Adm 1.2

2.11b)

f, g 1-1 $\Rightarrow$ $f \circ g$ 1-1

2.11a) f 1-1 $\Rightarrow ZW_f = D_{f^{-1}}$

$ZW_f = ZW_{f_-} \cup ZW_{f_+} \overset{TW}{=} D_{f_-^{-1}} \cup D_{f_+^{-1}} = \{y : 1 + \frac{1}{y} \geq 0 ; y \neq 0\}$

$ZW_f = (-\infty, -1] \cup (0, +\infty)$

c) $f(x) = \frac{x}{x-1} = 1 + \frac{1}{x-1}$

$y - 1 = \frac{1}{x-1} \quad / \cdot \frac{x-1}{y-1}$

$x - 1 = \frac{1}{y-1}$

$x = 1 + \frac{1}{y-1} = \frac{y}{y-1}$

$f^{-1}(y) = \frac{y}{y-1}$

~~$f[(-\infty, -1) \cup (-5, +\infty)$~~

~~$(y) =$~~

2.14) c) $f(x) = \underbrace{2}_{\text{nie zmienia okresu}} \overbrace{\sin 3x}^{g(x)} + \underbrace{3}_{\text{nie zmienia okresu}} \overbrace{\cos 2x}^{h(x)}$

$T_{\sin} = T_{\cos} = 2\pi$

$T_g = \sin(3(x+T)) = \sin(3x + \cancel{2\pi})$

$3T = 2\pi$

$T = \frac{2}{3}\pi$

$T_h = \cos(2x + 2T) = \cos(2x)$

$2T = 2\pi$

$T = \pi$

$T_f = NWW(T_g, T_h)$

$m \cdot T_g = n \cdot T_h \qquad m, n \in N$

$m \cdot \frac{2}{3}\pi = n \cdot \pi$

$\frac{m}{n} = \frac{3}{2} \overset{\min}{\Rightarrow} \begin{cases} m = 3 \\ n = 2 \end{cases}$

$T_f = 2 \cdot \pi = \cancel{2\pi}$

$a_n \nearrow$ od n_0-ego miejsca

d) $a_n = n - 3^n$

$a_{n+1} - a_n = (n+1) - 3^{n+1} - n + 3^n = 1 - 3^{n+1} + 3^n = 1 - 3 \cdot 3^n + 3^n =$

$1 - 3^n(3-1) = 1 - 3^n \cdot 2 < 0$

$(a_n) \searrow$ od 1-ego miejsca

3.2 b) $b_n = \frac{9^n}{n!}$

$$\frac{\frac{9^{n+1}}{(n+1)!}}{\frac{9^n}{n!}} = \frac{9^{n+1}}{(n+1)\cancel{n!}} \cdot \frac{\cancel{n!}}{9^n} = \frac{9}{n+1}$$

od $n=9$ maleje

3.6 a, m, f, o

a) $\lim\limits_{n\to\infty} \frac{4n-3}{6-5n} =$

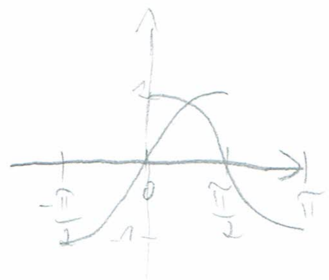
m) $a_n = \frac{2n + (-1)^n}{n} = 2 + \frac{(-1)^n}{n}$

Fakt: $\left.\begin{array}{l} a_n \xrightarrow{n\to\infty} 0 \\ (b_n) \text{ ogr} \end{array}\right\} \Rightarrow a_n b_n \xrightarrow{n\to\infty} 0$

2.11) $\arcsin = \left(\sin\restriction[-\pi/2, \pi/2]\right)^{-1}$ . . . arctg

$\arccos = \left(\cos\restriction[0, \pi]\right)^{-1}$ . . . arcctg

2.12.



$$\arcsin\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$$

$\sin x = -\frac{1}{\sqrt{2}}$
w przedziale

$$\arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$$

$\cos x = -\frac{1}{2}$

$$\mathrm{arctg}\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$$

$\mathrm{tg}\, x = -\frac{1}{\sqrt{3}}$

$$\mathrm{arcctg}(-\sqrt{3}) = \frac{5}{6}\pi$$

$\mathrm{ctg}\, x = -\sqrt{3}$

5.6 a)

$$\frac{2n+\sqrt{n+n^2}}{3n-\sqrt[3]{n\cdot n^5}}=\frac{2+\sqrt{\frac{1}{n}+1}}{3-\sqrt[3]{\frac{1}{n^2}-1}}=\frac{2+1}{3+1}=\frac{3}{4}$$

$\frac{1}{n} \to 0$, $\frac{1}{n^2} \to 0$

f) $\sqrt{4n^2+5n-7}-2n=\sqrt{n^2\left(4+\frac{5}{n}-\frac{7}{n^2}\right)}-2n=|n|\sqrt{4+\frac{5}{n}-\frac{7}{n^2}}$ $\frac{1}{4}$

~~f) $\sqrt{4n^2+5n-7}-2n=\left(\sqrt{4n^2+5n-7}-2n\right)\left(\sqrt{4n^2}\right.$~~

$$\lim_{n\to\infty} q^n \begin{cases} +\infty \text{ gdy } q>1 \\ =0 \text{ gdy } |q|<1 \\ =1 \text{ dla } q=1 \\ \text{nie istnieje dla } q\leq -1 \end{cases}$$