combination

	with no Repetation	w/ Unlimited Rep.	w/ Limited Rep
Permutation (arrange)	^nP_r $n\geq r$	n^r $n\geq r$ or $r\geq n$	$\frac{n!}{n_1!n_2!\cdots n_r!}$
combinations (selection)	^nC_r	$^{n-1+r}C_r$	Generating functs

Permutation :- (Select & arrange)

Note : outcomes of (i) games (ii) Dice (iii) Coin (iv) Answer key Possible are Related to w/ Unlimited Rep $\boxed{n^r}$

Note Default mode of (a) toys (b) books (c) Coins (d) Dice (e) Object (f) People (g) boys are distincts.

Note :- problems like MISSISSIPPI can be done in w/ Limited Rep. $\boxed{\frac{n}{n_1!n_2!n\ldots n_r!}}$

Note :- when asked for like not start with or end with then do problem $\boxed{\text{Total} - \text{start or end}}$

Note :- Multiset can be solved in w/ Limited Rep

Constraint ① Starting with, ending with, not starting with, not ending
② altogether, all not togther (at least 2 are separated)
③ No two object of a certain type togther
④ Alternative arrangement.
⑤ circular

$\boxed{\text{at least 1 this = total - no this}}$

containing element.

$\boxed{\text{Not all togher = Total - all togher}}$

No two object togther = arrange other type and fill object b/w them

If n B n G, n B $(n+1)$ G, $(n+1)$ B n G — only possible, else result is 0 — alternate

n B n G $\rightarrow 2 \times (n!)^2$

n B $(n+1)$ G $\rightarrow n!\,(n+1)!$

$(n+1)$ B n G $\rightarrow n!\,(n+1)!$

arrangement of n peoples in circular is $(n-1)!$ [Distinct]

" " n beads in " " $\frac{(n-1)!}{2}$ [Identical] (or) [can rotate circle]

Note:- (1) $\boxed{\text{Chocolate problem}}$ is also Unlimited rep. Combination

$$^{n+r-1}C_r$$

(no. of groups) → n; r ← (max sum of all group number)

Note (2) Non-negative integer sol^n is unlimited rep combination

$$a+b+c+d=10, \quad a,b,c,d \geq 0$$

Note (3) Identical Ball in Distinct Box (Distribution)

Note: outcome of Identical coins & dice — Unlimited rep combination

00|0|0|0|0

Note: Dice sum problem & Digit sum

$$x_1+x_2+x_3=12$$

$$1 \leq x_1 \leq 6$$

Distribution

Distn → Distn, Dist → Ident, Ide → Dist

Distn, Ident: Distribution

Dist: unlimited rep combination

Variation of ${}^{n-1+r}C_r$

① $1 \le x_1$, $1 \le x_2$, $1 \le x_3$, so put 1 to each then $r-3k$

② if x_1 contain upper and lower constraint then solve using Generating function.

③ $x_1+x_2+x_3 \le K \Rightarrow x_1+x_2+x_3+\underbrace{x_4}_{\text{virtual Box}} = K$

④ $x_1+x_2 = 10$ ide $- x_1$ $\quad \boxed{x_1 \times x_2 \times x_3}$

$= 15 \quad - x_2$

$= 12 \quad - x_3$

Que- distribution of n balls into r boxes.

(i) balls are Identical and Box are Identical

then Answer $= \boxed{P(n, r)}$ and $\sum_{K=1}^{r} P(n-r, K)$

(ii) balls are ~~dist~~ Identical and Box are Distinct

$x_1+x_2+x_3 \ldots x_r = n$

no. of positive $= \boxed{{}^{n-1}C_{r-1}}$

(iii) balls - Distinct and Box are Identical

$$\boxed{S(n,r) = \frac{1}{r!}\left({}^{r}C_0 r^n - {}^{r}C_1 (r-1)^n + {}^{r}C_2 (r-2)^n + \ldots + (-1)^{r-1}(1)^n\right)}$$

(iv) balls Distinct and box are distinct

= no. of onto functions n to r

$$= \boxed{r^n - \left({}^{r}C_1 (r-1)^n - {}^{r}C_2 (r-2)^n + \ldots - (-1)^{r-1}(1)^n\right)}$$

Dearrangement

$$\boxed{D_n = \sum_{r=2}^{n} (-1)^r \frac{n!}{r!}}$$ [completly Dearrangement]

$$\boxed{D_n = {}^nC_r \, D_{n-r}}$$ [exactly r object in correct place] [object must be Distinct]

Some formulae

${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

${}^nC_x = {}^nC_y \Rightarrow x=y$ or $x+y=n$

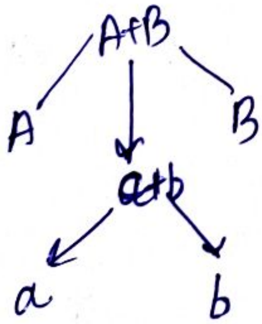
$n \cdot {}^{n-1}C_{r-1} \Rightarrow (n-r+1)\,{}^nC_{r-1}$

${}^nC_r = \frac{n}{r}\,{}^{n-1}C_{r-1}$

$\sum_{i=0}^{n} {}^nC_i = 2^n$, $\sum_{i=0}^{n/2} {}^nC_{2i} = 2^{n-1}$

$$\sum_{i=0}^{n/2} {}^nC_{2i+1} = 2^{n-1}$$

Selection of Distinct object from multiple group



$\boxed{{}^AC_a \times {}^BC_a}$

choosing a from A and b from B
A & B are distinct.

complement ~~version~~

${}^{A+B}C_{a+b} - \boxed{\text{condition}}$

Method to solve Limited Repetation

basic things

$$1+x+x^2+\dots x^n = \frac{1-x^{n+1}}{1-x}$$

$$1+x+x^2+\dots\dots = \frac{1}{1-x}$$

$$\sum_{r=0}^{\infty} {}^{n-1+r}C_r x^r = \frac{1}{(1-x)^n}$$

Selection of distinct from distinct set

⇒ When subset size specified then ~~answer~~ answer in form of ${}^nC_r$

⇒ when subset size is not ~~deffind~~ defind then anwser in form of 2^n.

e.g n distinct object in set.

(i) at least one element

$$\boxed{2^n - 1}$$

(ii) at least two element ⇒ $\boxed{2^n - [{}^nC_0 + {}^nC_1]}$

(iii) exactly three element ⇒ $\boxed{{}^nC_3}$

(iv) at least k element

$$\boxed{{}^nC_k + {}^nC_{k+1} \dots \dots {}^nC_n}$$

(v) at most k element

$$\boxed{{}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_k}$$

Greatest coefficient in a binomial expression

$(1+x)^n$ is ${}^nC_r$ for r.

n = even $\quad {}^nC_{n/2}$

n = odd $\quad {}^nC_{n-1/2}$ or ${}^nC_{n+1/2}$