Advanced Informatics and Control

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Introduction to Computer Vision in Quality Control

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Wrocław University of Technology
EUROPEAN SOCIAL FUND
Project co-financed from the EU European Social Fund

Introduction to Computer Vision in Quality Control

Lecture 5 – Finding objects

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Summary of Lecture 4

- Single image processing
- Simple operations on pairs of images
- Logical operations on binary images
- Simple statistics
- Histogram of gray levels

Summary of Lecture 5

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Methods of finding objects

Finding objects:

- parts for inspection,
- defects

is the most important task in visual inspection.

Fundamental classes of methods:

- Segmentation:
  - by thresholding,
  - watershed (morphological – later),
- Contouring,
- Template matching,
- Masks of geometrical objects,
- The Hough transform.

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Segmentation by thresholding

Segmentation by thresholding and finding contours are methods, which mark objects independently of their shapes. They use information on gray levels only.

Simple thresholding:

- Select threshold $T \in (0, 255)$ – by the histogram analysis or "automatically".
- For each pixel set:

$$y_{ij} = \begin{cases} 0 \text{ (black)} & \text{if} \quad f_{ij} \leq T, \\ 1 \text{ (white)} & \text{if} \quad f_{ij} > T \end{cases}$$

Note that have change the convention – now objects are black.

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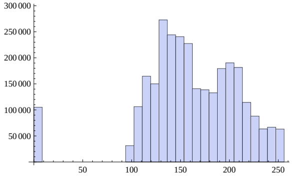
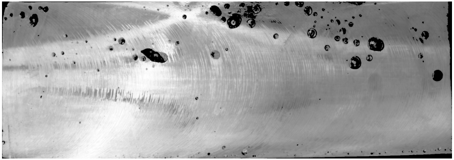
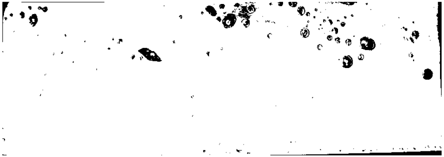
Thresholding – example 1

Select T by looking at the histogram – T = 50







All defects of interest are marked (but also badly cropped boundary).

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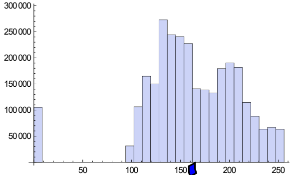
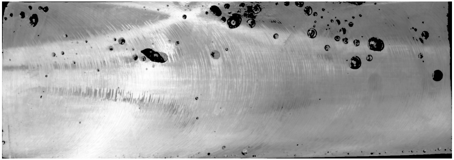
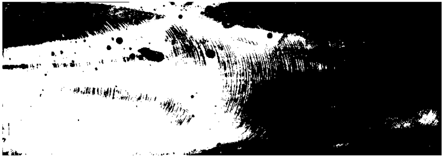
Automatic thresholding – a negative example

Select T using Otsu's method, which chooses the threshold to minimize the intraclass variance of the black and white pixels T = 163







It is still worth to study the methods of automatic threshold selection.

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Automatic threshold choice – assumptions:

Trying to apply methods of automatic threshold selection the following assumptions are usually (tacitly) made:

- object (defects) are black or dark gray,
- the background is white or light gray (or v.v.),
- the histogram is bi-modal (two loc. max.).

If these assumptions does not hold, one can expect bad choice of the threshold. Later, we discuss some remedies.

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Automatic threshold choice 1

Elementary method of selecting threshold (similar to 2-means, but not identical):

Step 1 **Set $n = 1$ and select a starting threshold T_0. Choose the accuracy $\epsilon \geq 1$.**

Step 2 **Calculate m_1 as the mean of those $f_{ij} \leq T_{n-1}$ and m_2 as the mean of those $f_{ij} > T_{n-1}$.**

Step 3 **Set $T_n = \text{Round}[(m_1 + m_2)/2]$. If $|T_n - T_{n-1}| < \epsilon$, then STOP, otherwise, $n := n + 1$ and go to 2.**

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Automatic threshold choice 2

Remarks on the elementary method:

- The term "elementary" is not pejorative one.
- The result depends on the starting point T_0.
- In our previous example: for $T_0 = 50$ provides quite good result $T = 81$, for $T_0 = 150$ provides $T = 165$, which is not very useful in our example.

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Automatic threshold choice 3

The well known clustering k-means method can be adopted as 2-means

- Step 1 **Set $n = 1$ and select two starting means $\mu_1 \neq \mu_2$ and set $T_0 = \text{Round}[(\mu_1 + \mu_2)/2]$ Choose the accuracy $\epsilon \geq 1$.**
- Step 2 **Calculate m_1 as the mean of those f_{ij}, which are closer to μ_1 then to μ_2. Calculate m_2 as the mean of those f_{ij}, which are closer to μ_2 then to μ_1**
- Step 3 **Set $T_n = \text{Round}[(m_1 + m_2)/2]$. If $|T_n - T_{n-1}| < \epsilon$, then STOP, otherwise, $n := n + 1$, $\mu_1 := m_1$, $\mu_2 = m_2$ and go to 2.**

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Automatic threshold choice 2

Remarks on 2-means method:

- The result depends on the starting points μ_1 and μ_2.
- More time-consuming than the elementary method.
- Slow convergence can be met.
- In our previous example: for $\mu_1 = 25$ and $\mu_2 = 153$ provides quite good result T $= 83$ (already in the first iteration), however, for $\mu_1 = 77$ and $\mu_2 = 204$ provides T $= 163$ (after 10 iterations), which is not very useful in our example.

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Selecting threshold using Bayesian classifier

Extended version – based on the ideas from Davies.

Assume that p.d.f.'s of gray levels of objects $k = 1$ and the background $k = 2$ are Gaussian:

$$g_k(x) = \frac{1}{\sqrt{2\,\pi}\,\sigma_k}\,\exp\left[-\frac{(x-\mu_k)^2}{2\,\sigma_k^2}\right]$$

Let p_1 and p_2 be a priori probabilities of objects and background, $p_1 + p_2 = 1$. Then, the classifier, which minimizes the error probability has the form: classify a pixel with gray level x to class "o", if $p_1\, g_1(x) \geq p_2\, g_2(x)$ and to "b", otherwise.

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Selecting threshold using Bayesian classifier 2

Point T, for which

$$\mathrm{p}_1\,\mathrm{g}_1(\mathrm{x}) = \mathrm{p}_2\,\mathrm{g}_2(\mathrm{x}) \qquad (*)$$

provides the class separating threshold. For Gaussian distributions equation (*) has two solutions, but only one of them is relevant:

$$\mathrm{T} = \frac{\pm\sqrt{\sigma_1^2\sigma_2^2\left(\left(\sigma_1^2-\sigma_2^2\right)\log\left(\frac{\mathrm{p1}^2\sigma_1^2}{\mathrm{p2}^2\sigma_2^2}\right)+\left(\mu_1-\mu_2\right)^2\right)}+\mu_2\sigma_1^2-\mu_1\sigma_2^2}{\sigma_1^2-\sigma_2^2}$$

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Selecting threshold using Bayesian classifier 3

If $\sigma_1^2 = \sigma_2^2$, which rarely happens in practice, the above simplifies to:

$$\mathrm{T} = \frac{1}{2}\left(\frac{\sigma^2 \log\left(\frac{\mathrm{p1}^2}{\mathrm{p2}^2}\right)}{\mu_1 - \mu_2} + \mu_1 + \mu_2\right)$$

If, additionally, $\mathrm{p}_1 = \mathrm{p}_2$, then

$$\mathrm{T} = \frac{1}{2}(\mu_1 + \mu_2)$$

Industrial Image Processing 5
Selecting threshold using Bayesian classifier 4

Remarks:

- In industrial practice subsequent images are very similar. It is not difficult to estimate p_k, μ_k and σ_k, $k = 1, 2$ from test images.
- The role of a priori probabilities is crucial – they can compensate the impact of "too heavy" background distribution. Other methods do not provide such a possibility and the threshold is frequently too close to the "heavier" distribution.

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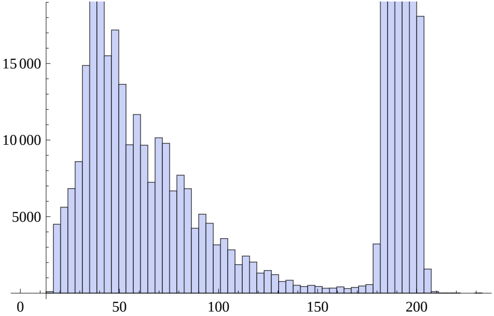
Selecting threshold using Bayesian classifier – example 1



"By visual inspection" one would select $T \in [160, 170]$.

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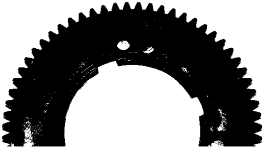
Selecting threshold using Bayesian classifier – example 2



Object select using T = 123, which is provided by the Otsu method, the elementary method and 2-means method. Not satisfactory – additional white places on the bottom.

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Selecting threshold using Bayesian classifier – example 3

Although the second distribution ("tower") is not Gaussian, we apply the Bayesian classifier. Parameters estimated from the same image:

- $\mu_1 = 58.4$, $\sigma_1^2 = 687.8$.
- $\mu_2 = 191.9$, $\sigma_2^2 = 33.3$.
- $p_1 = 0.45$, $p_2 = 0.55$.

Solving (*) we obtain $T = 231$, which is rejected and $T = 166$, which the correct solution.

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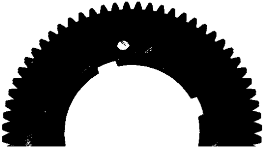
Selecting threshold using Bayesian classifier – example 4

Using the empirical Bayes solution $T = 166$ we obtain:



which is much better than earlier segmentations.

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Other statistical methods of selecting threshold

Let n_l be the number of pixel having gray level $l \in \{0, 1, \ldots, 255\}$. Clearly $\sum_{l=1}^{255} n_l = N \cdot M$ – the total number of pixels. Define:

- $\hat{p}_l = n_l/(N \cdot M)$,
- $\hat{\pi}_0(t) = \sum_{l=1}^{t} \hat{p}_l, \quad \hat{\pi}_1(t) = 1 - \hat{\pi}_0(t)$,
- $\hat{\mu}_0(t) = \hat{\pi}_0^{-1}(t) \sum_{l=0}^{t} l\, \hat{p}_l$
- $\hat{\mu}_1(t) = \hat{\pi}_1^{-1}(t) \sum_{l=t+1}^{255} l\, \hat{p}_l$
- $\hat{\mu} = \sum_{l=0}^{255} l\, \hat{p}_l$ – grand mean.

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The Otsu method:

The Otsu method – select $\hat{\mathsf{T}}$, for which the inter-class variance:

$$\hat{\sigma}_{\mathsf{IC}}(\mathsf{t}) = \hat{\pi}_0(\mathsf{t})\left(\hat{\mu}_0(\mathsf{t}) - \hat{\mu}\right)^2 + \hat{\pi}_1(\mathsf{t})\left(\hat{\mu}_1(\mathsf{t}) - \hat{\mu}\right)^2$$

attains max. w.r.t. to $\mathsf{t} \in \{0, 1, \ldots, 255\}$.

We have seen examples how it works.

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Maximum entropy principle

We retain the same definitions as above. Additionally define:

- $H_0(t) = -\sum_{l=0}^{t} \left(\frac{\hat{p}_l}{\hat{\pi}_0(t)}\right) \cdot \ln\left(\frac{\hat{p}_l}{\hat{\pi}_0(t)}\right)$
- $H_1(t) = -\sum_{l=t+1}^{255} \left(\frac{\hat{p}_l}{\hat{\pi}_1(t)}\right) \cdot \ln\left(\frac{\hat{p}_l}{\hat{\pi}_1(t)}\right)$
- $H_{sum}(t) = H_0(t) + H_1(t)$.

Select $\tilde{T}$, for which $H_{sum}(t)$ attains max. w.r.t. to $t \in \{0, 1, \ldots, 255\}$.

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Maximum entropy principle – justification

For a discrete probability distribution $q_l \geq 0$, $l = 1, 2, \ldots, Q$, the entropy H is defined as

$$H = -\sum_{l=1}^{Q} q_l \cdot \ln(q_l). \qquad (**).$$

In the above the sequence $\hat{p}_l$, $l = 0, 1, \ldots, 255$ is divided for two distributions by a tentative threshold t. Namely:

$$\hat{p}_l/\hat{\pi}_0(t), \qquad l = 0, 1, \ldots, t \qquad \text{(Left)}$$

$$\hat{p}_l/\hat{\pi}_1(t), \qquad l = (t+1), 1, \ldots, 255 \qquad \text{(Right)}$$

Note that they are normalized by their sums $\hat{\pi}_0(t)$, $\hat{\pi}_1(t)$, respectively.

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Maximum entropy principle – justification 2

For random vector $Y = (Y_1, Y_2)$, say, the mutual information $I(Y)$ between its components (also called the Kullback-Leibler distance between the distribution of Y and the distribution of its components) has the form: $I(Y) = H(Y_1) + H(Y_2) - H(Y)$ and serves as the measure of independence between $Y_!$ and Y_2 (Hastie et all).
Thus, in our case the maximization of $H_{sum}(t)$ is equivalent to the maximization of the degree of independence between the left and right distributions.

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Maximum entropy principle – example 1

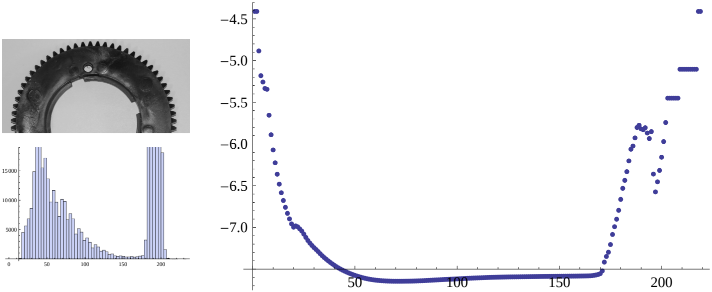

Figure: Histogram and the corresponding $\mathbf{H_{sum}(t)}$ – local max of $\mathbf{H_{sum}(t)}$ at $\mathbf{\tilde{T} = 174}$ is clearly visible.

Remark: in practice max. $\mathbf{H_{sum}(t)}$ should be restricted to more narrow range of gray levels (40–210, say), since for $\mathbf{t < 40}$ and $\mathbf{t > 210}$ the entropy of almost all image gray level range is calculated, which is larger than the entropy of its parts.

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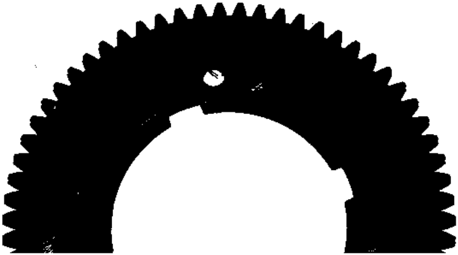
Maximum entropy principle – example 2



Threshold $\tilde{T} = 174$ – provided by the maximum entropy method – almost perfectly selects the object.

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Segmentation by thresholding – Discussion 1

- The elementary method, 2-means, Bayesian threshold and maximum entropy method are time consuming.
- They are applicable off-line to select the threshold, which can be used on-line, but only when:
  - very similar objects (with possible small defects) are inspected,
  - the same illumination conditions are kept.
- The histogram of a typical image is bi-modal and one threshold is sufficient for proper selection of objects.

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Segmentation by thresholding – Discussion 2

If these conditions are not met, we can select one of the following approaches:

- apply a simpler method of selecting the threshold,
- divide image into sub-images and select thresholds, which are local in space,
- use more than one threshold to each pixel,
- use more than one threshold to each pixel, taking into account gray levels of its neighbors (thresholding with hysteresis).

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Segmentation by thresholding – Discussion 3

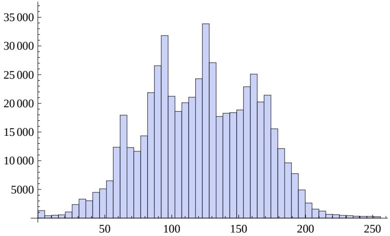
All the above topics will be shortly discussed next time.

Additionally, the following aspects of the segmentation by thresholding will be considered:

- adaptive thresholding,
- correction of a non-uniform background,
- the impact of noise on thresholding,
- consequences of more than two modes.

Image Processing 5
Segmentation by thresholding – Discussion 3

To illustrate consequences of more than two modes, recall the following histogram:



Further reading

- Otsu, N., "A Threshold Selection Method from Gray-Level Histograms," IEEE Transactions on Systems, Man, and Cybernetics, Vol. 9, No. 1, 1979, pp. 62-66.
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- Pratt, W.K., *Digital image processing*, New York, Wiley, 1991.