Reccurance Relation

→ LRCCH :- linear Relation constant coeficent homogenou

e.g

$$a_n = 2a_{n-1} + 3a_{n-2} \quad , \quad a_0 = 1 \; , \; a_1 = 2$$

constant coefficent | linear Relation | homogenous {terms are same type}

Solving steps:- ① 'a' side all term of same type

e.g. $\boxed{a_n - 2a_{n-1} - 3a_{n-2} = 0}$,

② convert relation into power of t^{th} as per ~~ak~~ a_n

$\boxed{t^2 - 2t - 3t^0 = 0}$

③ Solve roots

$$t^2 - 3t + t - 3 = 0 \Rightarrow \boxed{t = 3, -1}$$

rules for roots

① if roots α_1, α_2 are same then eqn will be $a_n = (C_1 + C_2 n)\alpha^n$

② if roots α_1, α_2 are differ then

$$a_n = C_1\alpha_1^n + C_2\alpha_2^n$$

③ $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$ and $\beta_1 = \beta_2 = \beta$

$$a_n = (C_1 + C_2 n + C_3 n)\alpha^n + (\beta_1 + n\beta_2)\beta^n$$

④ put roots into eqn

$$a_n = C_1 3^n + C_2(-1)^n$$

⑤ find value of C_1 and C_2 by a_0 and a_1.

e.g $a_1 = C_1 3^1 + C_2(-1)^1 \Rightarrow 2 = 3C_1 - C_2$

$a_0 = C_1 3^0 + C_2(-1)^0 \Rightarrow \underset{(+)}{1} = \underset{(+)}{C_1} + \underset{(+)}{C_2}$

$3 = 4C_1 \Rightarrow \boxed{C_1 = 3/4}$

$\boxed{C_2 = 1/4}$

$$\boxed{a_n = \left(\tfrac{3}{4}\right)3^n + \left(\tfrac{1}{4}\right)(-1)^n}$$

LRCCIH ⇒ LRCC In-Homogenous

e.g $a_n = 5a_{n-1} - 6a_{n-2} + 5$

[Different type other than a_{n-1}, a_{n-2} and so on]

These type of equation can be divided into two parts (i) Homogenous (a_n^h) (ii) particular (a_n^p)

$a_n = a_n^h + a_n^p$ eq(2) {Solution will be combination of a_n}

Solving of a_n^h are same as before.

e.g $\underbrace{\boxed{a_n - 5a_{n-1} + 6a_{n-2}}}_{a_n^h \text{ eq}^n} = \underbrace{\boxed{5}}_{a_n^p \text{ eq}^n}$ — eq(1)

~~Solving of a_n^p~~ :- There are some rules at Table 1

RHS	Trivial
c	d
c_1n+c_2	d_1n+d_2
$c_1n^2+c_2n+c_3$	$d_1n^2+d_2n+d_3$
ca^n	da^n
$(c_1n+c_2)a^n$	$(d_1n+d_2)a_n$

~~after solvin~~ after solving a_n^h, $a_n^n = c_1 2^n + c_2 3^n$

(1) convert a_n^p into d from table.

$a_n^p = d$

rule to convert :-

if a_n^p got collision with some of term in a_n^h then multiply it with n

e.g $a_n = \underbrace{c_1(1)^n}_{\text{constant}} + c_2(1)^n + \underbrace{d}_{\text{constant}}$

there is collistion, so multiple a_n^p with n.

so $a_n^p = dn$

again $a_n = c_1(1)^n + \underbrace{c_2n(1)^n}_{\text{same type}} + dn$ [both have n]

there is collision, so multiply a_n^p with n.

$a_n^p = dn^2$

again $a_n = c_1(1)^n + c_2n(1)^n + dn^2$

NO collision

② Now after checking collision make find a_n^p ③

$$a_n^p = d \xrightarrow[eq(1)]{} eq(3)$$

③ Replace a_n^p in eq(1) $\{2a_{n-1} + 3a_{n-3} \ldots\}$

rules of replacement

let $a_n^h = 2a_{n-1} + 3a_{n-3}$

if $a_n^p = d$, then $a_n^p = 2d + 3d$

if $a_n^p = nd$, then $a_n^p = 2(n-1)d + 3(n-3)d$

if $a_n^p = a^n d$ then $a_n^p = 2a^{n-1}d + 3a^{n-3}d$

$$d - 5d + 6d = 5 \Rightarrow \boxed{d = \frac{5}{2}}$$

so $\boxed{a_n^p = d = \frac{5}{2}}$

and $a_n = c_1 2^n + c_2 3^n + \frac{5}{2}$

④ find value of c_1 and c_2.

Variation of a_n^p

(i) If a_n^p is power form eg $a_n = 5a_{n-1} + 6a_{n-2} + 3 \cdot 2^n$ (Power form)

then do as simple.

(ii) If a_n^p is Guardic form eg $a_n = 5a_{n-1} + \underbrace{n^2 + n + 2}_{\text{(Guardic form)}}$

then apply subsitution method

$$S_n = 1 + 2 + \ldots n = \frac{n(n+1)}{2}$$

$$S_n = 1^2 + 2^2 + \ldots n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$S_n = 1^3 + 2^3 + 3^3 \ldots n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Other type of reccurence

LRVC :- Linear ~~relation~~, variable coefficient

$n a_n = 5 n a_{n-1} - 5 a_{n-1} + 3$

[variable coefficient]

[convert LRVC $\rightarrow$ LRCC if possible.]

$n a_n = \cancel{a_{n-1}}\ 5(n-1) a_{n-1} + 3$

Put $n a_n = b_n$, so

$\boxed{b_n = 5 b_{n-1} + 3}$ $\Rightarrow$ solve as per previous methods

NLR :- Non-linear relation

$a_n^2 = 5 a_{n-1}^2 + 6$ [NLR $\rightarrow$ LR]

put $b_n = a_n^2$

$\boxed{b_n = 5 b_{n-1} + 6}$ $\rightarrow$ solve as per previous methods

$a_n^2 = 8 a_{n-1}$

Put log both side

$2 \log a_n = \log 8 + \log a_{n-1}$

Put $b_n = \log a_n$

$\boxed{2 b_n = b_{n-1} + \log 8}$

$a_n = a_{n/2} + 5$

Put $n = 2^k$

$a_{2^k} = a_{2^{k-1}} + 5$

Put $a_{2^k} = b_k$

$\boxed{b_k = b_{k-1} + 5}$