4) f) $\dfrac{(1+i)^{22}}{(1-i\sqrt{3})^{6}}$

$\dfrac{\sqrt{3}}{2}$ $\dfrac{\sqrt{2}}{2}$ $\dfrac{1}{2}$

$\sqrt{3}$ 1 $\dfrac{1}{\sqrt{3}}$

$r_1 = \sqrt{1+1} = \sqrt{2}$ $r_2 = 2$

$\varphi_1 = \dfrac{\pi}{4}$ $\varphi_2 = \dfrac{\pi}{3}$

$$\frac{\sqrt{2}\, e^{i 22 \cdot \frac{\pi}{4}}}{\frac{\sqrt{3}}{2} e^{i 6 \frac{\pi}{3}}} = \ldots \cdot e^{i\left(\frac{11}{2}\pi - 2\pi\right)} = \ldots \quad e^{\frac{7\pi}{2}} = 2\pi \ \frac{3\pi}{2}$$

$$\cdot \left(\cos \frac{3}{2}\pi + i \sin \frac{3}{2}\pi\right) = -\frac{\sqrt{2}^{22}}{\left(\frac{\ldots}{2}\right)^{6}} i = -\frac{2^{11}}{\ldots} i$$

$= -2^{5} i = -32i$

4.1 a) $z^2 = \overline{z}$

$(a+bi)$

O ??

$e^{2\varphi i} = e^{(2k\pi - \varphi)i}$

$2\varphi = 2\pi - \varphi$

$3\varphi = 2\pi k$

$\varphi = \frac{2\pi k}{3}$

$k=0 \quad z = 1$

$k=1 \quad z = -\frac{1}{2} + \frac{\sqrt{3}}{2} i$

$k=2 \quad z = -\frac{1}{2} - \frac{\sqrt{3}}{2} i$

b) $|z^4| = 2 \qquad 1, 0$

$e^{i0} r^4 = re^{i\varphi}$

$\varphi = 0$

c) $z^3 \cdot (\overline{z})^2 = -1 \qquad e^{-2(\varphi - 2k\pi)}$

$e^{3(\varphi+2k\pi)i} \cdot e^{2(2k\pi - \varphi)i} = -1 =$

$e^{3(\varphi+2k\pi)i - 2(\varphi - 2k\pi)i} = -1$

$e^{3\varphi i + 6k\pi i - 2\varphi i + 4k\pi i} = e^{(\varphi + 2k\pi)i} = -1$

$e^{\varphi i} =$

4.2) a) $(1-i)^{12} = \left(\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)\right)^{12} = 2^6\left(\cos 3\cdot 7\pi + i\sin 3\cdot 7\pi\right) = 2^6(\cos\pi + i\sin\pi) = -64$

b) $(1+\sqrt{3}i)^8 = \left(2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right)^8 = 2^8\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = -2^7 + 2^7\sqrt{3}i$

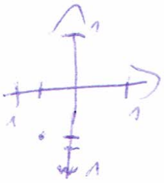
c) $(2\sqrt{3} - 2i)^{30} = \left(4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)\right)^{30} = 4^{30}(\cos\pi + i\sin\pi) = -4^{30}$

d) $\cancel{(1+i)^4(1+i)^6 = (1+i)^6 = (1+i)^6\left((1+i)^4 - 1\right) =}$

d) $(1+i)^{10}-(1+i)^{6} = \left(\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right)^{10} - \left(\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right)^{6} =$

$2^{5}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right) - 2^{3}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right) = i2^{5} + i2^{3} = 40i$

e) $\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^{12} = (\cos 16\pi + i\sin 16\pi) = 1$



$\frac{4\pi}{3}+\pi$

$\frac{1}{4}+\sqrt{\frac{3}{4}}$

f) $\frac{(1+i)^{22}}{(1-i\sqrt{3})^{6}} = \frac{\left(\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right)^{22}}{2^{6}\left(\cos\frac{10\pi}{6}+i\sin\frac{10\pi}{6}\right)} = \frac{-2^{11}i}{2^{6}i} = -2^{5}i = -32i$

$\frac{3\pi}{2}+\frac{\pi}{6} = \frac{9\pi}{6}+\frac{\pi}{6} = \frac{10\pi}{6}$

4.3) ~~$\mathrm{Re}\left(\frac{(1+3i)(2i+2)}{3+i}\right) = \mathrm{Re}\,\frac{2i-2-6-6i}{3i} = \mathrm{Re}\,\frac{-8-4i}{3+i} = \mathrm{Re}\,\frac{-4(2+i)}{3+i}$~~

~~Re~~

4.4) a) $2i$ ~~b) $2i\sqrt{2i}$ $(\sqrt{2i})^{3}$~~ c)

b) $\sqrt{-8i}$ $\sqrt{i} = i^{\frac{1}{2}} = \sqrt[4]{-1}$

$\sqrt{-8\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)} = 2\sqrt{2}i\left(\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}\right)$

a) $2z + (3-i)\bar{z} = 5+4i$

$2(a+bi) + (3-i)(a-bi) = 5+4i$

$2a + 2bi + 3a - 3bi - ai - b = 5+4i$

$(2a+b) + (-a-b)i$

$2a+b = 5$
$-a-b = 4$

$a = 9$

$b = -13$

b) $z+i = \overline{z+i}$

$a+bi+i = \overline{a+bi+i}$

$a+i(b+1) = \overline{a+i(b+1)}$

$a+i(b+1) = a-i(b+1)$

$a = a$

$b+1 = -b-1$ $z = a-i;\ a \in \mathbb{R}$

$2b = 2$

$b = 1$

~~4. b) $z^2+4z+5=0$~~

c) $z \cdot \bar{z} + (z - \bar{z}) = 3+2i$

$(a+bi)(a-bi) + (a+bi - a+bi) = 3+2i$

$(a^2 - abi + abi + b^2) + (2bi) = 3+2i$

$a^2+b^2 + 2bi = 3+2i$

$2b = 2$
$b = 1$

$a^2+1 = 3$
$a^2 = 2$
$a = \sqrt{2} \lor a = -\sqrt{2}$

$\sqrt{2}+i \lor -\sqrt{2}+i$

e) $\frac{1+i}{z} = \frac{2-3i}{\bar{z}}$ $\qquad z \neq 0$

$(1+i)\bar{z} = (2-3i)z$

$(1+i)(a-bi) = (2-3i)(a+bi)$

$a-bi+ai+b = 2a+2bi-3ai+3b$

$\cancel{a(1+i)+b(1-i)=a(}$

$(a+b)+(a-b)i = (2a+3b)+(2b-3a)i$

$a+b = 2a+3b$

$a-b = 2b-2a$

$\cancel{a,b \in \emptyset}$

Grah vodičení

$a = 2b$

$-a = b$

$\cancel{+2b=}$

$a = 2b$

$a = -b$

f) $\frac{2+i}{z-1+4i} = \frac{1-i}{2z+i}$

$(1-i)(a+bi-2+4i) = (2+i)(2a+2bi+i)$

$a+bi-2+4i-ai+b+2i+4 = 4a+4bi+2i+2ai \mp 2b-1$

$a+bi \cancel{+4i} -ai+b-4a-4bi-2ai+2b = +2i-1+2-4i-2i-4$

$(a-4a+b+2b)+i(-a+b-4b-2a) = (-1+2-4)+i(2-4-2)$

$(-3a+3b)+i(-3a-3b) = -3-4i$

$$\begin{aligned} -3a+3b &= -3 \\ -3a-3b &= -4 \\ \hline -6a &= -7 \end{aligned}$$

$a = \frac{7}{6}$

$b = \frac{1}{6}$

$z = \frac{7-i}{6}$

$\cancel{1}$.6) $z^2 = t$

$t^2 - 2t + 4 = 0$

$\Delta = 4 - 4 \cdot 1 \cdot 4 = -12$

$t_1 = \frac{2 - \sqrt{-12}}{2}$ $\qquad$ $t_2 = \frac{2 + \sqrt{-12}}{2}$

$t_1 = \frac{\cancel{2} - \cancel{2}i\sqrt{3}}{\cancel{2}}$ $\qquad$ $t_2 = \frac{\cancel{2} + \cancel{2}i\sqrt{3}}{\cancel{2}_1}$

$(a+bi)^2 = a^2 + abi - b^2$

$a^2 - b^2 + abi = 1 - 1i\sqrt{3}$

$a^2 - b^2 = 1 \land ab = \sqrt{3}$

$(a-b)(a+b) = 1 \land ab = \sqrt{3}$

4.7) $\cancel{a + a + bi = 8 + 4i}$ $\sqrt{a^2+b^2} + a + bi = 8 + 4i$

$\cancel{2a + bi =}$

$a +$

$x^2 - y^2 + 2xyi = -9 + 40i$

4.9/h) $(z+2)^2 = (\bar{z}+2)^2$

$$\left(\frac{z+2}{\bar{z}+2}\right)^2 = 0$$

$$\frac{z+2}{\bar{z}+2} = \sqrt{1} = \{1, -1\}$$

$z+2 = \bar{z}+2 \quad \vee \quad z+2 = -\bar{z}-2$

$z - \bar{z} = 0 \quad \vee \quad z+\bar{z} = -4$

$x+iy-x+iy = 0 \quad \vee \quad x+iy+x-iy = -4$

$2iy = 0 \qquad 2x = -4$

$y = 0 \qquad x = -2$

$z = x \quad \vee \quad z = -2+iy$

i) $z^2 - (6+i)z + 11 - 7i = 0$

$\Delta = -9+40i$

$\sqrt{-9+40i} = x+iy$

$x^2+y^2 =$

4.8) ALGA $\cos^3\varphi = (\cos\varphi)^3$

4.7)

$$\sqrt{a^2+b^2} + a + bi = 8 + 4i$$

$$\begin{cases} \sqrt{a^2+b^2} + a = 8 \\ b\cancel{i} = 4\cancel{i} \end{cases}$$

$$\sqrt{a^2+16} + a = 8$$

$$\frac{a^2+16-a^2}{\sqrt{a^2+16}-a} = 8$$

$$\frac{16}{\sqrt{a^2+16}-a} = 8$$

$$16 = 8\left(\sqrt{a^2+16}-a\right)$$

$$\begin{cases} \sqrt{a^2+16} - a = 2 \\ \sqrt{a^2+16} + a = 8 \end{cases}$$

$$2\sqrt{a^2+16} = 10$$

$$\sqrt{a^2+16} = 5$$

$$|a^2+16| = 25$$

$$a^2+16 = 25 \vee a^2+16 = -25$$

$$a^2 = 9 \quad \vee a^2 \neq -41$$

$$a = 3 \vee -3 \quad \cancel{\vee\ a = i\sqrt{41} \vee -i\sqrt{41}}$$

e) $\sqrt{-11-60i} = a+bi$ $\qquad \sqrt{\sqrt{11^2+60^2}} = \sqrt{61}$

$-11-60i = (a+bi)^2$

$a^2-b^2+2abi = -11-60i$

$\sqrt{61} = \sqrt{a^2+b^2}$

$61 = a^2+b^2$

$$\begin{cases} a^2-b^2+2abi = -11-60i \\ a^2+b^2 = 61 \end{cases}$$

$2a^2+2abi = 50-60i$

$a^2+abi = 25-30i$

$a^2=25$

$a=5 \lor a=-5$

$ab=-30$

$b=\pm 6$

$5-6i \lor -5+6i$

4.5 g) $z^2-4z+13=0$

$\Delta z = 16-4\cdot 13 = -36 = \sqrt{6i}^2$

$z = \frac{4\pm 6i}{2} = 2 \pm 3i$

h) $(z+2)^2 = (\bar{z}+2)^2$

$(a+bi+2)^2 = (a-bi+2)^2$

$\cancel{a^2}+abi+\cancel{2a}+abi-\cancel{b^2}+2bi+\cancel{2a}+2bi+\cancel{4} = \cancel{a^2}-abi+\cancel{2a}-abi-\cancel{b^2}-2bi+\cancel{2a}-2bi+\cancel{4}$

$6abi-\cancel{b^2} = -6abi$

$6abi = -6abi$

$a=0 \lor b=0$

$$\sqrt{-8i} = \sqrt{-8(0-1i)} = \sqrt{8\left(\cos\tfrac{3\pi}{2} + i\sin\tfrac{3\pi}{2}\right)} = 2\sqrt{2}\left(\cos\tfrac{3\pi}{4} + i\sin\tfrac{3\pi}{4}\right)$$

$r(\cos\varphi + i\sin\varphi)$ $\frac{1}{\sqrt{2}}$ $\frac{1}{\sqrt{2}}$

$\cos\varphi = 0$ $\quad \varphi = \frac{3\pi}{2}$
$\sin\varphi = -1$

$$2\sqrt{2}\cdot\left(-\frac{1}{\sqrt{2}} + \frac{1i}{\sqrt{2}}\right) = -2+2i = \pm 2(i-1)$$

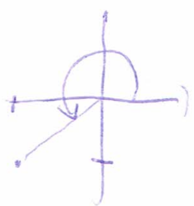
7) $\sqrt{-8-6i} = \sqrt{} = a+bi$

$\cancel{r(\cos\varphi + i\sin\varphi)}$

$\cancel{8(-1-\tfrac{3}{4}}$

$-8-6i$

$r = \sqrt{64+36} = 10$



$\sin\varphi = \frac{6}{10} = \frac{3}{5}$

$\cos\varphi = \frac{4}{5}$

$\sqrt{64+36}$

$\cancel{\sqrt{a^2+b^2} = \sqrt{10}}$

$\cancel{a^2+b^2 = 10}$

$(a+bi)^2 = -8-6i$

$a^2 - b^2 + 2abi = -8-6i$

$a^2 + b^2 = \cancel{\sqrt{}}10$

$2a^2 + 2abi = 2-6i$

$2a^2 = 2 \iff a = \cancel{\sqrt{2}}\, 1$

$2ab = -\cancel{6}\, 3$

$b = \frac{-3}{1} = -3$

$1 + -3i$

$\pm(1-3i)$