac{3i - 3 - \sqrt{27} + \sqrt{27}i}{2} = -\frac{3+3\sqrt{3}}{2} + \frac{3+3\sqrt{3}}{2} i$

(2 pkt)

b) Obliczyć $\left|\frac{3i-\sqrt{27}}{1-i}\right| = \frac{|3i-\sqrt{27}|}{|1-i|} = \frac{\sqrt{9+27}}{\sqrt{2}} \frac{6}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$ //

(1.5 pkt)



c) Obliczyć Arg $\left(\frac{3i-\sqrt{27}}{1-i}\right) = \underbrace{\mathrm{Arg}(3i-\sqrt{27})} - \underbrace{\mathrm{Arg}(1-i)}_{7\pi/4} + 2k\pi =$

$\cos\varphi = -\frac{\sqrt{27}}{6} = -\frac{\sqrt{3}}{2}$, $\sin\varphi = 3/6 = \frac{1}{2}$ $\Rightarrow \varphi = \pi - \pi/6 = \frac{5\pi}{6}$

(1.5 pkt)

$= \frac{5\pi}{6} - \frac{7\pi}{4} + 2k\pi = \frac{10\pi}{12} - \frac{21\pi}{12} + 2k\pi \overset{k=1}{=} -\frac{11\pi}{12} + \frac{24\pi}{12} = 13\pi/12$ //

Zadanie 3

Przedstawić na płaszczyźnie zespolonej zbiory spełniające poniższe warunki

a) $|iz+1-i| < |z+1+i|$.

(2.5 pkt)

b) $0 < \mathrm{Arg}(-2z) \leqslant \frac{\pi}{2}$.

(2.5 pkt)

Zad 1 cd. (1) $z + i = 2z + iz \Rightarrow z_1 = \frac{i}{1+i} \frac{1-i}{1-i} = \frac{1+i}{2} = \frac{1}{2} + \frac{1}{2}i$ //

$z(-1-i) = -i$

(2) $z + i = -z + 2iz$

$z(2-2i) = -i$

$z_2 = -i/2-2i \cdot \frac{2+2i}{2+2i} = \frac{2-2i}{8} = \frac{1}{4} - \frac{1}{4}i$ //

(4) $\cancel{z} + i = \cancel{z} - 2iz$

$2iz = -i$

$z_4 = -1/2$ //

(3) $z + i = -2z - iz \Rightarrow z(3+i) = -i \Rightarrow z = \frac{-i}{3+i} \cdot \frac{3-i}{3-i} =$

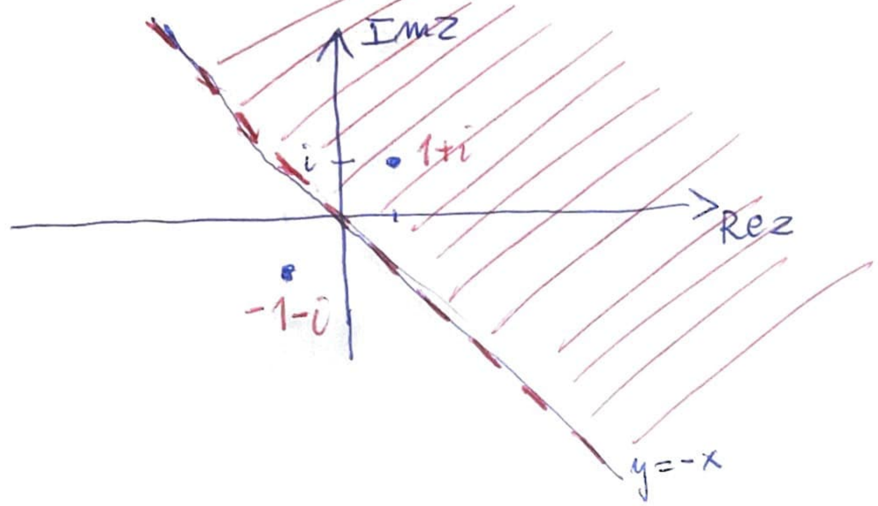
$z_3 = \frac{-1-3i}{10} = -1/10 - 3/10\, i$ //

a)

$$|iz+1-i| = \left|i\left(z+\frac{1-i}{i}\right)\right| = \underset{=1}{|i|}\left|z+\frac{1-i}{i}\cdot\frac{i}{i}\right| =$$

$$= \left|z+\frac{1+i}{-1}\right| = |z-(1+i)|$$

$$|z-(1+i)| \leq |z-(-1-i)|$$



b)

$$0 < \operatorname{Arg}(-2z) \leq \frac{\pi}{2}$$

$$0 < \operatorname{Arg}(-2) + \operatorname{Arg}(z) + 2k\pi \leq \frac{\pi}{2}$$

$$0 < \pi + \operatorname{Arg}(z) + 2k\pi \leq \frac{\pi}{2} \quad \Big| -\pi - 2k\pi$$

$$-\pi - 2k\pi < \operatorname{Arg}(z) \leq -\frac{\pi}{2} - 2k\pi \quad \leftarrow k=-1$$

$$\pi < \operatorname{Arg}(z) \leq \frac{3\pi}{2}$$