Hasse Diagram

① convert partial order to Hasse diagram

(i) NO self loops
(ii) NO arrow (direction is upward)
(iii) NO Edge to represent transitivity

e.g. $A = \{1, 2, 3, 4\}$ $R = \{\langle 1,1\rangle, \langle 2,2\rangle, \langle 3,3\rangle, \langle 4,4\rangle$
$\langle 1,2\rangle, \langle 2,3\rangle, \langle 1,3\rangle \}$

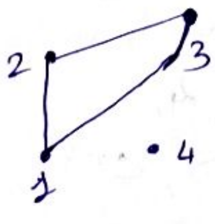
Step ① start Draw from minimal ~~dig~~, element

minimal element :- It does not appear on ~~at~~ RHS of any pair expect self pair.

so minimal elements are $\{1, 4\}$

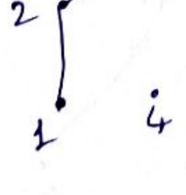
(draw minimal at same level)

Step ② Draw rest pairs



(remove the (1,3) because it is transitive)

Step ③ Remove transitive



[# note :- Remove the transistion pair from the relation.]

eg $A = \{1, 2, 3, 4\}$ $R = \{(1,1), (2,2), (3,3), (4,4), (5,5)$
$(2,1), (1,3), \cancel{(2,3)}, (1,5), \cancel{(2,5)}$
$(4,1), \cancel{(4,3)}, \cancel{(4,5)}\}$

Step ① find the minimal element

2 4

Step ② Remove all the transistive pair

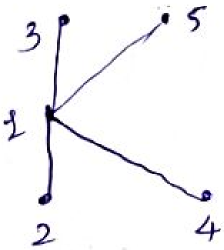
$(2,1)\ (1,3) \rightarrow (2,3)$ [Remove]

$(2,1), (1,5) \rightarrow (2,5)$ [Remove]

$(4,1), (1,3) \rightarrow (4,3)$ [Remove]

$(4,1), (1,5) \rightarrow (4,5)$ [Remove]

Step ③ Draw all the edges.



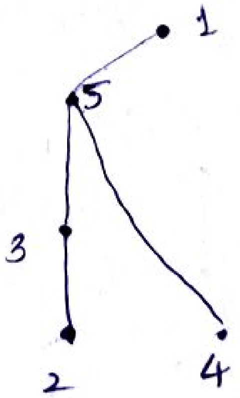
e.g $A = \{1, 2, 3, 4\}$

$R = \{\langle 1,1\rangle, \langle 2,2\rangle, \langle 3,3\rangle, \langle 4,4\rangle$
$\langle 5,5\rangle, \langle 2,3\rangle, \langle 3,5\rangle, \langle 5,1\rangle$
$\cancel{\langle 2,1\rangle}, \cancel{\langle 2,5\rangle}, \cancel{\langle 3,1\rangle}, \langle 4,5\rangle,$
$\cancel{\langle 4,1\rangle}\}$

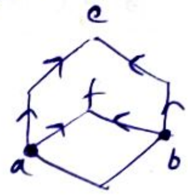
Step ① minimal elements 2, 4,

Step ② Remove transitive $\langle 2,5\rangle, \langle 2,1\rangle, \langle 3,1\rangle, \langle 4,1\rangle$

Step ③ Draw all the pairs



DM- Lattice -from Gate @ zeal



Upper bound :- all common upper nodes

$up(\{a,b\}) = \{e,f\}$

lower bound :- all common lower node of set S.

Least upper Bound of $A = \{a,b\}$

(i) find all upper bound of A

(ii) check if lower element is compare.able or not, if compareble. then small element will be least upper Bound.

Greatest lower Bound :- of $A = \{a,b\}$

(i) find all lower bound of A.

(ii) check if Greatest element is compareable.

Check for Lattice :-

(i) make all pairs of uncompareable element

(ii) find LUB and GLB of each pair if any of ~~not~~ not-exist than S is not lattice

Note :- single element in minimal set called minimum.

Note :- NO elements pair are complement to each other if they appear on same chain.

Note :- only greater (1) and leaster (0) are complement to each other in single chain.

Complement lattice :- ① find all non-comparable pairs.

② find at least one ~~pair~~ complement element

$a \wedge a' = 0$ && $a \vee a' = 1$

for all pairs.

Distributive lattice :-

If $\forall a, b, c \in S$

$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$$
$$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$$

Note:- If any element have more than or equal two complement then it not Distributive lattice

two complement $\Longrightarrow$ not distributive lattice

Note: If not contains  or  type sublattice then lattice is distributive.