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Permutations and Combinations



SYNOPSIS

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- Permutations
- Combinations

INTRODUCTION

The concept of permutation and combination is used to select or arrange some items out of a group according to certain predetermined conditions. This concept helps find the number

of ways to arrange, select, or reject these items. The concepts of permutations and combinations derive from the principles of factorials the fundamental principle of counting.

FACTORIAL

The factorial of a natural number n , denoted by $n!$ And is the product of all natural numbers from 1 up to n .

$$n! = 1 \times 2 \times 3 \times 4 \times \dots \times (n-1) \times n$$

OR

$$n! = n \times \{(n-1) \times (n-2) \times (n-3) \times (n-4) \times \dots \times 1\}$$

The factorials of the first few natural numbers given as follows:

1!	1	1
2!	2×1	2
3!	$3 \times 2 \times 1$	6
4!	$4 \times 3 \times 2 \times 1$	24
5!	$5 \times 4 \times 3 \times 2 \times 1$	120
6!	$6 \times 5 \times 4 \times 3 \times 2 \times 1$	720
7!	$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$	5040
8!	$8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$	40320
9!	$9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$	362880
10!	$10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$	3628800

Example: What is the value of $5! \times 3!$?

Solution: First expand both the factorials,

$$= (5 \times 4 \times 3 \times 2 \times 1) \times (3 \times 2 \times 1)$$

$$= 120 \times 6 = 720$$

Example: What is the value of $\frac{(12! \times 8!)}{(9! \times 10!)}$?

Solution: First of all, simplify the above given factorials,

$$\frac{12 \times 11 \times 10! \times 8!}{9 \times 8! \times 10!} = \frac{12 \times 11}{9}$$

$$= \frac{132}{9} = \frac{44}{3}$$



PERMUTATIONS

If there are n objects and one is supposed to arrange r ($r \leq n$) objects out of these, then the number of ways in which this can be done is written as nP_r and is read as 'the number of permutations of n objects taken r at a time'.

The number of permutations is given as:

$${}^nP_r = \frac{n!}{(n-r)!}$$

Example: In how many ways can you arrange two books out of three?

Solution: ${}^3P_2 = \frac{3!}{(3-2)!}$

$$= \frac{3!}{1!} = 3 \times 2 = 6$$

Some specific cases related to permutations are:

If $r = 0$,

$${}^nP_0 = \frac{n!}{(n-0)!} = 1$$

If $r = 1$,

$${}^nP_1 = \frac{n!}{(n-1)!} = \frac{n \times (n-1)!}{(n-1)!} = n$$

If $r = n$,

$${}^nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} = n!$$

If $r = n - 1$,

$${}^nP_{n-1} = \frac{n!}{(n-n+1)!} = \frac{n!}{1!} = n!$$

Example: Find the value of 5P_5 ?

Solution: As we know, ${}^nP_n = n!$

So, ${}^5P_5 = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Permutations of repeated objects

If n objects are to be arranged among themselves and these contain p identical objects of one kind, q identical objects of another kind, r identical objects of still another kind, and so on, the total number of ways in which they can be arranged is given by,

$$\frac{n!}{p!q!r!....}$$

Example: How many words can be formed by arranging the letters of the word 'weeded'?

Solution: Total number of letters in the word 'weeded' = 6.

D occurs 2 times and E occurs 3 times. This is a case where n objects (6 in this case) are to be arranged among themselves and they contain 2 identical objects of one kind (D) and 3 identical objects of another kind (E).

So, number of words = $\frac{6!}{2!3!} = 720/12 = 60$.

Permutations with repetitions

The number of permutations of n different objects taken r at a time, when repetitions are allowed is n^r .

Example: How many four-digit numbers can be formed using the digits 4, 5, 6, 7, and 9 if repetition of digits is allowed?

Solution: Here, there are five different objects, i.e., the digits 4, 5, 6, 7, and 9; $n = 5$

Since repetition is allowed, the first digit can be taken in five different ways, the second can be taken in five different ways, and so on.

So, the number of four-digit numbers is $5 \times 5 \times 5 \times 5 = 625$.

COMBINATIONS

If there are n objects, and r out of them ($r \leq n$) are to be selected, then the number of ways in which this can be done is nC_r and is read as 'the number of combinations of n objects taken r at a time'. The number of combinations is given as:

$${}^nC_r = \frac{n!}{(n-r)!r!}$$

Example: In how many different ways can you select 2 students out of 3?

Solution: It can be written as 3C_2 .

$${}^3C_2 = \frac{3!}{(3-2)!2!} = 3.$$

The specific cases of combinations are:

$${}^nP_r = {}^nC_r \times r!$$

If $r = 0$,

$${}^nC_0 = \frac{n!}{(n-0)! \times 0!} = 1$$



If $r = 1$,

$${}^nC_1 = \frac{n!}{(n-1)! \times 1!} = n$$

If $r = n$,

$${}^nC_n = \frac{n!}{(n-n)! \times n!} = 1$$

If $r = n - 1$,

$${}^nC_{n-1} = \frac{n!}{(n-n+1)! \times (n-1)!} = \frac{n!}{1! \times (n-1)!} = n$$

Example: Find the value of 7C_2 ?

$$\begin{aligned} \text{Solution: } {}^7C_2 &= \frac{7!}{(7-2)!2!} \\ &= \frac{7!}{5!2!} = \frac{5040}{120 \times 2} = 21. \end{aligned}$$

Chapter Summary



$$n! = 1 \times 2 \times 3 \times 4 \times \dots \times (n-1) \times n$$

$$\text{Permutation} = {}^nP_r = \frac{n!}{(n-r)!}$$

Some specific cases related to permutations are:

If $r = 0$,

$${}^nP_0 = \frac{n!}{(n-0)!} = 1$$

If $r = 1$,

$${}^nP_1 = \frac{n!}{(n-1)!} = \frac{n \times (n-1)!}{(n-1)!} = n$$

If $r = n$,

$${}^nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} = n!$$

If $r = n - 1$,

$${}^nP_{n-1} = \frac{n!}{(n-n+1)!} = \frac{n!}{1!} = n!$$

$$\text{Combination} = {}^nC_r = \frac{n!}{(n-r)!r!}$$

The specific cases of combinations are:

$${}^nP_r = {}^nC_r \times r!$$

If $r = 0$,

$${}^nC_0 = \frac{n!}{(n-0)! \times 0!} = 1$$

If $r = 1$,

$${}^nC_1 = \frac{n!}{(n-1)! \times 1!} = n$$

If $r = n$,

$${}^nC_n = \frac{n!}{(n-n)! \times n!} = 1$$

If $r = n - 1$,

$${}^nC_{n-1} = \frac{n!}{(n-n+1)! \times (n-1)!} = \frac{n!}{1! \times (n-1)!} = n$$



PRACTICE QUESTIONS

1. If the ratio between $\frac{n!}{2!(n-2)!}$ and $\frac{n!}{!(n-4)!}$ is 2:1, then find the value of n .
A. 5
B. 4
C. 3
D. 2
2. Find the value of n in $(n+1)! = 12(n-1)!$
A. 2
B. 3
C. 4
D. 5
3. Find the value of n in $(n+2)! = 60(n-1)!$
A. 1
B. 2
C. 3
D. 4
4. If ${}^nP_3 = 120$, find the value of n .
A. 3
B. 4
C. 5
D. 6
5. If ${}^{10}P_r = 720$, then find the value of r .
A. 2
B. 3
C. 4
D. 5
6. How many numbers of four digits can be formed with digits 2, 4, 6, 7, and 8? (Repetition of digits is not allowed.)
A. 100
B. 110
C. 120
D. 130
7. How many numbers between 200 and 800 can be made with digits 0, 1, 2, 3, 4, and 5? (Repetition of digits not allowed.)
A. 50
B. 60
C. 70
D. 80
8. A gentleman has 5 friends to invite. In how many ways can he send invitation cards to them, if he has three servants to carry the cards?
A. 243
B. 247
C. 351
D. 498
9. In how many ways 3 prizes can be given away to 6 boys when each boy is eligible for any of the prizes?
A. 195
B. 200
C. 216
D. 248
10. In how many ways can 7 boys be seated at a round table so that 3 particular boys are next to each other?
A. 12
B. 121
C. 11
D. 144
11. In how many ways can 7 boys be seated at a round table so that 2 particular boys are separated.
A. 480
B. 490
C. 500
D. 510
12. How many different letter arrangements can be made from the letters of the word ARISE?
A. 100
B. 120
C. 140
D. 160
13. How many different letter arrangements can be made from the letters of the word RECOVER?
A. 1,230
B. 1,240
C. 1,250
D. 1,260



14. How many permutations can be made out of the letters of the word TRIANGLE, which begin with *T* and end with *E*?
- A. 720
B. 740
C. 760
D. 800
15. In how many ways can the letters of the word MOTHER be arranged so that all the vowels come together?
- A. 100
B. 120
C. 140
D. 160
16. In how many ways can the letters of the word DIRECTOR be arranged so that the three vowels never come together?
- A. 12,000
B. 15,000
C. 18,000
D. 20,000
17. There are 10 points in a plane out of which 4 are collinear. Find the number of straight lines formed by joining them.
- A. 40
B. 45
C. 50
D. 55
18. In how many different ways can the letters of the word TRUST be arranged?
- A. 60
B. 70
C. 80
D. 90
19. In how many different ways can the letters of the word ATTEND be arranged?
- A. 320
B. 360
C. 400
D. 420
20. In how many different ways can the letters of the word BANKING be arranged?
- A. 2,500
B. 2,520
C. 2,550
D. 3,000
21. In how many different ways can the letters of the word PEANUT be arranged?
- A. 700
B. 710
C. 720
D. 730
22. A committee of five members is to be formed out of 4 students, 3 teachers, and 2 sports coaches. In how many ways can the committee be formed if the committee should consist of 2 students, 2 teachers, and 1 sports coach?
- A. 6
B. 18
C. 36
D. 48
23. A committee of five members is to be formed out of 4 students, 3 teachers, and 2 sports coaches. In how many ways can the committee be formed if any five can be selected?
- A. 123
B. 124
C. 125
D. 126
24. In how many different ways can the letters of the word REPLACE be arranged?
- A. 2,550
B. 2,540
C. 2,530
D. 2,520
25. In how many ways can a group of 5 men and 2 women be made out of a total of 7 men and 3 women?
- A. 63
B. 64
C. 65
D. 66
26. How many different words can be formed with the letters of the word ALLAHABAD?
- A. 7,560
B. 7,550
C. 7,540
D. 7,530
27. In how many different ways can the letters of the word SOFTWARE be arranged in



- such a way that the vowels always come together?
- A. 4,320
B. 4,330
C. 4,340
D. 4,350
- 28.** In how many different ways can a group of 4 men and 4 women be formed out of 7 men and 8 women?
- A. 2,400
B. 2,450
C. 2,500
D. 2,550
- 29.** In how many different ways can the letters of the word MIRACLE be arranged?
- A. 5,020
B. 5,040
C. 5,080
D. 5,160
- 30.** A committee of 12 persons is to be formed from 9 women and 8 men. In how many ways can this be done if at least 5 women have to be included in a committee?
- A. 6,061
B. 6,062
C. 6,063
D. 6,064
- 31.** A committee of 12 persons is to be formed from 9 women and 8 men. In how many of these committees the women are in majority?
- A. 2,700
B. 2,701
C. 2,702
D. 2,703
- 32.** A committee of 5 members is to be formed out of 3 trainees, 4 professors, and 6 research associates. In how many different ways can this be done if the committee should have all 4 professors and 1 research associate or all 3 trainees and 2 professors?
- A. 8
B. 10
C. 12
D. 14
- 33.** A committee of 5 members is to be formed out of 3 trainees, 4 professors, and 6 research associates. In how many different ways can this be done if the committee should have 2 trainees and 3 research associates?
- A. 40
B. 50
C. 60
D. 70
- 34.** In how many different ways can the number 25,69,774 be arranged, using each digit only once in each arrangement, such that the digits 6 and 5 are at the extreme ends in each arrangement?
- A. 12
B. 24
C. 36
D. 48
- 35.** Two girls and 4 boys are to be seated in a row in such a way that the girls do not sit together. In how many different ways can it be done?
- A. 120
B. 240
C. 360
D. 480
- 36.** In how many different ways can the letters of the word DRASTIC be arranged in such a way that the vowels always come together?
- A. 660
B. 680
C. 700
D. 720
- 37.** In how many different ways can the letters of the word CASUAL be arranged?
- A. 360
B. 180
C. 90
D. 45
- 38.** A dinner party is to be fixed for a group of 100 persons. In this party 50 people do not prefer fish, 60 prefer chicken, and 10 do not prefer either chicken or fish. Find the number of people who prefer both fish and chicken?



- A. 20
B. 30
C. 40
D. 50
- 39.** In how many different ways the letters of the word RECTITUDE can be arranged so that vowels come together?
A. 4,320
B. 4,330
C. 4,340
D. 4,350
- 40.** A person can go from Mumbai to Delhi by rail, road, or air. There are five routes to reach Manali from Delhi. Only one road goes from Manali to Ladakh. If Alina wants to go to Ladakh again but one of the roads between Delhi and Manali is closed, in how many different ways can she go from Mumbai to Ladakh?
A. 12
B. 24
C. 36
D. 48
- 41.** Munim likes to have passwords of eight characters, the first four being different vowels and the last four being different digits. How many different passwords can he have?
A. 5,37,800
B. 6,04,800
C. 7,28,800
D. 8,94,500
- 42.** A family of five goes in a five-seater car. In how many different ways can they sit?
A. 100
B. 120
C. 140
D. 160
- 43.** How many words can be formed by arranging the letters of the word HELPING?
A. 5,000
B. 5,040
C. 5,080
D. 5,120
- 44.** Using all the letters of the word LINEAR, how many different words can be formed that start and end with a vowel?
A. 121
B. 144
C. 169
D. 181
- 45.** Using all the letters of the word LINEAR, how many different words can be formed that start with a vowel but end with a consonant?
A. 214
B. 215
C. 216
D. 217
- 46.** In how many different ways can the letters of the word READING be arranged in such a way that the vowels always come together?
A. 700
B. 710
C. 720
D. 730
- 47.** Lakhan has 15 DVDs out of which 3 are of the movie Mother India, 5 are of the movie Sholay and the rest are all different. In how many ways can he arrange the DVDs on a shelf?
A. 1234567890
B. 1244985690
C. 1816214400
D. 1327528930
- 48.** In how many ways can 5 people be seated around a circular table?
A. 20
B. 22
C. 24
D. 26
- 49.** In how many ways can 6 different beads be arranged to form a necklace?
A. 30
B. 40
C. 50
D. 60
- 50.** In how many ways can four cards be selected from a pack of cards such that exactly one of them is an ace?
A. ${}^4C_2 \times {}^{52}C_4$
B. ${}^4C_3 \times {}^{52}C_3$
C. ${}^4C_4 \times {}^{48}C_4$
D. ${}^4C_1 \times {}^{48}C_3$



SOLUTIONS

$$1. \quad (A) \quad \frac{n! / 2!(n-2)!}{n! / 4!(n-4)!} = \frac{2}{1}$$

$$\frac{n!}{2!(n-2)!} \times \frac{4!(n-4)!}{n!} = \frac{2}{1}$$

$$(n-2)(n-3) = 3 \times 2$$

$$n-2 = 3 \text{ and } n-3 = 2$$

So, $n = 5$

$$2. \quad (B) \quad \frac{(n+1) \times n \times (n-1)!}{(n-1)!} = 12$$

$$(n+1) \times n = 12 \text{ or } (n+1) \times n = 4 \times 3$$

Hence, $n = 3$

$$3. \quad (C) \quad \frac{(n+2) \times (n+1) \times n \times (n-1)!}{(n-1)!} = 60$$

$$(n+2)(n+1)n = 60 \text{ or } (n+2)(n+1)n = 5 \times 4 \times 3$$

Hence, $n = 3$

$$4. \quad (D) \quad \frac{n!}{(n-3)!} = 120$$

$$\frac{n(n-1)(n-2)(n-3)!}{(n-3)!} = 120$$

$$n(n-1)(n-2) = 120 = 6 \times 5 \times 4$$

Hence, $n = 6$

$$5. \quad (B) \quad \frac{10!}{(10-r)!} = 720 = 10 \times 9 \times 8$$

$$\frac{10!}{(10-r)!} = \frac{10 \times 9 \times 8 \times 7!}{7!}$$

$$\frac{10!}{(10-r)!} = \frac{10!}{7!}$$

$$(10-r)! = 7!$$

So, $r = 3$

6. (C) There are five numbers and number of places to be filled up = 4

So, required number of numbers is 5P_4

$${}^5P_4 = \frac{5!}{(5-4)!} = 120$$

7. (D) Any number between 200 and 800 will be of three digits and the first number must be fulfilled by 2, 3, 4, or 5 because if we start from 0 it will be lower than 200.

So, required number of ways to fill first place = 4

And, required number of ways to fill remaining two places = 5P_2

$$\text{Required number of numbers} = 4 \times {}^5P_2$$

$$= 4 \times \frac{5!}{(5-2)!} = 80$$

8. (A) Invitation cards may be sent to each of 5 friends by any one of the three servants in 3 ways.

$$\text{So, required ways} = 3 \times 3 \times 3 \times 3 \times 3 = 3^5 = 243.$$

9. (C) Each of the three prizes can be given away to any of the 6 boys in 6 ways.

$$\text{So, the required number of ways} = 6^3 = 216 \text{ ways.}$$

10. (D) Let the 3 particular boys be taken together as one unit, then the number of units will be 5.

They can sit around the table in 4! Ways, and for each of this arrangement, 3 can be interchanged in 3! Ways.

$$\text{Hence, total number of arrangements} = 4! \times 3! = 24 \times 6 = 144$$

11. (A) Total ways in which 7 boys can be seated at a round table = $(7-1)! = 6!$

Total ways in which 2 particular boys sit together while arranging 7 boys = $5! \times 2!$

The arrangements that the two persons are separated = $6! - 5! \times 2! = 480$ ways.

12. (B) In word ARISE all 5 letters are different. Hence, total number of permutations = 5P_5

$$= \frac{5!}{(5-5)!} = 120$$

13. (D) In word RECOVER, 2 letters E and R have come two times.

$$\text{Total number of permutation} = \frac{7!}{2!2!} =$$

1260 ways.

14. (A) Total letters in word TRIANGLE = 8

Two letters T and E have fixed positions, so remaining letters = 6

$$\text{Number of permutations} = 6! = 720.$$



- 15. (B)** Number of vowels in word MOTHER = (O, E) = 2
Total letters in MOTHER after leaving vowels = M, T, H, R = 4
As all vowels come together, so we will count them as 1, then we have only $4 + 1 = 5$ letters.
Arrangements of these 5 letters = $5! = 120$.
- 16. (C)** Total number of letters = 8 and letter R occurring twice. Number of vowels = 3
Total number of arrangements when there is no restriction = $\frac{8!}{2!}$
When three vowels are together, taking them as one letter, we have only $5 + 1 = 6$ letters.
These 6 letters can be arranged in $\frac{6!}{2!}$ ways, since R occurs twice.
Number of arrangements when 3 vowels are together = $\frac{6!}{2!} \times 3!$
Required number = $\frac{8!}{2!} - \frac{6!}{2!} \times 3! = 18000$.
- 17. (A)** Let us suppose that the 10 points are such that no three of them are collinear. Now a straight line will be formed by any two of these 10 points. Thus forming a straight line amounts to selecting two of the 10 points.
Now out of 10 points 2 can be selected in ${}^{10}C_2$ ways.
Number of straight lines formed by 10 points when now let the four points become collinear, then 4C_2 straight lines formed by them will reduce to only one straight line.
Required number of lines formed = ${}^{10}C_2 - {}^4C_2 + 1 = 45 - 6 + 1 = 40$.
- 18. (A)** The word TRUST consists of five letters in which T comes twice.
Number of arrangements = $\frac{5!}{2!} = 60$.
- 19. (B)** The word ATTEND consists of six letters in which T comes twice.
Number of arrangements = $\frac{6!}{2!} = 360$.
- 20. (B)** The word BANKING consists of seven letters in which N comes twice.
Number of arrangements = $\frac{7!}{2!} = 2,520$.
- 21. (C)** The word PEANUT consists of six distinct letters.
Number of arrangements = $6! = 720$.
- 22. (C)** Required number of combinations = ${}^4C_2 \times {}^3C_2 \times {}^2C_1 = 36$.
- 23. (D)** Required number of combinations = ${}^9C_5 = 126$.
- 24. (D)** The word REPLACE consists of seven letters in which E comes twice.
Number of arrangements = $\frac{7!}{2!} = 2520$.
- 25. (A)** There are 7 men and 3 women. We have to select 5 men out of 7 and 2 women out of 3. This can be done in ${}^7C_5 \times {}^3C_2$ ways.
The number of ways of making the selection = ${}^7C_5 \times {}^3C_2 = 63$.
- 26. (A)** There are 9 letters in the word ALLAHABAD out of which 4 are As and 2 are L.
So, required number of words = $\frac{9!}{4!2!} = 7,560$.
- 27. (A)** There are 8 letters in the word SOFTWARE, including 3 vowels (O, A, E) and 5 consonants (S, F, T, W, R).
Considering three vowels as one letter, we have six letters which can be arranged in ${}^6P_6 = 6!$ ways.
But corresponding to each way of these arrangements, the vowels can be put together in $3!$ ways.
So, the required number of words = $6! \times 3! = 4320$.
- 28. (B)** 4 men out of 7 men and 4 women out of 8 women can be chosen in ${}^7C_4 \times {}^8C_4 = 2,450$.
- 29. (B)** The word MIRACLE has 7 distinct letters.
So, the number of arrangements = $7! = 5,040$.
- 30. (B)** There are 9 women and 8 men. A committee of 12 consisting of at least 5 women can be formed by choosing.
5 women and 7 men
6 women and 6 men



7 women and 5 men

8 women and 4 men

9 women and 3 men

So, total number of ways of forming the committee = ${}^9C_5 \times {}^8C_7 + {}^9C_6 \times {}^8C_6 + {}^9C_7 \times {}^8C_5 + {}^9C_8 \times {}^8C_4 + {}^9C_9 \times {}^8C_3$
 $= 126 \times 8 + 84 \times 28 + 36 \times 56 + 9 \times 70 + 1 \times 56 = 6,062$.

- 31. (C)** There are 9 women and 8 men. A committee of 12, consisting of at least 5 women, can be formed by choosing.

5 women and 7 men

6 women and 6 men

7 women and 5 men

8 women and 4 men

9 women and 3 men

As, women are in majority in 3rd, 4th, and 5th case.

So, total number of such committees = ${}^9C_7 \times {}^8C_5 + {}^9C_8 \times {}^8C_4 + {}^9C_9 \times {}^8C_3$
 $= 36 \times 56 + 9 \times 70 + 1 \times 56 = 2,702$.

- 32. (C)** Number of combinations =

$${}^4C_4 \times {}^6C_1 + {}^3C_3 \times {}^4C_2$$

$$1 \times 6 + 1 \times 6 = 12.$$

- 33. (C)** We have to select 2 trainees out of 3 and 3 research associates out of 6.
 So, the number of combinations = ${}^3C_2 \times {}^6C_3 = 60$.

- 34. (D)** Case 1 when arrangement is 6----5
 Four empty places can be filled by 2, 9, 7 and 4 in $4! = 24$ ways.
 Case 1 when arrangement is 5----6
 Four empty places can be filled by 2, 9, 7 and 4 in $4! = 24$ ways.
 Required number of arrangements = $24 + 24 = 48$ ways.

- 35. (D)** 4 boys can be seated in row in ${}^4P_4 = 4!$ ways.
 Now in the 5 gaps 2 girls can be arranged in 5P_2 ways.
 Hence, the number of ways in which no two girls sit together = $4! \times {}^5P_2 = 480$ ways.

- 36. (D)** There are 7 letters in the word DRASTIC including 2 vowels (A, I) and 5 consonants

(D, R, S, T, C), considering two vowels as one letter, we have 6 letters which can be arranged in $6!$ ways. But corresponding to each way of the arrangements, the vowels can be put together in $2!$ ways.

$$= 6! \times \frac{2!}{2!} = 720 \text{ ways.}$$

- 37. (A)** The word CASUAL has 6 letters in which letter A comes twice.

So, number of arrangements = $\frac{6!}{2!} = 360$ ways.

- 38. (A)** 50 out of 90 do not prefer fish.

So, the required answer = $60 - 40 = 20$.

- 39. (A)** There are 9 letters in the word RECTITUDE including 4 vowels (E, I, U, E) and 5 consonants (R, C, T, T, D), and letters T and E come twice.

So, number of arrangements = $\frac{6! \times 4!}{2! \times 2!} = 4,320$ ways.

- 40. (A)** Mumbai to Delhi = 3 ways (rail, road, air)
 Delhi to Manali = 5 ways (5 routes)
 Manali to Ladakh = 1 way (one road)
 So, total ways to reach Ladakh from Mumbai are $3 \times 5 \times 1 = 15$ ways.
 Since one road between Delhi and Manali is closed, there are four possible roads.
 Hence, total ways = $3 \times 4 \times 1 = 12$ ways.

- 41. (B)** The first 4 characters of Munim's password are to be chosen from the 5 vowels. Since there are 5 vowels available, the first vowel can be chosen in 5 different ways. Now, as all four vowels need to be different, the second vowel can be chosen only in 4 ways, the third in 3 ways, and the last in 2 ways. Similarly, the four digits have to be different. The first digit can be chosen from the 10 digits in 10 ways, the second can be chosen from the remaining 9 digits in 9 ways. Similarly, the third and fourth digit can be chosen in 8 and 7 ways, respectively. Hence, the four numbers can be chosen from the 10 digits from 0 to 9 in 10, 9, 8, and 7 ways, respectively.



Thus the number of different passwords that Munim can have = $5 \times 4 \times 3 \times 2 \times 10 \times 9 \times 8 \times 7 = 604800$.

- 42. (B)** Number of seats = 5 and number of people = 5.

So, required arrangement = ${}^5P_5 = 5! = 120$ ways.

- 43. (B)** The word HELPING has 7 distinct letters. So, the number of arrangements = $7! = 5040$.

- 44. (B)** The word LINEAR has 3 vowels = I, E and A.

If a word starts and ends with a vowel, the two letters to occupy the first and the last positions can be selected and arranged in ${}^3P_2 = 6$ ways.

The remaining 4 letters can be arranged among themselves in ${}^4P_4 = 4! = 24$ ways.

Hence, the number of words that start and end with a vowel = $24 \times 6 = 144$.

- 45. (C)** The word LINEAR has 3 vowels (I, E, A) and 3 consonants (L, N, R).

If a word starts with a vowel but ends with a consonant, its first letter can be selected from I, E, and A in 3 ways. Its last letter can be selected from L, N, and R in 3 ways. The remaining four letters can be arranged in $4!$ ways.

Hence, the number of words that start with a vowel but end with a consonant = $3 \times 3 \times 4! = 9 \times 24 = 216$.

- 46. (C)** The word READING has 7 different letters.

The vowels (E, A, I) can be arranged among themselves in $3! = 6$ ways

Since E, A, and I are considered to be together, consider these as one letter.

Hence, letters to be arranged are R, D, N, G, vowels. These can be arranged in $5! = 120$ ways

So, the required number of ways = $120 \times 6 = 720$ ways.

- 47. (C)** There are 15 DVDs out of which 3 are of one kind and 5 are of another kind.

So, the total number of arrangements possible = $\frac{15!}{3! \times 5!} = 1816214400$.

- 48. (C)** n objects can be arranged in a circle in $(n - 1)!$ ways.

Similarly, 5 people can be seated around a circular table in $(5 - 1)! = 4! = 24$ ways.

- 49. (D)** n objects can be arranged in a necklace in $\frac{1}{2} \times (n - 1)!$ ways

Thus, 6 beads can form a necklace in $\frac{1}{2} \times (6 - 1)! = \frac{5!}{2} = 60$ ways.

- 50. (D)** An ace can be selected from 4 aces in 4C_1 ways.

Since exactly one card is an ace, the remaining three cards can be selected from the $52 - 4 = 48$ non-ace cards in ${}^{48}C_3$ ways.

So, total number of selections = ${}^4C_1 \times {}^{48}C_3$.



SYNOPSIS

- Introduction
- General terms of Probability
- Types of Events
- Probability
- Addition Theorem of Probability
- Conditional Probability
- Odds

Probability is a method of expressing the event of something occurring or not happening. The higher the probability, the greater the odds of an event occurring and vice versa.

SOME GENERAL TERMS OF PROBABILITY

Deterministic Experiment: The experiment that gives a definite result is called a deterministic experiment. For example, rolling a fair die: each number on a six-sided die has the same odds ($1/6$) of coming up.

Random Experiment: A random experiment is an action that gives one or more results. For example, we toss a coin three times and watch the heads/tails sequence. The sample space in this case could be described as follows: $S = \{(H, H, H), (H, H, T), (H, T, H), (T, H, H), (H, T, T), (T, H, T), (T, T, H), (T, T, T)\}$.

Outcome: The result of a random experiment is called an outcome.

Sample Space: The sample space (S) is the set of all possible outcome of an event. The number of elements in the sample space is denoted by $n(S)$. For example, when a single die is thrown, it has 6 outcomes since it has 6 faces. Therefore, the sample is given as $S = \{1, 2, 3, 4, 5, \text{ and } 6\}$.

Unbiased Experiment: A random experiment having equally likely outcomes is called an unbiased experiment.

TYPES OF EVENT

Simple Event: If an event contains only one sample point, then it is called a simple event. Consider the sample space of the experiment of rolling an unbiased die,

$$S = \{1, 2, 3, 4, 5, 6\}$$

Any subset of S is called an *event*.

The event E of getting a perfect cube when an unbiased die is rolled once is an example of a simple event, i.e., $E = \{1\}$.

Impossible Event: An impossible event is one in which the sample set is empty, i.e., it contains no sample points. For example, getting 7 by throwing a die is an impossible event.

Certain Event: If the event is sure to happen, it is a certain event. For example, getting a value less than 7 on throwing an unbiased die.

Complementary Event: Let A be an event in the sample space S . Then A is a subset of S . The complement of A is $(S - A)$ It is represented as A' .

Combination of Events: The *union* of the events A and B of a sample space ($A \cup B$) is the event that either A or B or both take place.

The *intersection* of the events A and B of a sample space ($A \cap B$) is the event that both A and B take place.

Exhaustive Events: If two events A and B of a sample space S are such that $(A \cup B) = S$, then A and B are called exhaustive events.

If A and B are exhaustive, $(A \cup B) = S$

$$\text{So, } n(A \cup B) = n(S)$$

Independent Events: If the happening of one event, A has no effect on the other event B , then A and B are said to be independent of each other.

Example: When three unbiased dice are rolled, what type of events are these —‘getting a total greater than 2’ and ‘getting a total greater than 20’?

Solution: When three dice are rolled, the minimum total is $1 + 1 + 1 = 3$ and the maximum total is $6 + 6 + 6 = 18$.



Hence, the event getting a total greater than 3° degrees will always happen and is hence a 'certain event'. On the other hand, the event 'getting a total greater than 18°' will never happen and hence, an 'impossible event'.

PROBABILITY

Probability refers to the occurrence of an event occurring. If S is an unbiased experiment's sample space and E is an event, then the probability that it occurs is,

$$P(E) = \frac{n(E)}{n(S)}$$

Example: Consider tossing an unbiased coin. Let E be the event that the result is heads. Let E' be an event that the result is tails. Then, $n(S) = 2$; $n(E) = 1$ and $n(E') = 1$

$$\text{So, } P(E) = \frac{1}{2} \text{ and } P(E') = \frac{1}{2}$$

So when a coin is tossed, the probability of occurrence of heads is $\frac{1}{2}$. Similarly, the probability of occurrence of tails is $\frac{1}{2}$.

Example: Two unbiased coins are tossed. What is the probability that both the tosses give different results?

Solution: When a coin is tossed the result is either heads (H) or tails (T).

Tossing two coins gives 4 outcomes, i.e., $S = \{HH, HT, TH, TT\}$

Hence, $n(S) = 4$

Now, let A be the event that both the tosses give different results. The first coin shows heads and the second shows tails or the first coin shows tails and the second shows heads.

So, $A = \{HT, TH\}$ and $n(A) = 2$

$$\text{Hence, } P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

ADDITION THEOREM OF PROBABILITY

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If A and B are mutually exclusive events, $n(A \cap B) = 0$.

So, $P(A \cap B) = 0$

$$\text{Hence, } P(A \cup B) = P(A) + P(B)$$

Example: A bag contains 20 coins numbered 1 to 20. At random, a coin is chosen. What is the probability that the coin will come up with a number that is a multiple of 3 or 5?

Solution: Multiples of 3 from 1 to 20 = 3, 6, 9, 12, 15, 18, i.e., 6 values.

Multiples of 5 from 1 to 20 = 5, 10, 15, 20, i.e., 4 values.

Multiples of both 3 and 5 = 15, i.e., only 1 value.

So, number of multiples of 3 or 5 = 6 + 4 - 1 = 9

There are 20 numbers in all.

Hence, required probability = $\frac{9}{20}$

CONDITIONAL PROBABILITY

Conditional probability is the probability of an event or outcome that depends on the occurrence of a preceding event or outcome.

Let A and B be two events defined on a sample space S .

Let $P(B) > 0$. Then the conditional probability of A given B , is denoted by $P(A/B)$ and is defined by,

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

Example: There is a 0.03 probability that it is Friday and a student is absent. As only school is open only for 5 days, there is a 0.2 probability that it is Friday. Given that it is Friday, what is the probability that a student will be absent?

Solution:

$$P(\text{Absent} | \text{Friday}) = \frac{P(\text{Absent} \cap \text{Friday})}{P(\text{Friday})} = \frac{0.03}{0.2}, \text{ i.e., } 0.15 = 15\%$$

ODDS

$$\text{Odds in Favour} = \frac{\text{Number of favourable cases}}{\text{Number of unfavourable cases}}$$

$$\text{Odds Against} = \frac{\text{Number of unfavourable cases}}{\text{Number of favourable cases}}$$

Example: A boy randomly picks a marble from his bag containing 10 yellow, 18 orange, 6 red, and 20 blue marbles. What are the odds that his marble is blue or orange?



Solution: Total marbles $10 + 18 + 6 + 20 = 54$
So, total events = ways of picking 1 marble out of 54 = 54
Number of favourable events = ways of picking a blue or orange marble = $20 + 18 = 38$.

And, the number of unfavourable events = $54 - 38 = 16$.

Hence, odds of getting a blue or orange marble = $38:16 = 19:8$.

PRACTICE QUESTIONS

1. A bag contains seven balls, four of which are red and three of which are blue. How probable is it that a blue ball will be chosen?
A. $4/7$
B. $3/7$
C. $2/7$
D. $1/7$
2. A dice is tossed. Calculate the probability of getting a multiple of three.
A. $1/12$
B. $1/9$
C. $1/6$
D. $1/3$
3. What is the probability of throwing a number larger than 4 using regular dice with numbered faces ranging from 1 to 6?
A. $1/3$
B. $1/4$
C. $1/5$
D. $1/6$
4. Three balls are picked at random from a bag containing four white and five black balls. Calculate the probability that all three are black.
A. $1/14$
B. $2/21$
C. $5/42$
D. $1/42$
5. Two dice are thrown at the same time. What is the probability of having an even-numbered doublet?
A. $1/12$
B. $1/3$
C. $1/4$
D. $1/6$
6. Tickets with numbers ranging from 1 to 20 are mixed together and a ticket is selected at random. What is the probability that the ticket will contain a number that is a multiple of 3 or 7?
A. $1/5$
B. $2/5$
C. $3/5$
D. $4/5$
7. A word is made up of nine letters, five consonants, and four vowels. At random, three letters are picked. What is the probability that more than one vowel will be chosen?
A. $({}^3C_2 \times {}^4C_2 + {}^4C_3)/{}^9C_3$
B. $({}^2C_2 \times {}^3C_2 + {}^1C_2)/{}^9C_3$
C. $({}^4C_2 \times {}^5C_1 + {}^4C_3)/{}^9C_3$
D. $({}^1C_3 \times {}^3C_1 + {}^4C_3)/{}^9C_3$
8. From a group of three men, two women, and four children, four people will be picked at random. What is the probability of choosing one man, one woman, and two children?
A. $1/7$
B. $3/7$
C. $5/7$
D. $2/7$
9. From a group of three men, two women, and four children, four people will be picked at random. What is the probability of selecting exactly two children?
A. $8/21$
B. $3/7$
C. $10/21$
D. $11/21$



10. From a group of three men, two women, and four children, four people will be picked at random. What is the probability of choosing two women?
- A. $1/6$
B. $1/3$
C. $1/2$
D. $2/3$
11. What are the chances that a leap year with 53 Sundays is chosen at random?
- A. $1/7$
B. $4/7$
C. $6/7$
D. $2/7$
12. If the year chosen is not a leap year, what are the chances of 53 Sundays?
- A. $2/7$
B. $1/7$
C. $4/7$
D. $3/7$
13. From a typical deck of 52 playing cards, a single card is chosen at random. What are the chances of picking a king or a club?
- A. $1/13$
B. $2/13$
C. $3/13$
D. $4/13$
14. There are 17 boys and 13 girls in a maths class of 30 students. Four boys and five girls received an A on a unit test. What are the chances of picking a female or an 'A grade student' at random from the class?
- A. $7/15$
B. $8/15$
C. $17/30$
D. $19/30$
15. From a class of 50 males, a teacher selects one student at random. What is the probability that the chosen student is a boy?
- A. $1/2$
B. 1
C. $1/4$
D. $1/3$
16. There are 5 white and 8 black balls in each bag. Two consecutive draws of three balls are made, with the balls not being replaced before the second draw. What is the probability that the first draw will provide three white balls and the second draw will yield three black balls?
- A. $7/429$
B. $5/666$
C. $8/671$
D. $13/456$
17. A has a $1/3$ probability of hitting the target, but B has a $2/5$ probability of hitting it. If each of A and B shoots at the target, what is the probability that the target will be hit?
- A. $1/5$
B. $2/5$
C. $3/5$
D. $4/5$
18. Six red, five green, and eight blue balls are contained in a basket. What is the probability that all four balls are red or that any two of the four are green if four balls are chosen at random?
- A. $437/3474$
B. $675/3214$
C. $925/3876$
D. $435/3475$
19. Six blue, two red, four green, and three yellow balls are contained in a basket. What is the probability that two balls are taken at random and both are green or both are yellow?
- A. $1/35$
B. $2/35$
C. $3/35$
D. $4/35$
20. Six blue, two red, four green, and three yellow balls are contained in a basket. What is the probability that at least one of the five balls chosen at random is blue?



- A. $137/141$
B. $37/41$
C. $137/143$
D. $37/43$
- 21.** Six blue, two red, four green, and three yellow balls are contained in a basket. What is the probability that two balls are drawn at random and both turn out to be blue?
A. $1/7$
B. $1/9$
C. $1/11$
D. $1/13$
- 22.** Six blue, two red, four green, and three yellow balls are contained in a basket. What is the probability that two balls are red and two are green if four balls are chosen at random?
A. $1/455$
B. $2/455$
C. $3/455$
D. $4/455$
- 23.** There are six blue, two red, four green, and three yellow balls in a basket. What is the probability that none of the three balls chosen at random are yellow?
A. $6/13$
B. $43/91$
C. $44/91$
D. $3/13$
- 24.** Three blue and four red balls are contained in a basket. What is the probability that three balls are picked at random from the basket and all three are blue or red?
A. $1/3$
B. $1/5$
C. $1/6$
D. $1/7$
- 25.** Four green, five blue, two red, and three yellow marbles are contained in an urn. What is the probability that both or at least one marble will be red if two marbles are picked at random?
A. $23/91$
B. $25/91$
C. $27/91$
D. $29/91$
- 26.** Four green, five blue, two red, and three yellow marbles are contained in an urn. What is the probability that at least one marble will be yellow if three marbles are picked at random?
A. $197/364$
B. $199/364$
C. $201/364$
D. $50/91$
- 27.** Four green, five blue, two red, and three yellow marbles are contained in an urn. What is the probability that there are equal numbers of marbles of each colour if 8 marbles are picked at random?
A. $50/1001$
B. $60/1001$
C. $70/1001$
D. $80/1001$
- 28.** Four green, five blue, two red, and three yellow marbles are contained in an urn. What is the probability that none of the three marbles picked at random are green?
A. $20/91$
B. $30/91$
C. $40/91$
D. $50/91$
- 29.** Four green, five blue, two red, and three yellow marbles are contained in an urn. What is the probability that two blue marbles and two red marbles are picked at random?
A. $10/1001$
B. $20/1001$
C. $30/1001$
D. $40/1001$
- 30.** There are 13 white and 7 black balls in each bag. At random, two balls are picked. What's the probability they're the same colour?
A. $1/2$
B. $99/190$
C. $99/199$
D. $95/199$



- 31.** What is the probability that one card picked at random from a well-shuffled pack of 52 playing cards will be a black king?
- A. $1/13$
B. $2/13$
C. $3/13$
D. $4/13$
- 32.** What is the probability of rolling two dice and obtaining the number 4 consecutively?
- A. $1/9$
B. $1/18$
C. $1/27$
D. $1/36$
- 33.** Three red balls, two blue balls, and one black ball are contained in a bag. What are the chances of getting a blue ball out of the bag?
- A. $1/6$
B. $1/2$
C. $1/3$
D. $1/4$
- 34.** What is the probability of getting an even number when a fair die is thrown? What's the probability you'll obtain 4 or a greater number?
- A. $1/3$
B. $1/4$
C. $1/2$
D. $1/6$
- 35.** Two dice are thrown at the same time. What is the probability that the sum of the thrown numbers is less than 2?
- A. $1/9$
B. $1/36$
C. 0
D. $1/18$
- 36.** Two dice are thrown at the same time. What is the probability that the sum of the thrown numbers is 9?
- A. $1/18$
B. $1/3$
C. $1/9$
D. $1/12$
- 37.** Two dice are thrown at the same time. What is the probability that the sum of the thrown numbers is even?
- A. $1/2$
B. $1/3$
C. $1/4$
D. $1/5$
- 38.** What is the probability of receiving the same number on both dice if two dice are thrown at the same time?
- A. $1/3$
B. $1/6$
C. $1/9$
D. $1/12$
- 39.** What is the probability of receiving the same face on a coin tossed twice?
- A. 1
B. $1/4$
C. $3/4$
D. $1/2$
- 40.** What is the probability of receiving the same face if three coins are tossed simultaneously?
- A. $1/2$
B. $1/3$
C. $1/4$
D. $1/5$
- 41.** In a class of 20, 13 students passed the statistics exam. What is the probability of a student passing statistics if they are chosen at random from this group?
- A. $7/20$
B. $11/20$
C. $12/20$
D. $13/20$
- 42.** What is the probability of getting two consecutive numbers on a pair of dice if both are rolled at the same time?
- A. $1/9$
B. $1/6$
C. $2/9$
D. $5/18$



- 43.** What is the probability of getting the same face on a dice if you roll it three times?
A. $1/9$
B. $1/18$
C. $1/27$
D. $1/36$
- 44.** Two dice are rolled at the same time. What is the probability that the sum of the faces will be more than three?
A. $4/9$
B. $5/9$
C. $7/12$
D. $11/12$
- 45.** What is the probability of getting at least one 6 if a fair die is thrown twice?
A. $11/27$
B. $11/36$
C. $1/18$
D. $7/36$
- 46.** Ram tosses a die into the air. What are the chances that it won't land on the same side two times in a row?
A. $1/6$
B. $1/2$
C. $5/7$
D. $1/3$
- 47.** What are the chances of getting a sum of 10 if a pair of dice is rolled?
A. $1/12$
B. $1/3$
C. $1/6$
D. $1/9$
- 48.** Ravi will win the game if he gets a 4 on the die roll. What is the probability?
A. $1/3$
B. $1/4$
C. $1/5$
D. $1/6$
- 49.** Calculate the probability of getting a multiple of two when you roll a die.
A. $2/3$
B. $1/2$
C. $1/6$
D. $1/3$
- 50.** Three dice are thrown at the same time. What are the chances of all three faces being the same on the top?
A. $1/36$
B. $1/27$
C. $1/18$
D. $1/9$

SOLUTIONS

- 1. (B)** Number of favourable outcomes = 3
Total number of outcomes = 7
Probability of one ball = $3/7$
- 2. (D)** $S = \{1, 2, 3, 4, 5, 6\}$
Total number of outcomes = 6
Total number of favourable event = $\{3, 6\} = 2$
Probability of getting a multiple of 3 = $2/6 = 1/3$
- 3. (A)** Total number of outcomes = 6
Total number of favourable event = $\{5, 6\} = 2$
Probability of getting a number greater than 4 = $2/6 = 1/3$
- 4. (C)** 3 balls can be selected from 9 balls in $n(S) = {}^9C_3$ ways
If A be the event of getting 3 black balls in $n(A) = {}^5C_3$ ways
Required probability $P(A) = \frac{n(A)}{n(S)} = \frac{5}{42}$
- 5. (A)** Here, $n(S) = 36$
And doublet of even number, $n(A) = \{(2, 2) (4, 4) (6, 6)\} = 3$
So, probability = $3/36 = 1/12$
- 6. (B)** Let A be the required event then, $A = \{3, 6, 7, 9, 12, 14, 15, 18\}$
and, $n(A) = 8$; $n(S) = 20$
Required probability = $8/20 = 2/5$
- 7. (C)** Three letters can be chosen out of 9 letters in 9C_3 ways



More than one vowel can be chosen in the following way. 2 vowels and one consonant or 3 vowels.

If A be the required event, then, $n(A) = ({}^4C_2 \times {}^5C_1 + {}^4C_3)/{}^9C_3$

Hence, this is the required probability.

8. (D) Out of 9 persons, 4 can be selected in ${}^9C_4 = 126$ ways.

Hence, $n(S) = 126$

Let A be the required event then, $n(A) = {}^3C_1 \times {}^2C_1 \times {}^4C_2 = 36$

Hence required probability = $36/126 = 2/7$

9. (C) Out of 9 persons, 4 can be selected in ${}^9C_4 = 126$ ways.

Hence, $n(S) = 126$

Let A be the required event then, $n(A) = {}^4C_2 \times {}^5C_2 = 60$

Hence required probability = $60/126 = 10/21$

10. (A) Out of 9 persons, 4 can be selected in ${}^9C_4 = 126$ ways.

Hence, $n(S) = 126$

Let A be the required event then, $n(A) = {}^2C_2 \times {}^7C_2 = 21$

Hence required probability = $21/126 = 1/6$

11. (D) A leap year has 366 days so it has 52 complete weeks and 2 more days. The two days can be Sunday and Monday, Monday and Tuesday, Tuesday and Wednesday, Wednesday and Thursday, Thursday and Friday, Friday and Saturday, Saturday and Sunday, i.e., $n(S) = 7$.

Out of these 7 cases, cases favourable for more Sundays are (Sunday and Monday, Saturday and Sunday), i.e., $n(E) = 2$

Hence, required probability = $2/7$

12. (B) When the year is not a leap year, it has 52 complete weeks and 1 more day that can be Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, $n(S) = 7$

Out of these 7 cases, cases favourable for one more Sunday is $n(E) = 1$.

Hence, required probability = $1/7$

13. (D) Here, $n(S) = 52$

So, $P(\text{King or Club}) = P(\text{King}) + P(\text{Club}) - P(\text{King and Club}) = 4/52 + 13/52 - 1/52 = 16/52 = 4/13$

14. (C) $n(S) = 30$

$P(\text{Girl or A}) = P(\text{Girl}) + P(A) - P(\text{Girl and A}) = 13/30 + 9/30 - 5/30 = 17/30$

15. (B) As all the students are boys, so required probability is,
 $= 50/50 = 1$

16. (A) Let A be the event of drawing 3 white balls in the first draw and B be the event of drawing 3 black balls.

Hence, required probability = $P(A \cap B) = P(A) \times P(B/A)$

$P(A) = 5/143$

After drawing 3 white balls in the first draw 10 balls are left in the bag, out of which 8 are black balls.

So, $P(B/A) = 7/15$

Hence, $P(A \cap B) = P(A) \times P(B/A) = 5/143 \times 7/15 = 7/429$

17. (C) Let A and B are the events of hitting the target and A' and B' are not hitting the target.

So, $P(A) = 1/3$; $P(A') = 2/3$; $P(B) = 2/5$; $P(B') = 3/5$

Required probability = A hits the target and B does not hit the target

Or

B hits the target and A does not hit the target

Or

A hits the target and B hits the target

$= (1/3 \times 3/5) + (2/5 \times 2/3) + (1/3 \times 2/5) = 3/5$

18. (C) Number of balls = 19

Ways of selecting 4 balls out of 19 = ${}^{19}C_4 = 3,876$

Selecting 4 red balls or any two green balls out of the four = $15 + 910 = 925$

Hence, required probability = $925/3,876$

19. (C) Total numbers of the balls in the basket = 15

Number of ways of selecting 2 balls out of 15 balls = ${}^{15}C_2 = 105$

Favourable number of cases = ${}^4C_2 + {}^3C_2 = 9$

Hence, required probability = $9/105 = 3/35$

20. (C) Total numbers of the balls in the basket = 15

Number of ways of selecting 5 balls out of 15 balls = ${}^{15}C_5 = 105$



Let no blue ball be selected

So, number of ways of selecting 5 balls out of 9 balls without blue balls = 9C_5
Hence, required probability = $1 - {}^9C_5 / {}^{15}C_5$
 $= 1 - 6/143 = 137/143$

- 21. (A)** Total numbers of the balls in the basket = 15

Number of ways of selecting 2 balls out of 15 balls = ${}^{15}C_2 = 105$

Favourable number of cases = ${}^6C_2 = 15$

Hence, required probability = $15/105 = 1/7$

- 22. (B)** Number of ways of selecting 4 balls out of 15 balls = ${}^{15}C_4 = 1365$

Favourable number of cases = ${}^2C_2 + {}^4C_2 = 6$

Hence, required probability = $6/1365 = 2/455$

- 23. (C)** Number of ways of selecting 3 balls out of 15 balls = ${}^{15}C_3 = 455$

Favourable number of cases = ${}^{12}C_3 = 220$

Hence, required probability = $220/455 = 44/91$

- 24. (D)** Number of possible outcomes = ${}^7C_3 = 35$

Favourable number of cases = ${}^4C_3 + {}^3C_3 = 4 + 1 = 5$

Hence, required probability = $5/35 = 1/7$

- 25. (B)** Total number of marbles in the urn = $4 + 5 + 2 + 3 = 14$

Total possible outcome = Selection of 2 marbles out of 14 marbles = ${}^{14}C_2 = 91$

Favourable number of cases = ${}^2C_2 + {}^2C_1 \times {}^{12}C_1 = 1 + 2 \times 12 = 25$

Hence, required probability = $25/91$

- 26. (B)** Total possible outcome = ${}^{14}C_3 = 364$

When no marble is yellow, favourable number of cases = ${}^{11}C_3 = 165$

So, probability that no marble is yellow = $165/364$

Hence, required probability = $1 - 165/364 = 199/364$

- 27. (B)** Total possible outcome = ${}^{14}C_6 = 3003$

Favourable number of cases = ${}^4C_2 \times {}^5C_2 \times {}^2C_2 \times {}^3C_2 = 180$

Hence, required probability = $180/3003 = 60/1001$

- 28. (B)** Total possible outcome = ${}^{14}C_3 = 364$

No ball is green. So, selection of 3 marbles out of 5 blue, 2 red and 3 yellow marbles = ${}^{10}C_3 = 120$

Hence, required probability = $120/364 = 30/91$

- 29. (A)** Total possible outcome = ${}^{10}C_3 = 1,001$

Favourable outcome = ${}^5C_2 \times {}^2C_2 = 10$

Hence, required probability = $10/1,001$

- 30. (B)** Total possible outcome = ${}^{20}C_2 = 190$

Total favourable outcome = ${}^{13}C_2 + {}^7C_2 = 78 + 21 = 99$

Hence, required probability = $99/190$

- 31. (C)** Total possible outcome = ${}^{52}C_1 = 52$

Favourable number of cases = ${}^{12}C_1 = 12$

Required probability = $12/52 = 3/13$

- 32. (D)** Total outcome for a pair of dice = 36

Favourable outcome = 1

Hence, required probability = $1/36$

- 33. (C)** Total number of balls = 6

Favourable outcome = 2 blue balls

Probability of blue ball = $2/6 = 1/3$

- 34. (C)** For even number,

Total outcome = 6

Favourable outcome (even numbers) = 3

Hence, required probability = $3/6 = 1/2$

For 4 or higher number,

Total outcome = 6

Favourable outcome (4, 5, and 6) = 3

Hence, required probability = $3/6 = 1/2$

- 35. (C)** Total outcome = 36

Favourable outcome (sum is less than 2) = 0

Hence, required probability = $0/6 = 0$

- 36. (C)** Total outcome = 36

Favourable outcome, sum is 9 {(4, 5) (5, 4) (3, 6) (6, 3)} = 4

Hence, required probability = $4/36 = 1/9$

- 37. (A)** Total outcome = 36

Favourable outcome (even sum) = 18

Hence, required probability = $18/36 = 1/2$

- 38. (B)** Total outcome = 36

Favourable outcome (same number) = {(1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6)} = 6

Hence, required probability = $6/36 = 1/6$



- 39. (D)** Total outcome (2^2) = 4
Favourable outcome (same face appearing)
= {(H, H) (T, T)} = 2
Hence, required probability = $2/4 = 1/2$
- 40. (C)** Total outcome (2^3) = 8
Favourable outcome (same face appearing)
= {(H, H, H) (T, T, T)} = 2
Hence, required probability = $2/8 = 1/4$
- 41. (D)** Total outcome (number of students) = 20
Favourable outcome (passed in statistics)
= 13
Hence, required probability = $13/20$
- 42. (D)** Total outcome = 36
Favourable outcome (two consecutive numbers) = {(1, 2) (2, 3) (3, 4) (4, 5) (5, 6) (2, 1) (3, 2) (4, 3) (5, 4) (6, 5)} = 10 ways
Hence, required probability = $10/36 = 5/18$
- 43. (D)** Total outcome (6^3) = 216
Favourable outcome (having same face) = {(1, 1, 1) (2, 2, 2) (3, 3, 3) (4, 4, 4) (5, 5, 5) (6, 6, 6)} = 6 ways
Hence, required probability = $6/216 = 1/36$
- 44. (D)** Total outcome = 36
Favourable outcome (sum less than or equal to 3) = {(1, 1) (2, 1) (1, 2)} = 3
Hence, required probability = $3/36 = 1/12$
- 45. (B)** Total outcome = 36
Favourable outcome (at least one 6) = {(1, 6) (2, 6) (3, 6) (4, 6) (5, 6) (6, 6) (6, 1) (6, 2) (6, 3) (6, 4) (6, 5)} = 11
Hence, required probability = $11/36$
- 46. (C)** Total outcome = 36
Possible outcome with same side = {(1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6)} = 6 outcome
So, probability of same side = $6/36 = 1/6$
Hence, probability of not landing on same side = $1 - 1/6 = 5/6$
- 47. (A)** Total outcome = 36
Favourable outcome (getting a sum of 10) = {(4, 6) (5, 5) (6, 4)} = 3
Hence, required probability = $3/36 = 1/12$
- 48. (D)** Total outcome = 6
Favourable outcome (rolling a 4) = 1
Hence, required probability = $1/6$
- 49. (B)** Total outcome = 6
Favourable outcome (multiple of 2) = {2, 4, 6} = 3
Hence, required probability = $3/6 = 1/2$
- 50. (A)** Total outcome (6^3) = 216
Favourable outcome (having all the three same faces on the top) = {(1, 1, 1) (2, 2, 2) (3, 3, 3) (4, 4, 4) (5, 5, 5) (6, 6, 6)} = 6 outcome
Hence, required probability = $6/216 = 1/36$



SYNOPSIS

- Introduction
- 2D Shapes
- Quadrilaterals
- Area
- Perimeter
- Formulae Related to Two-Dimensional Figures
- Circles
- Triangle
- Polygons
- Regular Polygon
- Regular Hexagon

INTRODUCTION

This chapter deals with the concepts related to all the two-dimensional (2D) shapes that also includes the area and perimeter of all 2D figures. In this chapter, you will revise how to calculate the perimeter and area of squares, rectangles, triangles, and circles. The perimeter of a shape is the distance all the way around the sides of the shape. The area of a shape is the flat space inside the shape. You will also learn how to calculate the areas of different 2D figures, as well as investigate the effect on the perimeter and area of a shape when its dimensions are changed.

2D SHAPES

The 2D shapes are also known as flat shapes. These are the shapes having two dimensions only. It has length and breadth. It does not have thickness. The two different measures used for measuring the flat shapes are area and the perimeter. Two-dimensional shapes are the shapes that can be drawn on the piece of paper. Some of the examples of 2D shapes are square, rectangle, circle, and triangle.

QUADRILATERALS

Any four-sided polygon is called as a quadrilateral in general.

Different quadrilaterals and their properties

1. Rectangle

- a. Adjacent sides are at right angles to each other.
- b. Opposite sides are equal and parallel.
- c. Diagonals are equal.

2. Square

- a. All four sides are equal.
- b. All four angles are equal to 90° .
- c. The diagonals are equal.
- d. Diagonals are perpendicular bisector of each other.

3. Rhombus

- a. All sides are equal.
- b. All angles are not necessarily equal to 90° .
- c. All diagonals are perpendicular bisectors of each other but are not equal.

4. Kite

- a. The adjacent sides are equal.
- b. The diagonals are perpendicular to each other.
- c. The longer diagonal bisects the shorter one.

5. Trapezium

A quadrilateral in which one pair of lines is parallel.

6. Isosceles Trapezium

The non-parallel lines are equal.

7. Parallelogram

- a. Opposite sides are equal and parallel.
- b. Opposite angles are equal.
- c. Diagonals bisect each other.
- d. Sum of adjacent angles is 180° .

*Rectangle, Square, and Rhombus are special cases of parallelogram.



AREA

An area is a quantity that expresses the extent of a 2D figure or shape or planar lamina in the plane. Lamina shapes include 2D figures that can be drawn on a plane, e.g., circle, square, triangle, rectangle, trapezium, rhombus, and parallelogram. Area of shapes such as circle, triangle, square, rectangle, parallelogram, etc., are the region occupied by them in space.

Polygon shape: A polygon is a 2D shape that is formed by straight lines. The examples of polygons are triangles, hexagons, and pentagons. The names of shapes describe how many sides exist in the shape. For instance, a triangle consists of three sides and a rectangle has four sides. Hence, any shape that can be formed using three straight lines is known as a triangle and any shape that can be drawn by linking four lines is known as a quadrilateral. The area is the region inside the boundary/perimeter of the shapes which is to be considered.

PERIMETER

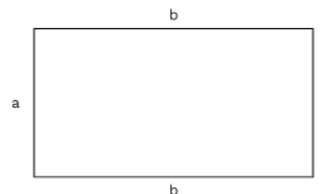
A perimeter is a closed path that surrounds a 2D shape. The Perimeter of a shape is defined as the total distance around the shape, it is the length of the outline or boundary of any 2D geometric shape. The word perimeter has been derived from the Greek word 'peri' meaning around, and 'metron' which means measure. Perimeter is the total length of the sides of a two-dimensional shape.

We often find perimeter when we have to put fencing around our field or when have to calculate the dimensions of a park. We use ruler to measure the length of the sides of a small regular shape. The perimeter is determined by adding the lengths of the sides/edges of the shape. For small irregular shapes, we can use a string of thread and place it exactly along the boundary of the shape, once. The total length of the string used along the boundary is the perimeter of the shape. The perimeter of all polygons can be determined by adding the lengths of their sides/edges.

FORMULAE RELATED TO TWO-DIMENSIONAL FIGURES

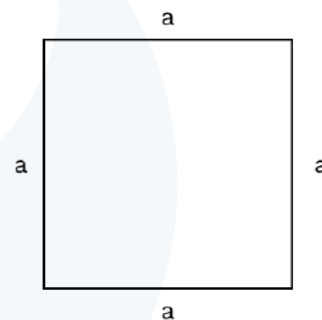
1. Rectangle

- Area of rectangle = $a \times b$
- Perimeter of a rectangle = $2(a + b)$
- Length of diagonal = $\sqrt{a^2 + b^2}$



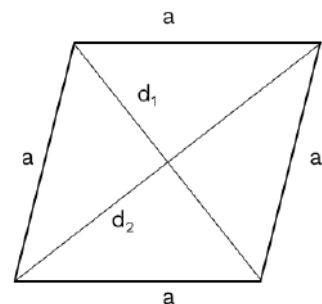
2. Square

- Area of square = a^2
- Perimeter of square = $4a$
- Length of diagonal = $a\sqrt{2}$



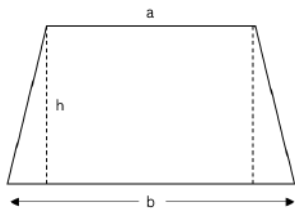
3. Rhombus

- Area = $\frac{1}{2} \times d_1 \times d_2$
- Side = $\frac{1}{2} \sqrt{d_1^2 + d_2^2}$



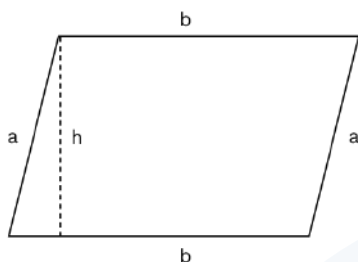
4. Trapezium

Area of trapezium = $\frac{1}{2} \times (a + b) \times h$



5. Parallelogram

Area of parallelogram = $b \times h$

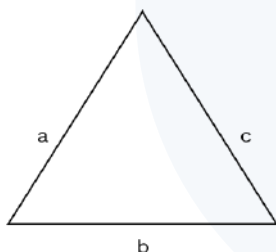


6. Triangle

a. Area of triangle = $\frac{1}{2} \times b \times h$

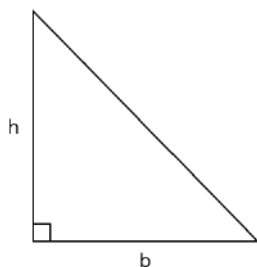
b. Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$;

where $s = \frac{a+b+c}{2}$



7. Right triangle

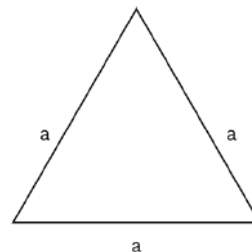
Area = $\frac{1}{2} \times b \times h$



8. Equilateral Triangle

a. Area = $\frac{\sqrt{3}}{4} a^2$

b. Perimeter = $3a$

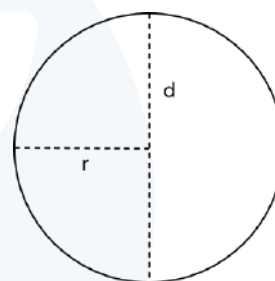


CIRCLES

A circle is a set of points in a plane, which are at a constant distance from a fixed point in the plane. The fixed point is then known as the centre and the fixed distance is called the radius of the circle.

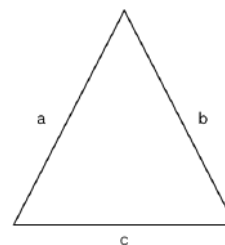
Circumference (perimeter) of a circle = $2\pi r = \pi d$, where r is the radius and d the diameter.

Area of circle = πr^2



TRIANGLE

A triangle is a plane figure bounded by three straight lines, or it is defined as a polygon of three sides. A vertex of a triangle is a point where two straight lines meet. Thus, in a triangle there are three vertices. The symbol $\triangle ABC$ is used to denote the triangle with vertices A, B, and C. The three straight lines AB, BC, and CA are called the sides and three angles $\angle BAC$, $\angle ABC$ and $\angle BCA$ are called the angles of the $\triangle ABC$.



Area of triangle = $\frac{1}{2} \times \text{base} \times \text{height}$

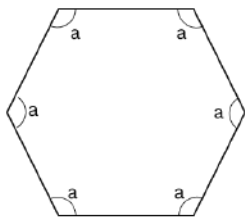


POLYGONS

A figure bounded by three or more sides is called a polygon. There are different polygons with different number of sides which are as follows:

NUMBER OF SIDES	NAME
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

A polygon is regular, if all its sides as well as angles are equal, else it is an irregular polygon.



Regular Polygon

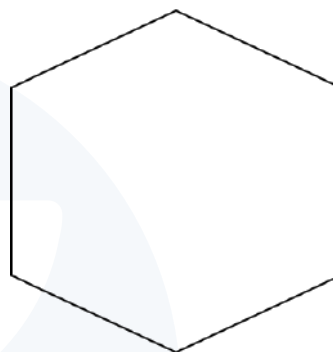


Irregular Polygon

REGULAR POLYGON

- Interior angle + exterior angle = 180°
- Perimeter = number of sides \times length of side(s)
- Each interior angle = $\frac{(n-2) \times 180}{n}$; where n is the number of sides
- Each exterior angle = $\frac{360^\circ}{n}$
- Sum of exterior angles = 360°

REGULAR HEXAGON



- Perimeter = $6 \times \text{Side}$
- Area = $\frac{3\sqrt{3}}{2} \times (\text{side})^2$

PRACTICE QUESTIONS

- Find the area of rhombus whose diagonals are 8 cm and 10 cm.
A. 40 cm^2
B. 36 cm^2
C. 44 cm^2
D. 38 cm^2
- Find the area of an equilateral triangle whose side is 6 cm.
A. $6\sqrt{3} \text{ cm}^2$
B. $9\sqrt{3} \text{ cm}^2$
C. $9\sqrt{6} \text{ cm}^2$
D. $8\sqrt{3} \text{ cm}^2$
- In a four-sided field, the longer diagonal is 220 m. The perpendicular that is drawn from the opposite vertices to the longer diagonal are 40.4 m and 50.8 cm. Find the area of the field.
A. $10,032 \text{ m}^2$
B. $10,080 \text{ m}^2$
C. $10,400 \text{ m}^2$
D. $10,060 \text{ m}^2$
- If the perimeter of the square is 16 cm, find its area.
A. 10 cm^2
B. 16 cm^2
C. 14 cm^2
D. 18 cm^2



5. If the area of a circle is 314 cm^2 , find the circumference of the circle.
A. 60 cm
B. 58.4 cm
C. 61.4 cm
D. 62.8 cm
6. The perimeter of a rectangle is 326 cm and its length is 98 cm. Find the area of the rectangle.
A. 6,370
B. 6,820
C. 7,420
D. 6,680
7. The breadth of a rectangle is 12 cm and its perimeter is 50 cm. Find the length of the rectangle.
A. 15 cm
B. 18 cm
C. 14 cm
D. 13 cm
8. The area of a rectangle is $1,029 \text{ cm}^2$ and if the sides of the rectangle are in the ratio 3 : 7, then find its perimeter.
A. 120
B. 180
C. 140
D. 160
9. The length and breadth of a garden is 10 m and 7 m, respectively. If the cost of fencing is Rs. 40/m, then what will be the cost for fencing the garden?
A. 1,400
B. 1,360
C. 1,340
D. 1,260
10. The perimeter of a rectangle is 50 m and the sides of the rectangle are in the ratio 3 : 2. Find the area of the rectangle.
A. 600 m^2
B. 580 m^2
C. 640 m^2
D. 560 m^2
11. The perimeter of the square hall is 200 ft. If the cost of flooring is Rs. 50 per m^2 , then find the total cost of flooring the hall.
A. 72,000
B. 68,000
C. 70,000
D. 75,000
12. Find the area of a square if the length of the diagonal of the square is 10 m.
A. 40 m^2
B. 50 m^2
C. 54 m^2
D. 46 m^2
13. Find the area of the right-angled triangle whose hypotenuse is 15 cm and one of the sides is 12 cm.
A. 60 cm^2
B. 48 cm^2
C. 50 cm^2
D. 54 cm^2
14. Find the area of a circle whose diameter is 20 cm.
A. 300
B. 310
C. 314
D. 306
15. The area of the triangle is $3,125 \text{ cm}^2$, and the base and height are in the ratio 5 : 2. What is the height of the triangle?
A. 40
B. 48
C. 50
D. 46
16. The perimeter of a triangle is 96 cm. If two sides of the triangle are 40 cm and 24 cm, then find the area of the triangle.
A. 384
B. 380
C. 428
D. 364
17. The base and height of a parallelogram field is 54 m and 24 m, respectively. Find the cost of levelling the field at the rate of Rs. 5 per m^2 .
A. 6,110
B. 6,850
C. 7,420
D. 6,480



- 18.** The perimeter of one square is 24 m and that of another is 32 m. Perimeter of a square whose area is equal to the sum of the areas of the two squares will be:
- A. 40
B. 38
C. 44
D. 36
- 19.** Find the area of a triangle whose sides are 3 cm, 4 cm, and 5 cm, respectively.
- A. 8 cm^2
B. 4 cm^2
C. 5 cm^2
D. 6 cm^2
- 20.** The area of a parallelogram is 192 cm^2 , if the height of the parallelogram is one-third of the base, then find the height of the parallelogram.
- A. 6
B. 12
C. 8
D. 10
- 21.** The perimeter and area of rhombus are 180 cm and 315 cm^2 , respectively. Find the altitude of the rhombus.
- A. 10 cm
B. 7 cm
C. 8 cm
D. 6 cm
- 22.** The side of a square shaped pool is 10 m and if the cost of flooring is Rs. 100 per m^2 then, find the total cost of flooring the pool.
- A. 12,000
B. 10,000
C. 8,000
D. 6,000
- 23.** The area of a square is increased by 60 cm^2 when its side is increased by 4 cm. Find the side of the square.
- A. 5.5
B. 8
C. 4.5
D. 5
- 24.** The length and breadth of a field are 36 m and 21 m, respectively. Poles are required to be fixed all along the boundary at a distance of 3 m apart. The number of required poles will be?
- A. 40
B. 48
C. 38
D. 36
- 25.** If the length of the diagonal of the square is 5.2 cm, then find the area of the square.
- A. 14 cm^2
B. 13.50 cm^2
C. 14.42 cm^2
D. 13.52 cm^2
- 26.** The perpendicular and area of a right-angled triangle is 20 cm and 10 cm^2 . Find the base of the triangle.
- A. 1 cm
B. 3 cm
C. 4 cm
D. 5 cm
- 27.** The perimeter of the rectangle is 50 cm and its breadth is 10 cm. Find its area.
- A. 100 cm^2
B. 150 cm^2
C. 140 cm^2
D. 160 cm^2
- 28.** The perimeter of a square garden is 400 m and the cost of gardening is Rs. 3 per m^2 , then find the total cost for gardening the garden.
- A. Rs. 1,000
B. Rs. 2,800
C. Rs. 4,000
D. Rs. 3,000
- 29.** Find the area of the rhombus whose length of the diagonal is 10 cm.
- A. 40 cm^2
B. 38 cm^2
C. 50 cm^2
D. 46 cm^2
- 30.** Find the height of the triangle whose area is 100 cm^2 and length of the base is 20 cm.
- A. 10
B. 8
C. 14
D. 6



SOLUTIONS

1. **(A)** Area of rhombus = $\frac{1}{2} \times d_1 \times d_2$
 $\text{Area} = \frac{1}{2} \times 8 \times 10$
 $= 40 \text{ cm}^2$, hence, the option (A) is correct.
2. **(B)** Area of equilateral triangle = $\frac{\sqrt{3}}{4} a^2$
 $= \frac{\sqrt{3}}{4} \times 6 \times 6 = 9\sqrt{3}$
The area of equilateral triangle is $9\sqrt{3} \text{ cm}^2$, thus, option (B) is correct.
3. **(A)** Area of the field = $\frac{1}{2} \times \text{diagonal} \times \text{sum of the perpendicular from the opposite vertices on to this diagonal}$
 $= \frac{1}{2} \times 220 \times (40.4 + 50.8)$
 $= 10,032 \text{ m}^2$, thus, option (A) is correct.
4. **(B)** Perimeter of square = $4a$
 $4a = 16$, $a = 4$
Area of square = a^2
 $= 4 \times 4 = 16 \text{ cm}^2$
5. **(D)** Area of a circle = πr^2
 $= 3.14 \times r^2 = 314$
 $= r^2 = 100$, $r = 10 \text{ cm}$
Circumference of a circle = $2\pi r$
 $= 2 \times 3.14 \times 10 = 62.8 \text{ cm}$
The circumference of the circle is 62.8 cm; hence, option D is correct.
6. **(A)** Given: Perimeter = 326 cm, Length = 98 cm
Perimeter = $2(l + b)$
 $326 = 2(98 + b)$
 $326 = 196 + 2b$
 $2b = 130$
Breadth = 65 cm
Area = Length \times Breadth
 $\text{Area} = 98 \times 65 = 6370 \text{ cm}^2$, hence, option (A) is correct.
7. **(D)** Given: Breadth = 12 cm, Perimeter = 50 cm
Perimeter = $2(l + b)$
 $50 = 2(l + 12)$
 $50 = 2l + 24$
 $2l = 26$, $l = 13$
The length of the rectangle is 13 cm, hence, option (D) is correct.
8. **(C)** Given: Area = $1,029 \text{ cm}^2$
 $\text{Area} = l \times b$
Let the length and breadth be $7x$ and $3x$
 $1029 = 3x \times 7x$
 $1029 = 21x^2$
 $x^2 = 49$, $x = 7$
Length = $7(7) = 49$
Breadth = $3(7) = 21$
Perimeter = $2(l + b)$
 $= 2(49 + 21)$
 $= 140 \text{ cm}$, hence, option (C) is correct.
9. **(B)** Given: Length = 10 m, Breadth = 7 m
Perimeter = $2(l + b)$
 $= 2(10 + 7)$
 $= 34 \text{ m}$
Cost of fencing = $34 \times 40 = \text{Rs. } 1,360$, hence, option (B) is correct.
10. **(A)** Let the length and breadth be $3x$, and $2x$ respectively
Perimeter = $2(l + b)$
 $50 = 2(3x + 2x)$
 $50 = 6x + 4x$
 $50 = 10x$, $x = 10$
Length = $10(3) = 30 \text{ m}$, Breadth = $10(2) = 20 \text{ m}$
Area = $l \times b$
 $= 30 \times 20 = 600 \text{ m}^2$, hence, option (A) is correct.
11. **(D)** Given: Perimeter = 200 ft
Perimeter = $4a$
 $200 = 4a$, $a = 50 \text{ ft}$
Area = side \times side
 $= 50 \times 50 = 2,500 \text{ ft}^2$
Cost of flooring = $2,500 \times 50 = \text{Rs. } 75,000$, hence, option (D) is correct.
12. **(B)** Given: Diagonal = 10 m
Diagonal = $a\sqrt{2}$
 $10 = a\sqrt{2}$
 $100 = 2a^2$
 $a^2 = 50$
The area of the square is 50 m^2 , hence, option (B) is correct.
13. **(D)** $AB^2 = AC^2 - BC^2$
 $= (15)^2 - (12)^2$
 $= 225 - 144$



$$AB^2 = 81$$

$$AB = 9 \text{ cm}$$

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 12 \times 9$$

$$= 54 \text{ cm}^2, \text{ hence, option (D) is correct.}$$

- 14. (C)** Given: Diameter = 20 cm, radius = 10 cm

$$\text{Area of circle} = \pi r^2$$

$$= 3.14 \times 10 \times 10$$

$$= 314 \text{ cm}^2, \text{ hence, option (C) is correct.}$$

- 15. (C)** Let the base and height be $5x$ and $2x$, respectively.

$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$3125 = \frac{1}{2} \times 5x \times 2x$$

$$3125 = 5x^2$$

$$625 = x^2$$

$$x = 25$$

$$\text{Height of triangle} = 2(25) = 50 \text{ cm, thus, option (C) is correct.}$$

- 16. (A)** Given: Perimeter = 96 cm, $a = 40$ cm,

$$b = 24 \text{ cm}$$

$$96 = 40 + 24 + c$$

$$c = 32 \text{ cm}$$

$$S = \frac{(a+b+c)}{2}$$

$$= \frac{(32+24+40)}{2} = \frac{96}{2}$$

$$= 48 \text{ cm}$$

$$\text{Area of triangle} = \sqrt{[s(s-a)(s-b)(s-c)]}$$

$$= \sqrt{[48(48-40)(48-24)(48-32)]}$$

$$= \sqrt{(48 \times 8 \times 24 \times 16)}$$

$$= 384 \text{ cm}^2, \text{ hence, option (A) is correct.}$$

- 17. (D)** Area of parallelogram = base \times height

$$= 54 \times 24 = 1296 \text{ m}^2$$

$$\text{Cost of levelling} = 1,296 \times 5 = 6,480$$

$$\text{The total cost of levelling is Rs. 6,480, hence, option (D) is correct.}$$

- 18. (A)** Perimeter of first square = 24 m

$$\text{Side of first square} = 24/4 = 6$$

$$\text{Perimeter of second square} = 32 \text{ m}$$

$$\text{Side of second square} = 32/4 = 8$$

$$\text{Area of first square} = 6 \times 6 = 36 \text{ m}^2$$

$$\text{Area of second square} = 8 \times 8 = 64 \text{ m}^2$$

$$\text{Sum of areas} = 36 + 64 = 100 \text{ m}^2$$

$$\text{Side of square will be} = \sqrt{100} = 10 \text{ m}$$

$$\text{Perimeter} = 4 \times 10 = 40 \text{ m, hence, option (A) is correct.}$$

- 19. (D)** $a = 3$ cm, $b = 4$ cm, $c = 5$ cm

$$S = \frac{(a+b+c)}{2} = \frac{(3+4+5)}{2} = 6 \text{ cm}$$

$$\text{Area of triangle} = \sqrt{[s(s-a)(s-b)(s-c)]}$$

$$= \sqrt{[6(6-3)(6-4)(6-5)]}$$

$$= \sqrt{36} = 6 \text{ cm}^2, \text{ hence, option (D) is correct.}$$

- 20. (C)** Let the height and base of parallelogram be x cm and $3x$ cm respectively.

$$\text{Area} = 192 \text{ cm}^2$$

$$\text{Area of parallelogram} = \text{base} \times \text{height}$$

$$192 = 3x(x)$$

$$192 = 3x^2$$

$$x^2 = 64, x = 8$$

$$\text{The height of the parallelogram is 8 cm; hence, option (C) is correct.}$$

- 21. (B)** Perimeter = 180 cm

$$\text{Side} = 180/4 = 45 \text{ cm}$$

$$\text{Area} = b \times h$$

$$315 = 45 \times h$$

$$h = 7 \text{ cm, hence, option (B) is correct.}$$

- 22. (B)** Area = side \times side

$$= 10 \times 10 = 100 \text{ m}^2$$

$$\text{Total cost of flooring} = 100 \times 100 = \text{Rs. 10,000}$$

$$\text{Total cost of flooring is Rs. 10,000, hence option (B) is correct.}$$

- 23. (A)** Let the side be x cm

$$\text{Therefore, } (x+4)^2 - x^2 = 60$$

$$= x^2 + 8x + 16 - x^2 = 60$$

$$x = 5.5 \text{ cm, hence, option (A) is correct.}$$

- 24. (C)** Perimeter = $2(l \times b)$

$$= 2(36 + 21) = 144 \text{ m}$$

$$\text{Number of poles required} = 144/3 = 38, \text{ hence, option (C) is correct.}$$

- 25. (D)** Area = $\frac{1}{2} \times \text{diagonal} \times \text{diagonal}$

$$= \frac{1}{2} \times 5.2 \times 5.2$$

$$= 13.52 \text{ cm}^2, \text{ hence, option (D) is correct.}$$

- 26. (A)** Given: Perpendicular = 20 cm, area = 10 cm^2

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{perpendicular}$$

$$10 = \frac{1}{2} \times \text{base} \times 20$$

$$\text{Base} = 1 \text{ cm, hence, option (A) is correct.}$$

- 27. (B)** Perimeter = $2(l + b)$

$$\text{Given: Perimeter} = 50 \text{ cm, Breadth} = 10 \text{ cm}$$



$$50 = 2(l + 10)$$

$$50 = 2l + 20$$

$$l = 15$$

$$\text{Area} = l \times b$$

$= 15 \times 10 = 150 \text{ cm}^2$, hence, option (B) is correct.

28. (D) Perimeter $= 4 \times \text{side}$

$$400 = 4 \times \text{side}$$

$$\text{Side} = 100 \text{ m}$$

$$\text{Area} = 100 \times 100 = 10,000 \text{ m}^2$$

Total cost for gardening $= 10,000 \times 3 =$
Rs. 3,000, hence, option (D) is correct.

29. (C) Area $= \frac{1}{2} \times \text{diagonal}^2$

$$= \frac{1}{2} \times 10 \times 10$$

$= 50 \text{ cm}^2$, hence, option (C) is correct.

30. (A) Area $= \frac{1}{2} \times \text{base} \times \text{height}$

Given: Area $= 100 \text{ cm}^2$, Base $= 20 \text{ cm}$

$$100 = \frac{1}{2} \times 20 \times h$$

$$200 = 20 h$$

$h = 10 \text{ cm}$, hence, option (A) is correct.



24 Surface Area and Volume



SYNOPSIS

- Introduction
- Surface Area
- Total Surface Area
- Curved Surface Area
- Volume
- Formulae Related to Three-Dimensional Figures
- Summary

INTRODUCTION

This topic deals with the mensuration which means measurement of lengths in a geometric figure that deals with the relation between lengths, area and volume of two-dimensional (2D) and three-dimensional (3D) figures. Mensuration deals with relating lengths of 2D figures with their area and perimeter. For 3D figures or solids, how volume and surface area is related to the measure of its sides:

Area = Product of Sides; Volume = Base area \times side length (Height)

SURFACE AREA

The surface area and volume can be calculated for any 3D geometrical shape. The surface of any area is the region occupied by the surface of an object. The volume is the amount of space available in an object. We have different types of shapes such as a hemisphere, sphere, cube, cuboid, cylinder, etc. All 3D shapes have area and volume. But in 2D shapes such as square, rectangle, triangle, circle, etc., 2D, we can only measure the area. The area occupied by a 3D object by its outer surface is called the surface area. It is measured in square units.

The area is of two types:

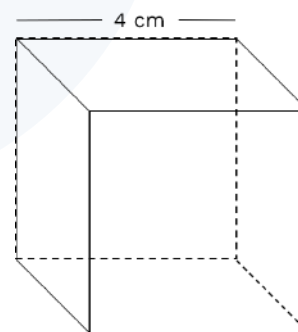
- Total surface area
- Curved surface area/lateral surface area

TOTAL SURFACE AREA

The area including the base(s) and the curved portion corresponds to the overall surface area. It is the amount of the area enclosed by the object's surface. If the form has a curved base and surface, so the sum of the two regions would be the total area. The total surface area can be defined as 'the total area covered by an object including its base as well as the curved part. If an object has both the base and curved area then the total surface area will be equal to the sum of a base and curved area'.

The total surface area is the total area occupied by an object. For example, take cuboid as an example the cuboid has 6 faces, 12 edges, and 8 vertices. The sum of all those 6 areas will be our total surface area of the particular shape.

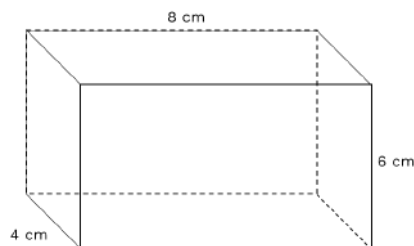
Example: Given below figure is a cube whose side is 4 cm. Find the total surface area of the cube.



Solution: Given side = 4 cm

$$\begin{aligned}\text{Total surface area of cube} &= 6a^2 \text{ sq. units} \\ &= 6 (4)^2 \text{ cm}^2 \\ &= 96 \text{ cm}^2\end{aligned}$$

Example: Given below figure is a cuboid having its dimensions: Length = 4 cm, Height = 6 cm and Breadth = 8 cm. Find the total surface of the cuboid.

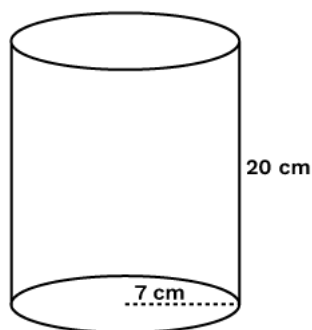


Solution: Given $L = 4$ cm, $B = 8$ cm, $H = 6$ cm
Total surface area = $2[(l \times b) + (l \times h) + (b \times h)]$
 $= 2[(4 \times 8) + (4 \times 6) + (8 \times 6)]$
 $= 2[(32) + (24) + (48)]$
 $= 2(104)$
 $= 208 \text{ cm}^2$

CURVED SURFACE AREA

Curved surface area, except its centre, corresponds to the area of only the curved portion of the shape (s). For shapes such as a cone, it is often called the lateral surface area. The lateral surface area can be defined as ‘the area which includes only the curved surface area of an object or lateral surface area of an object by excluding the base area of an object’. The lateral surface area is also known as the curved surface area. Most of the shapes or objects refer to the curved surface area, the shape or object-like cylinder refers to it as a lateral surface area. In simple, ‘The area which is visible to us is called a lateral surface area’.

Example: Given below figure is a cylinder and height and radius of the cylinder is 7 cm and 20 cm, respectively. Find the curved surface area of the cylinder.



Solution: Given, $r = 7$ cm, $H = 20$ cm

Curved surface area = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 7 \times 20$$
$$= 880$$

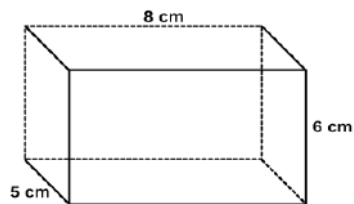
VOLUME

The volume is the amount of space in a certain 3D object. The total amount of space that an object or substance occupies is called volume. It is measured in cubic units. We have already learnt about volumes of certain figures (objects) in earlier classes. Recall that solid objects occupy space. The measure of this occupied space is called the volume of the object.

If an object is solid, then the space occupied by such an object is measured and is termed the volume of the object. On the other hand, if the object is hollow, then interior is empty, and can be filled with air, or some liquid that will take the shape of its container. In this case, the volume of the substance that can fill the interior is called the capacity of the container. In short, the volume of an object is the measure of the space it occupies, and the capacity of an object is the volume of substance its interior can accommodate. Hence, the unit of measurement of either of the two is cubic unit.

So, if we were to talk of the volume of a cuboid, we would be considering the measure of the space occupied by the cuboid. Further, the area or the volume is measured as the magnitude of a region. So, correctly speaking, we should be finding the area of a circular region, or volume of a cuboidal region, or volume of a spherical region, etc. But for the sake of simplicity, we say, find the area of a circle, volume of a cuboid or a sphere even though these mean only their boundaries.

Example: Given below figure is a cuboid, if length, breadth, and height of the cuboid is 8 cm, 5 cm, 6 cm, respectively, then find the volume of the cuboid.

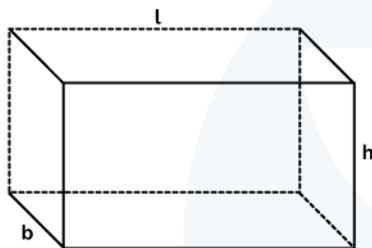


Solution: Given, $l = 8$ cm, $h = 6$ cm, $b = 4$ cm
 Volume = $l \times b \times h$
 $= 8 \times 4 \times 6$
 $= 192 \text{ cm}^3$

Formulae related to three-dimensional figures

1. Cuboid

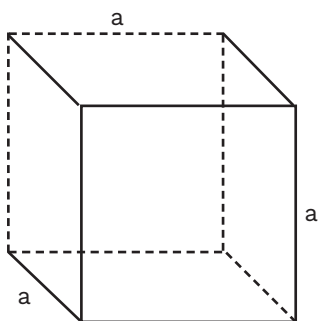
- Volume = $(l \times b \times h)$ cu. units
- Surface area = $2(lb + bh + lh)$ sq. units
- Longest diagonal = $\sqrt{l^2 + b^2 + h^2}$



2. Cube

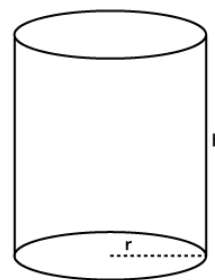
Each edge is 'a' unit.

- Volume = a^3 cu. units
- Total surface area = $6a^2$ sq. units
- Longest diagonal = $\sqrt{3a^2} = \sqrt{3}a$



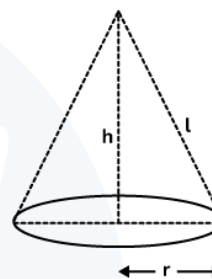
3. Right circular cylinder

- Curved surface area = $2\pi rh$
- Area of each circular face = πr^2
- Total surface area = $2\pi r(r + h)$
- Volume = $\pi r^2 h$



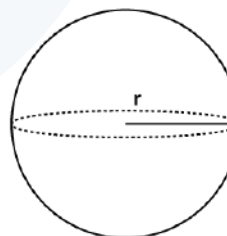
4. Right circular cone

- Curved surface area = πrl ;
 where $l = \sqrt{h^2 + r^2}$;
- Total surface area = $\pi r^2 + \pi rl = \pi r(r + l)$
- Volume = $\frac{1}{3} \pi r^2 h$



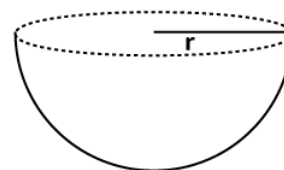
5. Sphere

- Volume = $\frac{4}{3} \pi r^3$
- Total surface area = $4\pi r^2$



6. Hemisphere

- Volume = $\frac{2}{3} \pi r^3$
- Curved surface area = $2\pi r^2$
- Total surface area = $2\pi r^2 + \pi r^2 = 3\pi r^2$





PRACTICE QUESTIONS

1. What is the total surface area of a cuboid whose dimensions are 25 cm, 10 cm, and 2 cm?
A. 250 cm^2
B. 400 cm^2
C. 500 cm^2
D. $1,000 \text{ cm}^2$
2. The diameter of the base of a right circular cylinder is 56 cm and its height is 20 cm. What is the curved surface of the cylinder?
A. $3,520 \text{ cm}^2$
B. $1,120 \text{ cm}^2$
C. $1,760 \text{ cm}^2$
D. $3,000 \text{ cm}^2$
3. A conical tent has floor area of 616 m^2 and a straight height of 48 m. If canvas costs Rs. $42/\text{m}^2$, find the cost of making the tent.
A. Rs. 46,200
B. Rs. 92,400
C. Rs. 88,704
D. Rs. 69,300
4. A solid sphere of radius 21 mm is melted to form 27 equal spherical balls. What is the surface area of each of the smaller spheres?
A. 204 mm^2
B. 308 mm^2
C. 412 mm^2
D. 616 mm^2
5. How much water can be stored in a cone with its base radius 7 m and height 12 m?
A. 204 m^3
B. 308 m^3
C. 412 m^3
D. 616 m^3
6. The volume of a cube is 343 cm^3 , find the edge of the cube.
A. 7 cm
B. 14 cm
C. 21 cm
D. 3.5 cm
7. Find the area of the cloth required to cover a cylindrical vessel of height 1 m and diameter 140 cm.
A. 3.74 m^2
B. 7.48 m^2
C. 74.8 m^2
D. 37.4 m^2
8. Two cubes each of volume 125 cm^3 are joined from end to end then, what is the total surface area of the resulting cuboid?
A. 125 cm^2
B. 200 cm^2
C. 250 cm^2
D. 300 cm^2
9. Find the total surface area of a cylinder whose base radius is 7 cm and height is 14 cm.
A. 564 cm^2
B. 231 cm^2
C. 462 cm^2
D. 924 cm^2
10. If the radius of a dome of a building is 7 m, then find the cost of whitewashing the dome from outside if cost of whitewashing is Rs. $5/\text{m}^2$.
A. Rs. 770
B. Rs. 1,540
C. Rs. 1,410
D. Rs. 1,630
11. How many spherical balls of 2 cm radius can be made out of a solid cube whose edge measures 44 cm?
A. 2,542
B. 5,084
C. 1,271
D. 7,626
12. Find the volume of a right circular cone whose diameter of base is 6 cm and height is 14 cm.
A. 33 cm^3
B. 66 cm^3
C. 132 cm^3
D. 44 cm^3
13. If the volume of a metallic cube is 125 m^3 , then find the cost of painting the cube if cost of painting is Rs. $10/\text{m}^2$.
A. Rs. 3000
B. Rs. 2250
C. Rs. 1800
D. Rs. 1500



- 14.** What is the curved surface area of a hemisphere of radius 14 cm?
 A. 616 cm^2
 B. 924 cm^2
 C. $1,232 \text{ cm}^2$
 D. $1,848 \text{ cm}^2$
- 15.** The volume and radius of base of a right circular cylinder is 770 cm^3 and 7 cm respectively. Find the height of the cylinder.
 A. 3 cm
 B. 4 cm
 C. 5 cm
 D. 6 cm
- 16.** If the surface area of a sphere is equal to its volume, then find the surface area of the sphere.
 A. 113
 B. 85
 C. 97
 D. 109
- 17.** The dimensions of a cuboid are 10 m, 12 m, and 8 m. Find the total cost of covering the cuboid with a sheet, if the cost of sheet is Rs. 12/m².
 A. Rs. 11,520
 B. Rs. 10,840
 C. Rs. 10,080
 D. Rs. 12,960
- 18.** The dimensions of a box are 10 cm, 10 cm, and 5 cm. Calculate the length of the longest stick that can be put into the box.
 A. 14 cm
 B. 15 cm
 C. 16 cm
 D. 18 cm
- 19.** Find the water holding capacity of a conical flask having height 21 cm and radius of base 7 cm.
 A. $1,078 \text{ cm}^3$
 B. $1,144 \text{ cm}^3$
 C. 796 cm^3
 D. $1,012 \text{ cm}^3$
- 20.** The radius of a hemisphere is 14 m. Find the cost of painting it if the cost of painting is Rs. 10/m².
 A. Rs. 9,240
 B. Rs. 12,320
 C. Rs. 16,240
 D. Rs. 18,480

SOLUTIONS

- 1. (C)** Total surface area of cuboid = $l \times b \times h$
 $= 25 \times 10 \times 2$
 $= 500 \text{ cm}^2$, hence, option (C) is correct.
- 2. (A)** Curved surface area of cylinder = $2\pi rh$
 Given: Base = 56 cm, Height = 20 cm
 $= 2 \times \pi \times 28 \times 20$
 $= 3520 \text{ cm}^2$, hence, option (A) is correct.
- 3. (B)** Floor area = $\pi r^2 = 616$
 $= r^2 = \frac{616 \times 7}{22} = 196 \text{ m}$
 $r = 14 \text{ m}$
 Slant height (l) = $\sqrt{r^2 + h^2} = \sqrt{14^2 + 48^2}$
 $= 50 \text{ m}$
 Surface area = $\pi rl = \frac{22}{7} \times 14 \times 50 = 2,200 \text{ m}^2$

Cost of tent making = $2,200 \times 42 = 92,400$
 Hence, option (B) is correct.

- 4. (D)** Volume of the larger sphere = $\frac{4}{3} \times \pi \times (21)^3$
 Volume of each smaller sphere = $\frac{4 \times \pi \times (21)^3}{3 \times 27}$
 Let the radius of each small sphere be r then
 $= \frac{4}{3} \pi (r)^3 = \frac{4}{3} \pi \left(\frac{21}{3}\right)^3$
 $r = 7 \text{ mm}$
 Surface area of each smaller sphere = $4\pi r^2$
 $= 4 \times \frac{22}{7} \times 7 \times 7 = 616 \text{ mm}^2$, hence, option (D) is correct.



5. (D) Given: Radius = 7 m, Height = 12 m

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 12 = 616 \text{ m}^3, \text{ hence,}$$

option (D) is correct.

6. (A) Volume of cube = side³

$$\text{Side}^3 = 343 \text{ cm}$$

$$\text{Side} = (7 \times 7 \times 7)^{\frac{1}{3}}$$

Side = 7 cm, hence, option (A) is correct.

7. (B) Required cloth = Total surface area of cylinder

$$\text{Total surface area of cylinder} = 2\pi r(h + r)$$

$$= 2 \times \frac{22}{7} \times \frac{70}{100} \left(1 + \frac{70}{100}\right)$$

$$= 2 \times 22 \times 0.1 \times 1.7 = 7.48 \text{ m}^2$$

Hence, option (B) is correct.

8. (C) Volume = side³

$$125 = \text{side}^3$$

$$\text{Side} = 5 \text{ cm}$$

After joining 2 cubes, its length will be 5 + 5 = 10 cm

$$\text{Total surface area of a cube} = 2(lb + bh + lh)$$

$$= 2(10 \times 5 + 5 \times 5 + 10 \times 5)$$

$$= 2(50 + 25 + 50)$$

$$= 250 \text{ cm}^2, \text{ hence, option (C) is correct.}$$

9. (D) Total surface area of cylinder = $2\pi r(r + h)$

Given: Radius = 7 cm, Height = 14 cm

$$= 2 \times \frac{22}{7} \times 7(7 + 14)$$

$$= 44 \times 21 = 924 \text{ cm}^2$$

Hence, option (D) is correct.

10. (B) Given: Radius = 7 m

$$\text{Surface area of dome} = 2\pi r^2$$

$$= 2 \times \frac{22}{7} \times 7 \times 7 = 308 \text{ m}^2$$

Cost of whitewashing = $308 \times 5 = \text{Rs. } 1,540$, hence, option (B) is correct.

11. (A) Volume of cube = side³

$$= (44)^3 = 85,184 \text{ cm}^3$$

$$\text{Volume of sphere} = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 2 \times 2 \times 2 = 33.5 \text{ cm}^3$$

$$\text{Number of spherical balls} = \frac{85,184}{33.5} =$$

$$2,542.8 \approx 2,542, \text{ hence, option (A) is correct.}$$

12. (C) Volume of cone = $\frac{1}{3} \pi r^2 h$

Given: Diameter = 6 cm, radius = 3 cm, Height = 14 cm

$$= \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 14$$

$$= 132 \text{ cm}^3, \text{ hence, option (C) is correct.}$$

13. (D) Volume = side³

$$125 = \text{side}^3$$

$$\text{Side} = 5 \text{ m}$$

$$\text{Surface area} = 6(\text{side}^2)$$

$$= 6 \times 25 = 150 \text{ m}^2$$

Cost of painting = $150 \times 10 = \text{Rs. } 1500$, hence, option (D) is correct.

14. (C) Curved surface area = $2\pi r^2$

Given: Radius = 14 cm

$$= 2 \times \frac{22}{7} \times 14 \times 14$$

$$= 1,232 \text{ cm}^2, \text{ hence, option (C) is correct.}$$

15. (C) Given: Volume = 770 cm³, Radius = 7 cm

$$\text{Volume} = \pi r^2 h$$

$$770 = \frac{22}{7} \times 7 \times 7 \times h$$

$$h = \frac{770}{154} = 5 \text{ cm}$$

The height of the cylinder is 5 cm, hence, option (C) is correct.

16. (A) Let 'r' be the radius of the sphere

Surface area of sphere = volume of sphere

$$4\pi r^2 = \frac{4}{3}\pi r^3$$

$$r = 3 \text{ cm}$$

$$\text{Surface area} = 4 \times \frac{22}{7} \times 3 \times 3$$

$$= 113.04 \approx 113, \text{ hence, option (A) is correct.}$$

17. (A) Given: Dimensions = 10 m, 12 m and 8 m

$$\text{Total surface area} = l \times b \times h$$



$$= 10 \times 12 \times 8$$

$$= 960$$

$$\text{Total cost of covering} = 960 \times 12 = \text{Rs. } 11,520$$

Hence, option (A) is correct.

- 18. (B)** Given: Dimensions = 10 cm, 10 cm, 5 cm
Length of longest stick = Diagonal of cuboid

$$\text{Required length} = \sqrt{10^2 + 10^2 + 5^2}$$

$$= \sqrt{225} = 15 \text{ cm}$$

The longest stick that can be put into box is 15 cm, hence, option (B) is correct.

- 19. (A)** Given: Height = 21 cm, Radius = 7 cm

$$\text{Volume} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 21$$

$$= 1,078 \text{ cm}^3, \text{ hence, option (A) is correct.}$$

- 20. (D)** Surface area = $3 \pi r^2$

$$= 3 \times \frac{22}{7} \times 14 \times 14$$

$$= 1,848 \text{ m}^2$$

$$\text{Total cost of painting} = 1,848 \times 10 = \text{Rs. } 18,480, \text{ hence, option (D) is correct.}$$



25 Angles and Triangles



SYNOPSIS

- Definition of triangle
- Properties of a triangle
- Types of triangles
 - On the basis of length
 - Equilateral triangle
 - Isosceles triangle
 - Scalene triangle
 - On the basis of angle
 - Acute angle triangle
 - Right angle triangle
 - Obtuse angle triangle
- Area and perimeter of triangle and its types
- Some Important Theorems Related to Triangles

DEFINITION OF A TRIANGLE

A triangle can be defined as a polygon with three vertices and three edges.

In a triangle, the sum of all the three internal angles must be equal to 180° . This is called the *angle sum property of a triangle*.

A triangle is a two-dimensional closed shape. It's a polygon having three sides. Straight lines run along all three sides. The vertex is the intersection of two straight lines. As a result, there are three vertices in the triangle. An angle is formed by each vertex.

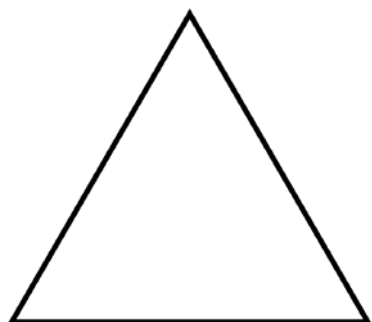


Fig. 5.1

PROPERTIES OF A TRIANGLE

- There are three edges and three sides of a triangle
- Sum of interior angles of a triangle is equal to 180° .
- Sum of exterior angles of a triangle is equal to 360° .
- The sum of two sides of a triangle will always be greater than the third side.
- The difference between any two sides of a triangle is always less than the length of the third side.
- The shortest side of a triangle will always be opposite to the smallest interior angle.
- The longest side of a triangle will always be opposite to the largest interior angle.

TYPES OF TRIANGLE

Triangles can be categorized on two bases, i.e., on the basis of length of sides and on the basis of angle.

On the basis of length of sides

There are three types of triangles based on the length of its sides. They are as:

1. **Equilateral triangle:** The triangle whose all the three sides are equal is called an equilateral triangle. Due to equal sides, it also has equal interior angles, each measuring 60° .

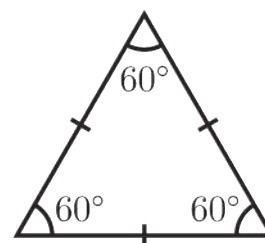


Fig. 5.2

2. **Isosceles Triangle:** The triangle that has two equal sides are called isosceles triangles.



The interior angles opposite to the two equal sides are also equal.

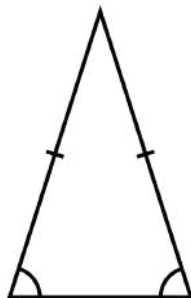


Fig. 5.3

- 3. Scalene Triangle:** A triangle that has all sides of different lengths and thus no equal interior angles, is called a scalene triangle.

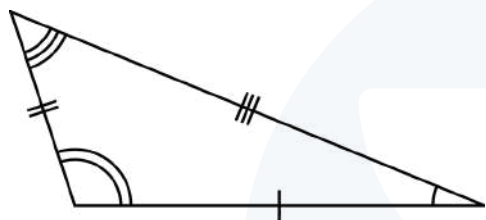


Fig. 5.4

On the basis of angles

There are three types of triangles on the basis of their angles. They are as follows:

- 1. Acute Angle Triangle:** The triangle which has all of its angle smaller than 90° is called an acute angle triangle.

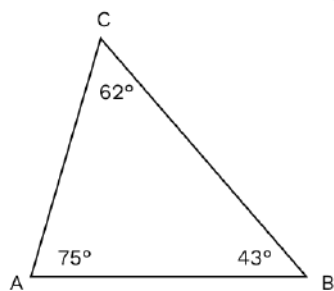


Fig. 5.5

- 2. Obtuse Angle Triangle:** The triangle that has one of its angles more than 90° is called an obtuse angle triangle.

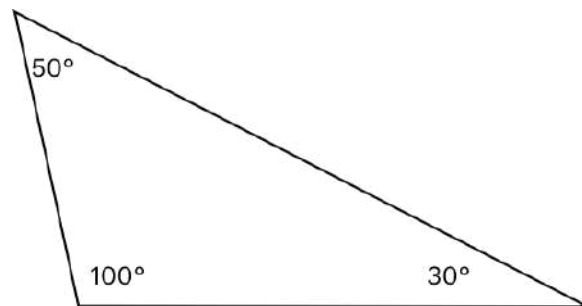


Fig. 5.6

- 3. Right-Angle Triangle:** The triangle whose one angle is 90° is called a right-angle triangle.

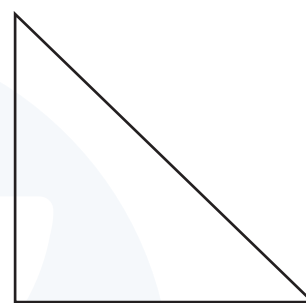


Fig. 5.7

AREA AND PERIMETER OF TRIANGLES AND ITS TYPES

Perimeter of a triangle = Sum of all sides of a triangles

Area of a triangle = Half of product of base and height
 $= \frac{1}{2} \times \text{Base} \times \text{Height}$

Area of triangle using Heron's formula

If the height of a triangle is not given, we use Heron's formula to calculate the area of the given triangle.

For this we need the semi perimeter of the triangle.

$$S = \frac{\text{Perimeter of the triangle}}{2}$$

Consider a triangle with sides a , b , and c . Then,

$$S = \frac{a+b+c}{2}$$

Then the area of the triangle is given by:

$$A = \left[(s-s)(s-a)(s-b)(s-c) \right]$$

Area of isosceles triangle using only sides

$$\frac{1}{2} \sqrt{a^2 - \frac{b^2}{4}} \times b$$

Here a = length of the equal side

b = base of the triangle

h = height of the triangle

PERIMETER OF ISOSCELES TRIANGLE

The perimeter of the isosceles triangle;

$$P = 2a + b$$

Here a = length of the equal side

b = length of the unequal side

AREA OF ISOSCELES RIGHT-ANGLE TRIANGLE

$$A = \frac{1}{2} \times a^2$$

SOME IMPORTANT THEOREMS RELATED TO TRIANGLE

- Two triangles are similar, if
 - Their corresponding angles are equal and
 - Their corresponding sides are in the same ratio (or proportion).
- The ratio of any two corresponding sides in two equiangular triangles is always the same.
- If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

- If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.
- If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.
- If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.
- The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

PYTHAGORAS THEOREM

In a right-angle triangle, sum of the square of the hypotenuse is equal to the sum of the square of the other two sides.

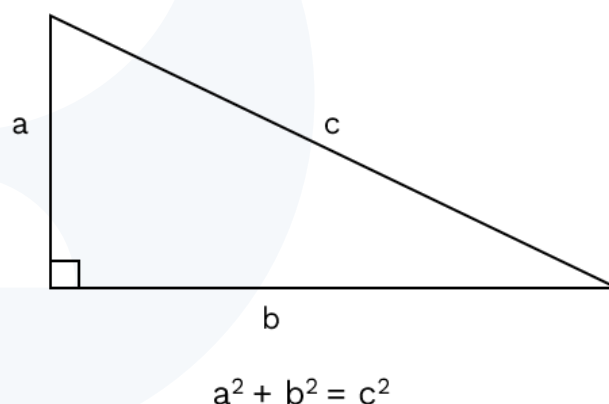


Fig. 5.8

Here, c is the hypotenuse and a and b are the other two sides of the triangle.

- If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.
- In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.



PRACTICE QUESTIONS

1. Which of the three sides cannot be used to construct a triangle?
A. 2, 4, 7
B. 3, 4, 5
C. 5, 2, 5
D. 8, 6, 10
2. Calculate the perimeter of a triangle with 13 cm on each side.
A. 38 cm
B. 39 cm
C. 40 cm
D. 41 cm
3. In a triangle with a perimeter of 55 cm and two sides of 17 cm each, what is the missing side length?
A. 23 cm
B. 22 cm
C. 21 cm
D. 20 cm
4. What is the area of a triangle with a 26 cm base and a 12 cm height?
A. 153 cm^2
B. 154 cm^2
C. 155 cm^2
D. 156 cm^2
5. One side of an equilateral triangle is 15 cm long. What is the triangle's perimeter?
A. 45 cm
B. 47 cm
C. 43 cm
D. 41 cm
6. The lengths of the two short sides of a right triangle are 6 and 8. What is the triangle's perimeter?
A. 23 cm
B. 24 cm
C. 25 cm
D. 26 cm
7. The residences of Tuku, Bobbie, and Duggu form a triangle. Bobbie's residence is 9 feet distant from Tuku's. Bobbie's residence is 13 feet away from Duggu's. Tuku's residence is 15 feet distant from Duggu's. What is the perimeter of the triangle formed by their houses?
A. 35 feet
B. 36 feet
C. 37 feet
D. 38 feet
8. What is the perimeter of a right triangle with a height 56 and a 33 cm base?
A. 151 cm
B. 152 cm
C. 153 cm
D. 154 cm
9. The triangle's three sides are 12 cm, 15 cm, and 18 cm. Calculate the area of the triangle.
A. 75.9 cm^2
B. 85.9 cm^2
C. 95.9 cm^2
D. 105.9 cm^2
10. Find the base of the triangle with an area of 102 m^2 and a height of 12 m.
A. 14 cm
B. 15 cm
C. 16 cm
D. 17 cm
11. Find the height of the triangle with a 14-unit base and a 147-square-unit area.
A. 20 units
B. 21 units
C. 22 units
D. 23 units
12. A triangle's base and height are in the ratio 13:12 and its area is 312 m^2 . What is the sum of triangle's height and base?
A. 50
B. 54
C. 56
D. 58
13. Find the area of a triangle whose sides are 10 cm, 24 cm, and 26 cm.
A. 110 cm^2
B. 120 cm^2
C. 130 cm^2
D. 140 cm^2



14. The area of triangle whose base is 8 cm is equal to the area of a circle of radius 7 cm. What is the height of the triangle?
- A. 18.5 cm
B. 28.5 cm
C. 38.5 cm
D. 48.5 cm
15. A right angled triangle has its area equal to that of a square whose side is 6 cm. What is the altitude of the triangle, if it is twice its base?
- A. 42 cm
B. 32 cm
C. 22 cm
D. 12 cm
16. What is the area of a triangle having sides 4 cm, 7 cm, and 9 cm?
- A. 13.41 cm^2
B. 15.43 cm^2
C. 17.45 cm^2
D. 19.47 cm^2
17. A triangular field's base is three times its height. Find the base and height of the field if the cost of cultivating the field at Rs. 28 per hectare be Rs. 364.
- A. 249:90
B. 259:91
C. 269:92
D. 279:93
18. The ratio of the areas of two triangles is 4:3, while the ratio of their heights is 3:4. Calculate their base ratio.
- A. 18:6
B. 17:7
C. 16:9
D. 15:8
19. The base of a triangle is 17 cm and height is 14 cm. The height of another triangle of double the area having the base 20 cm is?
- A. 23.8 cm
B. 33.8 cm
C. 43.8 cm
D. 53.8 cm
20. A triangle having the base AB is ABC . D is a point on AB where AB equals 8 cm and DB equals 6 cm. What is the ratio of area of triangle ADC to that of the area of triangle ABC ?
- A. 5 cm
B. 6 cm
C. 7 cm
D. 8 cm
21. The area of right angled triangle is 20 times its base. What is the height?
- A. 40 cm
B. 41 cm
C. 42 cm
D. 43 cm
22. If a triangle's area is 1176 cm^2 and the base-to-height ratio is 3:4, what is the triangle's altitude?
- A. 55 cm
B. 56 cm
C. 57 cm
D. 58 cm
23. A triangle's three sides are 9 cm, 40 cm, and 41 cm, respectively. Then its area is?
- A. 160 cm^2
B. 170 cm^2
C. 180 cm^2
D. 190 cm^2
24. The sides of a triangle are in the ratio of $1/4 : 1/6 : 1/8$. If the perimeter is 52 cm, then the length of the small side is?
- A. 15 cm
B. 14 cm
C. 13 cm
D. 12 cm
25. A triangle has a surface area of 216 cm^2 and sides in the ratio 3:4:5. What is the triangle's perimeter?
- A. 71 cm
B. 72 cm
C. 73 cm
D. 74 cm
26. The sides of a triangle are 6 cm, 8 cm, and 10 cm. The area of the triangle formed by joining the mid-points of the sides of this triangle is?



- A. 4 cm^2
B. 5 cm^2
C. 6 cm^2
D. 7 cm^2
- 27.** One side of a right angle triangle is two times the other, and the hypotenuse is 10 cm. The area of the triangle is?
A. 50 cm^2
B. 40 cm^2
C. 30 cm^2
D. 20 cm^2
- 28.** A right angled triangle has a perimeter of 60 cm. It has a 26-cm hypotenuse. What is the triangle's area?
A. 120 cm^2
B. 122 cm^2
C. 124 cm^2
D. 126 cm^2
- 29.** If an isosceles right triangle's perimeter is $(6+3\sqrt{2})$ m, what is the triangle's area?
A. 2.5 m^2
B. 3.5 m^2
C. 4.5 m^2
D. 5.5 m^2
- 30.** A triangle's perimeter is 30 cm and its area is 30 cm^2 . What is the length of the shortest side of the triangle if the greatest side is 13 cm?
A. 2 cm
B. 3 cm
C. 4 cm
D. 5 cm

SOLUTIONS

- 1. (A)** We already know that the total of any two triangle sides must be bigger than the third side. Option (A) does not meet this requirement.
- 2. (B)** The triangle is an equilateral triangle because all three sides are of the same length.
Perimeter = $3 \times \text{side}$
 $39 = 3 \times 13$
- 3. (C)** The triangle is isosceles triangle as only two sides are equal here.
So, perimeter = $2L + B$
 $55 = 2 \times 17 + B$
 $B = 21 \text{ cm}$
- 4. (D)** Area of triangle = $\frac{1}{2} \times b \times h$
 $= \frac{1}{2} \times 26 \times 12$
 $= 156 \text{ cm}^2$
- 5. (A)** Perimeter of triangle = $3 \times 15 = 45 \text{ cm}$.
- 6. (B)** For perimeter, first we have to find the length of the third side.
Using Pythagoras theorem to find the third side,
 $a^2 + b^2 = c^2$
 $6^2 + 8^2 = c^2$
Hence, (C) or third side = 10
Now, perimeter = $6 + 8 + 10 = 24 \text{ cm}$.
- 7. (C)** The formula for calculating the perimeter of a triangle is to add the lengths of the three sides together.
The three sides of this triangle are 9ft, 13ft, and 15ft in length. When you combine these three sides together, you get a perimeter of 37 feet.
- 8. (D)** Using the Pythagoras theorem,
 $(56)^2 + (33)^2 = C^2$
 $3136 + 1089 = C^2$
So, third side = 65 cm
Perimeter = $56 + 33 + 65 = 154 \text{ cm}$.
- 9. (C)** Perimeter = $13 + 15 + 18 = 46 \text{ cm}$
Semi perimeter = $\frac{46}{2} = 23 \text{ cm}$
Area = $\sqrt{23(23-13)(23-15)(23-18)}$
 $= \sqrt{23 \times 10 \times 8 \times 5}$
 $= \sqrt{9200} = 95.9 \text{ cm}^2$
- 10. (D)** Area = $\frac{1}{2} \times b \times h$
 $102 = \frac{1}{2} \times b \times 12$
Base = $\frac{102}{6} = 17 \text{ cm}$.
- 11. (B)** Area = $\frac{1}{2} \times b \times h$
 $147 = \frac{1}{2} \times 14 \times h$
Height = $\frac{147}{7} = 21 \text{ units}$.
- 12. (A)** Let's take height and base is $12x$ and $13x$



$$\begin{aligned}\text{Area} &= \frac{1}{2} \times b \times h \\ 312 &= \frac{1}{2} \times 13x \times 12x \\ 312 &= 78x^2\end{aligned}$$

$$x = 2$$

$$\text{So, height} = 12 \times 2 = 24$$

$$\text{and, base} = 13 \times 2 = 26$$

- 13. (B)** Firstly we have to find the semiperimeter

$$\text{of this triangle} = \frac{10 + 24 + 26}{2} = 30 \text{ cm}$$

$$\begin{aligned}\text{Now, area of triangle} &= \sqrt{30(30-10)(30-24)(30-26)} \\ &= \sqrt{30 \times 20 \times 6 \times 4} = 120 \text{ cm}^2\end{aligned}$$

- 14. (C)** Area of circle $= \pi r^2$
 $= 22/7 \times 7 \times 7 = 154 \text{ cm}^2$

$$\text{Area of triangle} = \frac{1}{2} \times b \times h$$

$$154 = \frac{1}{2} \times 8 \times h$$

$$\text{Height} = 38.5 \text{ cm.}$$

- 15. (D)** Area of square $= (6)^2 = 36 \text{ cm}^2$

Since the altitude of the triangle is twice the base, let the base and altitude is b and $2b$, respectively.

$$\text{Area of triangle} = \frac{1}{2} \times b \times h$$

$$36 = \frac{1}{2} \times b \times 2b$$

$$b = 6$$

$$\text{So, altitude} = 2b = 12 \text{ cm.}$$

- 16. (A)** Firstly we have to find the semi-perimeter

$$\text{of this triangle} = \frac{4 + 7 + 9}{2} = 10 \text{ cm}$$

$$\begin{aligned}\text{Now, area of triangle} &= \sqrt{10(10-4)(10-7)(10-9)} \\ &= \sqrt{10 \times 6 \times 3 \times 1} = 13.41 \text{ cm}^2\end{aligned}$$

- 17. (D)** Area of the field $= \text{Total cost/Rate} =$
 $364/28 = 13 \text{ hectare} = 1,30,000 \text{ m}^2$

$$\text{Let altitude be } x \text{ and base be } 3x$$

$$\text{Then, } \frac{1}{2} \times 3x \times x = 1,30,000$$

$$x = 93$$

$$\text{Hence, base} = 279 \text{ and altitude} = 93$$

- 18. (C)** Let the bases of two triangles be x and y and their heights be $3h$ and $4h$, respectively.

$$\text{Then, } (\frac{1}{2} \times x \times 3h) / (\frac{1}{2} \times y \times 4h) = 4/3$$

$$x/y = 4/3 \times 4/3 = 16/9$$

$$\text{Hence, required ratio} = 16:9$$

- 19. (A)** Area of first triangle $= \frac{1}{2} \times 17 \times 14 = 119 \text{ cm}^2$

$$\text{So the area of second triangle} = 238 \text{ cm}^2$$

$$\text{Hence, } 238 = \frac{1}{2} \times 20 \times h$$

$$\text{Height} = 23.8 \text{ cm.}$$

- 20. (B)** Here, semi-perimeter $= \frac{1}{2} (8 + 6 + 10) =$
 12 cm

$$\text{Area} = \sqrt{12 \times 4 \times 6 \times 2} = 24 \text{ cm}^2$$

$$\text{Now, } \frac{1}{2} \times 8 \times h = 24$$

$$\text{So, height} = 6 \text{ cm.}$$

- 21. (A)** Area $= \frac{1}{2} \times b \times h$

$$20 \times b = \frac{1}{2} \times b \times h$$

$$\text{So, height} = 40 \text{ cm.}$$

- 22. (B)** Let base be $3x$ and height is $4x$

$$\text{Then, } \frac{1}{2} \times 3x \times 4x = 1176$$

$$12x^2 = 2,352$$

$$x = 14 \text{ cm}$$

$$\text{So, altitude} = 4 \times 14 = 56 \text{ cm}$$

- 23. (C)** It is a right angle triangle. So,

$$\text{Area} = \frac{1}{2} \times 40 \times 9 = 180 \text{ cm}^2$$

- 24. (D)** Ratio of sides $= \frac{1}{4} : \frac{1}{6} : \frac{1}{8} = 6:4:3$

$$\text{Perimeter} = 52 \text{ cm. So, sides are } 24 \text{ cm, } 16 \text{ cm, and } 12 \text{ cm}$$

$$\text{Hence, length of the smallest side} = 12 \text{ cm}$$

- 25. (B)** Let sides be $3x \text{ cm, } 4x \text{ cm, and } 5x \text{ cm.}$

$$\text{So, semiperimeter} = 6x \text{ cm}$$

$$\text{Area} = \sqrt{6x \times 3x \times 2x \times x} = 6x^2 \text{ cm}^2$$

$$216 = 6x^2$$

$$\text{So, } x = 6$$

$$\text{Hence, sides are } 18 \text{ cm, } 24 \text{ cm, and } 30 \text{ cm.}$$

$$\text{So, perimeter} = 18 + 24 + 30 = 72 \text{ cm.}$$

- 26. (C)** It is a right angle triangle with base 6 cm and height 8 cm.

$$\text{So, area} = \frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2$$

$$\text{Hence, area of required triangle} = \frac{1}{4} \times 24 = 6 \text{ cm}^2$$

- 27. (D)** Let the sides be x and $2x$

$$\text{Then, } (x)^2 + (2x)^2 = (10)^2$$

$$x^2 = 20$$

$$\text{Area} = \frac{1}{2} \times x \times 2x = x^2 = 20 \text{ cm}^2$$

- 28. (A)** Perimeter $= b + h + 26$

$$60 - 26 = b + h \text{ or } (b + h)^2 = 34^2$$

$$\text{Also, } b^2 + h^2 = 26^2$$

$$\text{So, } (b + h)^2 - (b^2 + h^2) = 34^2 - 26^2$$

$$2bh = (34 + 26)(34 - 26)$$

$$bh = 240 \text{ or } \frac{1}{2} bh = 120$$

$$\text{Hence, area} = 120 \text{ cm}^2$$



- 29. (C)** Let the sides be a , a and b meters.
Then, $2a + b = 6 + 3\sqrt{2}$ and $b^2 = a^2 + a^2 = 2a^2$
 $b = \sqrt{2}a$
So, $2a + \sqrt{2}a = 6 + 3\sqrt{2}$
 $a = 3$
Hence, area = $\frac{1}{2} \times 3 \times 3 = 4.5 \text{ m}^2$
- 30. (D)** Let the smallest side be x

Then, the other sides are 13 cm. and $(17 - x)$ cm
Here, $a = 13$, $b = x$ and $c = (17 - x)$ with semi-perimeter = 15
So, Area = $\sqrt{15 \times 2 \times (15 - x) \times (x - 2)}$
 $30^2 = 30 \times (15 - x) (x - 2)$
 $x = 5$
Hence, smallest side = 5 cm.





SYNOPSIS

- Definition of Circle
- Parts of Circle
 - Radius
 - Diameter
 - Sector
 - Segment
 - Arc
 - Secant
 - Tangent
 - Chord
 - Centre
- Circle Formulas
 - Areas and perimeter
- Properties of Circle
- Some Important Theorems

DEFINITION OF CIRCLE

Circle may be defined as a collection of points in a plane which are equidistant from a fixed point in the same plane.

A circle divides the plane it exists on into 3 parts. They are: (i) inside the circle, also known as the circle's interior; (ii) the circle; and (iii) outside the circle, also known as the circle's exterior. The circular region is made up of the circle and its inside.

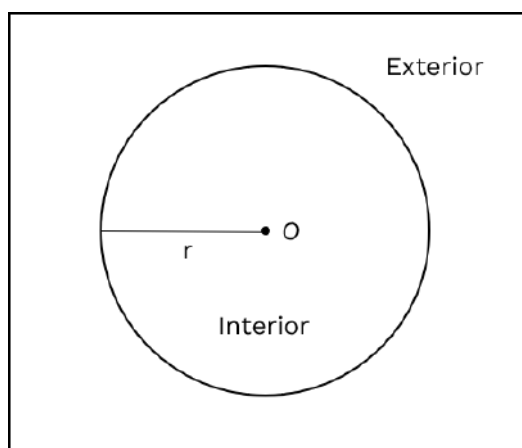


Fig. 6.1

Fig. 9.1 shows the interior of a circle, the exterior of a circle and the circle.

PARTS OF A CIRCLE

- **Centre:** The fixed point as shown in Fig. 6.1 (represented by O) is called as the centre of the circle.
- **Radius:** The fixed distance between the center of the circle and all the points forming the circle is defined as the radius of the circle. It is generally represented by r (see Fig. 6.1).
- **Chord:** A line segment whose both the endpoints lie on the circle is called a chord.
- **Diameter:** A line segment which has both its endpoints on the circle and passes through the center is called the diameter of the circle. It is generally denoted by d and is twice the size of the radius of the circle.

Diameter is the longest chord of the circle.

$$d = 2r$$

$$r = d/2$$

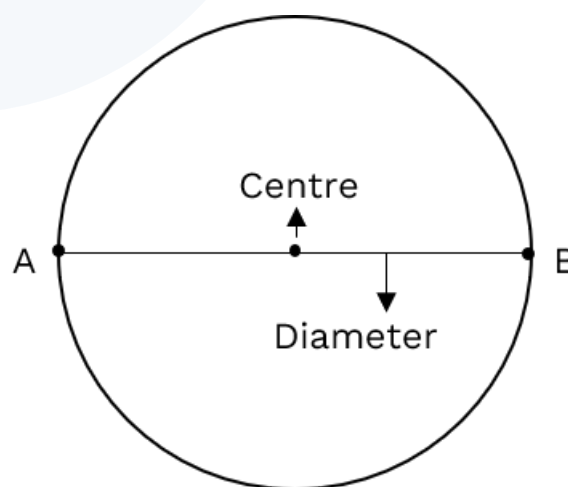


Fig. 6.2



- **Arc:** Arc is basically a part of the circumference of the circle. It is denoted by ' \frown ' or ' \smile '.
- **For example:** In the Fig. 6.3, AB is the arc and is read as 'arc AB'.

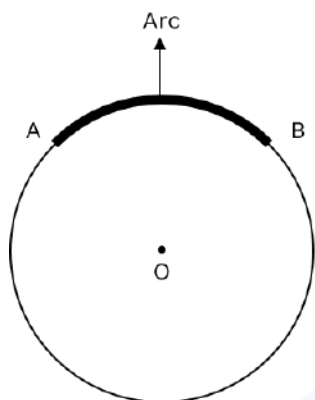


Fig. 6.3

- **Secant:** Secant can be defined as an extended chord which cuts the circle at two points.
- **Segment:** The region bounded by a chord and an arc between the two endpoints of the chord. It may be both in interior as well as exterior of the circle.
- **Sector:** The region in the interior of the circle bound by two radii and an arc formed between the two end points of the radii on the circle.
- **Tangent:** A line segment touching the circle at one and only point on the exterior is called a tangent.

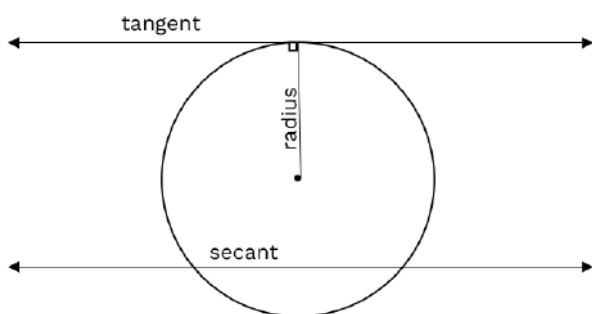


Fig. 6.4

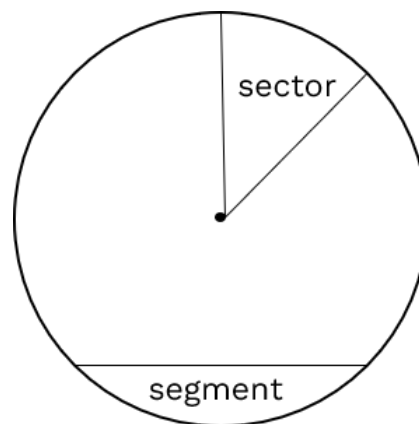
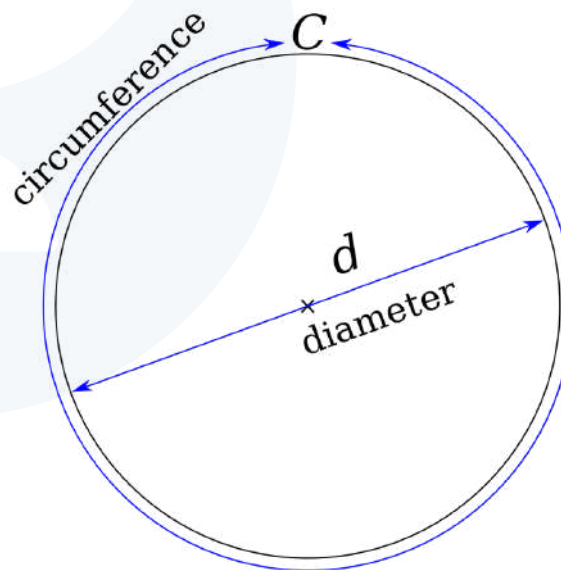


Fig. 6.5

Fig. 6.4 shows the sector and segment of a circle. Tangent and secant can be clearly seen in Fig. 6.5.

CIRCLE FORMULAS

- **Circumference of a circle:** Circumference of a circle is defined as the distance around the circle.



Circumference of a circle (C) = $2\pi r = \pi d$

- **Area of a circle:** The total amount of space occupied by a circle is defined as the area of the circle.
Area of the circle = πr^2
- **Circumference of a semicircle:** Circumference of a semicircle is half the area of a circle.
Area of a semicircle = $\frac{1}{2}\pi r$

$$\frac{\theta}{360^\circ}$$



- **Area of sector:** $\frac{\theta}{360^\circ} \times \pi r^2$ (when θ is in degrees)
- **Perimeter of a sector:** $2 \text{ Radius} + (\frac{\theta}{360^\circ} \times 2\pi r)$
- **Area of a segment:** Area of a segment in degrees = $\frac{1}{2} \times r^2 \times [(\frac{\pi}{180^\circ}) \theta - \sin \theta]$
Area of segment in radians = $\frac{1}{2} \times r^2 (\theta - \sin \theta)$
- **Perimeter of a segment:** The perimeter of a segment is the sum of chord length and arch length.
Perimeter of segment in degrees = $(\frac{\theta}{360^\circ} \times 2\pi r) + (2r \sin \frac{\theta}{2})$
Perimeter of segment in radians = $r\theta + (2r \sin \frac{\theta}{2})$

PROPERTIES OF A CIRCLE

- Two circles with equal radii are congruent to each other.
- Diameter is the longest chord of a circle and is double the radius in length.

- Diameter of a circle divides it into two equal parts.
- Two circles with equal radii are similar circles.

SOME IMPORTANT THEOREMS

- Angle made by equal chords at the centre are equal
- If two angles made by chords at the centre are equal the chords are equal.
- A perpendicular from the centre of a circle bisects the chord in two equal halves.
- There can be one and only one circle passing through three non-collinear points.
- Two equal chords of the circle are at equal distance from the centre.
- Two chords which are at equal distance from the centre are equal in length.
- The angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.
- Angles in the same segment of a circle are equal.

PRACTICE QUESTIONS

- Calculate the area of a circle with a radius of 6.
A. 28π
B. 36π
C. 42π
D. 46π
- Calculate the circumference of a circle with a radius of 49.
A. 306
B. 307
C. 308
D. 309
- Calculate the diameter of a circle with a circumference of 17π .
A. 14
B. 15
C. 16
D. 17
- Find the area of a sector of a 28-cm-radius circle with a 90-degree central angle.
A. 616 cm^2
B. 626 cm^2
C. 636 cm^2
D. 646 cm^2
- What is the area of the segment if the area of a sector is 128 sq. ft. and the area of the enclosed triangle is 67 square feet?
A. 61 sq. ft.
B. 71 sq. ft.
C. 81 sq. ft.
D. 91 sq. ft.
- AB subtends a 72-degree angle to the centre of a circle with a radius of 21 cm, given that arc. Determine the length of the arc AB.
A. 26 cm
B. 26.4 cm
C. 26.8 cm
D. 26.5 cm



7. If the area of the appropriate minor segment is 37 sq. units and the radius is 7 units, find the area of the major segment of a circle.
- A. 115 sq. units
B. 116 sq. units
C. 117 sq. units
D. 118 sq. units
8. If the central angle of a pizza slice is 60 degrees and the radius is 6 units, determine the area of the segment generated by removing the triangle part of the pizza slice.
- A. 3.19 sq. units
B. 4.27 sq. units
C. 3.67 sq. units
D. 3.27 sq. units
9. Calculate the length of an arc of a circle that forms a 150-degree angle with the centre of a circle with a radius of 27 cm.
- A. 70.45 cm
B. 70.55 cm
C. 70.65 cm
D. 70.75 cm
10. An arc is 70 meters long. Find the angle subtended by the arc if the radius of the circle is 21 m.
- A. 191 degrees
B. 191.08 degrees
C. 191.80 degrees
D. 192 degrees
11. Find the radius of an arc with a length of 127 cm and a 120-degree angle to the circle's centre.
- A. 6.44 cm
B. 6.54 cm
C. 6.64 cm
D. 6.74 cm
12. An equilateral triangle is the form of a wire. The triangle's sides are 14 cm long. A circle is formed by bending the wire. Calculate the circumference of the resulting circle.
- A. 44.63 square cm
B. 45.63 square cm
C. 46.63 square cm
D. 47.63 square cm
13. Calculate the length of an arc, having a radius of 17 m and an angle of 33.67 radians, in radians.
- A. 575.39 m
B. 574.39 m
C. 573.39 m
D. 572.39 m
14. If a wheel revolves 49 times to cover 343 meters, what is its radius?
- A. 3.83 m
B. 2.83 m
C. 0.83 m
D. 1.83 m
15. A ring's circumference and diameter differ by 28 cm. Calculate the ring's radius.
- A. 5.53 cm
B. 6.53 cm
C. 7.53 cm
D. 8.53 cm
16. 5:00 p.m. is displayed on a circular clock. The minute hand measures 14 units in length. When the time is 5:30 p.m., find the distance travelled by the tip of the minute hand.
- A. 44 units
B. 46 units
C. 48 units
D. 50 units
17. A circle's circumference is 576 yards. Calculate the circle's diameter.
- A. 183.43 yards
B. 185.43 yards
C. 187.43 yards
D. 189.43 yards
18. The shape of a race track is that of a circular ring. The track has a 47-yard inner radius and a 49-yard outside radius. Find the area of the race track.
- A. 601.42 sq. yards
B. 603.42 sq. yards
C. 605.42 sq. yards
D. 607.42 sq. yards
19. The area ratio of two circles is 16:25. Find the ratio of their radii using the area of circle formula.



- A. 1:5
B. 2:5
C. 4:5
D. 8:5
- 20.** If the radius is 28 cm and the angle is 108° , calculate the area of the sector.
A. 43.2 cm^2
B. 44.2 cm^2
C. 45.2 cm^2
D. 46.2 cm^2
- 21.** A man walks twice a day on a walking track that is shaped like a sector with an angle of 111° and a radius of 97 m. In a single day, determine the area traversed by the guy of the walking track.
A. 17642.68 m
B. 17742.68 m
C. 17852.68 m
D. 17967.68 m
- 22.** In a field, a horse is grazing. It's attached to a pole via a 7-m rope. The horse moves from point A to point B, forming a 72-degree arch. Find the horse's grazing place in the sector.
A. 26.8 sq. meter
B. 28.8 sq. meter
C. 30.8 sq. meter
D. 32.8 sq. meter
- 23.** If the area of the sector is 57 m and the part of a triangle in the sector is 23 m, find the area of the segment.
A. 31 sq. meter
B. 32 sq. meter
C. 33 sq. meter
D. 34 sq. meter
- 24.** If the angle subtended by a sector's arc at its centre is 180 degrees, the sector's area in square units is?
A. $\frac{\pi r^2}{2}$
B. $\frac{\pi r^2}{4}$
C. $\frac{3\pi r^2}{2}$
D. $\frac{2}{\pi r^2}$
- 25.** A bicycle's wheels have a diameter of 77 cm. To cover a distance of 147 meters, how many revolutions will each wheel make?
A. 41 revolutions
B. 51 revolutions
C. 61 revolutions
D. 71 revolutions
- 26.** Each wheel of a motorcycle has a radius of 0.280 m. If each wheel rotates 100 times, how far will the motorcycle travel? Assume that the motorcycle is on a straight path.
A. 146 m
B. 156 m
C. 166 m
D. 176 m
- 27.** To form a circle, a piece of wire in the shape of a rectangle with a length of 80 cm and a width of 35 cm is cut and folded. Calculate the radius and circumference of the completed circle.
A. 36.62 cm
B. 38.62 cm
C. 40.62 cm
D. 42.62 cm
- 28.** Calculate the perimeter of a 0.14-meter-radius circular flower garden.
A. 0.088 m
B. 0.88 m
C. 1.88 m
D. 2.88 m
- 29.** What is the area of a sector of a circle with a radius of 46 cm that is cut off by a 12.5 cm long arc?
A. 277.5 cm^2
B. 287.5 cm^2
C. 297.5 cm^2
D. 267.5 cm^2
- 30.** When a circular disc with a radius of 10 cm is partitioned into sectors with angles of 120 and 150, the ratio of the two sectors' areas is?
A. 1:5
B. 2:5
C. 3:5
D. 4:5



SOLUTIONS

1. **(B)** Area = πr^2
 $= \pi (6)^2$
 $= 36\pi$
2. **(C)** Circumference = $2\pi r$
 $= 2 \times \frac{22}{7} \times 49$
 $= 2 \times 22 \times 7 = 308.$
3. **(D)** Circumference = πd
 $17\pi = \pi \times d$
Hence, diameter = 17.
4. **(A)** Here, $r = 28$ cm and $\theta = 90$
 $l = \frac{\theta}{360} \times 2\pi r = \frac{90}{360} \times 2 \times \frac{22}{7} \times 28 = 44$ cm
Perimeter = $l + 2r = 44 + 2 \times 28 = 100$ cm
Area = $\frac{1}{2} \times 44 \times 28 = 616$ cm²
5. **(A)** Area of the segment = area of the sector - area of the triangle
 $= 128$ sq. ft. - 67 sq. ft.
 $= 61$ sq. ft.
Therefore, the area of the segment is 61 sq. ft.
6. **(B)** Here $r = 21$ cm, $\theta = 72$ degrees.
By substitution,
The length of an arc = $2\pi r \left(\frac{\theta}{360}\right)$
Length of the arc = $2 \times \frac{22}{7} \times 21 \times \frac{72}{360}$
 $= 26.4$ cm.
7. **(C)** Area of the major segment = area of the circle - area of the minor Segment
 $= \pi r^2 - 62$
 $= \left(\frac{22}{7}\right) \times 7 \times 7 - 37$
 $= 117$ sq. units
Therefore, the area of the major segment
 $= 117$ sq. units.
8. **(D)** The radius of pizza is, $r = 6$ units.
The central angle is, $\theta = 60$ degrees.
The area of the segment is,
 $r^2 \left[\pi \frac{\theta}{360} - \sin \frac{\theta}{2}\right]$
 $= 6^2 \left[3.142 \times \frac{60}{360} - \sin \frac{60}{2}\right]$
 ≈ 3.27 square units.
Therefore, the area of the segment of the pizza = 3.27 square units.
9. **(C)** The length of an arc = $2\pi r \left(\frac{\theta}{360}\right)$
 $= 2 \times 3.14 \times 27 \times 150/360$
 $= 70.65$ cm.
10. **(B)** The length of an arc = $2\pi r \left(\frac{\theta}{360}\right)$
 $70 = 2 \times 3.14 \times 21 \times \left(\frac{\theta}{360}\right)$
 $70 = 131.88 \times \frac{\theta}{360}$
Multiply both sides by 360 to remove the fraction.
 $25200 = 131.88\theta$
Divide both sides by 131.88
 $\theta = 191.08$ degrees.
11. **(D)** The length of an arc = $2\pi r \left(\frac{\theta}{360}\right)$
 $127 = 2 \times 3.14 \times r \times \frac{120}{360}$
 $127 = 18.84 r$
Divide both sides by 18.84
 $r = 6.74$ cm.
So, the radius of the arc is 6.74 cm.
12. **(A)** Perimeter of the triangle = $3 \times$ side =
 $3 \times 14 = 42$ cm.
Perimeter of the triangle = circumference of the circle.
Thus, the perimeter of the triangle is also 42 cm.
Circumference of a circle = $2\pi r = 2 \times \frac{22}{7} \times r = 42$.
 $r = \frac{(42 \times 7)}{44} = 6.68$.
Therefore, the radius of the circle is 6.68 cm.
Area of a circle = $\pi r^2 = \frac{22}{7} \times (6.68)^2 = 44.63$ square cm.
Therefore, the area of a circle is 44.63 square cm.
13. **(D)** Arc length = $r \theta$
 $= 17 \times 33.67 = 572.39$ m.
14. **(C)** One rotation of the wheel = circumference of the wheel
49 rotations = 343 m
1 rotation = 7 m
So, circumference = 7 m
 $2\pi r = 7$ m
 $r = 7 \times \frac{7}{22} \times \frac{1}{2}$
 $r = 0.83$ m
15. **(B)** Circumference - Diameter = 28 cm
 $2\pi r - 2r = 28$ cm
 $2r(\pi - 1) = 28$ cm
 $2r\left(\frac{22}{7} - 1\right) = 28$ cm



$$2r \left(\frac{15}{7}\right) = 28 \text{ cm}$$

$$r = 28 \times \frac{7}{15} \times \frac{1}{2}$$

$$r = 6.53 \text{ cm}$$

- 16. (A)** At 5:30 p.m., the minute hand covers half of the circle. As a result, the minute hand's travel distance is actually half of the circumference.

Distance = πr (where r is the length of the minute hand).

As a result, the distance travelled is $\frac{22}{7} \times 14 = 22 \times 2 = 44$ units.

As a result, the total trip distance is 44 units.

- 17. (A)** Circumference = $2\pi R$

$$576 = 2 \times 3.14 \times R$$

$$576 = 6.28R$$

Divide both sides by 6.28 to get,

$$R = 91.71$$

Therefore, the radius of the circle is 91.71 yards. But, since the diameter is twice the radius of a circle, the diameter is equal to 183.43 yards.

- 18. (B)** $R = 49$ yd, $r = 47$ yd.

Let the area of outer circle be A_1 and the area of inner circle be A_2

$$\text{Area of race track} = A_1 - A_2 = \pi R^2 - \pi r^2 = \pi (49^2 - 47^2) = \frac{22}{7} \times 192 = 603.42 \text{ square yards.}$$

Therefore, the area of the race track is 603.42 square yards.

- 19. (C)** It is given that $A_1 : A_2 = 16 : 25$

$$\text{Area of a Circle} = \pi r^2$$

$$\pi R_1^2 : \pi R_2^2 = 16 : 25$$

Taking square roots of both sides,

$$R_1 : R_2 = 4 : 5$$

Therefore, the ratio of the radii = 4:5

- 20. (D)** Area of sector = $\frac{108}{360} \times \frac{22}{7} \times 28 \times 28 = 46.2 \text{ cm}^2$

- 21. (B)** Area of the sector = $\frac{111}{360} \times \frac{22}{7} \times 97 \times 97 = 8871.34$ square meter

Area covered by the man of the walking track in a day = $8871.34 + 8871.34 = 17742.68$ m.

- 22. (C)** Area of sector = $\frac{72}{360} \times \frac{22}{7} \times 7 \times 7 = 30.8$ square meter

- 23. (D)** Area of segment = Area of sector – area of triangle
= $57 - 23 = 34$ square meter.

- 24. (A)** It's a semicircle, since the central angle is 180 degrees.

As a result, the area of this sector is $\frac{1}{2}$ the area of the circle = $\frac{\pi r^2}{2}$

- 25. (C)** Circumference = $\frac{22}{7} \times 77 = 242$ cm
To get the number of revolutions of the wheel, divide the distance covered by the circumference of the wheel.

We need to convert 147 meters to cm before dividing, so we multiply 147 by 100 to get 14,700 cm. Therefore,

$$\text{Number of revolutions} = \frac{14,700 \text{ cm}}{242 \text{ cm}} = 61 \text{ revolutions (approx).}$$

- 26. (D)** Circumference = $2 \times \frac{22}{7} \times 0.280 = 1.76$ m.
To find the distance travelled, multiply the circumference of the wheel by the number of revolutions taken.

$$\text{Distance} = 1.76 \times 100 = 176 \text{ m}$$

Therefore, the distance travelled is equal to 176 meters.

- 27. (A)** The circumference of the circle formed = the perimeter of the rectangular wire.
Perimeter of a rectangle = $2(80 + 35) \text{ cm} = 230 \text{ cm}$.

Therefore, the circumference of the circle will be 230 cm.

$$\text{Circumference} = 2 \pi R$$

$$230 \text{ cm} = 2 \times \pi \times R$$

$$230 \text{ cm} = 2 \times 3.14 \times R$$

$$R = 36.62 \text{ cm}$$

So, the radius of the circle will be 36.62 cm.

- 28. (B)** Circumference = $2 \times \frac{22}{7} \times 0.14 = 0.88$ m.

- 29. (B)** Area = $\frac{1}{2} \times IR = \frac{1}{2} \times 12.5 \times 46 = 287.5 \text{ cm}^2$

- 30. (D)** Here, $\frac{120}{360} = \frac{1}{3}$

$$\text{and, } \frac{150}{360} = \frac{5}{12}$$

So, sector with angle 120 and 150 is part $\frac{1}{3}$ and $\frac{5}{12}$.

Hence, Ratio of area of two sectors = Ratio of central angle = $120:150 = 4:5$.

27 Trigonometry



SYNOPSIS

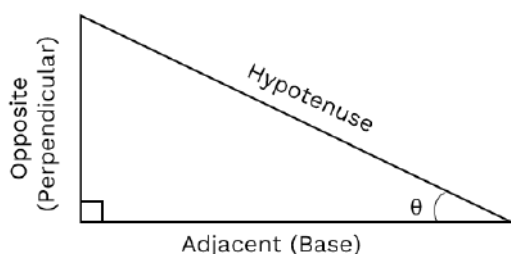
- Some Important Definitions
- Pythagoras Theorem
- Trigonometric Ratios
- Reciprocal Relationships
- Trigonometric Table
- Relation between Degree and Radian
- Table for Degree and Radian Relation
- Trigonometric Identities
- Pythagoras Identities
- Heights and Distances

INTRODUCTION

Trigonometry can be defined as the branch of mathematics which deals with the relation between the side lengths and angles of a right-angled triangle.

SOME IMPORTANT DEFINITIONS

- **Hypotenuse:** Hypotenuse is the largest side of the triangle and is opposite to the right angle of the triangle.
- **Adjacent:** The side of the triangle on which the right angle lies is called as the adjacent of the triangle. It is also referred as base of the triangle.
- **Opposite:** Side perpendicular to the base of the right-angled triangle. It is also referred as perpendicular for trigonometric calculations.



PHTYAGORAS THEOREM

Pythagoras theorem states that the square of the hypotenuse in a right-angled triangle is equal to the sum of squares of the other two sides.

$$\text{Hypotenuse}^2 = \text{Adjacent side}^2 + \text{Opposite side}^2$$

TRIGONOMETRIC RATIOS

$\sin\theta$	$\frac{\text{Opposite side}}{\text{Hypotenuse}}$
$\tan\theta$	$\frac{\text{Opposite side}}{\text{Adjacent side}}$
$\cos\theta$	$\frac{\text{Adjacent side}}{\text{Hypotenuse}}$
$\text{Cosec}\theta$	$\frac{\text{Hypotenuse}}{\text{Opposite side}}$
$\sec\theta$	$\frac{\text{Hypotenuse}}{\text{Adjacent side}}$
$\cot\theta$	$\frac{\text{Adjacent side}}{\text{Opposite side}}$

RECIPROCAL RELATIONSHIP

$$\text{Cosec}\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\text{Cosec}\theta \times \sin\theta = 1$$

$$\sec\theta \times \cos\theta = 1$$

$$\cot\theta \times \tan\theta = 1$$



TRIGONOMETRIC TABLE

Angles	0°	30°	45°	60°	90°	180°	270°	360°
Sinθ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
Cosθ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
Tanθ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	∞	0
Cosecθ	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	∞	-1	∞
Secθ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	-1	∞	1
Cotθ	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	∞	0	∞

RELATION BETWEEN DEGREE AND RADIAN

$$2\pi \text{ radian} = 360^\circ$$

TABLE FOR DEGREE AND RADIAN RELATION

DEGREE	30°	45°	60°	90°	180°	270°	360°
RADIAN	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π

TRIGONOMETRIC IDENTITIES

If an equation involves trigonometric ratios of an angle and is true for all the values of an angle, it is called trigonometric identity.

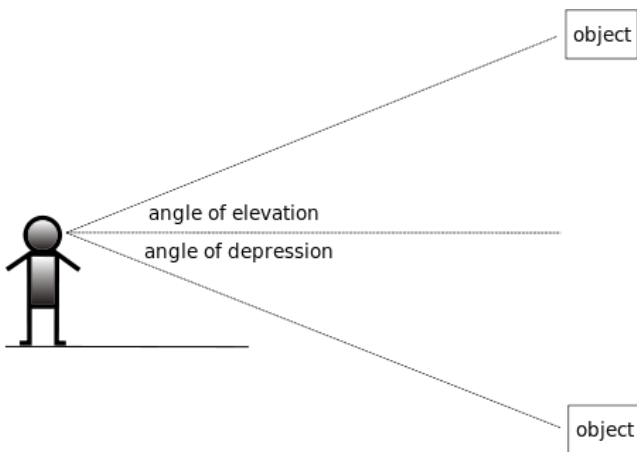
PYTHAGORAS IDENTITIES

$$\begin{aligned}\sin^2\theta + \cos^2\theta &= 1; \\ \tan^2\theta + 1 &= \sec^2\theta; \\ \cot^2\theta + 1 &= \operatorname{cosec}^2\theta;\end{aligned}$$

HEIGHT AND DISTANCE

Trigonometry is best used in real-life for finding height and distance.

- **Angle of Elevation:** The angle formed by the line of sight with the horizontal, when the point being observed is above the horizontal level is called as the angle of elevation.
- **Angle of Depression:** The angle formed by the line of sight with the horizontal when the point being observed is below the horizontal level is called as the angle of depression.



Points to Remember

- In a right angled triangle, if the angles are in the ratio $45^\circ: 45^\circ:90^\circ$, then the sides are in the ratio $1:1:2$.
- In a right angled triangle, if the angles are in the ratio $30^\circ:60^\circ:90^\circ$, then the sides are in the ratio $1:3:2$.

PRACTICE QUESTIONS

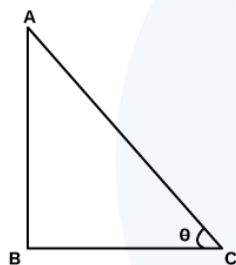
- The length of the shadow of a tree is equal to its height. What is the angle of the elevation of the light?
 - 30°
 - 60°
 - 90°
 - 45°
- During a cyclone, a tree in the coastal area broke in such a way that its top touched the ground making an angle of 60° . If the initial height of the tree is 15 m, at what height is the tree broken?
 - $15(1 + \sqrt{3})$ m
 - $\frac{15\sqrt{3}}{2 + \sqrt{3}}$ m
 - $\frac{30\sqrt{3}}{(2 + \sqrt{3})}$ m
 - 5 m
- The angle of elevation of a building from a point P is 45° . If P is 20 m away from the foot of the pole, then what is the height of the pole?
 - $20\sqrt{2}$ m
 - 10 m
 - 20 m
 - 15 m
- There are two points P and Q situated 18m and 32m away from the foot of a tower. The angles of elevations of the top of the tower from these two points are complementary. What is the height of the tower?
 - 20 m
 - 24 m
 - 36 m
 - 32 m
- The angle of elevation of the top of a tower is 30° from the top of a 5 m high platform. If the height of the tower is 45 m, what is the distance between the tower and the platform?
 - $13\sqrt{3}$ m
 - $40\sqrt{3}$ m
 - $15\sqrt{4}$ m
 - $24\sqrt{5}$ m
- There are 2 cars 200 m apart, the angles of depression of the cars from the top of a building are 45° and 30° towards east. What is the height of the building?
 - 100 m
 - 173 m
 - 200 m
 - 273 m
- The angle of elevation of an under construction building at a certain point 150 m from its base is 30° . If, as per the actual construction plan, the angle of elevation at the same point of the fully constructed building is to be 45° , then the building has to be raised by how many metres?
 - 59.4 m
 - 61.4 m
 - 62.4 m
 - 63.4 m



8. What is the height of the pillar when from the top of a large statue, which has the height of 90 metres, the angles of depression of the top and bottom of a pillar are 30° and 60° ?
- A. 30 m
B. 45 m
C. 60 m
D. 75 m
9. What is the height of a large plastic structure when the plastic structure got broken at a point and its top touches the ground at a distance 20 m from the base of the structure making an angle 30° with the ground?
- A. $\frac{40}{\sqrt{3}}$ m
B. $20\sqrt{3}$ m
C. $40\sqrt{3}$ m
D. 30 m
10. When the flood light's elevation is 30° , the shadow of a house is 15 m. How long must have been the shadow if the flood light's elevation would have been 60° ?
- A. 3 m
B. 4 m
C. 5 m
D. 6 m

SOLUTIONS

1. (D)



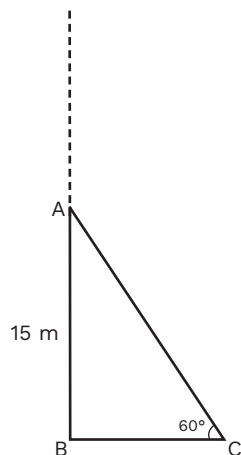
Let AB is the tree and BC be the shadow of the tree.

Given, $AB = BC$

$$\tan \theta = AB/BC = 1$$

$$\theta = 45^\circ$$

2. (B)



Let the tree be BAC after falling. Total length $BAC = 15$ m

$$BA + AC = 15$$

Let the height at which tree is broken is x .

$$BA = x; AC = 15 - x$$

$$\sin 60^\circ = AB/AC$$

$$\frac{\sqrt{3}}{2} = \frac{x}{15 - x}$$

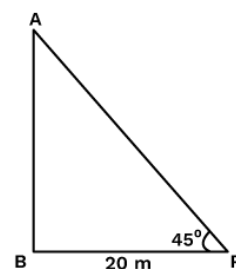
$$\sqrt{3}(15 - x) = 2x$$

$$15\sqrt{3} - \sqrt{3}x = 2x$$

$$15\sqrt{3} = 2x + \sqrt{3}x$$

$$x = \frac{15\sqrt{3}}{2 + \sqrt{3}}$$

3. (C)

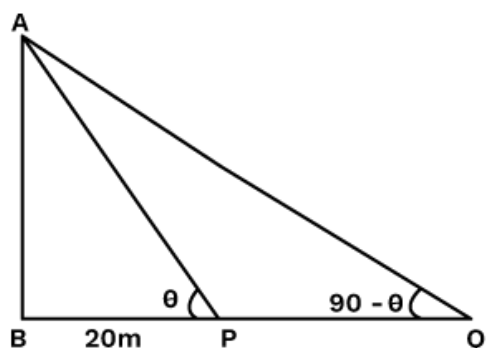


$$\tan 45^\circ = AB/BP$$

$$1 = \frac{AB}{20}$$

$$AB = 20 \text{ m}$$

4. (B)



Let angle at P = θ and height of the tower be h .

Then angle at P = $(90 - \theta)$

$$\tan \theta = \frac{AB}{BP} = \frac{h}{18}$$

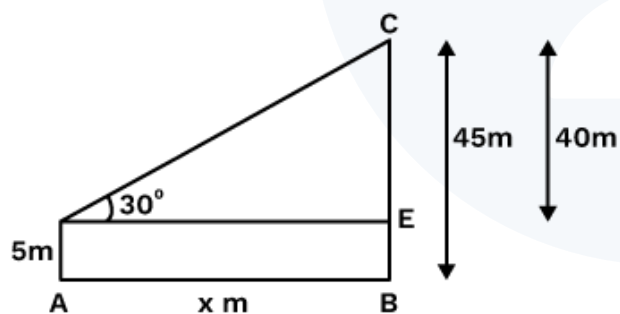
$$\tan(90 - \theta) = \frac{h}{32}; \cot \theta = \frac{h}{32}$$

$$\tan \theta \times \cot \theta = 1 = \frac{h}{18} \times \frac{h}{32}$$

$$18 \times 32 = h^2$$

$$h = 24 \text{ m}$$

5. (B)



In $\triangle DEC$,

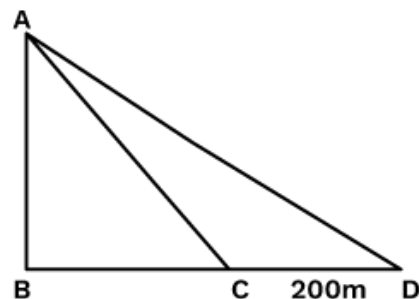
$$\tan 30^\circ = \frac{CE}{DE}$$

$$\tan 30^\circ = \frac{40}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{40}{x}$$

$$x = 40\sqrt{3} \text{ m}$$

6. (B)



Given,

$$\angle ACB = 45^\circ$$

$$\angle ADB = 30^\circ$$

Distance between two cars, i.e.,

$$CD = 200 \text{ m}$$

Then, $AB = ?$

Let $BC = x \text{ m}$

In $\triangle ABC$,

$$\tan 45^\circ = AB/BC$$

$$(\because \tan 45^\circ = 1)$$

$$1 = \frac{AB}{x}$$

$$AB = x \text{ m}$$

$$\text{In } \triangle ABD, \tan 30^\circ = AB/BD$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{x + 200}$$

$$(\because \tan 30^\circ = \frac{1}{\sqrt{3}})$$

$$x = \sqrt{3} \text{ m}, AB = 200$$

From equations (i) and (ii),

$$AB = \sqrt{3} AB - 200$$

$$\sqrt{3} AB - AB = 200$$

$$0.732 AB = 200$$

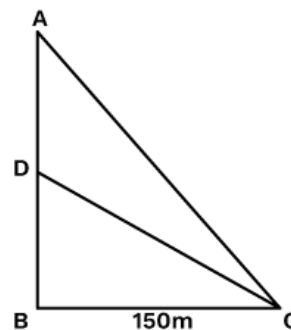
$$(\because \sqrt{3} = 1.732)$$

$$AB = \frac{200}{0.732} = 273.22$$

$$= 273 \text{ m}$$

Hence, option (D) is correct.

7. (D)





Given, $BC = 150$ m

$$\angle ACB = 30^\circ$$

and, $\angle DCB = 45^\circ$

Then, $AD = ?$

In $\triangle ABC$, $\tan 30^\circ = AB/BC$

$$\frac{1}{\sqrt{3}} = \frac{AB}{150}$$

$$AB = \frac{150}{\sqrt{3}} = 86.6 \text{ m}$$

In $\triangle DBC$, $\tan 45^\circ = DB/BC$

$$1 = \frac{DB}{150}$$

$$DB = 150$$

$$AD + AB = 150$$

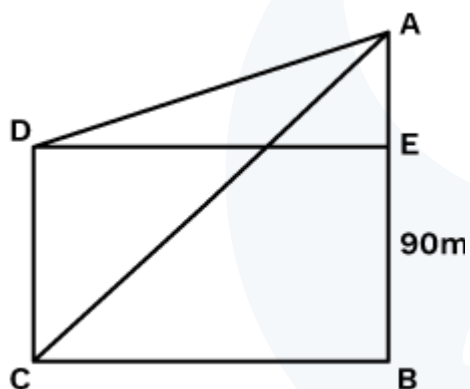
$$[\because DB = AD + AB]$$

$$AD = 150 - AB$$

$$= 150 - 86.6 = 63.4 \text{ m}$$

Hence, option (D) is correct.

8. (C)



Given, $AB = 90$ m

$$\angle ADE = 30^\circ$$

And $\angle ACB = 60^\circ$

Then, $DC = ?$

Ratio of angles,

$$\tan 30^\circ / \tan 60^\circ = (AE/ED) / (AB/BC)$$

$$[\because ED = BC]$$

$$\frac{1}{\sqrt{3}} = \frac{AE}{90}$$

$$\frac{1}{3} = \frac{AE}{90}$$

$$AE = 30 \text{ m}$$

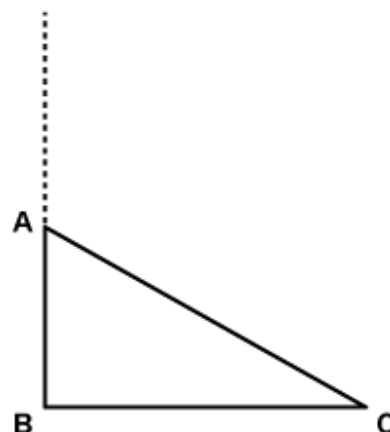
Now, $DC = EB$

$$= AB - AE$$

$$= 90 - 30 = 60 \text{ m}$$

Hence, option (C) is correct.

9. (B)



Given, $BC = 20$ m

$$\angle ACB = 30^\circ$$

Total height of the large plastic structure is $(AB + CA) = ?$

In $\triangle ABC$, $\tan 30^\circ = AB/BC$

$$\frac{1}{\sqrt{3}} = \frac{AB}{20}$$

$$AB = \frac{20}{\sqrt{3}} \text{ m}$$

Now, $\cos 30^\circ = BC/AC$

$$\frac{\sqrt{3}}{2} = \frac{20}{AC}$$

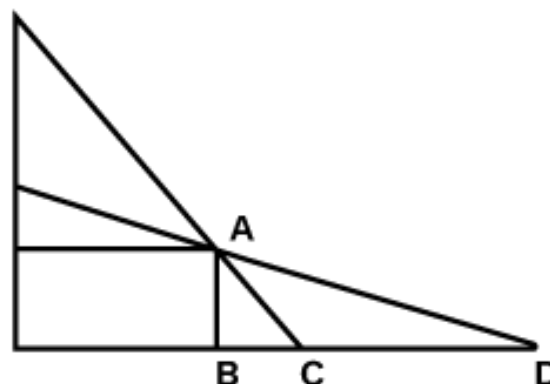
$$AC = \frac{40}{\sqrt{3}} \text{ m}$$

$$\text{So, } AB + CA = \frac{20}{\sqrt{3}} + \frac{40}{\sqrt{3}} = \frac{60}{\sqrt{3}}$$

$$= 20\sqrt{3} \text{ m}$$

Hence, option (B) is correct.

10. (C)





Given, $\angle ADB = 30^\circ$ and $\angle ACB = 60^\circ$

When the flood light's elevation is 30° , the shadow of house is 'BD = 15 m' and when the flood light's elevation is 60° , the shadow of house is 'BC = ?'

Let, BC = x m

In $\triangle ABD$, $\tan 30^\circ = \frac{AB}{BD}$

$$\frac{1}{\sqrt{3}} = \frac{AB}{15}$$

$$AB = \frac{15}{\sqrt{3}}$$

....(i)

In $\triangle ABC$, $\tan 60^\circ = AB/BC$

$$\sqrt{3} = AB/x$$

$$AB = x\sqrt{3}$$

...(ii)

From equations (i) and (ii), we get

$$x\sqrt{3} = \frac{15}{\sqrt{3}}$$

$$x = 5 \text{ m}$$

Hence, option (C) is correct.



28 Data Interpretation



SYNOPSIS

- Table
- Pie chart
- Line graph
- Bar chart
- Mixed graph
- Caselet
- Most common types of questions and important formulae
- Some important tricks useful in data interpretation
- Summary

INTRODUCTION

Data interpretation is analysing the provided data and calculating the required value asked based upon that data. Data in competitive questions can be in various forms like tables, pie charts, bar graphs, line graphs, etc.

TABLES

In tabular data interpretations (DIs), data are divided systematically into horizontal rows and vertical columns followed by certain questions based on that data that are to be answered by students. For example, the table given below represents the loss of three companies in the period of 3 years from 2018 to 2020.

YEAR COMPANY	2018 (LOSS %)	2019 (LOSS %)	2020 (LOSS %)
A	18	11	7
B	13	17	11
C	7	14	15

Comprehending the Data Table

If a question asks to calculate the increase/decrease in loss of B's revenue from 2019 to 2020 when total revenue for the years is 2 crores:

1. Identify loss in 2019, i.e., the intersection of B row and 2019 column, 17%

2. Similarly, loss in 2020, 11%

3. Finally, calculate the change in loss = 17% of 2 crores – 11% of 2 crores

Note: Pay attention to the units.

**Illustration:**

Directions (Questions 1–4): Study the given table carefully and answer the questions that follow:

STATES	TOTAL NUMBER OF PEOPLE WHO VOTED	PERCENTAGE OF PEOPLE WHO VOTED FOR DIFFERENT PARTIES FROM RESPECTIVE STATES			
		A	B	C	D
P	80	25	20	15	40
Q	100	24	33	21	22
R	200	32	20	17	31
S	250	30	10	20	50

1. In state Q, the total number of people who voted for parties A and C together is what percent less than the total number of people who voted for B and D?

- A. $16\frac{2}{11}$ B. $15\frac{2}{11}$
C. $18\frac{2}{11}$ D. $17\frac{3}{11}$

Solution: Total number of people who voted for parties A and C together in Q

$$= 100 \times \frac{24}{100} + 100 \times \frac{21}{100}$$
$$= 24 + 21 = 45$$

Total number of people who voted for parties B and D together in Q

$$= 100 \times \frac{33}{100} + 100 \times \frac{22}{100}$$
$$= 33 + 22 = 55$$

$$\text{Required percentage} = \frac{(55 - 45)}{55} \times 100 = 18\frac{2}{11}$$

2. What is the average number of people who voted in states P, Q, and S for party C?

- A. 23 B. 27
C. 29 D. 32

Solution: Number of people who voted in state

$$\text{P for party C} = 80 \times \frac{15}{100} = 12$$

Number of people who voted in state R for

$$\text{party C} = 200 \times \frac{17}{100} = 34$$

Number of people who voted in state S for

$$\text{party C} = 250 \times \frac{20}{100} = 50$$

$$\text{Required average} = \frac{(12 + 34 + 50)}{3} = 32$$

3. What is the difference between the total number of people who voted in state P for parties A and D together and the total number of people who voted in state R for the same party together?

- A. 63 B. 74
C. 85 D. 96

Solution: Total number of people who have voted in P for parties A and D together

$$= 80 \times \frac{25}{100} + 80 \times \frac{40}{100}$$
$$= 20 + 32 = 52$$

Total number of people who have voted in R for parties A and D together

$$= 200 \times \frac{32}{100} + 200 \times \frac{31}{100}$$
$$= 64 + 62 = 126$$

$$\text{Required difference} = 126 - 52 = 74$$

4. It is given that in S, 42% are females. And if 20% of these total females voted for party B, what is the number of male people who voted for party B from state S?

- A. 145 B. 132
C. 129 D. 117

Solution: In S, females = 42%

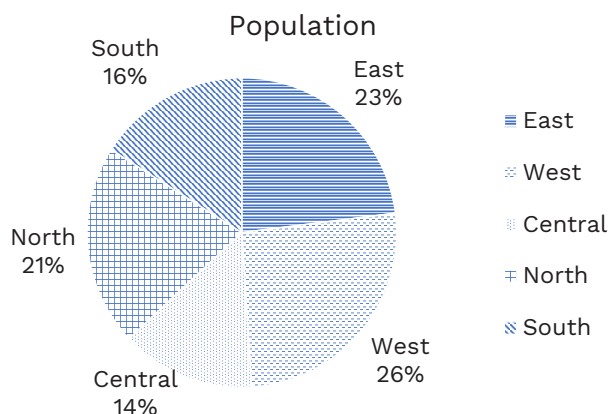
Males = 58%

Total number of male people in S who voted for

$$\text{party B} = \frac{58}{100} \times 250 = 145$$

PIE CHART

In pie charts, data are represented as sectors of circular charts, which are proportional to the quantities they represent. And the total quantity over the whole circle, i.e., 360° or 100%. For example, the following pie chart represents the distribution of population in various districts of Town X.



COMPREHENDING PIE CHARTS

1. If data are represented in degrees:

Value of any sector =

$$\frac{\text{Angle of any sector}}{360^\circ} \times \text{Total value}$$

2. If data are represented in percentage:

Value of any sector

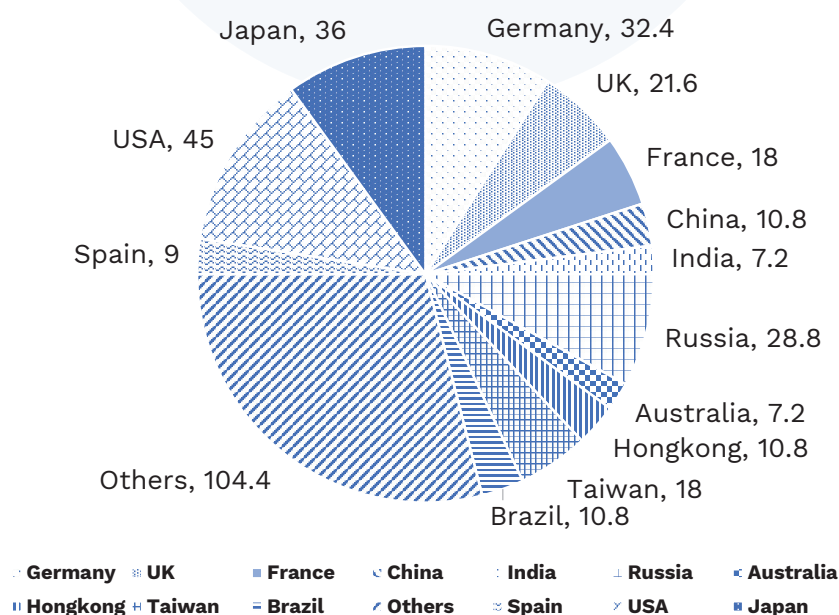
$$= \frac{\text{Per cent of the sector}}{100} \times \text{Total value}$$

Thus, from the above chart the population of any zone can be calculated using the later formula.

Illustration:

Directions (Questions 1–5): Study the given table carefully and answer the questions that follow:

Country-wise Exports (in degree) Total = 72000 billion





1. By how much does the value of the imports of the USA exceed that of Germany?
- A. 2326 billion B. 2789 billion
C. 2900 billion D. 2520 billion

Solution: Difference in the angles subtended by the USA and Germany = $45^\circ - 32.4^\circ = 12.6^\circ$
The difference in the imports of the USA and Germany = $\frac{12.6^\circ}{360^\circ} \times 72000 \text{ billion} = 2520 \text{ billion}$

2. The difference in the values of the imports of Japan and France is how many times that of the UK and Taiwan?
- A. 2 times B. 3 times
C. 4 times D. 5 times

Solution: The difference in the angles of the import of Japan and France
= $36^\circ - 18^\circ = 18^\circ$

The difference in the angles subtended by the UK and Taiwan
= $21.6^\circ - 18^\circ = 3.6^\circ$

Hence, the difference in the value of imports of Japan and France is 5 times that of the UK and Taiwan.

3. The value of the imports of the Organization of the Petroleum Exporting Countries (OPECs) organisation is how much more than the value of the imports of India and Australia put together, given that OPEC has a 20% share in the value of the imports of others?
- A. 2673 billion B. 2569 billion
C. 1296 billion D. 1325 billion

Solution: Value of imports of India and Australia
= $\frac{(7.2^\circ + 7.2^\circ)}{360^\circ} \times 72000 = 2880 \text{ billion}$

Value of imports of OPEC countries

$$= \frac{20}{100} \times \left(\frac{104.4^\circ}{360^\circ} \times 72000 \right) = 4176 \text{ billion}$$

Required difference = $4176 - 2880 = 1296 \text{ billion}$

4. If imports of developing countries accounted for 36% of the total worldwide imports, then what is the value of the imports of Japan as a percentage of the imports of the developing countries?
- A. 23.3% B. 27.7%
C. 29.9% D. 33.3%

Solution: Imports of developing countries = 36% of total

$$\text{Imports of Japan} = \frac{36^\circ}{360^\circ} \times 100 = 10\% \text{ of total}$$

$$\text{Required per cent} = \frac{10}{36} \times 100 = 27.7\%$$

5. What is the number of countries whose imports are more than the average imports per country?
- A. 6138.5 billion B. 5276.4 billion
C. 4829.3 billion D. 5142.8 billion

Solution: Average imports

$$= \frac{\text{Total imports}}{\text{Number of countries}} = \frac{72000}{14} = 5142.8 \text{ billion}$$

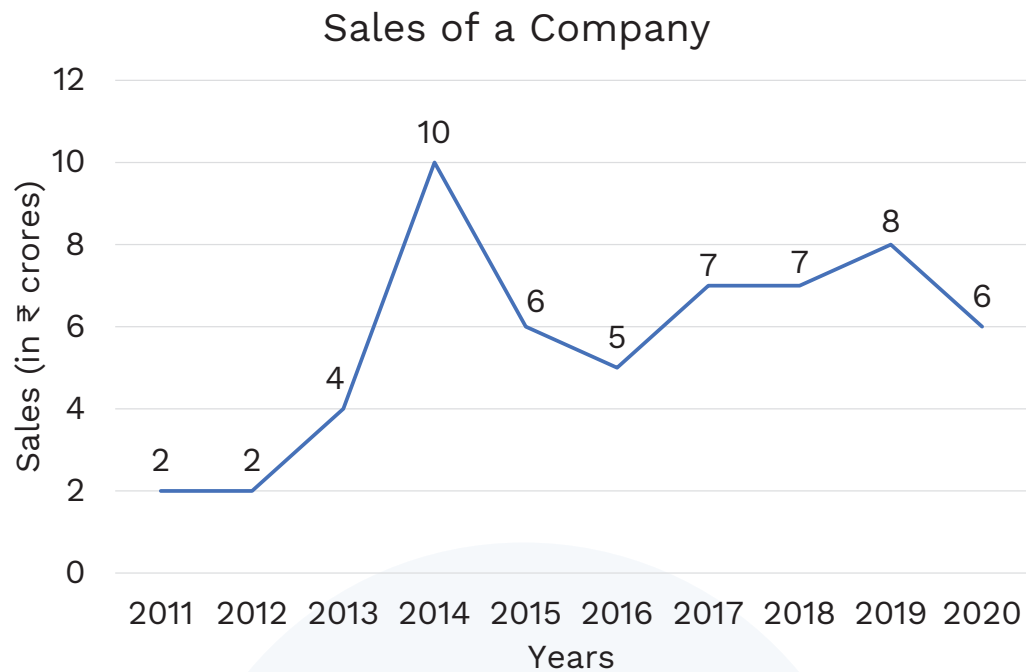
After calculating imports of each country individually, only USA, Japan, Germany, Russia, and Others (i.e., 5) have imports greater than average.

LINE GRAPHS

In a line graph, data are distributed on X and Y-axes, which represent a variation of quantity with respect to these perimeters over axes.

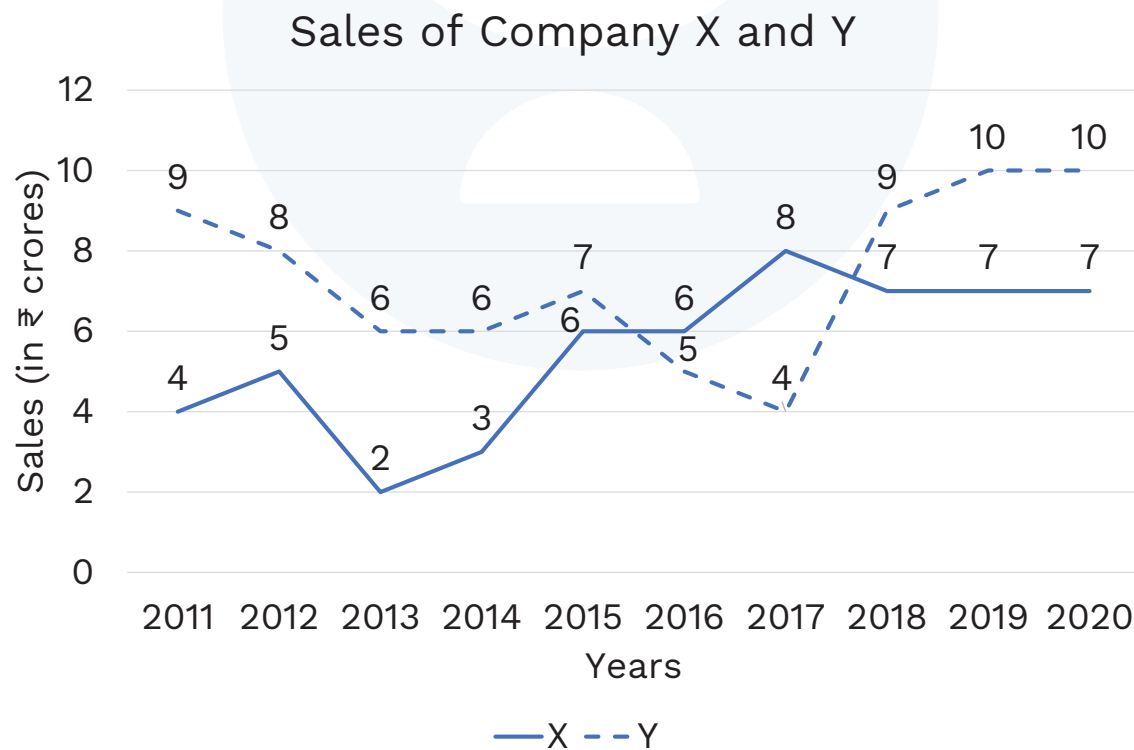
Types of line graph

1. **Single-line graph:** Used for a single variable. For example, the following graph represents the sales of a company in the years 2011–2020.



2. Two-variable graph:

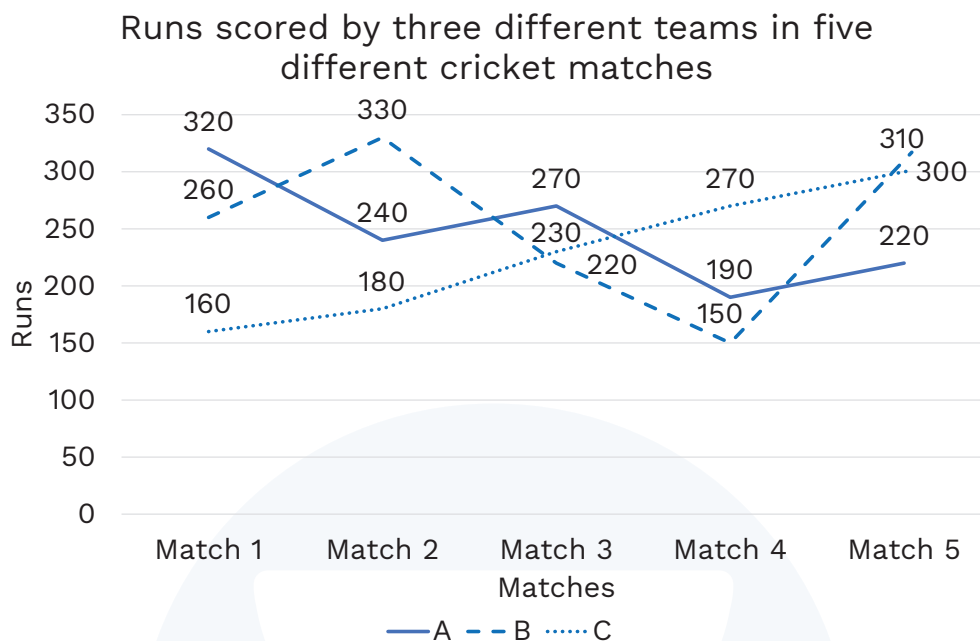
For example, the following graph represents the sales of two companies X and Y in the years 2011–2020.





3. Three-variable graph:

For example, the following graph represents the runs scored by three different countries in five different cricket matches.



COMPREHENDING LINE GRAPH

If a question asks, the sales of company X in 2015 is by how much per cent more than the sales of company Y in 2016:

1. Identify sales of company X in the year 2015 from the blue line
2. Similarly, sales of company Y in the year 2016 from the orange line

3. Finally, calculate increase = 6 crores – 5 crores

Illustration:

Directions (Questions 1–5): Study the given line graph carefully and answer the questions that follow:





1. What is the difference between the total number of articles sold by both the shops together on the 2nd day and that by both the shops together on the 4th day?

A. 230 B. 270
C. 290 D. 320

Solution: Total number of articles sold on 2nd day = $120 + 90 = 210$

And the total number of articles sold on the 4th day = $240 + 200 = 440$

Required difference = $440 - 210 = 230$

2. The number of articles sold by shop Y on the 1st day is what per cent of the number of articles sold by the same shop on the 5th day?

A. $25\frac{1}{4}$ B. $22\frac{3}{4}$
C. $22\frac{1}{4}$ D. $25\frac{3}{4}$

Solution: Required percentage = $\frac{60}{270} \times 100 = \frac{600}{27}$
 $= 22.25$ or $22\frac{1}{4}$

3. What is the average number of articles sold by shop Y on the 1st and 4th day?

A. 130 B. 170
C. 190 D. 220

Solution: Average number of articles sold by shop Y on 1st and 4th day = $\frac{60 + 200}{2} = \frac{260}{2} = 130$

4. What is the respective ratio between the total number of articles sold by shop X on the 4th and 5th day together and by the same shop on the 2nd and 3rd day?

A. 3:2 B. 5:1
C. 7:3 D. 9:5

Solution: Total articles sold on 4th and 5th day = $240 + 300 = 540$

And total articles sold on 2nd and 3rd day = $120 + 180 = 300$

Required ratio = $540 : 300 = 9 : 5$

5. The number of articles sold by shop X on the 3rd day is what per cent more than that sold by the same Y on the same day?

A. 30% B. 20%
C. 40% D. 10%

Solution: Required percentage

$$= \frac{180 - 150}{150} \times 100 = 20\%$$

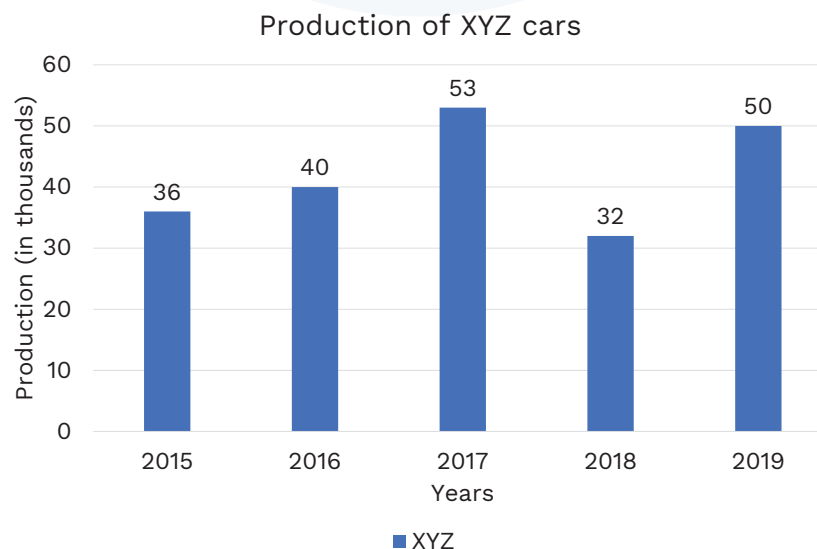
BAR GRAPH

A bar chart represents data in the form of rectangles whose lengths are proportional to the data they represent. And one axis represents variables while the other represents a parameter that is changing in the graph

Types of bar charts

1. Simple bar graph

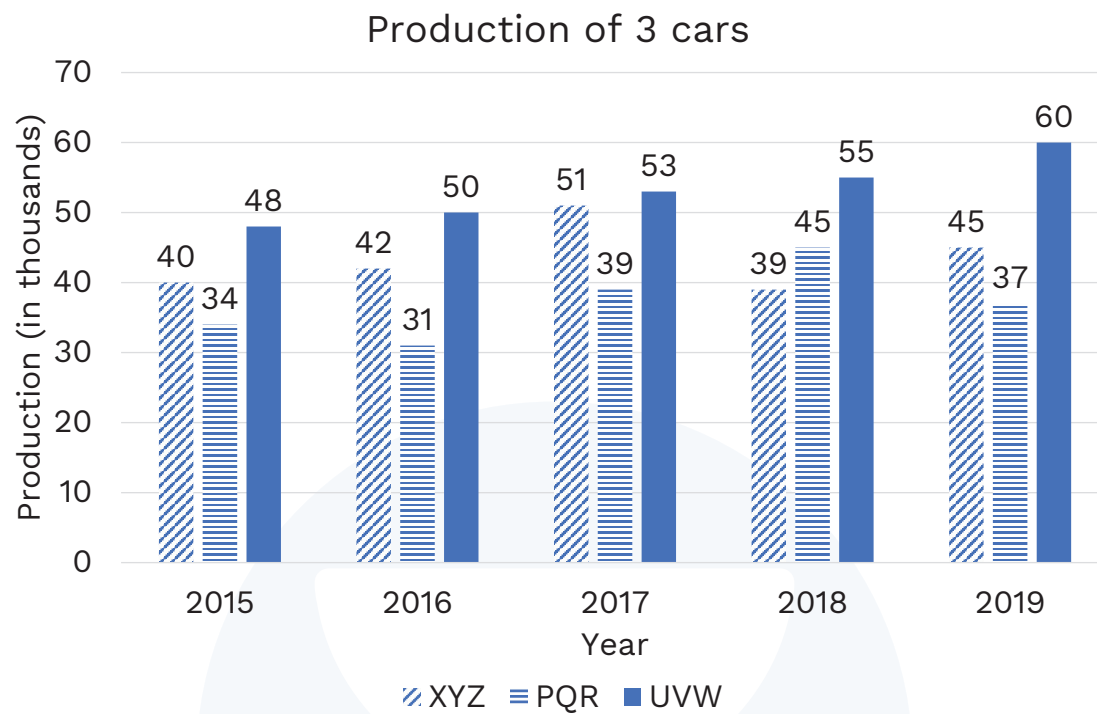
For example, the following bar graph represents the production of XYZ cars in five different years.





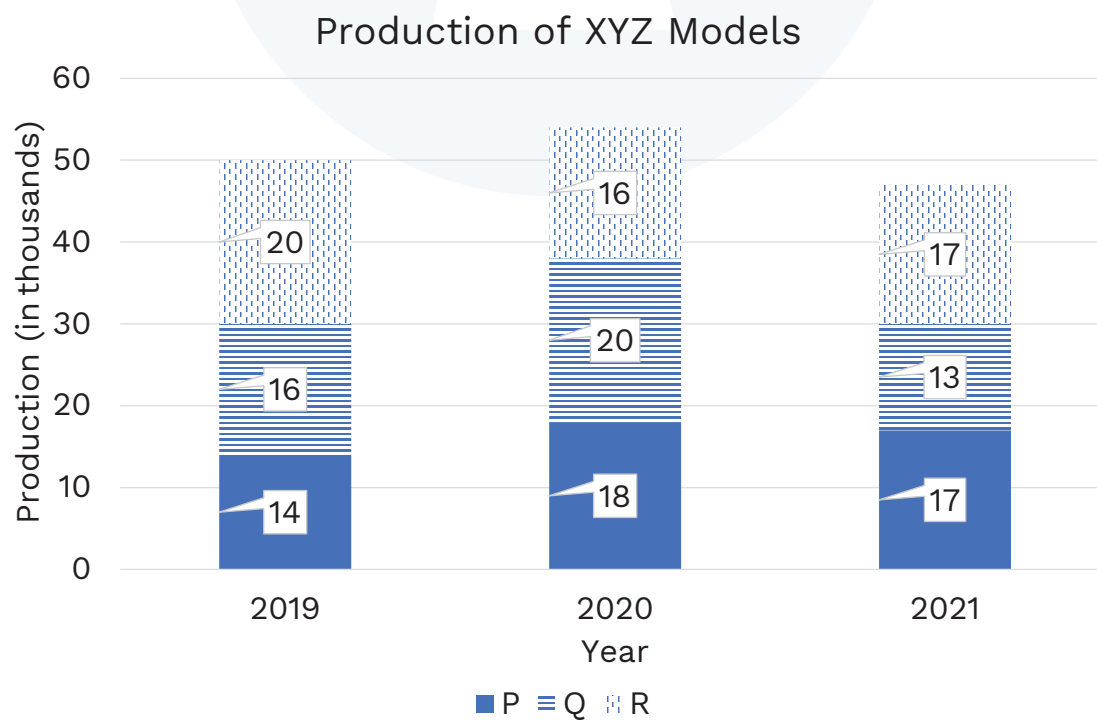
2. Grouped bar graph

For example, the following bar graph represents the production of XYZ, PQR, and UVW cars in five different years.



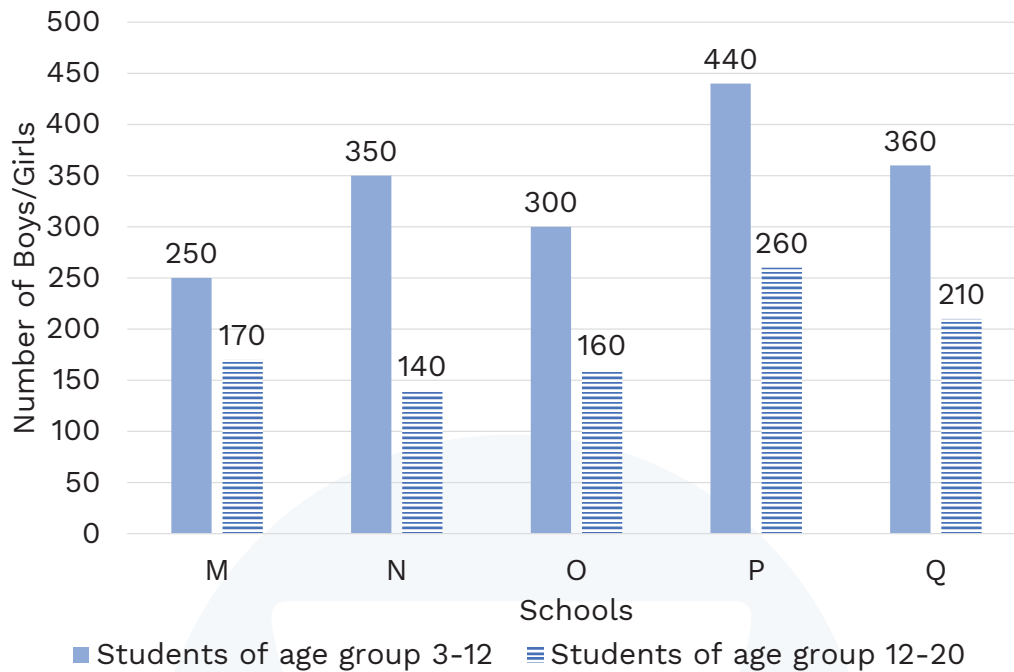
3. Stacked bar graph

For example, the following bar graph represents the production of various models of XYZ cars over three years 2019, 2020 and 2021.



**Illustration:**

Directions (Questions 1–5): Study the given table carefully and answer the questions that follow:



1. What is the difference between the total number of students of age group 3-12 studying in N and O together and the total number of students of age group 12-20 in the same schools together?

A. 210 B. 350
C. 490 D. 570

Solution: Total students of age group 3-12
= 350 + 300 = 650

Total students of age group 12-20
= 140 + 160 = 300

Required difference = 650 – 300 = 350

2. The number of students of age group 3-12 and students of age group 12-20 together in school N is what per cent less than that in school P?

A. 30% B. 70%
C. 50% D. 20%

Solution: Total students in N = 140 + 150 = 290

Total students in P = 260 + 440 = 700

Required per cent = $\frac{700 - 290}{700} \times 100 = 58.57\%$

3. The number of students of age group 3-12 and students of age group 12-20 studying in class X of Q school is 20% less than those in class XII of the same school. How many students study in class X then?

A. 456 B. 472
C. 489 D. 423

Solution: Total students of Q school in class XII
= 210 + 380 = 590

Number of students in class X
= 590 – 590 × $\frac{20}{100}$ = 472

4. The number of students of age group 3-12 studying in Q school is what per cent more than the number of students of age group 3-12 studying in M?

A. 48% B. 52%
C. 54% D. 56%

Solution: Students of age group 3-12 in Q = 380

Students of age group 3-12 in M = 250

Required percentage = $\frac{380 - 250}{250} \times 100 = 52\%$

5. What is the respective ratio between the number of students of age group 3-12 and students of age group 12-20 together studying in M and that in P?

A. 21 : 23 B. 23 : 27
C. 27 : 29 D. 29 : 31

Solution: Required ratio = $(170 + 250) : (160 + 300)$
= $420 : 460 = 21 : 23$

MIXED GRAPH

Mixed graphs present data in a combination of two or more forms of data presentation. These

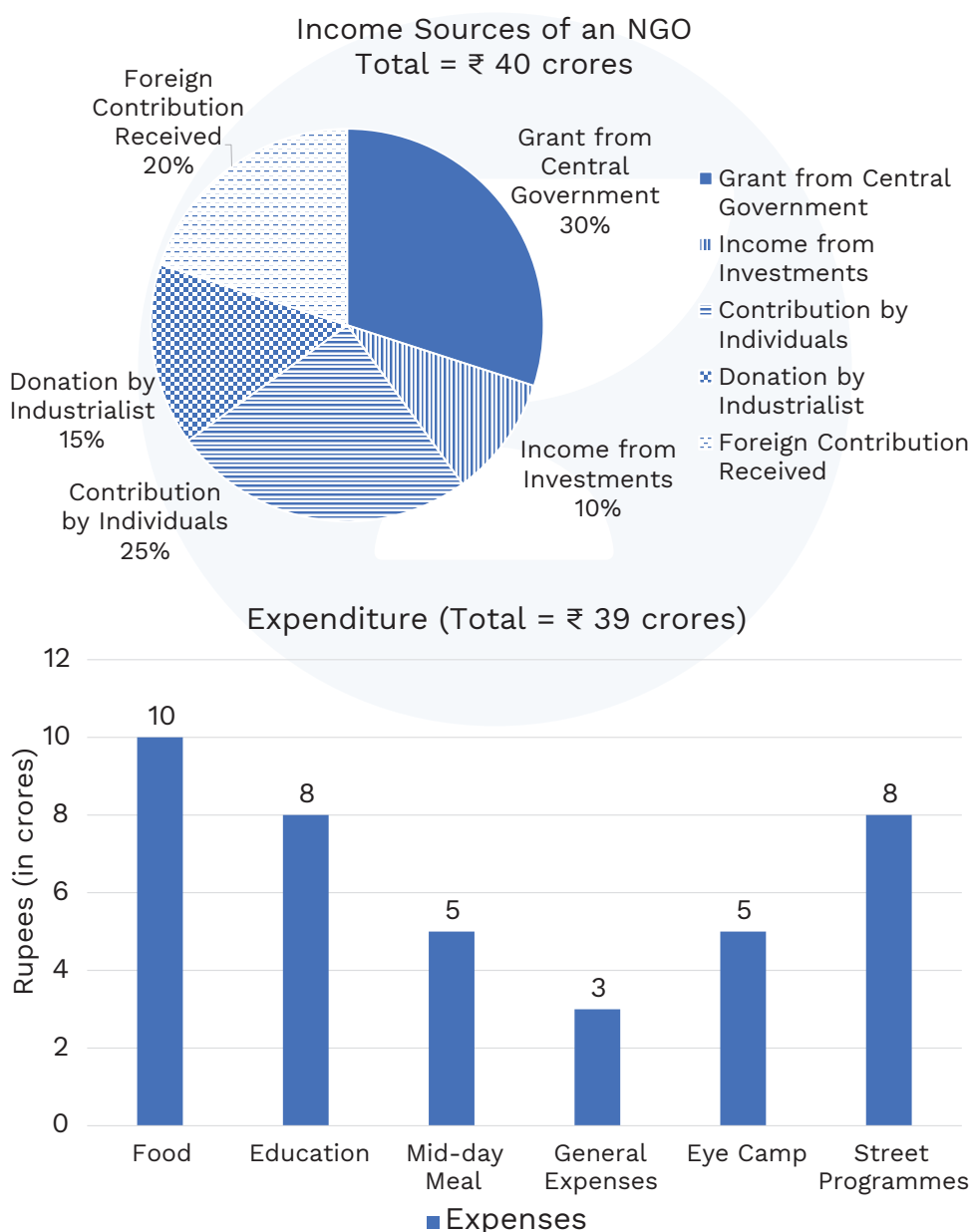
types of Data interpretation questions require the questions to be solved using both the interdependent data.

Types of mixed graphs

- Any two graphs/charts together

Illustration:

Directions (Questions 1–5): The following pie chart represents the different income sources of an NGO and the bar graph represents the expenditure of this NGO over different heads. Answer the questions that follow:





1. What percentage of money is saved?

- A. 2.3% B. 2.5%
C. 2.7% D. 2.9%

Solution: Saving = Total income – total expenditure

$$= 40 - 39 \text{ crore} = 1 \text{ crores}$$

$$\text{Per cent of money saved} = \frac{1}{40} \times 100 = 2.5\%$$

2. If the industries stop donating and the expenditure pattern remains the same, then what will be the decrease in money spent on the mid-day meal programme?

- A. 0.69 crores B. 0.73 crores
C. 0.76 crores D. 0.63 crores

Solution: Decreased income = 40 crores –

$$\frac{15}{100} \times 40 \text{ crores}$$

$$= 34 \text{ crores}$$

Percentage of expenditure on mid-day meal

$$= \frac{5}{39} \times 100 = 12.8\%$$

New expenditure after a gradual decrease in

$$\text{expenditure with respect to income} = \frac{39}{40} \times 34$$

$$= 33.15 \text{ crores}$$

New expenditure on mid-day meal

$$= \frac{12.8}{100} \times 33.15 = ₹ 4.24 \text{ crores}$$

Required decrease in expenditure on mid-day meal = 5 – 4.24 crores = 0.76 crores

3. What is the ratio of expenditure on food and mid-day meal programmes together to that of a grant from the central government?

- A. 2 : 1 B. 3 : 2
C. 4 : 3 D. 5 : 4

Solution: Total expense on food and mid-day meal = (10 + 5) = 15 crores

Grant from central government

$$= \frac{30}{100} \times 40 = 12 \text{ crores}$$

$$\text{Required ratio} = 15 : 12 = 5 : 4$$

4. The general expenses are how many times income from the investment?

- A. 0.25 times B. 0.50 times
C. 0.75 times D. 1 time

Solution: Income from investment =

$$\frac{10}{100} \times 40 = 4 \text{ crores}$$

4 crores is 0.75 times of 3 crores (i.e., general expense)

5. If in the next year, a grant from the central government increases by 10%, foreign contribution decreases by 10% and other income amounts and expense patterns remain the same. What is the per cent increase in expenditure on Food?

- A. 1% B. 2%
C. 3% D. 4%

Solution: Initially, amount of grant from central

$$\text{government} = \frac{30}{100} \times 40 \text{ crores} = 12 \text{ crores}$$

$$\text{After 10\% increase} = \frac{110}{100} \times 12 = 13.2 \text{ crores}$$

Similarly new foreign contribution

$$= \frac{8}{100} \times 90 = 7.2 \text{ crores}$$

New income

$$= 40 + (13.2 - 12) - (8 - 7.2) = 40.4 \text{ crores}$$

Gradual increase in expenditure

$$= \frac{39}{40} \times 40.4 = 39.9 \text{ crores}$$

Gradual increase in Food expense

$$= \frac{10}{39} \times 39.9 = 10.1 \text{ crores}$$

$$\text{Required per cent increase} = \frac{10.1 - 10}{10} \times 100 = 1\%$$

CASELET

Caselets DIs present information in the form of long paragraphs, instead of tables or graphs usually. And are followed by questions that are to be solved using that data. For example:

In data for 2020-2021 total minerals extracted in India were 10 lakh tonnes. The data recorded extraction of three major minerals: iron, aluminium, and sulphur. Of which Iron accounted for 70% of the total, while Aluminum and Sulphur were in the ratio of 2:1. In the 2018-2019 annual year these same were extracted 10% less than the total extraction



in 2019-2020 but were 10% more than the total for 2017-2018. While the total extraction in 2019-2020 was also 10% more than that in 2020-2021. And the data hypothesis is that the extraction of iron increases at a rate of 5% per annum.

Comprehending caselet

1. Construct the data into a simple table, if it can be converted for reference:

YEAR	EXTRACTION (IN LAKH TONNES)
2020-2021	10
2019-2020	11
2018-2019	9.9
2017-2018	9.91

2. Use data accordingly for each question.

Illustration:

Directions (Questions 1-5): Study the given table carefully and answer the questions that follow:

There are 1400 students in ABC University in the academic year 2021. The ratio of the boys to the girls in the University is 4 : 3. All the students are interested in different extracurricular activities: mooting, debating, dancing, research and arbitration and one student is interested in only one extra-curricular activity. The number of boys interested in research is 153. The ratio of the number of boys who are interested in mooting to the number of boys who are interested in dancing is 72 : 89. 32% of the students are interested in arbitration. The number of boys interested in debating is 12.5% to the total number of boys. The number of girls interested in arbitration is 142 which is 37 less than the number of girls enrolled in dancing. The number of girls interested in mooting is 46 more than the number of boys

interested in the same. The total number of students interested in research is 204.

1. The difference in the number of boys and girls who are interested in research is what percentage of the total number of students who are interested in debating?
A. 51% B. 53%
C. 57% D. 59%
2. Find the percentage of students of the university who are interested in mooting.
A. 16.25% B. 17.5%
C. 18.75% D. 19%
3. Find the ratio of the number of boys interested in dancing to the number of girls interested in the same.
A. 67 : 70 B. 83 : 80
C. 67 : 90 D. 83 : 67
4. The total number of girls in arbitration is what percentage of the total number of students in Arbitration?
A. 23% B. 27%
C. 29% D. 32%
5. Find the number of girls who are enrolled in Debating.
A. 68 B. 72
C. 76 D. 80

Solution 1: Total number of students = 1400

$$\text{Total number of boys} = \frac{4}{7} \times 1400 = 800$$

$$\text{Total number of girls} = 1400 - 800 = 600$$

$$\text{Number of boys interested in research} = 153$$

$$\text{Number of students interested in arbitration} = \frac{32}{100} \text{ of } 1400 = 448$$

$$\text{Number of girls interested in arbitration} = 144$$

$$\text{Number of boys interested in arbitration} = 448 - 144 = 304$$

$$\text{Number of boys interested in debating} = \frac{12.5}{100} \text{ of } 1400 = 100$$

$$\text{Number of girls interested in dancing} = 720$$

$$\text{Number of students interested in research} = 204$$

$$\text{Number of girls interested in research} = 204 - 153 = 51$$



Number of boys interested in mooting and dancing together = $800 - (100 + 153 + 304) = 242$
 Number of boys interested in Mooting = $= \frac{72}{(72 + 89)} \times 242 = 108$

Number of girls interested in mooting = 154
 Number of boys interested in dancing = $242 - 108 = 134$
 Number of girls interested in debating = $600 - (154 + 180 + 51 + 144) = 72$

ACTIVITY	BOYS	GIRLS	TOTAL
Mooting	108	154	262
Dancing	134	180	314
Debating	100	72	172
Research	153	51	204
Arbitration	304	144	448

Difference in boys and girls interested in Research = $(153 - 51) = 102$
 Required percentage = $\frac{102}{172} \times 100 = 59\%$

Solution 2: Required percentage

$$= \frac{262}{1400} \times 100 = 18.75\%$$

Solution 3: Required ratio = 134 : 180
 = 67 : 90

Solution 4: Required percentage

$$= \frac{144}{448} \times 100 = 32\%$$

Solution 5: B

COMMON TYPE OF QUESTIONS AND IMPORTANT FORMULAE

1. Growth rate

Finding the growth rate of quantities is one of the most asked questions in DI. It is the percentage increase/decrease in a quantity over a period of time.

Illustration:

Following is the table of sales of total sales of almonds in different years.

YEAR	2010	2011	2012	2013
Sale (In Tonne)	90	95	94	101

- By what per cent did sales grow over the years 2010–2013?

Solution: If quantity changes from A to B,

$$\text{Growth Rate} = \frac{B - A}{A} \times 100$$

Thus, here,

$$\text{Growth Rate} = \frac{101 - 90}{90} \times 10 = \frac{11}{9} \times 10 = 12.2\%$$

Note

- If B is greater than A, then growth is positive.
- If B is smaller than A, the growth rate is negative.

- What was the average annual growth rate of sales over the years 2010–2013?

Solution: average annual/monthly growth rate is an average growth rate over the years/months.

Average growth rate

$$= \frac{\text{Total growth rate}}{\text{No. of years or months elapsed}}$$

Hence, here,

average annual growth rate of sales

$$= \frac{12.2\%}{3} = 4.07\% \text{ per year}$$

Note

Though the number of years given is 4 (2010, 2011, 2012, 2013) the growth happens over a year, i.e., as in 2002–2003. Thus, the number of years elapsed in the time period will be 3 (2010–2011, 2011–2012, 2012–2013)



2. Quantity X is how much of Quantity Y

This type of questions include calculating, how much is quantity 1 in respect of quantity 2 or quantity 1 in respect of total quantity or ratio of two quantities, etc.

Illustration:

Following is the table of the number of students appearing for an entrance exam in different cities.

CITY	A	B	C	D	E	F
Number of students (in thousand)	9.4	13	11	14.7	15.6	12

1. The number of students appearing from city E is what per cent of the number of students appearing from city A?

Solution:

$$\text{Required percentage} = \frac{15600}{9400} \times 100 = 165.9\%$$

2. If the number of students appearing from city G is 70% more than students appearing

from city E, what is the number of students appearing for the entrance from city G?

Solution:

$$\text{Number of students} = 15600 + \frac{70}{100} \times 15600$$

$$\text{Or } 170\% \text{ of } 15600 = 26520$$

3. Average of all or some units

Illustration:

Following is the table units sold (in hundred) by five companies over different years

YEAR	A	B	C	D	E
2001	1.3	2.2	1.7	2.2	1.4
2002	2.0	1.6	1.5	1.9	1.7
2003	0.9	1.6	1.0	1.5	1.1
2004	0.4	1.3	1.4	1.2	2.5
2005	1.5	1.2	2.1	1.1	2.6

1. What is the average number of units sold by company D over all the years?

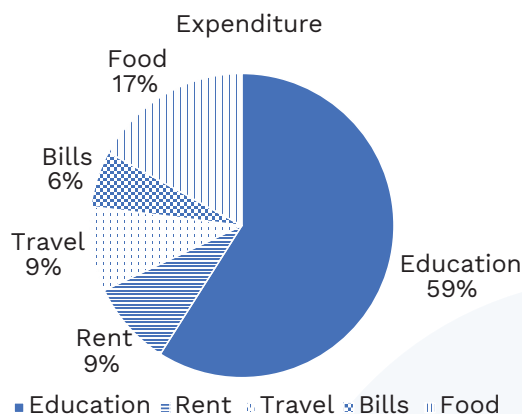
Solution: Average = $\frac{\text{Sum of units}}{\text{Number of years}}$

$$= \frac{220 + 190 + 150 + 120 + 110}{5} = 158$$

4. Calculating X if the total is given or vice versa

Illustration:

Following is the pie chart representing the percentage distribution of budget expenditure of a household



Which year has the highest quantity

Illustration:

Following is the line graph of export of sugar in different years



- Which year has the highest per cent increase in exports as compared to the preceding year?

Solution: Per cent change = $\frac{B - A}{A} \times 100$

$$2006 = \frac{200 - 300}{300} \times 100 = 33.33\% \text{ decrease}$$

- If the house has a total expenditure of ₹ 30 thousand, then how much did they spend on Travel?

Solution:

$$\text{Expenditure on travel} = \frac{9}{100} \times 30000 = ₹ 2700$$

- If 9000 were spent on bills, then what would have been the total expenditure of the house?

Solution:

$$\text{Amount spent on bills} = \frac{6}{100} \times \text{total}$$

$$\text{Total} = \frac{9000}{6} \times 100 = 150000$$

$$2007 = \frac{600 - 200}{200} \times 100 = 200\% \text{ increase}$$

$$2008 = \frac{450 - 600}{600} \times 100 = 25\% \text{ decrease}$$

$$2009 = \frac{600}{450} \times 100 = 33.33\% \text{ increase}$$

Thus, the highest per cent increase is in the year 2007, i.e., 200%



SOME IMPORTANT TRICKS USEFUL IN DATA INTERPRETATION

- If the data are given in text form, converting the data first into a simple table saves a lot of time and is easy to refer to in a shortage of time.
- If the quantity of the item is not mentioned in bar graphs or line graphs, etc., label the quantity beforehand so that you don't look back again and again at both axes.
- Use approximation if options are not close. ex: in question, you do not need to calculate $\frac{11}{9}$ first and then multiply it by 100 or in any other longer way. You can take an approximation of $\frac{100}{9}$ as 11.1 or 11, and thus

the answer can be calculated in one step as 11^2 .

Or,

In question $\frac{15600}{9400}$ could be approximated

$$\text{as } \frac{15000}{9000} \times 100 = \frac{10}{6} \times 100$$

Next, you know $\frac{1}{6}$ is 0.166%. Thus, $\frac{1000}{6} =$

166.6%, and $\frac{15600}{9400}$ will be near to 166.6%;

hence, one could ultimately choose 165.9% from the options.

- Write the calculated values at a separate place so that if any question refers to them again, you do not have to spend time again in calculating.

Chapter Summary



- If data are represented in degrees:
$$\text{Value of any sector} = \frac{\text{Angle of any sector}}{360^\circ} \times \text{Total Value}$$
- If data are represented in percentage:
$$\text{Value of any sector} = \frac{\text{Per cent of any sector}}{360^\circ} \times \text{Total Value}$$
- $$\text{GrowthRate} = \frac{B - A}{A} \times 100$$
- $$\text{Average} = \frac{\text{Sum of units}}{\text{Number of years}}$$

PRACTICE QUESTIONS

Data: The table lists the share of students (in per cent) per total population who had at least one smartphone available at home and the share of students (in per cent) who bought a new smartphone after 2020 in the ASER survey of

the year 2020 and 2021, categorised according to the level of education of their parents. Given that the total population of India is 1.5 billion.



PARENTS' EDUCATION	TOTAL FAMILIES	ASER 2020		ASER 2021	
		At least one smartphone at home	Bought a new smartphone for children's education since March 2020	At least one smartphone at home	Bought a new smartphone for children's education since March 2020
Low	50 billion	45	5	50	25
Medium	30 billion	60	10	65	25
High	0.5 billion	80	10	80	30
Nil	20 billion	60	10	65	25

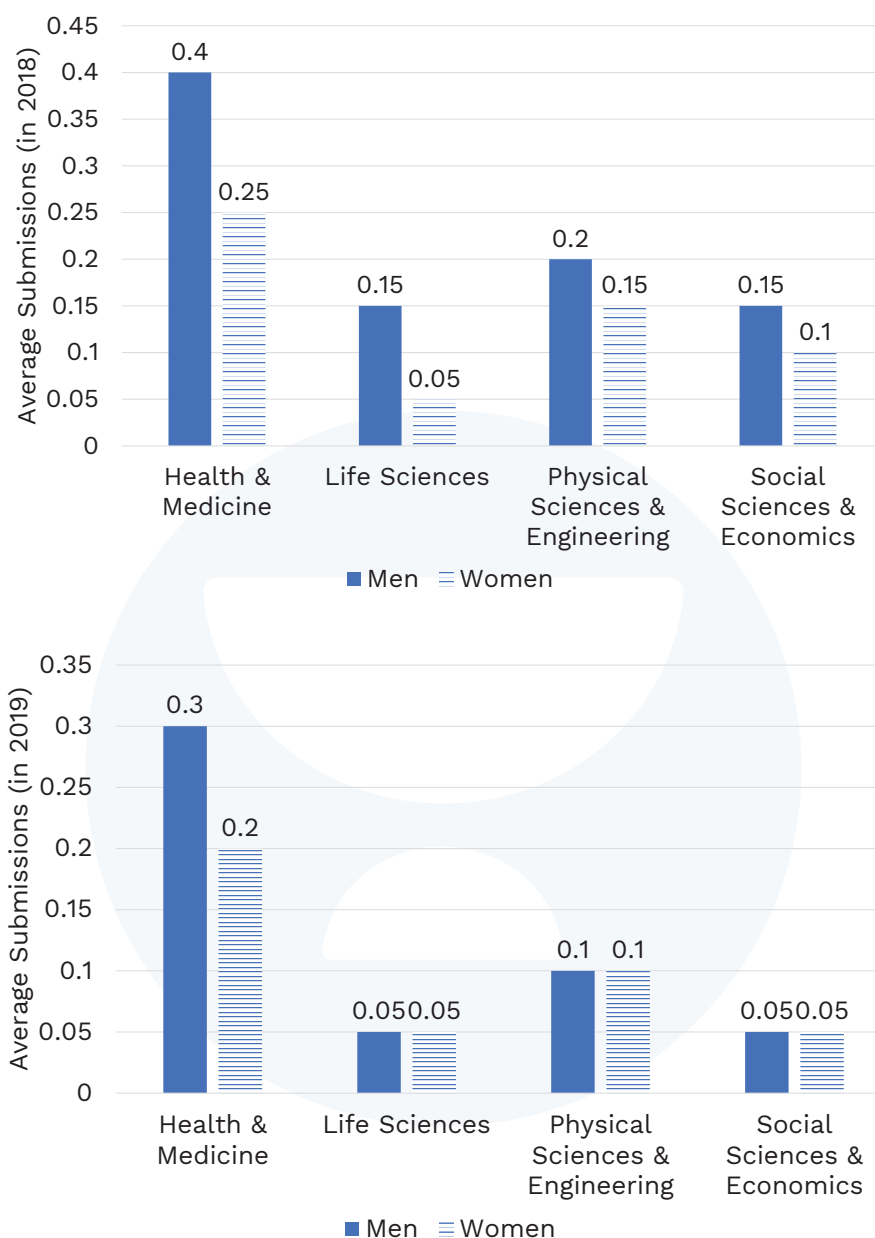
Source: The Hindu Data Point

- The total of people who had at least one smartphone at home in the 2020 survey is what per cent of total people who bought a new phone after march 2020 in the 2021 survey? (use approximate values)
 - 190%
 - 210%
 - 250%
 - 175%
- The number of nil parental education people who bought a new phone after march 2020 in the 2021 survey are what per cent less than the same in 2020 survey?
 - 60%
 - 50%
 - 80%
 - 30%
- What is the respective ratio of people who had at least one smartphone at home whose parents had low income in 2020 and people who didn't have even one smartphone at home in a high parental education home in the same year survey?
 - 1 : 225
 - 150 : 4
 - 225 : 1
 - 175 : 2
- What is the difference between the total number of people who had at least one smartphone in 2021 and people who bought a new phone after march 2020 in a medium parental education home in the 2020 survey?
 - 56.7 billion
 - 48.5 billion
 - 52 billion
 - 54.9 billion
- What is the difference between the total number of people who didn't even have at least one smartphone in 2020 and that in 2021 in a nil education family?
 - 1 billion
 - 1.5 billion
 - 3.5 billion
 - 2 billion



Data: The chart shows the average change in scholarly article submissions of men compared to the average number of submissions of women in 2018 and 2019 across various research areas.

Source: The Hindu Data Point

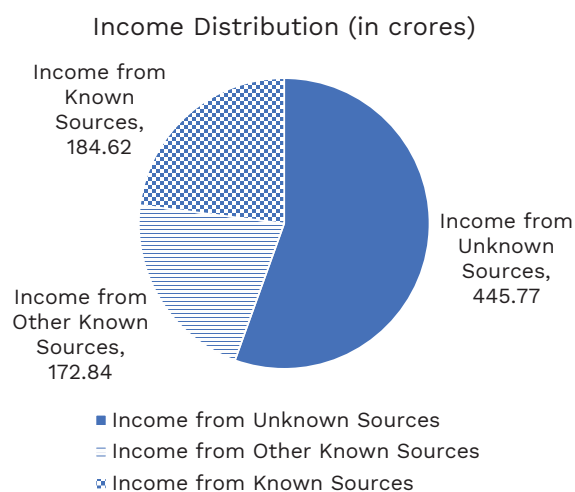


6. What is the ratio of the number of times when average submissions of women in any research area are higher/equal than men in the same to the number of times when average submissions of women in any research area are lower than men in both years?
- A. 2 : 7
B. 3 : 4
C. 3 : 5
D. 4 : 3
7. What is the difference between the total average submissions of men and women in all the research areas in 2018?
- A. 0.45
B. 0.35
C. 0.50
D. 1.25

8. How many times did women have at least 50% of the average submissions of men in respective research areas?
- 4 times
 - 3 times
 - 7 times
 - 9 times
9. Sum Average submission of men in life science area in both the years is approximately what per cent of sum Average submission of men in social science and economics area in both years?
- 100%
 - 50%
 - 75%
 - 25%
10. The area in which the percentage decrease of total average submission of men and women in 2019 was maximum from its preceding year?
- health and medicine
 - life science
 - physical sciences and engineering
 - social sciences and economics

Data: The pie chart shows the sources of income of select regional parties in FY20, The total income for the selected regional parties in FY20 amounted to ₹803.24 crores, of which 55% was from unknown sources. The table depicts the total income of select regional parties in FY20 against the share from unknown sources.

Source: The Hindu Data Point



PARTY	UNKNOWN INCOME (IN %)	TOTAL INCOME (IN CRORE)
ABC	25	10
BCD	45	15
CDE	55	20
DEF	90	20
EFG	20	50
FGH	70	60
GHI	55	90
HIJ	10	90
IJK	80	90
JKL	90	90
KLM	40	110
LMN	70	130

11. The unknown income of ABC is less than the unknown income of GHU by (approximately)
- 80%
 - 95%
 - 85%
 - 70%
12. What is the difference between the unknown income and the rest income of IJK?
- 54 Crores
 - 56 Crores
 - 52 Crores
 - 50 Crores
13. The total income from other known sources is what per cent of total income from known sources? (approximately)
- 85%
 - 69%
 - 93%
 - 75%

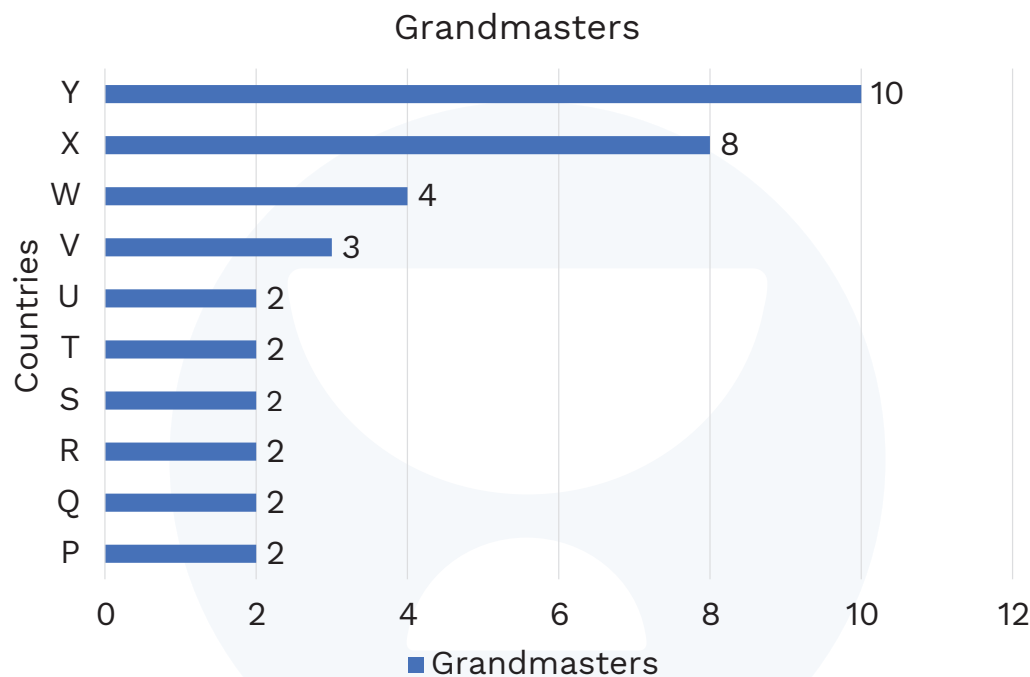


14. What is the total income from unknown sources of KLM, ABC, and GHU together?
- A. 89
 - B. 98
 - C. 96
 - D. 81

15. What is the difference between the income which is not unknown of LMN and that of JKL?
- A. 10 Crores
 - B. 20 Crores
 - C. 30 Crores
 - D. 40 Crores

Data: The Bar chart shows the number of Chess Grandmasters from different countries born after 2000.

Source: The Hindu Data Point



16. How many times are the number of grandmasters from Y than the number of grandmasters from P?
- A. 2
 - B. 3
 - C. 4
 - D. 5
17. The total grandmasters from T and V are what per cent of total grandmasters from the X and Q?
- A. 75%
 - B. 25%
 - C. 70%
 - D. 50%
18. What is the per cent difference in Grandmasters from W than that from S?

- A. 75%
 - B. 100%
 - C. 50%
 - D. 25%
19. What is the difference in the average grandmasters from P, Q, R, S and T taken together to the average of U, V, W, X, and Y?
- A. 35.5
 - B. 25.5
 - C. 20.5
 - D. 18.5
20. If there are 23 grandmasters from P who were born before 2000, then what is the percentage difference in the number of grandmasters born after 2000 than the number of grandmasters born before 2000 in P? (approximately)
- A. 97%
 - B. 80%
 - C. 91%
 - D. 89%



Data: 1.7 million migrants tried to illegally enter the US between FY 21. Of which, 0.15 million were unaccompanied minors. And 2500 of the 1.7 million were Indians. The line graph represents the share of individuals who were apprehended more than once by the Border Patrol out of total immigrants.

Source: The Hindu Data Point

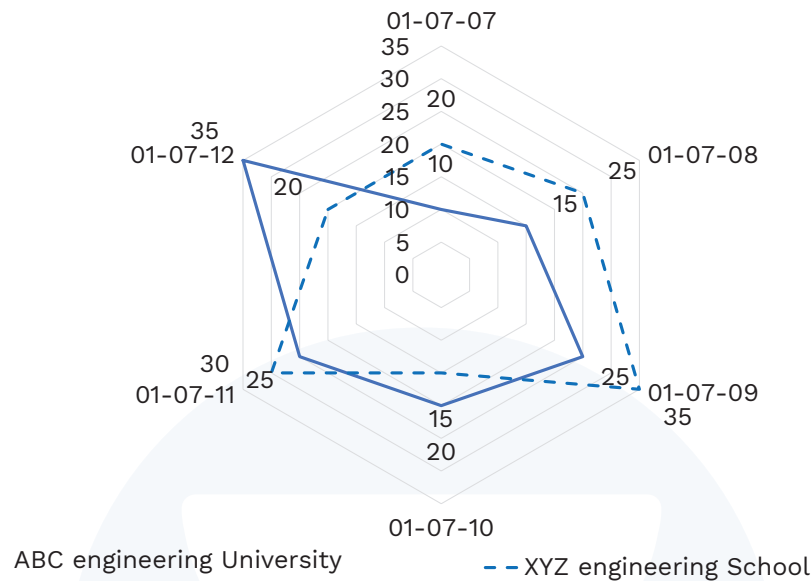


21. Which year has the highest per cent difference in repeated apprehenders as compared to the preceding year?
- A. FY 16
 - B. FY 21
 - C. FY 20
 - D. FY 17
22. What is the difference in people who crossed the US border in FY 19 and FY 21, if total immigrants in FY 19 were 90% of total immigrants in FY 21?
- A. 0.17 million
 - B. 0.15 million
 - C. 0.19 million
 - D. 0.13 million
23. Unaccompanied minors are what per cent of illegal immigrants in FY 21?
- A. 10%
 - B. 8.8%
 - C. 9%
 - D. 9.8%
24. If the number of repeated apprehenders in 2017 who were Indian were 2% of the total immigrants of the same year and were 100 less than in 2021, then what is the total number of illegal immigrants in FY 17.
- A. 0.13 million
 - B. 0.14 million
 - C. 0.12 million
 - D. 0.11 million
25. For how many years, repeated apprehender immigrants per cent were more than the average per cent overall years?
- A. 2 years
 - B. 3 years
 - C. 4 years
 - D. 5 years



Data: The radar graph below represents the number of students(in hundred) in ABC engineering university and XYZ engineering school in the starting of six different academic years.

Source: The Hindu Data Point



26. What was the difference between the number of students in ABC engineering school in the year 2010 and the number of students in XYZ engineering school in the year 2012?
- A. 500
B. 1200
C. 700
D. 1000
27. What is the sum of the number of students in ABC engineering university in the year 2007 and the number of students in XYZ engineering school in the year 2011 together?
- A. 2500
B. 2800
C. 3500
D. 3000
28. If 20% of the students in XYZ in the year 2010 were females, what was the number of male students in XYZ in the same year?
- A. 1000
B. 1100
C. 1200
D. 1300
29. What was the per cent increase in the number of students in ABC engineering university in the year 2011 as compared to the previous year?
- A. 33.3%
B. 14.28%
C. 75%
D. 66.6%
30. In which year was the difference between the number of students in ABC engineering university and XYZ highest?
- A. 2008
B. 2007
C. 2010
D. 2012



Data: The table shows the average spending on drugs per patient for both forms of treatment in the 10 high TB-burden countries in 2020.

Source: The Hindu Data Point

COUNTRY	FIRST-LINE TREATMENT (IN DOLLAR)	DRUG- RESISTANT TB TREATMENT (IN DOLLAR)
A	93	20287
B	46	2188
C	24	1723
D	45	1600
E	47	1352
F	67	1339
G	45	913
H	26	864
I	63	713
J	33	700

31. Among the given countries, which country spent the total most approximately to the average total spending of all countries?

- A. H
B. C
C. I
D. B

32. What is the difference between the total spending of the top five countries and that of the bottom five countries?

- A. 23423 dollars
B. 22656 dollars
C. 34567 dollars
D. 43513 dollars

33. How much will J spend next year if its total spending next year increases by 13%?

- A. 834.2 dollars
B. 849.6 dollars
C. 857.5 dollars
D. 828.2 dollars

34. How much is the total spending of H more than that of C?

- A. 678 dollars
B. 756 dollars
C. 857 dollars
D. 934 dollars

35. Which of the following countries has the lowest difference in respective spending over both types of treatment?

- A. J
B. I
C. H
D. B

SOLUTIONS

1. (B) People who had at least one smartphone in 2020

$$\begin{aligned} &= \frac{45}{100} \times 50 + \frac{60}{100} \times 30 + \frac{80}{100} \times 0.5 + \frac{60}{100} + 20 \\ &= 22.5 + 18 + 0.4 + 12 = 53 \text{ billion (approx.)} \end{aligned}$$

People who bought a new smartphone in 2021

$$\begin{aligned} &= \frac{25}{100} \times 50 + \frac{25}{100} \times 30 + \frac{30}{100} \times 0.5 + \frac{25}{100} \times 20 \\ &= 12.5 + 7.5 + 0.15 + 5 = 25 \text{ billion (approx.)} \end{aligned}$$



- Required percentage = $\frac{53}{25} \times 100 = 210\%$
2. (A) People who bought a new phone after march 2020 in 2021 = $\frac{10}{100} \times 20 = 2$ billion
People who bought a new phone after march 2020 in 2020 = $\frac{25}{100} \times 20 = 5$ billion
Required percentage = $\frac{5-2}{5} \times 100 = 60\%$
3. (C) People who had at least one smartphone at home whose parents had low income in 2020 = $\frac{45}{100} \times 50 = \frac{45}{2}$ billion
People who didn't have even one smartphone at home in high parental education home in 2020 = $\frac{20}{100} \times 0.5 = \frac{1}{10}$ billion
Ratio = $\frac{45}{2} : \frac{1}{10} = \frac{225}{1} = 225:1$
4. (D) Total number of people who had at least one smartphone in 2021 = $\frac{50}{100} \times 50 + \frac{65}{100} \times 30 + \frac{80}{100} \times 0.5 + \frac{65}{100} \times 20$
= $25 + 19.5 + 0.4 + 13 = 57.9$ billion
People who bought a new phone after march 2020 in medium parental education home = $\frac{10}{100} \times 30 = 3$ billion
Required difference = $57.9 - 3 = 54.9$ billion
5. (A) Total number of people who didn't even had at least one smartphone in 2020 = $\frac{40}{100} \times 20 = 8$ billion
Total number of people who didn't even had at least one smartphone in 2021 = $\frac{35}{100} \times 20 = 7$ billion
Required difference = $8 - 7 = 1$ billion
6. (C) Number of times when average submissions of women in any research area are higher than men = 3
Number of times when average submissions of women in any research area are lower than men = 5
Required ratio = 3:5
7. (B) Total average submissions of men = $0.4 + 0.15 + 0.2 + 0.15 = 0.9$
Total average submissions of women = $0.25 + 0.05 + 0.15 + 0.1 = 0.55$
Required difference = $0.9 - 0.55 = 0.35$
8. (C) Self-explanatory. 7 times
9. (A) Sum average submission of men in life science area in both years = $0.15 + 0.05 = 2$
Sum average submission of men in social science and economics area in both years = $0.15 + 0.05 = 2$
Required per cent = $\frac{2}{2} \times 100 = 100\%$
10. (D) Percentage decrease of total average submission of men and women than its preceding year in one area
Total submission in 2019 –
= $\frac{\text{Total submission in 2018}}{\text{Total submission in 2018}} \times 100$
Percentage change in health and medicine area = $\frac{0.5-0.65}{0.65} \times 100 = 23\%$ decrease
Percentage change in life science area = $\frac{0.1-0.2}{0.2} \times 100 = 50\%$ decrease
Percentage change in physical sciences and engineering area = $\frac{0.2-0.35}{0.35} \times 100 = 42\%$ decrease
Percentage change in social sciences and economics area = $\frac{0.1-0.25}{0.25} \times 100 = 60\%$ decrease
Thus, the maximum percentage decrease is in social sciences and economics.
11. (B) Unknown income of ABC = $\frac{25}{100} \times 10 = 2.5$ crores approx.
Unknown income of GHU = $\frac{55}{100} \times 90 = 50$ crores approx.



$$\text{Required per cent} = \frac{47.5}{50} \times 100 = 95\%$$

12. (A) Unknown income of IJK = $\frac{80}{100} \times 90 = 72$

crores

Rest income of IJK = $\frac{20}{100} \times 90 = 18$ crores

Required difference = $72 - 18 = 54$ crores

13. (C) Income from other known sources = 172 crores

Income from known sources = 184 crores

Required per cent = $\frac{172}{184} \times 100 = 93\%$

approximately

14. (C) Income from unknown sources of KLM = $\frac{40}{100} \times 110 = 44$ crores

Income from unknown sources of ABC = $\frac{25}{100} \times 10 = 2.5$ crores

Income from unknown sources of GHU = $\frac{55}{100} \times 90 = 49.5$ crores

Total = $44 + 2.5 + 49.5 = 96$

15. (C) Income that is not unknown of LMN = $\frac{30}{100} \times 130 = 39$ crores

Income which is not unknown of JKL = $\frac{10}{100} \times 90 = 9$ crores

Required difference = 30 crores

16. (D) Grandmasters from Y = 10

Grandmasters from P = 2

Self-explanatory 5 times.

17. (D) Total grand masters from T and V = $2 + 3 = 5$

Total grand masters from X and Q = $8 + 2 = 10$

Required per cent = $\frac{5}{10} \times 100 = 50\%$

18. (B) Grand masters from W = 4

Grand masters from S = 2

$$\text{Required per cent difference} = \frac{4 - 2}{2} \times 100$$

$$= 100\%$$

19. (B) Average grandmasters from P, Q, R, S, and T = 2

Average of U, V, W, X, and Y = 27.5

Required difference = 25.5

20. (B) Grandmasters from P who were born before 2000 = 23

Number of grandmasters born after 2000 in P = 2

Required difference per cent = $\frac{21}{13} \times 100 =$

$$91\%$$

21. (C) Per cent difference in repeated apprehenders as compared to the preceding year:

$$\text{FY 16} = \frac{12 - 14}{14} \times 100 = 0.14\%$$

$$\text{FY 17} = \frac{10 - 12}{12} \times 100 = 16.6\%$$

$$\text{FY 18} = \frac{11 - 10}{10} \times 100 = 10\%$$

$$\text{FY 19} = \frac{6 - 11}{11} \times 100 = 45.4\%$$

$$\text{FY 20} = \frac{26 - 6}{6} \times 100 = 333.3\%$$

$$\text{FY 21} = \frac{27 - 26}{26} \times 100 = 3.8\%$$

Thus, highest in FY 20

22. (A) People who crossed US border in the

$$\text{FY 19} = \frac{90}{100} \times 1.7 = 1.53 \text{ million}$$

People who crossed US border in the FY 21 = 1.7 million

Required difference = 0.17 million

23. (B) Required per cent = $\frac{0.15}{1.7} \times 100 = 8.8\%$

24. (C) Number of Indian repeated apprehenders in 2017 = 2500 - 100

2400 = total immigrants of the same year $\times 2/100$

Total immigrants of the same year = 0.12 million



- 25. (A)** Average per cent overall years = $(14 + 12 + 10 + 11 + 6 + 26 + 27)/7 = \frac{106}{7} = 15\%$
approximately
Thus, repeated apprehender immigrants per cent were more than the average per cent overall years for 2 FY.
- 26. (A)** The number of students in ABC engineering school in the year 2010 = 2000
Number of students in XYZ engineering school in the year 2012 = 1500
Difference = 500
- 27. (C)** Self-explanatory
- 28. (C)** Female students in XYZ in the year 2010 = $\frac{20}{100} \times 1500 = 300$
Male students = $1500 - 300 = 1200$
- 29. (D)** Number of students in ABC engineering university in the year 2011 = 25
Number of students in ABC engineering university in the year 2010 = 15
Per cent increase = $\frac{25 - 15}{15 \times 100} \times 100 = 66.6\%$
- 30. (D)** Year 2007 difference = 10
Year 2008 difference = 10
Year 2009 difference = 10
Year 2010 difference = 5
Year 2011 difference = 5
Year 2012 difference = 15
Thus, highest in 2012.
- 31. (D)** Average spending of all countries = $(20380 + 2234 + 1747 + 1645 + 1399 + 1406 + 958 + 890 + 958 + 890 + 776 + 733)/10 = 3216.8$ dollars
Thus, B spent total most approximate to the average total spending of all countries (i.e., 2234 dollars)
- 32. (B)** Total spending of top five countries = $20380 + 2234 + 1747 + 1645 + 1399 = 27412$ dollars
Total spending of bottom five countries = $1406 + 958 + 890 + 958 + 890 + 776 + 733 = 4756$ dollars
Required difference = 22656 dollars
- 33. (D)** J's spending next year = $\frac{113}{100} \times 733 = 828.2$ dollars
- 34. (C)** Total spending of H = 890 dollars
Total spending of C = 1747 dollars
Required difference = 857 dollars
- 35. (A)** Difference in both spendings:
A = 20194
B = 2142
C = 1699
D = 1555
E = 1305
F = 1292
G = 868
H = 868
I = 838
J = 650
Thus, lowest in J.



SYNOPSIS

- Introduction
- Types of numbers
- Factors
- HCF
- LCM
- Simplification
- Fractions
- Types of fractions
- Surds and indices
- Divisibility rule

INTRODUCTION

Number system basically comprises ten digits, i.e., **0, 1, 2, 3, 4, 5, 6, 7, 8, and 9**. When these digits are combined together, they form a particular number. And when we form a number using these digits, then each digit takes a particular place, and that particular place of the digits we call the *place value* of that particular digit in that particular number. But the value of these

particular digits remains the same wherever they are placed in a number and called as *face value* of that particular number.

Example:

Let us form a number using some of the digits given above, 23789 is an example of a number that we formed using digits 2, 3, 7, 8, and 9. Now, let us see the place value of these digits in this number, so here, the place value of each number is:

PLACE VALUE	TEN THOUSAND	THOUSANDS	HUNDREDS	TENS	ONES/UNITS
Digit	2	3	7	8	9

We use a number system in our day-to-day lives, for example, while playing cricket we count runs scored by a team, or how many wickets are taken by a team or by a particular bowler and so on.

After understanding the basic concept of the number system now comes the types of numbers in the number system.

TYPE OF NUMBERS

Natural numbers

Natural numbers are all positive counting numbers that begin with 1 and can be counted

indefinitely. Because natural numbers can never be negative, these numbers are also known as positive integers.

Example:

1, 2, 3, 4, ... 267, ... 4999, ... ∞ (infinity)

Whole numbers

If we add the number zero to the collection of natural numbers, we get the collection of whole numbers. Whole numbers are counting numbers that can be counted starting from digit 0 to infinity. Whole numbers include natural numbers that begin from 1 onwards.



Whole numbers include positive integers along with 0.

Example:

0, 1, 2, 3, 4, 5,...556, 721,... ∞ (infinity)

Even numbers

Even numbers are basically integers, positive integers as well as negative integers, which can be divided exactly or evenly by digit 2. If an integer is exactly divided by 2, it implies that the number in question has a remainder of 0 upon the division of 2.

Example:

2, 4, 6, 8,... 498, 8888,...

Odd numbers

Odd numbers are integers, positive integers as well as negative integers, which cannot be divided exactly or evenly by digit 2. If an integer is not exactly divisible by 2, it implies that the number in question has a remainder of 1 upon the division of 2.

Example:

1, 3, 5, 7, ...379, 689,...

Prime numbers

Prime numbers are those positive integers that have only two factors, 1 and the number itself. They are not divisible by any other numbers.

Example:

2, 3, 5, 11,...71,...1381,...

Note

- There are only 25 prime numbers up to 100, i.e., 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97.
- 2 is the only even prime number.

Co-prime numbers

Co-prime numbers is basically a set of two numbers that have HCF as 1, i.e., there are

no common factors between these two numbers.

Example:

5 and 24, 6 and 37, etc.

Composite numbers

Composite numbers are those numbers that have more than 2 factors excluding the number itself. Also, a composite number is a positive integer that can be formed by multiplying two smaller positive integers. Equivalently, it is a positive integer that has at least one divisor other than 1 and itself.

Example:

4, 6, 8, 9, 10,... 656, 7864,...

Integers

From negative infinity to positive infinity, integers are all positive and negative counting numbers. Integers are a type of number that consists of both positive and negative numbers. Integers, like whole numbers, do not include the fractional portion. Integers, on the other hand, are numbers that can be positive, negative or zero but not a fraction.

Zero is also an integer.

Example:

...-23, -498,... 890, 4567,...

Positive integers

Positive integers are all whole numbers that are larger than zero and do not include fractions or decimals. The positive integers lie on the right side of 0 on a number line.

Example:

1, 2, 3, 4,... 666, 879,...

Negative integers

Negative integers are numbers that have a value less than zero. They do not include fractions or decimals. The negative integers lie on the left side of 0 on a number line.

Example:

...-37, -44, -990, -8769...

**Note**

- Zero is defined as neither negative nor positive. The ordering of integers is compatible with the algebraic operations in the following way: if $a < b$ and $c < d$, then $a + c < b + d$. If $a < b$ and $0 < c$, then $ac < bc$.

Rational numbers

Rational numbers are those numbers that can be written in the form $\frac{p}{q}$, where $q \neq 0$. Also, rational numbers include all integers, zero, or fractions.

Example:

$\frac{5}{2}$, $\frac{11}{13}$, 0.7777, $-\frac{1}{2}$, etc.

Irrational numbers

Irrational numbers are those numbers that cannot be expressed in the form of $\frac{p}{q}$.

Example:

π , $\sqrt{7}$, 0.12122122212222...

Real numbers

The collection of all rational numbers and irrational numbers together makes up what we call the collection of real numbers. All the concepts that we have discussed above, about different types of numbers, fall under the category of real numbers. So, all of the above-given examples are examples of real numbers also.

Complex numbers

Complex numbers are those numbers that can be written in the form $a + bi$ where a and b are real numbers, and i is the square root of -1 .

Example:

..., $-7 + 2i$, 0, $1 + 3i$...

FACTORS

Factors of a number are the exact divisors of the provided numbers because they divide the

given number exactly with 0. For example, the number 15 is divisible by 1, 3, 5, and 15.

HCF

The Highest Common Factor is the full form of HCF. HCF is also known as Greatest Common Factor (GCF) or Greatest Common Divisor (GCD). The largest positive integer that divides all the provided integers with zero remainders is the greatest common factor (GCF, GCD, or HCF) for a subset of whole numbers.

Example: Consider two numbers 15 and 18.

Factors of 15 = 1, **3**, 5, and 15

Factors of 18 = 1, 2, **3**, 6, 9, and 18

The common factors of both 15 and 18 are 1 and 3. Here, 3 is the highest common factor of both numbers. Hence, 3 is the HCF of 15 and 18.

LCM

LCM stands for Lowest or Least Common Multiple in its full form. The smallest positive integer divisible by all the provided numbers is the LCM of two or more numbers.

Example:

Consider two numbers: 7 and 21.

The multiples of 7 are:

$$7 \times 1 = 7,$$

$$7 \times 2 = 14,$$

$$7 \times 3 = \mathbf{21},$$

$$7 \times 4 = 28, \text{ and so on...}$$

The multiples of 21 are:

$$21 \times 1 = \mathbf{21},$$

$$21 \times 2 = 42,$$

$$21 \times 3 = 63,$$

$$21 \times 4 = 84, \text{ and so on...}$$

21 is the lowest common multiple from all the above multiples of 7 and 21. So, 21 is the LCM of 7 and 21.

SOME OTHER IMPORTANT CONCEPTS**Simplification**

As we know that in GATE and other engineering entrance examinations, fast and quick calculations are very important, and the simplification concept is one of the concepts



of quantitative aptitude, which helps you in solving questions fast and quick. Every question in the math section of your exam can be solved quickly with the help of this concept, and this makes this concept a very important one.

Finding an answer to a difficult calculation including division, multiplication, square roots, and cube roots, plus, and minus is known as simplification. When a calculation is given, it is sometimes observed that one of the numbers is missing. We must either approximate the given values or do basic operations to determine the missing number. We have to simplify the calculation when all the numbers are given with some operations between them.

Now, let's discuss some of the basic rules related to simplification:

1. Rule 1: Always replace 'of' by multiplication and '/' by division.

Example:

Find $\frac{1}{8}$ of 72.

Solution:

$$\frac{1}{8} \times 72 \text{ or } \frac{72}{8}$$

Therefore, 9 is the answer.

2. Rule 2: Always apply the rule of BODMAS in every number operation.

B	Bracket
O	Open, of
D	Division
M	Multiplication
A	Addition
S	Subtraction

So, whenever in any question when there is more than one type of calculation then we have to apply the rule of BODMAS while solving the question.

Example:

$$2 + 6 - (1 \times 4) \div 4 \times 5$$

Solution:

By applying the rule of BODMAS here, First, we will solve what is inside the bracket, i.e. $1 \times 4 = 4$

So, now the operation is $2 + 6 - 4 \div 4 \times 5$

Now, we will divide the number, i.e., $-4 \div 4 = -1$

Then, $2 + 6 - 1 \times 5$

Now, we will multiply the numbers, i.e. $-1 \times 5 = -5$

So, $2 + 6 - 5$

The next step is to add the number, i.e. $2 + 6 = 8$

So, $8 - 5$

Therefore, the answer is 3.

Fraction

Fractions are portions of a whole or a collection that are equal in size. Each part of a whole divided into equal pieces is a fraction of the total. Fractions are numbers that represent a piece of a larger whole. When an object or a set of objects is divided into equal pieces, each individual part becomes a fraction. Like most cases, a fraction is written as $\frac{1}{21}$, $\frac{50}{17}$, $\frac{7}{19}$, and

so on. The numerator represents the total number of equal parts into which the whole is divided, and the denominator represents the total number of equal parts into which the whole is divided. The total number of equal pieces removed is the numerator. For example, in the fraction $\frac{37}{4}$, the numerator is 37, and the denominator is 4.

We can also convert decimal numbers into fractions by putting 1 and the number of zeroes equal to the number of digits after the decimal, in the denominator.

Examples

If you divide a pizza into two equal portions, each half equals half of the total pizza,

$$0.2 = \frac{2}{10}, 1.879 = \frac{1879}{1000}, \text{ etc.}$$

**Example:**

What is the greatest fraction among:

$$\frac{11}{16}, \frac{7}{8}, \frac{13}{20}, \text{ and } \frac{31}{40}$$

Solution:

If the denominator of some fractions is the same then the fraction with the largest numerator will be the largest fraction. So, to make the denominator equal to each fraction take LCM of 16, 8, 20, and 40 which is 80.

$$\text{Now, } \frac{80}{16} = 5 = \frac{11}{16} \times \frac{5}{5} = \frac{55}{80}$$

$$\frac{80}{8} = 10 = \frac{7}{8} \times \frac{10}{10} = \frac{70}{80}$$

$$\frac{80}{20} = 4 = \frac{13}{20} \times \frac{4}{4} = \frac{52}{80}$$

$$\frac{80}{40} = 2 = \frac{31}{40} \times \frac{2}{2} = \frac{62}{80}$$

$$70 > 62 > 55 > 52 \Rightarrow \frac{7}{8} > \frac{31}{40} > \frac{11}{16} > \frac{13}{20}$$

Therefore, the greatest fraction is $\frac{7}{8}$.

Types of fraction**1. Unit fraction:**

In a fraction, the numerator with 1 is called a unit fraction. *For example,* $\frac{1}{2}, \frac{1}{3}, \frac{1}{99}$, etc.

2. Proper fraction:

If a numerator's value is less than the denominator's value, it is called a proper fraction. *For example,* $\frac{2}{7}, \frac{8}{11}, \frac{73}{79}$, etc.

3. Improper fraction:

If a numerator's value is greater than the denominator's value, then it is called an improper fraction. *For example,* $\frac{6}{5}, \frac{3}{2a}, \frac{97}{95}$, etc.

4. Mixed fraction:

If a fraction consists of a whole number with a proper fraction, it is called a mixed fraction. *For example,* $5\frac{3}{4}, 10\frac{1}{2}, 22\frac{1}{3}$, etc.

5. Like fractions:

The fractions with the same denominator are called like fractions. *For example,* $\frac{3}{2}, \frac{5}{2}, \frac{99}{2}$, etc.

6. Unlike fractions

The fractions with different denominators are called unlike fractions. *For example,* $\frac{3}{2}, \frac{4}{7}, \frac{67}{3}$, etc.

7. Equivalent fractions:

If two fractions result in the same value, after simplification, then they are equivalent to each other. *For example,* $\frac{2}{3}, \frac{4}{6}$ (when we simplify it, it will become $\frac{2}{3}$), $\frac{6}{9}$ (when we simplify it, it will become $\frac{2}{3}$), etc.

Surds and indices

Surds are the square roots ($\sqrt{\quad}$) of numbers that cannot be reduced to a single whole or rational number. It is impossible to portray it accurately in a fraction. Surds are also the root values that cannot be expressed in whole numbers. Indices are a value's power or exponent.

Examples:

$\sqrt{2} \approx 1.414$, and therefore, we leave it as a surd $\sqrt{2}$.

There are some rules for surds and indices, which are:

1. Rules for indices:

- $X^0 = 1$
- $X^a \times X^b = X^{a+b}$
- $\frac{X^a}{X^b} = X^{a-b}$
- $(X^a)^b = X^{ab}$
- $(X^y)^a = X^a Y^a$
- $\left(\frac{X}{Y}\right)^a = \frac{X^a}{Y^a}$

2. Rules for surds:

- $\sqrt[x]{a} = a^{\frac{1}{x}}$
- $\sqrt[x]{ab} = \sqrt[x]{a} \times \sqrt[x]{b}$
- $\sqrt[x]{\frac{a}{b}} = \frac{\sqrt[x]{a}}{\sqrt[x]{b}}$
- $(\sqrt[x]{a})^x = a$



e. $\sqrt[y]{\sqrt[x]{a}} = \sqrt[xy]{a}$

f. $(\sqrt[x]{a})^y = \sqrt[xy]{a^y}$

Example:

Find the value of $(\sqrt{64})^{\frac{1}{3}}$.

Solution:

Removing the root from the base value and putting it in the form of power, we get,

$$\Rightarrow \left(64^{\frac{1}{2}}\right)^{\frac{1}{3}} = (64)^{\frac{1}{6}}$$

$$(4^3)^{\frac{1}{6}} = 4^{\frac{3}{6}} = \sqrt{4} = 2$$

Therefore, the answer is 2.

Divisibility rule

Divisibility rule is when a number is divided by another number or digit without leaving a remainder, it is said to be divisible. To make division simple we use this rule, and here are some of the rules of few numbers:

1. **For 2:** If the unit or ones digit of any number is 0, 2, 4, 6, and 8, then that number is divisible by 2. For example, 2378, 666, and 3675344. As we see here the last digits of the numbers are 8, 6, and 4, respectively, so these numbers are divisible by 2.
2. **For 3:** A number is only divisible by 3 when the sum of all the digits of the number is divisible by 3. For example, 333 ($3 + 3 + 3 = 9$), 33455 ($3 + 3 + 4 + 5 + 5 = 20$). As we see here the sum of the digits of the first number is 9, which is divisible by 3, so 333 is also divisible by 3, but the sum of the digits of the second number is 20, which is not divisible by 3, so the number 33455 will also not be divisible by 3.
3. **For 4:** If the last digit of a number is 0, or the last two digits are divisible by 4, then that number is divisible by 4. For example, 2508, and 2506. As we see here, the last digits of the number 2508 are 08. Since 08 is divisible by 4, then the number 2508 is also divisible

by 4. But 2506 is not divisible by 4 because the last two digits, 06, are not divisible by 4.

4. **For 5:** If a number has 0 or 5 at its unit's place then it is divisible by 5. For example, 2700, 23675, and 34583805. As we see here the last digit of the numbers are 0, 5, and 5 respectively, so these numbers are divisible by 5.
5. **For 6:** If a number is divisible by 2 and 3 both are also divisible by 6. For example, 36, 216, and 1296. As we see here, all the three numbers are divisible by 2 as well as 3. Hence, the numbers are divisible by 6 also.
6. **For 7:** The difference between twice the unit digit of the given number and the remaining part of the given number should be a multiple of 7, or it should be equal to 0. Let us learn this using an example: 161. The first step would be to double the number at units place, i.e., $1 \times 2 = 2$. Now subtract this from the remaining number $16 - 2 = 14$. Check if the reduced number is divisible by 7 or not. If yes then number 161 is also divisible by 7.
7. **For 8:** A number is only divisible by 8 when its last 3 digits, i.e., digits at hundred's, ten's, and unit's place, are divisible by 8, or if the last 3 digits are zero. For example, 4608, 36864, 294912. As we see here the last three digits of all the three numbers are divisible by 8. So, all three numbers are completely divisible by 8.
8. **For 9:** A number is only divisible by 9 if the sum of all its digits is divisible by 9. For example, 999 ($9 + 9 + 9 = 27$), 89919 ($8 + 9 + 9 + 1 + 9 = 36$). As we see here the sum of the digits of the first number is 27, which is divisible by 9, so 999 is also divisible by 9 likewise the sum of the digits of the second number is 36, which is also divisible by 9, so the number 89919 will also be divisible by 9.
9. **For 10:** If the last digit of any number is 0, then the number is divisible by 10. For example,



356821730, 2546270, and 9876540, etc. All these numbers are completely divisible by 10.

- 10. For 11:** If the difference between the sum of digits at odd places and the sum of digits at even places is either 0 or multiple of 11, then the number is divisible by 11. For example, 1331 ($1 + 3 = 4$; $3 + 1 = 4$, difference

between the sum is 0), 161051 ($1 + 1 + 5 = 7$; $6 + 0 + 1 = 7$, difference between the sum is 0). As we see here, the difference between the sum of digits at odd places and the sum of digits at even places of the numbers is 0; hence, these numbers are divisible by 11.

PRACTICE QUESTIONS

- $93 + 3 - 7 \div 43 \times 29 = ?$
A. 90.44
B. 91.27
C. 100
D. 98.77
- $213 + 711 \times 322 + 23 - 9 = ?$
A. 229769
B. 229169
C. 229669
D. 229969
- $1.8 \times 7 + 5 - 14 \div 15 + 3 = ?$
A. 11.99
B. 12
C. 11.27
D. 12.50
- $121 + 517 + 47 - 17 \div 17 \times 41 = ?$
A. 600
B. 650
C. 666
D. 644
- $2 - 7 + 3 + 4 - 2 \times 19 \div 12 = ?$
A. 1.166
B. 2.679
C. -2.679
D. -1.166
- $\frac{1}{3 + \frac{8}{4 + \frac{3}{5 + \frac{1}{4}}}} = ?$
A. $\frac{4}{33}$
B. $\frac{4}{67}$
C. $\frac{4}{19}$
D. $\frac{4}{21}$
- If $4x - 1 \times 6 \times \frac{82x}{2x} \times 3x - 1 = 12$, then the value of x is:
A. $\frac{1}{3}$
B. $\frac{1}{2}$
C. $\frac{1}{4}$
D. $\frac{1}{5}$
- If $Y = \sqrt{2} + 1$, then the value of $Y + \frac{1}{Y}$ is:
A. $2\sqrt{2}$
B. $\sqrt{2}$
C. $2\sqrt[3]{2}$
D. 2
- Which among the following is the greatest fraction? $\frac{2}{5}, \frac{3}{5}, \frac{1}{5}, \frac{7}{15}, \frac{4}{5}$
A. $\frac{2}{5}$
B. $\frac{4}{5}$
C. $\frac{3}{5}$
D. $\frac{7}{15}$



10. The sum of the first squares of the first ten natural numbers is:
A. 333
B. 388
C. 385
D. 365
11. A pile of coconuts is divided into groups of 2, 3 and 5, with one coconut left out each time. What is the smallest number of coconuts in the heap?
A. 62
B. 30
C. 35
D. 31
12. The remainder is 3 when n is divided by 4. When $2n$ is divided by 4, what is the remainder?
A. 2
B. 3
C. 4
D. 1
13. What is the unit digit in the number $(211)^{111}$?
A. 1
B. 2
C. 3
D. 4
14. What is the sum of the first 45 natural numbers?
A. 1000
B. 1022
C. 1035
D. 1099
15. What is the smallest value that $*$ must have in order for the number $451*603$ to be divisible by 9 exactly?
A. 9
B. 6
C. 7
D. 8
16. What is the smallest value that must be provided to $*$ in order for $63576*2$ to be divisible by 8?
A. 1
B. 3
C. 5
D. 2
17. Which of the following is divisible by 99 exactly?
A. 114345
B. 114344
C. 114343
D. 114342
18. What is the greatest five-digit number that can be divided by 99?
A. 99990
B. 90909
C. 99099
D. 90999
19. When a number is divided by 338, the remainder is 68; nevertheless, when the same amount is divided by 13, the remainder is?
A. 3
B. 5
C. 1
D. 7
20. Between 200 and 600, how many numbers are divisible by 4, 5 and 6?
A. 6
B. 8
C. 10
D. 12
21. What is the closest number to 99547 that is divisible by 687 exactly?
A. 99617
B. 99615
C. 99619
D. 99614



22. Six bells began to toll in sync, at 3, 6, 9, 12, 15 and 18 seconds intervals, respectively. How many times did they toll collectively in 30 minutes?
- A. 9
B. 10
C. 11
D. 12
23. What is the smallest five-digit number that can be divided by 11?
- A. 10000
B. 10001
C. 10009
D. 10010
24. The sum of a two-digit number's digits is 9. If the digits are flipped, the number falls by 45, thus what is the original number?
- A. 72
B. 63
C. 90
D. 54
25. Find the smallest integer that leaves a remainder of 2, 3 or 4 when divided by 3, 4 or 5.
- A. 56
B. 57
C. 58
D. 59
26. When a number is divided by 943, the remainder is 76. When you divide the same number by 23, what is the remainder?
- A. 4
B. 6
C. 9
D. None of the above
27. 2093 is the sum of two numbers. The quotient is 2 when the larger number is divided by the difference in the numbers, and the remainder is 118. What is the smaller of the two numbers?
- A. 734
B. 735
C. 736
D. 737
28. Between 500 and 600, how many numbers include the number 9 only once?
- A. 18
B. 15
C. 8
D. 9
29. When $(1923^{1924^{1925}})$ is divided by 1924, what is the remainder?
- A. 0
B. 2
C. 1
D. 11
30. A chain smoker had used up all of his cash. He couldn't afford to buy cigarettes because he didn't have any money left. As a result, he joined the stubs and smoked them together. To make a single cigarette, he needed four stubs. How many cigarettes could he smoke in total if he received a pack of ten cigarettes as a gift?
- A. 12
B. 13
C. 11
D. 14
31. How many of the numbers given below are divisible by 132?
264, 396, 462, 792, 968, 2178, 5184, 6336
- A. 7
B. 6
C. 5
D. 4
32. Which of the following numbers is divisible by 11 exactly?
- A. 415624
B. 415625
C. 415626
D. 415627
33. In total, how many three-digit numbers are divisible by six?



- A. 144
B. 150
C. 160
D. 164
- 34.** What is the smallest prime number?
A. 0
B. 1
C. 2
D. 3
- 35.** How many prime numbers are less than 50?
A. 12
B. 13
C. 14
D. 15
- 36.** What is the smallest three-digit prime number?
A. 333
B. 497
C. 459
D. None of the above
- 37.** Which of the following is even if a and b are odd numbers?
A. $a + b + 1$
B. $a + 3b$
C. $a + b$
D. $a + \frac{b}{3}$
- 38.** We obtain 269 as the quotient and 0 as the remainder when we divide an integer by 68. What will be the remainder if you divide the same number by 67?
A. 3
B. 2
C. 1
D. 0
- 39.** When we divide a number by 56, the remainder is 29. What is the remainder when you divide the same number by 8?
- A. 7
B. 6
C. 5
D. 4
- 40.** When we divide a number by 357, the remainder is 39. What is the remainder when you divide the same number by 17?
A. 3
B. 4
C. 5
D. 6
- 41.** We receive 3 as a remainder when we divide a number by 5. When the square of this number is divided by 5, what is the remainder?
A. 4
B. 5
C. 6
D. 7
- 42.** 1365 is the difference between the two numbers. We obtain 6 as the quotient and 15 as the remainder when we divide the larger number by the smaller. What is the smaller of the two numbers?
A. 278
B. 270
C. 275
D. 289
- 43.** The remainder in a division sum is 0. When a pupil multiplied the divisor by 12 instead of 21, the quotient was 35. What is the right quotient to use?
A. 12
B. 21
C. 20
D. 35
- 44.** The product of two numbers is 35, while the sum of two numbers is 12. What is the sum of these numbers' reciprocals?
A. $\frac{35}{12}$



- B. $\frac{1}{12}$
C. $\frac{1}{35}$
D. $\frac{12}{35}$
- 45.** A positive proper fraction and its reciprocal have a difference of $\frac{9}{20}$. What is the fraction?
- A. $\frac{5}{4}$
B. $\frac{4}{5}$
C. $\frac{20}{9}$
D. $\frac{1}{5}$
- 46.** We receive the same remainder when we divide 2272 by 3 digit number X as we do when we divide 875 by 3 digit number X . What is the sum of X 's digits?
- A. 9
B. 10
C. 11
D. 13
- 47.** On multiplying a number by 7 the product is a number each of whose digits is 3. The smallest such number is:
- A. 47617
B. 47619
C. 47666
D. 47615
- 48.** Which of the following integers is divisible by the difference of the squares of two successive even integers?
- A. 3
B. 4
C. 5
D. 6
- 49.** Which of the following integers is divisible by the difference of the squares of two successive odd integers?
- A. 8
B. 9
C. 13
D. 17
- 50.** If n is a natural number, then $(6n^2 + 6n)$ is always divisible by:
- A. 6
B. 12
C. Both 6 and 12
D. None of the above

SOLUTIONS

- 1. (B)** As there is no bracket in this question, first we will solve the division part of the sum, then the multiplication part, then the addition part and lastly the subtraction part.
- $$\begin{aligned}\text{So, } 93 + 3 - \frac{7}{43} \times 29 \\ &= 93 + 3 - 0.16 \times 29 \\ &= 93 + 3 - 4.72 \\ &= 96 - 4.72 \\ &= 91.279069767442.\end{aligned}$$
- Therefore, the correct option is B.
- 2. (B)** As there is no bracket in this question, first we will solve the division part of the sum, then the multiplication part, then the addition part and finally the subtraction part.
- $$\begin{aligned}&= 213 + 711 \times 322 + 23 - 9 \\ &= 213 + 228942 + 23 - 9 \\ &= 229155 + 23 - 9 \\ &= 229178 - 9 = 229169\end{aligned}$$
- Therefore, the correct option is B.
- 3. (C)** As there is no bracket in this question, first we will solve the division part of the



sum, then the multiplication part, then the addition part and finally the subtraction part.

$$\begin{aligned}
 &= 1.8 \times 7 + 5 - \frac{14}{1.5} + 3 \\
 &= 12.6 + 5 - \frac{14}{1.5} + 3 \\
 &= 12.6 + 5 - 9.3 + 3 \\
 &= 17.6 - 9.3 + 3 \\
 &= 8.27 + 3 \\
 &= 11.27
 \end{aligned}$$

Therefore, the correct option is C.

4. (D) As there is no bracket in this question, first we will solve the division part of the sum, then the multiplication part, then the addition part and finally the subtraction part.

$$\begin{aligned}
 &= 121 + 517 + 47 - \frac{17}{17} \times 41 \\
 &= 121 + 517 + 47 - 1 \times 41 \\
 &= 121 + 517 + 47 - 41 \\
 &= 638 + 47 - 41 \\
 &= 685 - 41 = 644
 \end{aligned}$$

Therefore, the correct option is D.

5. (D) As there is no bracket in this question, first we will solve the division part of the sum, then the multiplication part, then the addition part and finally the subtraction part.

$$\begin{aligned}
 &= 2 - 7 + 3 + 4 - 2 \times \frac{19}{12} \\
 &= 2 - 7 + 3 + 4 - \frac{38}{12} \\
 &= 2 - 7 + 3 + 4 - 3.16 \\
 &= -5 + 3 + 4 - 3.16 \\
 &= -2 + 4 - 3.16 \\
 &= 2 - 3.16666666666667 \\
 &= -1.166
 \end{aligned}$$

Therefore, the correct option is D.

6. (C) Solving the sum,

$$\begin{aligned}
 &= \frac{1}{3} + \frac{168}{96} \\
 &= \frac{4}{19}
 \end{aligned}$$

Therefore, the correct option is C.

7. (B) $2^{2x-2} \times 2^x \times 3^x \times \frac{2^{6x}}{2^x} \times 3^x - 1$

$$= 2^2 \times 3^1$$

$$2^{2x-2+x+x+6x-x} \times 3^{x-x+1} = 2^2 \times 3$$

$$X = \frac{1}{2}$$

Therefore, the correct option is B

8. (A) $Y + \frac{1}{Y} = \frac{Y^2 + 1}{Y}$

Now put the value of $Y = \sqrt{2} + 1$

$$\frac{[(\sqrt{2} + 1)^2 + 1]}{\sqrt{2} + 1}$$

$$= 2\sqrt{2}$$

Therefore, the correct option is A.

9. (B) If the denominator of some fractions is the same then the fraction with the largest numerator will be the largest fraction. So, to make the denominator equal to each fraction take LCM of 5, 5, 5, 15 and 5 which is 15.

$$\text{Now, } \frac{15}{5} = 3 \Rightarrow \frac{2}{5} \times \frac{3}{3} = \frac{6}{15}$$

$$\frac{15}{5} = 3 \Rightarrow \frac{3}{5} \times \frac{3}{3} = \frac{9}{15}$$

$$\frac{15}{5} = 3 \Rightarrow \frac{1}{5} \times \frac{3}{3} = \frac{3}{15}$$

$$\frac{15}{15} = 1 \Rightarrow \frac{7}{15} \times \frac{1}{1} = \frac{7}{15}$$

$$\frac{15}{5} = 3 \Rightarrow \frac{4}{5} \times \frac{3}{3} = \frac{12}{15}$$

$$12 > 9 > 7 > 6 > 3 \frac{4}{5} > \frac{3}{5} > \frac{7}{15} > \frac{2}{5} > \frac{1}{5}$$

Hence, the greatest fraction is $\frac{4}{5}$.

Therefore, the correct option is B.

10. (C) $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2$

$$\begin{aligned}
 &= 1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 + 81 + 100 \\
 &= 385
 \end{aligned}$$

Therefore, the correct option is C.

11. (D) Take the LCM of 2, 3, 5 which is 30



So, number of coconuts = $30 + 1 = 31$,
Therefore, the correct option is D.

- 12. (A)** $N = 4 \times Q + 3$ where Q is quotient

$$2n = 2 \times 4 \times Q + 6$$

$$2n = 4(2Q + 1) + 2$$

Hence, when $2n$ is divided by 4, the remainder is 2.

Therefore, the correct option is A.

- 13. (A)** Any power of 1 will give 1 as a unit digit.
Therefore, the correct option is A.

- 14. (C)** Add all the numbers starting from 1 to 45.
Therefore, the correct option is C.

- 15. (D)** Since the sum of all the digits of the number must be divisible by 9, as per the divisibility rule of 9. So, the smallest value of $*$ is 8.
Therefore, the correct option is D.

- 16. (B)** Since the sum of the last three digits of the number must be divisible by 8, as per the divisibility rule of 8. So, the smallest value of $*$ is 3.
Therefore, the correct option is B.

- 17. (A)** For a number to be divisible by 99, it should have to be divisible by both 9 and 11. So, applying the divisibility rule of both 9 and 11 we find that only 114345 is divisible by 99.
Therefore, the correct option is A.

- 18. (A)** For a number to be divisible by 99, it should have to be divisible by both 9 and 11. So, applying the divisibility rule of both 9 and 11 we find that only 99990 is divisible by 99.
Therefore, the correct option is A.

- 19. (A)** Let the number
 $N = 338X + 68 = 13 \times 26X + 13 \times 5 + 3$
 $\Rightarrow 13(26X + 5) + 3$

Hence, when the number is divided by 13, then the remainder is 13.

Therefore, the correct option is A.

- 20. (A)** Every such number must be divisible by the LCM of 4, 5 and 6 which is 60. Such numbers are 240, 300, 360, 420, 480 and 540.

Hence, there are only 6 such numbers.

Therefore, the correct option is A.

- 21. (B)** When 99547 is divided by 687, remainder is 619.

$$\text{Hence, the nearest number} = 99547 + 68 = 99615$$

Therefore, the correct option is B.

- 22. (C)** LCM of 3, 6, 9, 12, 15 and 18 is 180.

So, the bell will toll together after every 3 minutes.

And in 30 minutes, they will together $10 + 1$ (in the starting, i.e. at 0 minutes) = 11 times.
Therefore, the correct option is C.

- 23. (D)** Apply the divisibility rule of 11 and eliminate three options from the options.
Therefore, the correct option is D.

- 24. (A)** Let the unit digit be X and tens digit be Y
Then, number = $10Y + X$

Now, according to the question,

$$X + Y = 9 \quad \dots\dots\dots (1)$$

$$\text{And } 10X + Y = 10Y + X - 45$$

$$= 9X - 9Y = -45$$

$$= Y - X = 5 \quad \dots\dots\dots (2)$$

Adding equations (1) and (2)

$$2Y = 14 \text{ or } Y = 7, \text{ then, } X = 2$$

Hence, the number is 72.

Therefore, the correct option is A.

- 25. (D)** The smallest number divisible by 3, 4 and 5 is 60. Here, we see the difference between the divisor and remainder is the same for every divisor that is 1. So 59 is the answer.

Therefore, the correct option is D

- 26. (D)** Let the number be N
So, $N = 943Q + 76$

$$\Rightarrow 23 \times 41Q + 3 \times 23 + 7$$

$$\Rightarrow 23(41Q + 3) + 7$$

Hence, the remainder is 7.



Therefore, the correct option is D.

- 27. (D)** Let the numbers be X and Y
So, $X + Y = 2093$...Equation (1)
According to the question
$$\Rightarrow \frac{X}{X} - Y = 2 + \frac{118}{X} - Y$$
$$\Rightarrow 2Y - X = 11 \text{...Equation (2)}$$

Solving equations (1) and (2) we get $Y = 737$.
Therefore, the correct option is D.
- 28. (D)** All numbers between 500 and 600 in which 9 occurs only once are: 509, 519, 529, 539, 549, 559, 569, 579, 589. Hence there are 9 such numbers.
Therefore, the correct option is D.
- 29. (C)** The remainder is 1.
Therefore, the correct option is C.
- 30. (B)** 10 cigarettes give 10 stubs. From 10 stubs 3 more cigarettes can be made. 13 is the answer.
Therefore, the correct option is B.
- 31. (D)** For a number to be divisible by 132 it should have to be divisible by 3, 4 and 11 also. So, applying the divisibility rules of these three numbers on each of the given numbers, we find that there are only 4 such numbers (264, 396, 792 and 6336).
Therefore, the correct option is D.
- 32. (A)** Apply the divisibility rule of 11 on each option.
Therefore, the correct option is A.
- 33. (B)** There are 150 such numbers.
Therefore, the correct option is B.
- 34. (C)** 2 is the smallest prime number.
Therefore, the correct option is C.
- 35. (D)** There are 15 prime numbers less than 50 (2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47).
Therefore, the correct option is D.
- 36. (D)** None of the given numbers is the smallest prime number.

Therefore, the correct option is D.

- 37. (C)** The sum of two odd numbers is always even.
Therefore, the correct option is C.
- 38. (C)** Dividend = $68 \times 269 + 0 = 18292$
Dividing 18292 by 67 we get 1 as remainder.
Therefore, the correct option is C.
- 39. (C)** Let's assume the number is $56 + 29 = 85$
 $85 = 56 \times 1 + 29$
When it is divided by 8,
$$\Rightarrow \frac{85}{8} = 8 \times 10 + 5 \text{ or } 29 = 3 \times 8 + 5$$

Hence, the remainder is 5.
Therefore, the correct option is C.
- 40. (C)** Let x be the number and y be the quotient.
Then, $x = 357y + 39 =$
$$\Rightarrow (17 \times 21 \times y) + (17 \times 2) + 5 =$$
$$\Rightarrow 17 \times (21y + 2) + 5$$

Required number = 5.
Therefore, the correct option is C.
- 41. (A)** 4 will be the remainder in this case due to the square condition applied here.
Therefore, the correct option is A.
- 42. (B)** Let the smaller number be x . Then the larger number = $(x + 1365)$.
$$\Rightarrow x + 1365 = 6x + 15$$
$$\Rightarrow 5x = 1350$$
$$x = 270$$

Smaller number = 270.
Therefore, the correct option is B.
- 43. (C)** Number = (12×35)
Correct quotient = $420 \div 21 = 20$
Therefore, the correct option is C.
- 44. (D)** $\frac{12}{35}$ is reciprocal.
Therefore, the correct option is D.
- 45. (B)** Let the required fraction be X .
Then, $X - \frac{1}{X} = \frac{9}{20}$



$$\text{So, } 1 - \frac{x^2}{x} = \frac{9}{20}$$

$$\Rightarrow x = \frac{4}{5}$$

Therefore, the correct option is B.

- 46. (B)** Clearly, $(2272 - 875) = 1397$, is exactly divisible by x .

$$\text{Now, } 1397 = 11 \times 127$$

The required 3-digit number is 127, the sum of whose digits is 10.

Therefore, the correct option is B.

- 47. (B)** 47619 is such a number.

Therefore, the correct option is B.

- 48. (B)** Let the two consecutive even integers be $2n$ and $(2n+2)$.

Then,

$$(2n+2)^2 - (2n)^2 = (2n+2+2n)(2n+2-2n)$$

$$\Rightarrow 2(4n+2)$$

$$\Rightarrow 4(2n+1), \text{ which is divisible by 4.}$$

Therefore, the correct option is B.

- 49. (A)** Let the two consecutive odd integers be $(2n+1)$ and $(2n+3)$.

$$\text{Then, } (2n+3)^2 - (2n+1)^2$$

$$\Rightarrow (2n+3+2n+1)(2n+3-2n-1)$$

$$\Rightarrow (4n+4) \times 2$$

$$8(n+1), \text{ which is divisible by 8.}$$

Therefore, the correct option is A.

- 50. (C)** $(6n^2 + 6n) = 6n(n+1)$, which is always

divisible by 6 and 12 both, since $n(n+1)$ is always even.

Therefore, the correct option is C.

30 Percentages



SYNOPSIS

- Introduction
- Percentage chart
- Percentage difference formula
- Important percentage formulas
- Summary

INTRODUCTION

The percentage concept is used to calculate the proportion of a value in relation to its original value. Percentages are often used in business to calculate a company's profit or loss percentage. It is also used in schools and universities to indicate the grades that students have obtained. A percentage formula can be used to assess the students' grades. In general, a percentage is defined as a ratio of any value to the total value multiplied then by 100. The percentage symbol % is used to denote the percentage.

Percentage is a relative figure that represents one-hundredth of a quantity. One per cent (1%) equals one hundredth; two per cent (2%) equals two hundredths, and so on.

There are no dimensions to the %. It means they are integers with no dimensions. When we say 80% of a number, we mean 80 percentage of the whole. It can also be stated as a decimal or a fraction, for example, 0.76% or 0.17%. The proportion of marks earned by students in any subject is calculated during the examination by using the concept of percentage.

Example:

A student received a 78% on his exam. So, this percentage is derived by dividing a student's total marks into all subjects by the total maximum marks and then multiplying the result by 100.

Some other understanding-based examples of percentages are:

10% is equal to $\frac{1}{10}$ fraction.

20% is equal to $\frac{1}{5}$ fraction.

25% is equal to $\frac{1}{4}$ fraction.

50% is equal to $\frac{1}{2}$ fraction.

75% is equal to $\frac{3}{4}$ fraction.

90% is equal to $\frac{9}{10}$ fraction.

Example:

Find the 40% of 300.

Solution:

$$\frac{40}{100} \times 300 = 40 \times 3 = 120$$

Hence, 120 is 40% of 300.

Example:

In a class of 72 students, 87.5% of the students cleared the GATE examination. How many did not clear it?

Solution:

The number of students who did not clear the

$$\text{GATE exam} = 12.5 \times \frac{72}{100} = 9.$$

**Example:**

The population of a town increases at the rate of 3.7% each year. It is 31,110 now. What was it like last year?

Solution:

Let's assume last year it was x

$$\text{So, } 31,110 = x \left(1 + \frac{3.7}{100}\right)$$

Hence, $x = 30,000$.

Example:

Kyler needed Rs. 800 to cover her fees. Her brother provided 20% of the funding, and her mother provided 30% of the remaining funds. She had Rs. 200 in her bank account. How much extra (in Rs.) does she require?

Solution:

Required amount = 800

$$\text{From her brother she got} = 800 \times \frac{20}{100} = 160$$

$$\text{From her mother she got} = (800 - 160) \times \frac{30}{100} =$$

$$640 \times \frac{30}{100} = 192$$

From the bank she got Rs. 200

Now, she needs $800 - (160 + 192 + 200) = 800 - 552 = \text{Rs. } 248$ more

Example:

Allen, the barber, shaved 40% of his customers and gave a haircut to 80% of his customers. He charged Rs. 7 for a shave and Rs. 5 for a haircut. If 20% of customers who opted for a shave also had a haircut, what were Allen's earnings if he had 75 customers (in Rs.)?

Solution:

Total customers = 75

$$\text{Numbers of customers shaved} = 75 \times \frac{40}{100} = 30$$

Number of customers who got hair cut

$$= 75 \times \frac{80}{100} = 60$$

Hence, his total income = $(30 \times 7) + (60 \times 5) = 210 + 300 = \text{Rs. } 510$.

Percentage chart

The percentage chart is given here for fractions converted into percentages.

FRACTIONS	PERCENTAGE
$\frac{1}{2}$	50%
$\frac{1}{3}$	33.33%
$\frac{1}{4}$	25%
$\frac{1}{5}$	20%
$\frac{1}{6}$	16.66%
$\frac{1}{7}$	14.28%
$\frac{1}{8}$	12.50%
$\frac{1}{9}$	11.11%
$\frac{1}{10}$	10%
$\frac{1}{11}$	9.09%
$\frac{1}{12}$	8.33%
$\frac{1}{13}$	7.69%
$\frac{1}{14}$	7.14%
$\frac{1}{15}$	6.66%
$\frac{1}{16}$	6.25%
$\frac{1}{17}$	5.88%
$\frac{1}{18}$	5.55%
$\frac{1}{19}$	5.26%
$\frac{1}{20}$	5%



Percentage difference formula

If we are given two numbers suppose X and Y , and the question asked us to find the percentage difference between the two numbers, then we will apply the below-given formula to find the percentage difference quickly:

$$\text{Percentage difference} = \frac{|X - Y|}{\left(\frac{X + Y}{2}\right)} \times 100$$

Example:

If two numbers are given suppose 20 and 30, and then the percentage difference between these two numbers is:

Solution:

$$\text{Percentage difference} = \frac{|20 - 30|}{\left(\frac{20 + 30}{2}\right)} \times 100$$

On solving the above equation, $\frac{10}{25} \times 100 = 40$

Therefore, the percentage difference between 20 and 30 is 40%.

Some other important percentage formulas

1. Percentage formula in case of fraction:

$$\frac{\text{Numerator}}{\text{Denominator}} \times 100$$

Example:

Convert $\frac{3}{16}$ into a percentage.

Solution:

Using above given formula,

$$\frac{3}{16} \times 100 = 3 \times \frac{25}{4}$$

$$\Rightarrow \frac{75}{4} \% \text{ or } 18.75\%$$

2. Percentage change formula:

$$\frac{\text{New value} - \text{Original value}}{\text{Original value}} \times 100$$

Example:

Annie works in a supermarket for \$10.00 per hour. If her pay is increased to \$12.00, then what is her percentage increase in pay?

Solution:

Here, original value = 10, and new value = 12

$$\frac{12 - 10}{10} \times 100$$

$$\frac{2}{10} \times 100 = 20$$

- In general, a percentage is defined as a ratio of any value to the total value multiplied then by 100.
- The percentage symbol % is used to denote the percentage.

- Percentage difference = $\frac{|X - Y|}{\left(\frac{X + Y}{2}\right)} \times 100$

- Percentage formula in case of fraction:

Chapter Summary



$$\frac{\text{Numerator}}{\text{Denominator}} \times 100$$

- Percentage change formula:
$$\frac{\text{New value} - \text{Original value}}{\text{Original value}} \times 100$$

PRACTICE QUESTIONS

1. Aaron receives 35% of the maximum marks in an exam, while John receives 50% of the maximum marks. Aaron would have received the same percentage as John if he had received 45 more marks.

What is the exam's maximum score?

- A. 300
- B. 400
- C. 350
- D. 450



2. Peter received 20% of the maximum marks in an exam, but he failed by 30 marks. Peter received 40% of the same and was passed by ten marks. What were the minimum passing marks for the exam?
- A. 70
B. 80
C. 120
D. 60
3. Tom's pay is 12% more than Jerry's. Jerry's pay is lesser than Tom's by what percentage?
- A. 10.71%
B. 10%
C. 9.71%
D. 11.71%
4. Benett is 25% older than Cyril, his younger brother. Cyril is younger than Benett by what percentage?
- A. 20%
B. 25%
C. 30%
D. 16.66%
5. Patrick's pay is reduced by 16% initially and increased by 16% in the next month. Patrick's current income is approximately how much less than his starting salary?
- A. 2% less
B. 2.16% less
C. 2.56% less
D. 2.009% less
6. James's salary is boosted by 20% for the first month and then reduced by 20% for the second month. What is James's current salary in relation to his starting salary?
- A. 2% more
B. 3% less
C. 4% less
D. 5% more
7. A commission of 5% on the first Rs. 5000 and 2.5% on the rest of the selling price is charged by an agent. What was his commission if the selling price was Rs. 30,600?
- A. 850
B. 1015
C. 890
D. 1020
8. In a two-candidate election, 6% of voters did not cast their ballots. Won by 3000 votes, the winning candidate received 48% of the total votes cast. How many people cast ballots in the election?
- A. 12000
B. 10000
C. 15000
D. 20000
9. To be eligible for a scholarship, you must get at least 80% in an examination. Tony received 1005 marks but was just 13% of the way to receiving the scholarship. What was the highest possible score?
- A. 2000
B. 2500
C. 3000
D. 2800
10. Marcos has a particular quantity of mangoes, with 15% of them being rotten. He offers his friend 60% of the remaining mangoes, leaving him with 102 mangoes. How many mangoes did he have when he first started?
- A. 200
B. 300
C. 400
D. 500
11. In a two-candidate election, one candidate received 55% of the total valid votes, while 20% of the total votes cast were invalid. What was the number of legitimate votes



- received by the other candidate if the total votes cast were 7500?
- A. 2500
B. 2700
C. 3000
D. 3200
- 12.** A person's daily income has been boosted by 75%, and he now receives Rs. 75. How much did he earn on a daily basis before the raise?
- A. 41
B. 42.85
C. 45.85
D. 45
- 13.** What is 66% of 66.67?
- A. 22
B. 11
C. 33
D. 44
- 14.** When Albert's income is 25% more than Pinto's, how much less does Pinto's income have to be?
- A. 10%
B. 15%
C. 18%
D. 20%
- 15.** In an exam, a candidate receives 20% of the maximum score and fails by ten marks. Another candidate receives 42% of the maximum possible marks, which are 12 marks more than the passing grade. What is the exam's maximum score?
- A. 80
B. 90
C. 110
D. 100
- 16.** In a poll between A, B, and C, a total of 600 votes were cast. A received 30% of the vote, B received 360 votes, with the remaining votes going to C. In relation to his nearest rival, what percentage of the overall votes did the winner receive?
- A. 100%
B. 500%
C. 300%
D. 200%
- 17.** When two candidates, A and B, are running for office election. A received 60% of the total valid votes, 15% of the total 5,00,000 ballots cast was considered illegitimate. What is the total number of votes that B has received?
- A. 100000
B. 150000
C. 170000
D. 190000
- 18.** An accountant charges a fee of 14% of the transaction amount. How much did Rory pay the accountant if he completed a transaction of Rs. 14,00,000 for him?
- A. 200000
B. 196000
C. 190000
D. 193000
- 19.** A carpenter manufactured 50 chairs, 14 of which are defective. What is the percentage of chairs that are in good condition?
- A. 50%
B. 75%
C. 80%
D. 72%
- 20.** In an election between A and B, A received 65% of the vote and won the election by a margin of 2748 votes. What is the total number of votes cast if no vote is declared invalid?
- A. 9160
B. 9000
C. 9169
D. 9167
- 21.** In a contest between Kurian and Pablo, Kurian received 25% of the vote but was defeated. Pablo received 14000 votes more



- than Kurian in the election. What is the total number of votes cast?
- A. 25000
B. 27000
C. 26000
D. 28000
- 22.** Nathan is 22% more productive than Lyon. In comparison to Nathan, how inefficient is Lyon?
- A. 17.03%
B. 18.03%
C. 19.03%
D. 20.03%
- 23.** Simon's total salary was reduced by 35%. He spent 45% of the money left over, leaving him with Rs. 2500. What was his overall salary?
- A. 6991
B. 6992
C. 6993
D. 6994
- 24.** If A's salary is 25% more than B's salary, what percentage of B's salary is less than A's salary?
- A. 10%
B. 20%
C. 15%
D. 18%
- 25.** Only 75% of the total number of eligible voters voted in a presidential election. The victorious candidate received 58% of all votes cast and won by a margin of 12000 votes. What was the total number of voters who were eligible to vote?
- A. 125000
B. 100000
C. 150000
D. 175000
- 26.** Stark's pay is set at 80% of that of his boss. If he spends 40% of his salary on rent and 20% on food and is left with Rs. 2400, what is Stark's boss's salary?
- A. 6200
B. 6250
C. 6225
D. 6275
- 27.** In the market, the price of rice increases by 14%. A family's rice intake must be reduced by what percentage in order to avoid an increase in rice expenditure?
- A. 12.28%
B. 12.50%
C. 12.47%
D. 12.98%
- 28.** Walter's pay has been boosted by 10%. At this point, his pay is 10% less than his boss's. What percentage of Walter's pay is smaller than his boss's salary if his salary is now lowered by 10%?
- A. 17%
B. 18%
C. 19%
D. 20%
- 29.** Jessi's salary is reduced by 40%, then boosted by 20%, and lastly increased by 20%. What percentage of his beginning pay does he earn now?
- A. 12.6% less
B. 11.6% less
C. 14.6% less
D. 13.6% less
- 30.** Robert, Sandy and Phil ran for office election in the same election. Robert received 40% of the vote and so won the election. Sandy and Phil received 35% and 25% of the overall vote, respectively. What is the amount of votes Robert received in the election if Sandy received 2000 more than Phil?
- A. 8000
B. 8100
C. 8200
D. 8500



31. A total of 45,000 individuals took part in the exam, with 40% of them being female. What was the percentage of passed students if 70% of the boys and 75% of the girls passed the exam?
- A. 70%
B. 71%
C. 72%
D. 73%
32. A mine's lead ore provides 60% metal, with 0.75% of that being silver. How much silver can be extracted from 8000 kg of ore?
- A. 34 grams
B. 35 grams
C. 36 grams
D. 37 grams
33. What percentages will the volume of a cone rise if the height and base radius of the cone are both increased by 50%?
- A. 237%
B. 237.5%
C. 238%
D. 238.5%
34. When 80% of a number is added to 80, the outcome is the same number. Which of the following numbers is the correct answer?
- A. 400
B. 500
C. 600
D. 700
35. What percentage does the volume of a cone rise if the radius and height of the cone are increased by 200% and 100%, respectively?
- A. 1700%
B. 1800%
C. 2000%
D. 2500%
36. What percentage of 800 is $\frac{2}{3}$ less than 0.2%?
- A. 0.50
B. 0.73
C. 0.93
D. 1
37. Rehan's pay is equal to half of Mariyam's. Rehan's pay is equivalent to 80% of Denver. What is Mariyam's monthly income if the total compensation of the three for a month is Rs. 22000?
- A. 2000
B. 3000
C. 4000
D. 5000
38. By how much is $\frac{2}{3}$ less than 0.2% of 800?
- A. 72%
B. 73%
C. 74%
D. 75%
39. What is the increase in the area of a rectangle if the length and breadth of the rectangle are raised by 20% and decreased by 10%, respectively?
- A. 5%
B. 6%
C. 7%
D. 8%
40. What is the number if 41% of a number is less than 76% of the same number by 105?
- A. 200
B. 300
C. 400
D. 500
41. What is the increase in the area of a square if the sides of a square are increased by 20%?
- A. 33%
B. 44%
C. 55%
D. 66%



- 42.** A total of 4000 people applied for an engineering entrance exam. One-fourth of these were girls. What is the number of successful candidates if 80% of the girls and 95% of the boys did not make it through?
- A. 350
B. 450
C. 550
D. 650
- 43.** From a 6-litre sugar solution containing 4% sugar, 1 litre of water is evaporated. What is the sugar content of the remaining solution?
- A. 4%
B. 4.8%
C. 5%
D. 5.8%
- 44.** A fruit dealer has some bad oranges, about 2% of which are rotten. He sells 95% of the remaining oranges, leaving him with only 49. How many oranges did he start with?
- A. 100
B. 10000
C. 1000
D. 100000
- 45.** There are two papers in an examination. A student receives a 30% grade on the first paper out of 180. How much should he score in the second paper if he has to earn 50% overall and the second paper is worth 150 marks?
- A. 222
B. 333
C. 444
D. 111
- 46.** A fabric dealer claims to sell cloth at cost price, but his measurements are inaccurate. His scale reads 1 metre for 95 centimetres. What is his approximate % gain or loss?
- A. 5.25% gain
B. 5% gain
C. 5.50% gain
D. 5.75% gain
- 47.** A 20% increase in bus fares resulted in a 10% decrease in passenger numbers. Despite this, the bus depot's daily collection climbed by Rs. 160. Before the rise, how much money did they gather on a daily basis?
- A. 1000
B. 2000
C. 3000
D. 4000
- 48.** A 25% reduction in the price of rice allows a buyer to get 3 kg more for Rs. 189. What was the original per-kilogram price of rice?
- A. 19 per kg
B. 20 per kg
C. 21 per kg
D. 22 per kg
- 49.** At EPS, 35 boys and 25 girls took part in the annual sports day. 60% of the students obtained certificates. What is the percentage of girls who earned certificates if the total number of guys who received certificates was 12?
- A. 50%
B. 33.33%
C. 25%
D. 66.67%
- 50.** The diameter of a circle is expanded by 15%. What is the area's percentage increase?
- A. 32.25%
B. 32%
C. 32.50%
D. 32.75%



SOLUTIONS

1. **(A)** Let maximum marks be X
Difference = 15%
So, 15% of $X = 45$
 $\frac{15}{100} \times X = 45$
 $X = 300$
Therefore, option A is correct.
2. **(A)** Let total marks be X
So, $\frac{20X}{100} + 30 = \frac{40X}{100} - 10$
 $\frac{20X}{100} = 40$
 $X = 200$
Hence, passing marks = 70
Therefore, option A is correct.
3. **(A)** Let Jerry's salary be Rs. 100
So, Tom's salary be Rs. 112
 $112 - \frac{100}{112} \times 100$
 $= 10.71\%$
Therefore, option A is correct.
4. **(A)** $\frac{25}{125} \times 100$
 $= 20\%$
Therefore, option A is correct.
5. **(C)** Let Patrick's pay be Rs. 100
So, after 16% reduction = 84
And after 16% increment = $84 \times 1.16 = \text{Rs. } 97.44$
Hence, the difference = $100 - 97.44$
 $= \text{Rs. } 2.56$
Therefore, option C is correct.
6. **(C)** Let James's starting salary be 100
So, after 20% increment it becomes 120
and after 20% reduction = 120×0.80
 $= \text{Rs. } 96$
Hence, difference $100 - 96 = 4$
Therefore, option C is correct.
7. **(C)** Commission due = $\frac{5}{100} \times 5000 + \frac{2.5}{100}$
 $\times (30600 - 5000)$
 $= \text{Rs. } 890$
- Therefore, option C is correct.
8. **(C)** Voters who voted = $100 - 6 = 94\%$
So the losing candidate got = $94 - 48 = 46\%$
Difference = 2%
Let the total voters be X
 $2\% \text{ of } X = 3000$
On solving
 $X = 150,000$
Therefore, option C is correct.
9. **(B)** Difference = $80 - 13 = 67\%$
So, let's assume max marks be X
 $67\% \text{ of } X = 1005$
 $X = 1500$
Therefore, option B is correct.
10. **(B)** Let's assume Marcos have 100 mangoes at the beginning
So, the number of rotten mangoes = 15
And number of mangoes given to friend = $85 \times 0.60 = 51$
Remaining mangoes = $85 - 51 = 34$
According to question,
 $34X = 102$
So, $100X = \frac{102}{34} \times 100$
 $= 300$
Therefore, option B is correct.
11. **(B)** Valid votes = $7500 - \frac{20}{100} \times 7500$
 $= 6000$
So, $55\% \text{ of } 6000 = 3300$
Other candidate got = $6000 - 3300$
 $= 2700$
Therefore, option B is correct.
12. **(B)** Let his daily wage before increase be X
 $x + \frac{75x}{100} = 75$
 $x = 42.85$
Therefore, option B is correct.
13. **(D)** $\frac{200}{3} \times 66\%$
 $= 44$
Therefore, option D is correct.



- 14. (D)** Let Pinto's income be 100
So, Albert's income is 125
Hence, it has to be 20% less
Therefore, option D is correct.
- 15. (D)** Let the max marks be X
 $\frac{20X}{100} + 10 = \frac{42X}{100} - 12$
 $X = 100$
Therefore, option D is correct.
- 16. (D)** $A = \frac{30}{100} \times 600 = 180$ votes
 $B = 360$ votes
So, $C = 60$ votes.
Hence, winner B received 200% votes as compared to his rival A
Therefore, option D is correct.
- 17. (C)** Invalid votes = $\frac{15}{100} \times 500000 = 75000$
So, valid votes = 425000
 $A = \frac{60}{100} \times 425000$
 $= 255000$
So, $B = 170000$
Therefore, option C is correct.
- 18. (B)** Fee paid = $\frac{14}{100} \times 1400000$
 $= 196000$
Therefore, option B is correct.
- 19. (D)** Good chairs = $50 - 14 = 36$
Percentage = $\frac{36}{50} \times 100$
 $= 72\%$
Therefore, option D is correct.
- 20. (A)** A votes = 65%
B votes = $100 - 65 = 35\%$
Difference = $65 - 35 = 30\%$
So, $30\% = 2748$
So, $100\% = 9160$
Therefore, option A is correct.
- 21. (D)** Let total votes be X
Pablo votes = 75%
Kurian votes = 25%
So, $50\% \text{ of } X = 14000$
 $X = 28000$
Therefore, option D is correct.
- 22. (B)** $\frac{22}{122} \times 100$
 $= 18.03\%$
Therefore, option B is correct.
- 23. (C)** Remaining salary after reduction = 65%
45% of the remaining = $65 \times 0.45 = 29.25\%$
Remaining salary in hand = 35.75% which is equal to 2500
So, total salary = 6993
Therefore, option C is correct.
- 24. (A)** Let B's salary be 100
So, A's salary be 125
 $125 - \frac{100}{125} \times 100$
 $= 20\%$
Therefore, option A is correct.
- 25. (B)** Difference between votes = $58 - 42 = 16\%$ which is equal to 12000
So, $100\% = 75000$, i.e., voters voted, i.e., 75%
Hence, total eligible voters = 100000
Therefore, option B is correct.
- 26. (B)** Let's assume Stark's salary be 100
So, rent = 40
Food = 12
Remaining salary = 48 which is equal to 2400
So, Stark's total salary = 5000
Hence, his boss's salary = 6250
Therefore, option B is correct.
- 27. (A)** $\frac{14}{114} \times 100$
 $= 12.28\%$
Therefore, option A is correct.
- 28. (C)** Let's take Walter's salary to be 100.
So, his new salary = 110
Percentage smaller as compared with boss = 19%
Therefore, option C is correct.
- 29. (D)** Let's take salary be 100
After reduction = 60
After increment = 72
After another increment = 86.4
Difference = $100 - 86.4$
 $= 13.6$
Therefore, option D is correct.



30. (A) Sandy got 2000 votes more than Phil
 So $35 - 25\% = 2000$
 Let total votes be X
 So, 10% of $X = 2000$
 $X = 20000$
 So, votes received by Robert = 8000
 Therefore, option A is correct.

31. (C) Total students = 45000
 So, female = 18,000
 Then, males = $45000 - 18000 = 27000$
 70% of the boys = 18900
 75% of the girls = 13500
 So, total passed students = $18900 + 13500 = 32400$
 And, 32400 is 72% of 45000.
 Therefore, option C is correct.

32. (C) Silver obtained = $\frac{0.75}{100} \times \frac{60}{100} \times 8000$
 $= 36$ grams
 Therefore, option C is correct.

33. (B) New volume = $\frac{1}{3}\pi \left(\frac{3}{2}R\right)^2 \times \frac{3}{2}H$
 $= \frac{9}{8}\pi R^2H$
 Difference = $\left(\frac{9}{8} - \frac{1}{3}\right)\pi R^2H$
 $= \frac{19}{24}\pi R^2H$
 So, increase in volume = 237.5%
 Therefore, option B is correct.

34. (A) Let the number be X
 $\frac{80X}{100} + 80 = X$
 $X = 400$
 Therefore, option A is correct.

35. (A) The new radius and height are $3r$ and $2h$ respectively.
 So, the new volume will become = $6\pi r^2h$
 Now, increase in volume
 $= 6\pi r^2h - \frac{1}{3}\pi r^2h$
 $= \frac{17}{3}\pi r^2h$
 $17 \times 100 = 1700\%$
 Therefore, option A is correct.

36. (C) 0.2% of $800 = \frac{8}{5}$
 Difference = $\frac{8}{5} - \frac{2}{3}$
 $= 0.93$
 Therefore, option C is correct.

37. (C) Let Denver's salary be 100
 Rehab's salary = 80
 Mariyam's salary = 40
 Total = $100 + 80 + 40 = 220 = 22000$
 Hence, Mariyam's salary = 4000
 Therefore, option C is correct.

38. (D) 0.2% of $800 = \frac{8}{5}$
 Now, difference = $\frac{8}{5} - \frac{2}{3} = \frac{14}{15}$
 And, this difference is 75%
 Therefore, option D is correct.

39. (D) New area = $1.2L \times 0.9B$
 $= 1.08LB$
 Change in area = $\frac{1.08LB}{LB} \times 100$
 $= 8\%$
 Therefore, option D is correct.

40. (B) Difference = $76 - 41 = 35$
 35% of $X = 105$
 $X = 300$
 Therefore, option B is correct.

41. (B) Let the original side be X , then area = X^2
 Now, new area = $(1.2X)^2 = 1.44X^2$
 Percentage change = $1.44\left(\frac{X}{X}\right)^2 = 1.44$
 OR 44%
 Therefore, option B is correct.

42. (A) Number of girls who gave the exam = 1000 ($\frac{1}{4}$ of 4000)
 Number of boys = $4000 - 1000 = 3000$
 Now, as given 80% girls failed then 20% succeeded
 Similarly, 95% boys failed then 5% succeeded
 So, girls who got through = 20% of 1000 = 200
 Boys who got through = 5% of 3000 = 150



Hence, total successful candidates = 200
+ 150 = 350
Therefore, option A is correct.

43. (B) Sugar = $\frac{4}{100} \times 6$

= 0.24L

% of sugar in remaining = $\frac{0.24}{5} \times 100$

= 4.8%

Therefore, option B is correct.

44. (C) Let he had X oranges

Now, according to the question, $\frac{2}{100} X +$

$\frac{95}{100} (X - \frac{2}{100} X) + 49 = X$

$\frac{1}{50} X + \frac{19}{20} X - \frac{19}{1000} X - X = -49$

$20X + 950X - 19X - 1000X = -49 \times 1000$

$X = 1000$

Therefore, option C is correct.

45. (D) Total marks = 180 + 150 = 330

50% of 330 = 165

30% of 180 = 54

He should score = 165 - 54 = 111

Therefore, option D is correct.

46. (A) On every 95 cm, the trader gains 5 cm

So, on a trade of Rs. 95 his gain is Rs. 5

So, gain % = $\frac{5}{95} \times 100 = 5.25\%$

Therefore, option A is correct.

47. (B) Let the bus fare be X and number of passenger be Y

So, total collection = XY

Now, after the hike, total collection,

$\left(X + \frac{20}{100} X\right) \left(Y - \frac{10}{100} Y\right) = 1.08 XY$

Increase in collection = $1.08XY - XY = 0.08XY$

According to the question, $0.08XY = 160$

So, $XY = 2000$

Therefore, option B is correct.

48. (C) New price = $\frac{3}{4}$ Initial price and total

amount is constant that is Rs. 189

New quantity = $\frac{4}{3}$ Initial quantity

New quantity is 3 kg more than original quantity. Hence, original quantity was 9 kg

Then, original price = $\frac{189}{9} = \text{Rs. 21 per kg.}$

Therefore, option C is correct.

49. (D) Total students participating = 35 + 25 = 60

Total students receiving certificates = 60 ×

$0.6 = 36$

Total girls receiving certificates = 36 - 12

= 24

Percentage of girls receiving certificates =

$\frac{24}{36} \times 100 = 66.66\%$

Therefore, option D is correct.

50. (A) $\pi \times r^2 = \pi \times (1.15r)^2 = \pi \times (1.3225)$

Hence, percentage increase = 32.25%

Therefore, option A is correct.

PRACTICE QUESTIONS

Questions 1–5: Vishesh is living in Delhi and works in a reputed company. The details of the monthly expenditure of Vishesh are given here. He spends 19% of the total monthly expenditure on Rent and 5% of the total monthly expenditure on Transport. He spends 21% of the total monthly expenditure on Food and 15% of the total monthly expenditure on Education. He spends 30% of the total monthly expenditure on other expenses and he saves 10% of his total expenditure.

1. If Vishesh is earning Rs. 50,000 per month, then what's his total expenditure on rent, transport and education?

- A. 20000 B. 19000
C. 20500 D. 19500

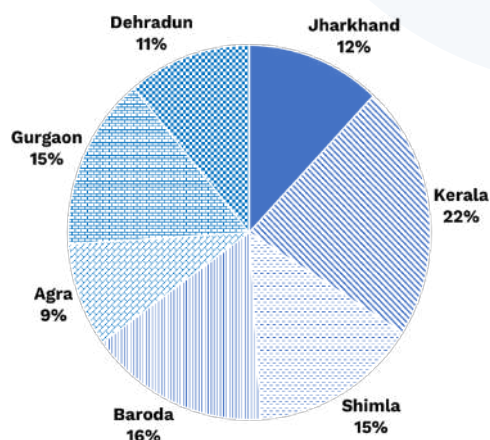
2. If Vishesh is earning Rs. 20,000 per month, then how much more does he spend on food than on education and transportation put together?

- A. 300 B. 200
C. 400 D. 250



3. In 'others,' Vishesh spends 40% on the accessories of his car which amounts to Rs. 3000. What's his total expenditure on transportation?
- A. 1050
B. 1200
C. 1200
D. 1250
4. If a pie chart is drawn by taking the savings as one part and total expenses as the second part, then what is the angle made by total expenses?
- A. 330°
B. 280°
C. 324°
D. 320°
5. Due to an increase in the price of fuel, Vishesh has to spend Rs. 300 more on transportation. This is 20% more than usual. If other expenses remain constant, find his new savings.
- A. 2700
B. 2600
C. 2500
D. 2400

Questions 6–10: The following pie gives the data of the total number of passengers travelling from Delhi to different districts = 12000



6. The number of passengers travelling from Delhi to Baroda is what percentage more than the number of passengers travelling to Jharkhand?
- A. 20%
B. 33%
C. $33\frac{1}{3}\%$
D. 30%
7. The number of passengers travelling from Delhi to Gurgaon is what percentage of the total number of passengers travelling from Delhi to Dehradun and Agra together?
- A. 70%
B. 75%
C. 72%
D. 68%
8. What is the average number of passengers travelling from Delhi to Shimla, Kerala and Baroda together?
- A. 2120
B. 2020
C. 2000
D. 2100
9. What will be the central angle for passengers travelling from Delhi to Baroda if it is put in the form of a pie-chart?
- A. 45%
B. 57%
C. 57.6%
D. 50%
10. Among the passengers who are travelling from Delhi to Kerala, 42% are women and 38% are men. What is the number of children?
- A. 428
B. 528
C. 280
D. 410

SOLUTIONS

1. **(D)** Total percentage of salary spent on rent, transport and education = $\{19 + 5 + 15\} = 39\%$

Amount spend on rent, transport and education = 39% of 50,000 = Rs. 19500
Therefore, option D is correct.
Therefore, option B is correct.



2. **(B)** The total percentage of salary spent on education and transportation together is $(15 + 5) = 20\%$
His expenditure is 21% on food.
Vishesh spends $21 - 20 = 1\%$ more on food.
Amount = 1% of 20,000 = Rs. 200
3. **(D)** Expenditure of Vishesh on car = 40% of 30% = 12%
Given 12% = 3000
(Transportation) 5% = x
 $x = (5 \times \frac{3000}{12}) = \text{Rs. } 1250$
Therefore, option D is correct.
4. **(C)** Vishesh spends 90% of his income
Angle made by this division = $\frac{90 \times 360}{100}$
 $= 324^\circ$
Therefore, option C is correct.
5. **(A)** Vishesh spends 5% on transportation
His new expenditure on transportation = 120% of 5% = 6%
Increase in expenditure = 1%
His new savings = 10% - 1% = 9%
 $= 1\% = 300$
 $9\% = x$
 $x = 9(300) = \text{Rs. } 2700$
Therefore, option A is correct.
6. **(C)** The number of passengers from Delhi to Baroda = 16%
Number of passengers from Delhi to Jharkhand = 12%

$$\text{Required \%} = \frac{16 - 12}{12} \times 100\% = \frac{4 \times 100}{12}$$

$$= 33 \frac{1}{3} \%$$

Therefore, option C is correct.

7. **(B)** Number of passengers travelling from Delhi to Gurgaon = 15%
Number of passengers travelling from Delhi to Dehradun and Agra = 11% + 9% = 20%

$$\text{Required \%} = \frac{15}{20} \times 100 = 75\%$$

Therefore, option B is correct.

8. **(A)** Total percentage of Shimla, Kerala and Baroda = $(15\% + 22\% + 16\%) = 53\%$
Therefore, 53% of 12000
 $= (50\% \text{ of } 12000 + 3\% \text{ of } 12000) = 6000 + 360 = 6360$

$$\text{Required average} = \frac{6360}{3} = 2120$$

Therefore, option A is correct.

9. **(C)** Central angle = $\frac{16}{100} \times 360^\circ = 57.6^\circ$

Therefore, option C is correct.

10. **(B)** Number of children travelling from Delhi to Kerala = $100\% - (42 + 38)\% = 100\% - 80\% = 20\%$

Total number of children travelling from Delhi to Kerala = 20% of 22% of 12000

$$= \frac{20}{100} \times \frac{22}{100} \times 12000 = 528$$

Therefore, option B is correct.

31 Averages



SYNOPSIS

- Introduction
- Average
- Properties of average
- Weighted average or grouped average
- Common types of questions asked
- Summary

INTRODUCTION

The word average is quite often used in our day-to-day life. The average salary in a company, the average score of a team, the average marks of a class, and many other similar phrases are some examples of average that we encounter in our daily routine.

Questions based on the concepts of average are being frequently asked in GATE, JEE Main, and other engineering entrance examinations. This topic is easy to understand, which makes it easy for the test takers to score more in these exams.

Average

The average is a mean or middle value of a set of data, which can be calculated by dividing the total sum of all values by the total number of values in the given data.

The formula for calculating average:

$$\text{Average} = \frac{\text{Total sum of values}}{\text{Number of values}}$$

Example:

If A, B, C, and D are the partners in a firm, the firm gains a profit of Rs. 1 lakh, which is to be divided equally among the partners. Then the average profit gained by a partner can be calculated as:

Solution:

The average profit of the partner is given by

$$\text{Average} = \frac{\text{Total sum of values}}{\text{Number of values}}$$

$$\text{Average profit} = \frac{1,00,000}{4} = \text{Rs. } 25,000$$

Therefore, each partner would get an average profit of Rs. 25,000.

Properties of average

1. In a given set of data, the average always lies between its maximum and minimum value.
2. If a constant is added or subtracted to each value in the data, then the average also increases or decreases respectively by the same constant.
3. If each value in the given set of data is multiplied or divided by a constant, then the average also gets multiplied or divided by the same constant.

Weighted average or grouped average

When the average of two or more groups with the same or the different number of values is given, and we have to find the average of the values of all the groups taken together, then the combined average of the groups is known as the weighted average or grouped average.

If there are two groups with 'A1' and 'A2' as the number of elements or values in them, let 'P1' and 'P2' be their averages, respectively. Then the combined average of these two groups taken together can be calculated by: weighted

$$\text{average} = \frac{A1P1 + A2P2}{A1 + A2}$$



Let us assume if there are ' n ' groups with ' $A_1, A_2, A_3, A_4, A_5, \dots, A_n$ ' as the number of elements or values in these groups and ' $P_1, P_2, P_3, P_4, P_5, \dots, P_n$ ' be the averages of these groups, respectively. Then the combined average of all these groups taken together can be calculated by:

weighted average

$$= \frac{A_1P_1 + A_2P_2 + A_3P_3 + A_4P_4 + A_5P_5 + \dots + A_nP_n}{P_1 + P_2 + P_3 + P_4 + P_5 + \dots + P_n}$$

Example:

The XYZ International School has two divisions in standard A, P and Q. The number of students in divisions P and Q is 42 and 35, respectively. The average weight of students in division P is 63 and that of students in division Q is 52. What is the average weight of standard A of XYZ International School?

Solution:

The average weight of the two divisions taken together is given by

$$\text{Weighted average} = \frac{A_1P_1 + A_2P_2}{A_1 + A_2}$$

$$\text{Weighted average} = \frac{42 \times 63 + 35 \times 52}{42 + 35}$$

Therefore, the average weight of standard A is 58.

Common types of questions asked

Case I

When a person is replaced by another person in a group, for example in cases of age, weight, height, etc. related problems.

1. If the average is increased:

The following formula will apply:

New person's weight = weight of person who left + (increase in the average \times total number of people)

Example:

The average weight of 8 persons is increased by 2.5 kg when one of them whose weight is 56 is replaced by a new man, the weight of the new man is:

Solution:

The new person's weight, if the average is increased, is given by

New person's weight = weight of person who left + (increase in the average \times total number of people)

$$\text{New person's weight} = 56 + (2.5 \times 8) = 76 \text{ kg}$$

2. If the average is decreased:

The following formula will apply:

New person's weight = weight of person who left + (decrease in the average \times total number of people)

Example:

The average weight of 10 persons is decreased by 2 kg when one of them whose weight is 50 is replaced by a new man; the weight of the new man is:

Solution:

The new person's weight if the average is decreased is given by

New person's weight = weight of person who left + (decrease in the average \times total number of people)

$$\text{New man's weight} = 50 + (-2 \times 10) = 30 \text{ kg}$$

Case II

When a person joins a group then there may be two cases:

1. If there is an increase in average, the following formula will apply:

New person's weight = earlier average + (increase in the average \times total number of people)

Example:

The average age of 10 students is 12 years. By the admission of a new student, the average age is increased to 13 years. The age of the new student is

Solution:

The average of new person's weight if there is an increase in average

New person's weight = earlier average + (increase in the average \times total number of people)



Age of a new student is $= 12 + (1 \times 11) = 23$ years

2. If there is an increase in average, the following formula will apply:

New person's weight = earlier average – (decrease in the average \times total number of people)

Example:

The average weight of 19 students is 15 kg. By the admission of a new student, the average weight is reduced to 14.8 kg. The weight of the new student is

Solution:

The average of new person's weight if there is a decrease in average

New person's weight = earlier average – (decrease in the average \times total number of people)

New student weight $= 15 - (0.2 \times 20) = 11$ kg

Case III

For calculating average speed in speed, time, and distance, the following formula will apply for calculating average speed:

Average speed $= \frac{2xy}{x+y}$ (wherein, x and y denote

for the different average speeds)

Example:

A covers a journey from Delhi to Jaipur by car at an average speed of 40 km/h and returns by scooter with an average speed of 24 km/h. What is his average speed during the whole journey?

Solution:

The average speed is given by

Average speed $= \frac{2xy}{x+y}$

Average speed $= \frac{2(40 \times 24)}{40 + 24} = \frac{1920}{64} = 30$ km/h

Case IV

Average when arithmetic progression (AP) series is odd:

The average of the odd AP series will be the term that is in the middle of the series.

Example:

The average for series 3, 6, 9, 12, 15, 18, 21, 24, and 27 is:

Solution:

The middle term of the series is 15. Therefore, its average will be 15.

Case V

Average when AP series is even:

If the AP series is even, then the average can be calculated from the average of two middle terms.

Example:

The average for series 2, 4, 6, 8, 10, and 12 is:

Solution:

The two middle terms of the series are 6 and 8. By calculating the average of both terms,

i.e., $\frac{6+8}{2} = 7$

Therefore, the average of the given series is 7.

Chapter Summary



- Average is an arithmetic mean of a given set of data.
- Average also changes accordingly if each value in the given set of data multiplies or divides by a constant.
- The average of two or more groups can be calculated by the weighted average.



PRACTICE QUESTIONS

1. The population of 5 cities is 35560, 27500, 30000, 25000, and 25600 people. What is the average population of a city?
A. 28742
B. 28562
C. 28732
D. 28730
2. The average weight of 48 employees is 70 kg. If the weight of two new employees is included, the average increases by 800 grams. What is the average weight of new employees?
A. 55 kg
B. 50 kg
C. 45 kg
D. 40 kg
3. The average of the first nine multiples of 5 is:
A. 23
B. 24
C. 25
D. 26
4. The average weight of 8 workers in a truck is increased by 1 kg when one of the workers, whose weight is 60 kg, is replaced by a new worker. What is the weight of the new worker?
A. 58 kg
B. 68 kg
C. 70 kg
D. 64 kg
5. Raj travels from Delhi to Gurgaon by train at an average speed of 80 km/h and returns by car with an average speed of 48 km/h. What is his average speed during the whole journey?
A. 60 km/h
B. 58 km/h
C. 54 km/h
D. 62 km/h
6. The average of 0.60204, 0.60203, 0.60202, and 0.60201 is:
A. 0.60203
B. 0.60204
C. 0.602025
D. 0.60201
7. The average mark of 40 students in a class is 60, and the average mark of 60 students in a different class is 40. What is the average of all the tests?
A. 48
B. 50
C. 46
D. 40
8. The average weight of 30 boys in a class of 40 is 50, and the average of remaining boys is 46. What is the average weight of the whole class?
A. 48
B. 50
C. 49
D. 40
9. The average of 40 numbers is 25. If two numbers 24 and 26 are removed, then the average of the remaining numbers will be:
A. 27
B. 24
C. 20
D. 25
10. The average number of persons crossing a toll plaza on the first four days of a week is 1163 and that on the last three days is 1660. What is the daily average for the whole week?
A. 1370
B. 1350
C. 1376
D. 1440
11. The average rainfall of the first three days of a week is 1 inch, and that of the next



- four days is 0.3 inch. What is the average rainfall for the whole week?
- A. 0.7 inch
B. 0.6 inch
C. 0.4 inch
D. 0.5 inch
- 12.** The average collection of a shop in the first three days is Rs. 2700, and that of the next three is Rs. 2900. If the average collection of the whole week is Rs. 2850, then what is the collection on the last day of the week?
- A. Rs. 3150
B. Rs. 3050
C. Rs. 2950
D. Rs. 3000
- 13.** The average of 3 numbers is 63. The first is twice the second and four times the third. Find the second number.
- A. 48
B. 54
C. 56
D. 50
- 14.** The average weight of 19 bags is 30 kg, and by adding one more bag, the average weight is reduced to 29.6 kg. What is the weight of the new bag?
- A. 28 kg
B. 20 kg
C. 26 kg
D. 22 kg
- 15.** The average weight of 8 students in a class is increased by 2.5 kg when one of them whose weight is 28 kg is replaced by the new student, what is the weight of the new student?
- A. 48
B. 50
C. 46
D. 40
- 16.** The average weight of 40 members in a joint family is 40 kg. If the weight of grandfather is included, the average weight increases by 500 grams. What is the weight of the grandfather?
- A. 58 kg
B. 60.5 kg
C. 60 kg
D. 55.5 kg
- 17.** The average mark of four siblings is 12. If the gap between their marks obtained is 4, then what are the least marks amongst all?
- A. 8
B. 5
C. 6
D. 4
- 18.** The average daily wage of 20 workers in a factory is Rs. 190 per day. If the daily wage of a new worker is added, the average daily wage becomes Rs. 200 per day. What is the daily wage of a newly added worker?
- A. 480
B. 500
C. 460
D. 400
- 19.** The average age of P, Q, R, and S ten years ago was 40 years. By including A, the present average age of all is 55 years. Find the present age of A.
- A. 75 years
B. 70 years
C. 76 years
D. 74 years
- 20.** A street vendor earned Rs. 1008 in 12 days. His average earnings for the first four days were Rs. 80 a day. What is his average earning for the remaining days?
- A. Rs. 88
B. Rs. 80
C. Rs. 86
D. Rs. 74



- 21.** The average height of 24 pillars in a class is 10 feet. If the height of the new pillar is included, the average height increases by 1 foot. The height of the new pillar is:
- A. 38
B. 35
C. 36
D. 40
- 22.** The average income of 4 employees in the office is Rs. 7350. One of the employees left the office, and therefore the average income was reduced to Rs. 6500. What is the income of the person who left the job?
- A. 9900
B. 9000
C. 9600
D. 9400
- 23.** Find the average of the given series: 4, 8, 12, 16, 20, 24, 28, 32, 36, and 40.
- A. 24
B. 25
C. 26
D. 22
- 24.** The average weight of 12 students in a class is 20 kg. If the teacher's weight is included, the average weight increases by 2 kg. What is the weight of the teacher?
- A. 48 kg
B. 44 kg
C. 46 kg
D. 40 kg
- 25.** The average height of 5 persons is 1.2 m. If the height of a new person is included, the average height increases by 100 cm. What is the height of the newly added person?
- A. 1.8 m
B. 1.4 m
C. 1.6 m
D. 1.4 m
- 26.** Find the average of the first six multiples of 9.
- A. 28
B. 31.5
C. 29
D. 30
- 27.** Arpit travelled from Kolkata to Jaipur by car at an average speed of 60 km/h. He returned to Kolkata by motorcycle at an average speed of 40 km/h. What is the average speed of the whole journey?
- A. 48
B. 50
C. 46
D. 40
- 28.** The average of 40 values is 20, and the average of 20 different values is 40. What is the average of all the values?
- A. 22
B. 24
C. 26
D. 20
- 29.** The average score of the first six batsmen is 15, and that of the next three batsmen is 18. Find the average score of nine batsmen.
- A. 14
B. 15
C. 16
D. 14
- 30.** The average daily expenditure on four vehicles is Rs. 100. If one of the vehicles gets sold, the average daily expenditure becomes Rs. 90. What is the daily expenditure on the vehicle which gets sold?
- A. 130
B. 160
C. 150
D. 140



SOLUTIONS

1. **(D)** Average

$$= \frac{35560 + 27500 + 30000 + 25000 + 25600}{5}$$

$$= \frac{143660}{5}$$

$$= 28732$$

Therefore, the correct option is D.

2. **(A)** Total weight of 48 employees

$$= 48 \times 70 = 3430 \text{ kg}$$

Total weight of all the employees including 2 new employees = $50 \times 70.8 = 3540 \text{ kg}$

Average weight of new employees

$$= (3540 - 3430) \text{ kg} = 110$$

$$= \frac{110}{2} = 55 \text{ kg}$$

The average weight of new employees is 55 kg.

Therefore, option A is correct.

3. **(C)** The first nine multiples of 5 are: 5, 10, 15, 20, 25, 30, 35, 40, and 45

The given numbers or multiples of 5 form an AP series and the middle term of the series is 25, therefore the average of the first nine multiples of 5 is 25.

Therefore, option C is correct.

4. **(B)** Weight of new crew member = Weight of person who left + (Increase in the average \times Total number of people)

$$\text{Weight of new worker} = 60 + (1 \times 8) = 68 \text{ kg}$$

The weight of the new worker is 68 kg.

Therefore, option B is correct.

5. **(A)** The average speed of the whole journey

$$= \frac{2xy}{x+y}$$

$$= \frac{2(80 \times 48)}{80 + 48}$$

$$= \frac{7680}{128} = 60 \text{ km/h}$$

The average speed of the whole journey is 60 km/h.

Therefore, option A is correct.

6. **(C)** Average

$$= \frac{0.60204 + 0.60203 + 0.60202 + 0.60201}{4}$$

$$= \frac{2.4081}{4} = 0.602025$$

The average of given four values is 0.60205.

Therefore, option C is correct.

7. **(A)** Average marks of all the students can

$$\text{be calculated by} = \frac{A1P1 + A2P2}{A1 + A2}$$

$$= \frac{(40 \times 60) + (60 \times 40)}{60 + 40}$$

$$= \frac{4800}{100} = 48$$

The average marks of all the students are 48.

Therefore, option A is correct.

8. **(C)** Average weight of whole class =

$$\frac{A1P1 + A2P2}{A1 + A2} = \frac{(30 \times 50) + (46 \times 10)}{30 + 10}$$

$$= \frac{1960}{40} = 49 \text{ kg}$$

The average weight of the whole class is 49 kg.

Therefore, option C is correct.

9. **(D)** Total sum of all numbers = $40 \times 25 = 1000$

$$\text{Total sum of remaining numbers} = 1000 - (24 + 26) = 950$$

$$\text{Average of remaining numbers} = \frac{950}{38} = 25$$

The average of remaining numbers will be 25.



Therefore, option D is correct.

- 10. (C)** Daily average for whole week

$$= \frac{A_1P_1 + A_2P_2}{A_1 + A_2}$$

$$= \frac{(1163 \times 4) + (1660 \times 3)}{4 + 3}$$

$$= \frac{(4652 + 4980)}{7} = \frac{9632}{7}$$

$$= 1376$$

The daily average for the whole week is 1376.

Therefore, option C is correct.

- 11. (B)** Average rainfall of whole week

$$= \frac{A_1P_1 + A_2P_2}{A_1 + A_2}$$

$$= \frac{(1 \times 3) + (0.3 \times 4)}{4 + 3}$$

$$= \frac{4.2}{7} = 0.6 \text{ inch}$$

The average rainfall of the whole week is 0.6 inch.

Therefore, option B is correct.

- 12. (A)** Total collection of first three days

$$= 2700 \times 3 = 8100$$

Total collection of next three days

$$= 2900 \times 3 = 8700$$

Total collection of first six days

$$= 8100 + 8700 = 16800$$

Collection of the last day = Total collection in a week – Total collection in first six days

$$\text{Collection of the last day} = (2850 \times 7) - 16800$$

$$= \text{Rs. } 3150.$$

The collection of the last day of a shop is Rs. 3150.

Therefore, option A is correct.

- 13. (B)** If the third number is x , the first number is $4x$ and the second is $2x$.

$$\text{The total sum of the numbers is} = 63 \times 3 = 189$$

$$= x + 2x + 4x = 7x = 189$$

$$x = \frac{189}{7} = 27$$

The numbers are

$$= (27 \times 4), (27 \times 2), 27 = 108, 54, 27$$

Therefore, option B is correct.

- 14. (D)** The weight of new bag = Earlier average – (Decrease in average \times Total number of bags)

$$\text{Weight of new bag} = 30 - (0.4 \times 20)$$

$$= 30 - 8 = 22 \text{ kg}$$

The weight of the new bag is 22 kg.

Therefore, option D is correct.

- 15. (A)** Weight of new student = Weight of student who left + (Increase in average \times Total number of students)

$$\text{Weight of new student} = 28 + (2.5 \times 8)$$

$$= 48 \text{ kg}$$

The weight of the new student is 48 kg.

Therefore, option A is correct.

- 16. (B)** Weight of grandfather = Earlier average + (Increase in average \times Total number of people)

The average weight of joint family = 40 kg

$$\text{Weight of grandfather} = 40 + (0.5 \times 41)$$

$$= 60.5 \text{ kg}$$

The weight of grandfather is 60.5 kg.

Therefore, option B is correct.

- 17. (C)** Let the minimum marks obtained be x
Total marks = $12 \times 4 = 48$

Then from the question,

$$x + (x + 4) + (x + 4 + 4) + (x + 4 + 4 + 4) = 48$$

$$\text{So, } 4x + 24 = 48, x = 6$$

The least mark obtained is 6.

Therefore, option C is correct.

- 18. (D)** The daily wage of newly added worker = Earlier average + (Increase in average \times Total number of people)

$$= 190 + (10 \times 21)$$

$$= \text{Rs. } 400$$

The daily wage of a newly added worker is Rs. 400 per day.

Therefore, option D is correct.

- 19. (A)** The total sum of ages of P, Q, R, and S ten years ago = $40 \times 4 = 160$

Total sum of ages of P, Q, R, and S at present

$$= 160 + 40 = 200$$

Total sum of ages of P, Q, R, S, and

$$A = 55 \times 5 = 275$$



Present age of A = $275 - 200 = 75$ years

The present age of A is 75 years.

Therefore, option A is correct.

- 20. (C)** Total earnings = 1008

Total earnings in first four days = $80 \times 4 = 320$

Total earnings for remaining days

= $1008 - 320 = 688$

Average for remaining days = $\frac{688}{8} = \text{Rs. } 86$

The average earning of the remaining days is Rs. 86.

Therefore, option C is correct.

- 21. (B)** Height of new pillar = Earlier average + (Increase in average \times Total number of pillars)

$$= 10 + (1 \times 25)$$

= 35 feet

The height of the new pillar is 35 feet.

Therefore, option B is correct.

- 22. (A)** Total income of 4 employees

= $7350 \times 4 = 29400$

The total income of 3 employees

= $6500 \times 3 = 19500$

Income of the person who left

= $29400 - 19500 = \text{Rs. } 9900$

The income of the person who left is Rs. 9900.

Therefore, option A is correct.

- 23. (D)** The two middle terms of the given series are 20 and 24

Average of two middle terms

$$= \frac{20 + 24}{2} = 22$$

The average of the given series is 22.

Therefore, option D is correct.

- 24. (B)** Weight of teacher = earlier average + (increase in average \times total number of people)

$$= 20 + (2 \times 12)$$

= 44 kg

The weight of the teacher is 44 kg.

Therefore, option B is correct.

- 25. (A)** Height of new person = earlier average + (increase in average \times total number of people)

$$= 1.2 + (0.1 \times 6)$$

= 1.8 m

The height of the newly added person is 1.8 m.

Therefore, option A is correct.

- 26. (B)** The first six multiples of 9 are: 9, 18, 27, 36, 45, and 54.

The middle terms of the formed series are 27 and 36.

The average of the series is = $\frac{27 + 36}{2}$

= 31.5

The average of the first six multiples of nine is 31.5.

Therefore, option B is correct.

- 27. (A)** The average speed of the whole journey

$$= \frac{2xy}{x + y}$$

$$= \frac{2(60 \times 40)}{60 + 40} = \frac{4800}{100}$$

= 48 km/h

The average speed of the whole journey is 48 km/h.

Therefore, option A is correct.

- 28. (D)** Grouped average of all values

$$= \frac{A_1P_1 + A_2P_2}{A_1 + A_2}$$

$$= \frac{(40 \times 20) + (20 \times 40)}{20 + 40}$$

= 20

The average of all the values is 20.

Therefore, option D is correct.

- 29. (C)** Average score of nine batsman

$$= \frac{A_1P_1 + A_2P_2}{A_1 + A_2}$$

$$= \frac{(15 \times 6) + (18 \times 3)}{6 + 3} = \frac{144}{9}$$

= 16 runs

The average score of nine batsmen is 16 runs.

Therefore, option C is correct.

- 30. (A)** Total expenditure on four vehicles =

$100 \times 4 = \text{Rs. } 400$



Total daily expenditure after one of the vehicles gets sold = $90 \times 3 = \text{Rs. } 270$

Daily expenditure of vehicle which gets sold = $400 - 270 = \text{Rs. } 130$

The daily expenditure of a vehicle that gets sold is Rs. 130.

Therefore, option A is correct.

PRACTICE QUESTIONS

Questions 1–5: The following table shows the data of sales of cars of different companies from 2015 to 2020.

COMPANY	2015	2016	2017	2018	2019	2020
A	440	480	470	500	520	510
B	400	410	415	415	420	430
C	380	390	390	400	420	495
D	360	380	400	415	440	500
E	480	440	440	420	425	435

- What is the average sale of cars in company C from the year 2016–2020?
A. 420
B. 419
C. 425
D. 415
- In which year the average sale of cars is the highest?
A. 2020
B. 2019
C. 2015
D. 2018
- If the average sale of cars of company D in 2015–2017 is 320 and that of 2018–2020 is 340. Find the average sale of cars of company D from 2015 to 2021.
A. 320
B. 319
C. 335
D. 330
- What is the average sale of cars in company B from 2016 to 2019?
A. 420
B. 419
C. 425
D. 415
- What is the average sale of cars in company A from 2015 to 2019?
A. 470
B. 480
C. 482
D. 450

Questions 6–10: There are five sections in the 12th standard which are A, B, C, and D. The number of students in section A is 40 and the average weight of the class is 45 kg. The total sum of weights of 30 students of section C is 1200 kg. The total number of students in section B is 20. The average weight of 35 students of section D is 30 kg. The average weight of section B is 30 kg.

- If the weight of the teacher is added, the average weight is 46 kg. Find the weight of the teacher.



- A. 86 kg
B. 79 kg
C. 75 kg
D. 85 kg
7. Find the average weight of students in section C.
A. 42 kg
B. 41 kg
C. 40 kg
D. 38 kg
8. If a student weighing 25 kg is replaced by a new student, the average weight of section D increases by 500 grams. What is the weight of the new student?
A. 42.5 kg
B. 42 kg
C. 41 kg
D. 40.5 kg
9. What is the average weight of the 12th standard?
A. 37
B. 37.2
C. 37.5
D. 35
10. What will be the average weight of section B, if a student weighs 30 kg removed from the class?
A. 30
B. 31
C. 32
D. 31.5

Questions 11–15: The following table represents the data of marks obtained by students in different subjects.

NAME OF STUDENTS	ENGLISH	HINDI	MATHS	SCIENCE
P	30	32	38	40
Q	36	32	30	36
R	42	36	34	30
S	40	40	42	26

11. What are the average marks obtained by P?
A. 37
B. 36
C. 37
D. 35
12. What are the average marks obtained by students in science subjects?
A. 32
B. 31
C. 33
D. 34
13. What is the combined average of marks obtained by students in Hindi and Maths?
A. 33
B. 34
C. 35
D. 35.5
14. What are the average marks obtained by students in English and Science?
A. 33
B. 35
C. 34
D. 33
15. Which student has scored the highest average marks?
A. R
B. S
C. Q
D. P



SOLUTIONS

1. **(B)** Average sale of C from 2016 to 2020

$$\begin{aligned} &= \frac{390 + 390 + 400 + 420 + 495}{5} \\ &= \frac{2095}{5} = 419 \end{aligned}$$

The average sale of company C from 2016 to 2020 is 419.

Therefore, option B is correct.

2. **(A)** Average sales for year 2015

$$\begin{aligned} &= \frac{440 + 400 + 380 + 360 + 480}{5} \\ &= \frac{2060}{5} = 412 \end{aligned}$$

Average sales for year 2016

$$\begin{aligned} &= \frac{480 + 410 + 390 + 380 + 440}{5} \\ &= \frac{2100}{5} = 420 \end{aligned}$$

Average sales for year 2017

$$\begin{aligned} &= \frac{470 + 415 + 390 + 400 + 440}{5} \\ &= \frac{2115}{5} = 423 \end{aligned}$$

Average sale for year 2018

$$\begin{aligned} &= \frac{500 + 415 + 400 + 415 + 420}{5} \\ &= \frac{2150}{5} = 430 \end{aligned}$$

Average sale for year 2019

$$\begin{aligned} &= \frac{520 + 420 + 420 + 440 + 425}{5} \\ &= \frac{2225}{5} = 445 \end{aligned}$$

Average sale for year 2020

$$\begin{aligned} &= \frac{510 + 430 + 495 + 500 + 435}{5} \\ &= \frac{2370}{5} = 474 \end{aligned}$$

The highest average sale of cars was in 2020.

Therefore, option A is correct.

3. **(D)** Average sale of company D from 2015

$$\begin{aligned} \text{to 2020} &= \frac{A1P1 + A2P2}{A1 + A2} \\ &= \frac{(320 \times 3) + (340 \times 3)}{3 + 3} = \frac{1980}{6} \end{aligned}$$

$$= 330$$

The average sale of company D from 2015 to 2020 is 330.

Therefore, option D is correct.

4. **(D)** The average sale of cars in B from 2016

$$\begin{aligned} \text{to 2019} &= \frac{410 + 415 + 415 + 420}{4} \\ &= \frac{1660}{4} = 415 \end{aligned}$$

The average sale of cars in company B from 2016 to 2019 is 415.

Therefore, option D is correct.

5. **(C)** Average sale of cars in A

$$\begin{aligned} &= \frac{440 + 480 + 470 + 500 + 520}{5} \\ &= \frac{2410}{5} = 482 \end{aligned}$$

The average sale of cars in company A from 2015 to 2019 is 482.

Therefore, option C is correct.

6. **(A)** Weight of teacher = earlier average + (increase in average \times total number of persons)

$$= 45 + (1 \times 41)$$

$$= 86 \text{ kg}$$

The weight of the teacher is 86 kg.

Therefore, option A is correct.

7. **(C)** The average weight of students in

$$\text{section C} = \frac{1200}{30}$$

$$= 40 \text{ kg}$$

The average weight of students in section C is 40 kg.

Therefore, option C is correct.



8. (A) Weight of new student = weight of person who left + (increase in average \times total number of people)
 $= 25 + (0.5 \times 35)$
 $= 25 + 17.5 = 42.5 \text{ kg}$

The weight of the new student is 42.5 kg.
 Therefore, option A is correct.

9. (B) Average weight of 12th standard
 $=$

$$\frac{A_1P_1 + A_2P_2 + A_3P_3 + A_4P_4 + A_5P_5 \dots + A_nP_n}{P_1 + P_2 + P_3 + P_4 + P_5 \dots P_n}$$

$$= \frac{(40 \times 45) + 1200 + (35 \times 30) + (20 \times 30)}{40 + 30 + 35 + 20}$$

$$= \frac{4650}{125} = 37.2 \text{ kg}$$

The total average of the 12th standard is 37.2 kg.
 Therefore, option B is correct.

10. (A) Total sum of weights of section B
 $= 20 \times 30 = 600 \text{ kg}$
 Total weight after removal of student
 $= 600 - 30 = 570 \text{ kg}$

$$\text{New average weight} = \frac{570}{19} = 30 \text{ kg}$$

The average of section B will be the same, i.e., 30 kg.

Therefore, option A is correct.

11. (D) Average marks of P = $\frac{30 + 32 + 38 + 40}{4}$
 $= \frac{140}{4} = 35$

The average marks obtained by P are 35.
 Therefore, option D is correct.

12. (C) Average marks in science
 $= \frac{40 + 36 + 30 + 26}{4}$

$$= \frac{132}{4} = 33$$

The average marks of students in science subjects are 33.
 Therefore, option C is correct.

13. (D) Average marks in Hindi
 $= \frac{32 + 32 + 36 + 40}{4}$

$$\text{Average marks in Hindi} = \frac{140}{4} = 35$$

$$\text{Average marks in Maths} = \frac{38 + 30 + 34 + 42}{4}$$

$$= \frac{144}{4} = 36$$

$$\text{Combined average} = \frac{35 + 36}{2} = 35.5$$

The combine average is 35.5.
 Therefore, option D is correct.

14. (B) Average marks in English
 $= \frac{30 + 36 + 42 + 40}{4}$

$$= \frac{148}{4} = 37$$

$$\text{Average marks in Science} = \frac{40 + 36 + 30 + 26}{4}$$

$$= \frac{132}{4} = 33$$

$$\text{Combined average} = \frac{37 + 33}{2} = 35$$

Average marks obtained by students in English and Science is 35, therefore, option B is correct.

15. (B) Average marks of P = $\frac{30 + 32 + 38 + 40}{4}$

$$= \frac{140}{4} = 35$$

$$\text{Average marks of Q} = \frac{36 + 32 + 30 + 36}{4}$$

$$= \frac{144}{4} = 36$$

$$\text{Average marks of R} = \frac{42 + 36 + 34 + 30}{4}$$

$$= \frac{142}{4} = 35.5$$

$$\text{Average marks of S} = \frac{40 + 40 + 42 + 26}{4}$$

$$= \frac{148}{4} = 37$$

S has scored the highest average marks.
 Therefore, option B is correct.



SYNOPSIS

- Introduction
- Ratio
- Calculating ratio
- Important concepts relating to ratio
- Properties of ratio
- Tips and tricks
- Proportion
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INTRODUCTION

This topic covers concepts related to ratio and proportion, and it is considered as a very important topic in Mathematics. In daily life also, the concept of ratio and proportion is frequently used. Questions related to ratio and proportion are often asked in GATE, JEE Main, and other engineering entrance examinations; therefore, test takers must be cleared with the concepts related to this topic.

Ratio

Basically, a ratio is a comparison of two quantities or numbers. It refers to the relationship between the two quantities and represents how many times one quantity is equal to the other quantity. Ratios are represented by the symbol of colon, i.e. (:).

The ratio of a to b is written as,

$$a : b = \frac{a}{b} = a \div b$$

In the ratio $a : b$, a and b are called the terms of the ratio, wherein ' a ' is the first term or known

as an antecedent, and ' b ' is the second term or known as consequent.

Example:

In the ratio $3 : 4$, 3 is the first term or antecedent, and 4 is the second term or consequent.

Calculating ratio

For calculating the ratio of two quantities of the same units, we can use the steps quoted hereunder.

Example:

If 30 kg of soil and 40 kg of cement are used to make a wall, let us calculate the ratio of soil and cement used to make a wall.

Solution:

Step 1: First find the quantities for which we have to determine the ratio, and in this case, it is 30 and 40.

Step 2: Write both the quantities in the fraction form, such as $\frac{30}{40}$.

Step 3: Thereafter, simplify the fraction, if possible. The simplified fraction will give the final ratio. Here, $\frac{30}{40}$ can be simplified into $\frac{3}{4}$.



Step 4: Therefore, the ratio of soil to cement is $\frac{3}{4}$.

Important concepts relating to ratio

1. To find the ratio of any two quantities, the quantities must be expressed in the same units.

Example:

4 m : 200 cm, therefore, for finding its ratio, both the units must be converted into the same. By converting any of the two units, we can find its ratio. 4 m = 400 cm; therefore the ratio will be 400 cm : 200 cm = 2 : 1

2. The ratio would remain same if the antecedent and the consequent in a ratio are multiplied or divided by a constant.

Example:

$$a : b = \frac{a}{b} = \frac{a \times y}{b \times y} = \frac{\frac{a}{y}}{\frac{b}{y}}, \text{ wherein } y = \text{constant}$$

3. If two different ratios are expressed in different units, then to find out the combined ratio of the two ratios, we have to compound the ratios. This is known as compounding.

The compound of $a : b$ and $p : q$ is $\frac{a \times p}{b \times q}$

Example:

To complete a task, 1 man works for 8 hours a day for 10 days, and 1 boy works for 12 hours a day for 20 days. How many boys can complete the same task that 1 man can do?

Solution:

To know how many boys are required to complete the same task that 1 man can do, we need to combine two different units, i.e., hours and days.

The ratio of days is 10 : 20, the ratio of hours is 8 : 12.

The compound ratio of days can be calculated

$$\begin{aligned} \text{as, } & \frac{a \times p}{b \times q} \\ &= \frac{10 \times 8}{20 \times 12} = \frac{80}{240} = \frac{1}{3} = 1 : 3 \end{aligned}$$

This ratio signifies the ratio of men to boys.

Therefore, 3 boys are required to complete the same task that one man can do in a day.

1. There should be significant order of terms in the ratio.

2. If there are two quantities in the ratio $x : y$, then the first quantity is $\frac{x}{x+y}$ times the

total and the second quantity is $\frac{y}{x+y}$

times the total of both the quantities.

Properties of ratio

To solve the questions of ratio and proportion, you need to remember the different sets of rules. Particularly while solving questions on proportion, you are required to remember the following set of rules. They are invertendo, alternado, componendo, dividendo, and componendo-dividendo.

1. This is the property of the ratio that if $a : b = c : d = e : f$, then the overall ratio of the series will be the sum of antecedent to the sum of all the consequents.

$$\text{If } \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{g}{h} = \dots,$$

$$\text{Ratio} = \frac{\text{Sum of numerators}}{\text{Sum of denominators}}$$

$$= \frac{a + c + e + g + \dots}{b + d + f + h + \dots}$$

2. In invertendo, if $a : b = c : d$, then $b : a = d : c$. In this law, you need to remember that if the ratio is in proportion then the inverse of that ratio is also the proportion to each other.

$$\frac{a}{b} = \frac{c}{d}$$

$$a \times b = c \times d$$

$$\frac{a \times d}{a \times c} = \frac{c \times b}{a \times c} \quad (\text{dividing both sides by } a \times c)$$

$$\frac{d}{c} = \frac{b}{a}$$

$$\text{If, } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{b}{a} = \frac{d}{c}$$



The inverse ratios of two equal ratios are equal. This property of ratio is called **invertendo**.

3. In this law, if $a : b :: c : d$, then $a : c :: b : d$. In this law, the property of continued proportion is used. Suppose you are given two ratios, and the values in both the ratios are proportional to each other. In this law, when you replace the denominator of the first ratio with the numerator of the second ratio, the two ratios remain proportional to each other.

$$\frac{a}{b} = \frac{c}{d}$$

$$a \times d = b \times c$$

$$\frac{a \times d}{c \times d} = \frac{b \times c}{c \times d} \text{ (dividing both sides by } c \times d \text{)}$$

$$\frac{a}{c} = \frac{b}{d}$$

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a}{c} = \frac{b}{d}$$

The ratios of the antecedents and the consequent of two equal ratios are equal. This property of ratio is called **alternado**.

4. In componendo, the basic rule that you need to remember is if $a : b :: c : d$, then $(a + b) : b :: (c + d) : d$. Thus, in componendo as you can see, you need to add the denominator to the numerator given in the ratios and then equate them. If you are using the rule on the left-hand side, then you also need to use the rule on the right-hand side.

$$\frac{a}{b} = \frac{c}{d}$$

$$\frac{a}{b} + 1 = \frac{c}{d} + 1 \text{ (adding 1 to both sides)}$$

$$\frac{a + b}{b} = \frac{c + d}{d}$$

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a + b}{b} = \frac{c + d}{d}$$

Therefore, this property of ratio is known as **componendo**.

5. In the law of dividendo, if $a : b :: c : d$, then $(a - b) : b :: (c - d) : d$. In dividendo, instead of addition, you are required to subtract the

denominator from the numerator in both ratios. Rest everything in this law is similar to the componendo.

$$\frac{a}{b} = \frac{c}{d}$$

$$\frac{a}{b} - 1 = \frac{c}{d} - 1 \text{ (subtracting 1 from both sides)}$$

$$\frac{a - b}{b} = \frac{c - d}{d}$$

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a - b}{b} = \frac{c - d}{d}$$

Therefore, this property of ratio is called **dividendo**.

6. If the ratio of any two numbers is equal to the ratio of another two numbers, then the ratios of the sum of numerator and denominator to the difference of numerator and denominator of both rational numbers are equal. This property of ratio is called componendo-dividendo.

$$\frac{a}{b} = \frac{c}{d}$$

$$\text{From componendo, } \frac{a + b}{b} = \frac{c + d}{d} \quad \dots (1)$$

$$\text{From dividendo, } \frac{a - b}{b} = \frac{c - d}{d} \quad \dots (2)$$

By dividing (1) by (2)

$$\text{We get, } \frac{a + b}{a - b} = \frac{c + d}{c - d}$$

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a + b}{a - b} = \frac{c + d}{c - d}$$

Therefore, this property of ratio is called **componendo-dividendo**.

Example:

$$\text{If } \frac{3a^3 + 5b^3}{3a^3 - 5b^3} = \frac{109}{19}, \text{ find the value of } \frac{5a + 2b}{5a - 2b}.$$

Solution:

$$\frac{3a^3 + 5b^3}{3a^3 - 5b^3} = \frac{109}{19}$$

By applying componendo-dividendo,

$$= \frac{\left[(3a^3 + 5b^3) + (3a^3 - 5b^3) \right]}{\left[(3a^3 + 5b^3) - (3a^3 - 5b^3) \right]} = \frac{109 + 19}{109 - 19}$$

$$= \frac{6a^3}{10b^3} = \frac{128}{90}$$



$$= \frac{a^3}{b^3} = \frac{64}{27}$$

$$= \frac{a}{b} = \frac{4}{3} \text{ (by taking cube root)}$$

$$= \frac{5a}{2b} = \frac{20}{6} = \frac{10}{3}$$

By applying componendo-dividendo,

$$\frac{5a + 2b}{5a - 2b} = \frac{13}{7}$$

Tips and Tricks

- In the ratio of $a : b$, if a and b are equal, then $a : b = 1$.
- In ratio $a : b$, if $a > b$, then $a : b > 1$.
- In ratio $a : b$, if $a < b$, then $a : b < 1$.
- To determine the ratio, quantities must be of similar units.

Proportion

When the ratio of two terms is equal to the ratio of two other terms, then these four terms are said to be in proportion. In simple words, the ratios are said to be in proportion when the two ratios are equal. If the two sets of given numbers are increasing or decreasing proportionally, then the ratios are said to be in proportion with each other.

To exemplify, if $\frac{a}{b} = \frac{c}{d}$ or $a : b = c : d$, then a , b , c , and d are in proportion.

Here, a and d are known as extremes, while b and c are known as means.

Example:

If $\frac{2}{3} = \frac{7}{14}$, then $2 : 3 = 7 : 14$.

1. Continued proportion:

The three quantities a , b , and c are said to be in continued proportion if the ratio between a and b is equal to the ratio between b and c .

Therefore, if $\frac{a}{b} = \frac{b}{c}$, then $b^2 = ac$, or we can say $b = \sqrt{ac}$

When quantities such as a , b , and c are in continued proportion, b is known as the geometric mean or mean proportion between a and c .

Example:

If $\frac{3}{x} = \frac{4}{14}$ are in continued proportion, then find the value of x .

Solution:

In continued proportion – $\frac{a}{b} = \frac{b}{c}$

Therefore, $x = 4$

2. Third proportion:

If a , b , and c are three quantities or numbers such that $a : b = b : c$, then c is called the third proportion to a and b .

Illustration: The third proportion of 6 and

15 is 'x' such that $\frac{6}{15} = \frac{15}{x}$

So, $6x = 225$; therefore, $x = 37.5$.

3. Fourth Proportion:

If a , b , c , and d are four quantities or numbers such that $a : b = c : d$, then d is called the fourth proportion of a , b , and c .

Example:

The fourth proportion of 10, 18, and 30 is a number 'y' such that $\frac{10}{18} = \frac{30}{y}$

So, $10y = 540$, therefore, $y = 54$.

4. Direct proportion:

If two quantities are related to each other in such a manner that an increase or decrease in one quantity will create a proportionate increase or decrease in the other quantity, then the two quantities are said to be in direct proportion. In simple words, if one quantity increases, the other quantity will increase proportionately, and if one quantity decreases, then the other quantity will also decrease proportionately.

Example:

The area of a circle increases with its increase in radius and decreases with its decrease in radius; therefore, the radius of a circle is directly proportional to its area.



If a and b are directly proportional to each other, then they will be denoted as $a \propto b$.

$a = kb$, where ' k ' is a non-zero constant, which is known as the constant of proportionality.

If a and b are directly proportional to each other, then $\frac{a_1}{b_1} = \frac{a_2}{b_2}$

Example:

The cost of salt varies directly as its weight. 12 kg of salt costs Rs. 138. Find the cost of 5 kg of salt.

Solution:

Let us take the cost of salt to be ' a ', and its weight is ' b '.

Therefore, we have $a \propto b$

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{138}{12} = \frac{a_2}{5}$$

$$a_2 = \frac{138 \times 5}{12}$$

$$a_2 = \text{Rs. } 57.5$$

The cost of 5 kg of salt is Rs. 57.5

5. Indirect proportion:

If two quantities are related to each other in such a manner that increases in one quantity will create a proportionate decrease in another quantity, and a decrease in one quantity will create a proportionate increase in another quantity, then the two quantities are said to be in indirect proportion. In simple words, if one quantity increases, the other quantity will decrease proportionately, and if one quantity decreases, then the other quantity will increase proportionately.

If a and b are inversely proportional to each other, then they will be denoted as $a \propto \frac{1}{b}$

$a = \frac{k}{b}$, where k is a non-zero constant,

which is known as the constant of proportionality.

If a and b are inversely proportional to each other, then $a_1 b_1 = a_2 b_2$

Example:

The number of days required to finish a work is inversely proportional to the number of workers employed. 15 men finish the work in 8 days. Find the number of men to be employed if the same job is to be completed in 6 days.

Solution:

Let us take the number of days to finish the work as ' x ' and the number of men employed

as ' y ', then $x \propto \frac{1}{y}$.

$$x_1 y_1 = x_2 y_2$$

$$8 \times 15 = 6 \times y_2$$

$$y_2 = \frac{8 \times 15}{6} = 20$$

20 men should be employed to finish the work in 6 days.

Some brief points to remember

1. If $A \propto B$ and $B \propto C$, then $A \propto C$.
2. If $A \propto C$ and $B \propto C$, then $A + B \propto C$ or $A - B \propto C$.
3. If $A \propto BC$, then $B \propto \frac{A}{C}$ and $C \propto \frac{A}{B}$.
4. If $A \propto B$ and $C \propto D$, then $AC \propto BD$.
5. If $A \propto B$, then $A^n \propto B^n$.
6. If $A \propto B$ and $A \propto C$, then $A \propto (B-C)$ and $A \propto (B+C)$.
7. If $A \propto B$, then $AX \propto BX$ where X is any constant, variable or quantity.



Chapter Summary



- The ratio is used to compare two quantities of the same kind, and when two or more ratios are equal, then they are said to be in proportion.
- Comparison can only be done between the quantities of the same units.
- Two ratios are said to be in proportion if they are equal.
- The ratio will remain the same after multiplication and dividing each term of the ratio by a constant.
- We can denote ratio and proportion in two ways, either by $\frac{a}{b} = \frac{c}{d}$ or by $a : b = c : d$.

PRACTICE QUESTIONS

1. What is the third proportion of 32 and 48?
A. 32
B. 48
C. 72
D. None of the above
2. What must be subtracted from the terms of the ratio 3 : 7 to make it 2 : 6?
A. 3
B. 5
C. 6
D. Data insufficient
3. The sum of the present ages of Abhay, Bhanu, and Chetan are 96 years. Eight years ago, their ages were in the ratio 1 : 2 : 3, what is the present age of Chetan?
A. 44
B. 45
C. 48
D. 40
4. What is the fourth proportion of 12, 28, and 40?
A. 28
B. 56
C. 12
D. 40
5. A person distributes chocolates among four children A, B, C, and D in the ratio $\frac{1}{6} :$
 $\frac{1}{8} : \frac{1}{10} : \frac{1}{12}$. What is the minimum number of chocolates before the distribution?
A. 52
B. 56
C. 57
D. 60
6. How many factors are there of 120?
A. 16
B. 12
C. 10
D. 18
7. If $6x = 8y$ and $5y = 9z$, then find the value of $\frac{z}{x}$.
A. $\frac{20}{50}$
B. $\frac{20}{38}$
C. $\frac{20}{40}$
D. $\frac{20}{48}$
8. If $a : b = \frac{3}{5}$, find $\frac{2a + 3b}{4a - b}$.
A. 3 : 1
B. 1 : 2
C. 3 : 2
D. 4 : 3



9. The ratio between the three angles in the triangle is 3 : 4 : 5. What is the difference between the smallest and the largest angle?
- A. 25°
B. 32°
C. 30°
D. 28°
10. If A has 10% less money than that of B who has 20% less than that of C. If C has Rs. 900, then what is the total amount of money A, B, and C have together?
- A. 2896
B. 2268
C. 3000
D. 2800
11. Rs. 9600 is distributed among Suresh, Ramesh, and Mahesh in the ratio of 6 : 5 : 4. What is the difference in the shares of Suresh and Mahesh?
- A. 1280
B. 1250
C. 1200
D. 1300
12. If two friends Vineet and Karan earn in the ratio of 5 : 6. If both of them earn more Rs. 100, then their earnings will be in the ratio 7 : 8. What were Vineet's initial earnings?
- A. 260
B. 240
C. 250
D. 248
13. A person distributed papers among Kartik, Ajay, Vijay, and Deepak in the ratio of 5 : 2 : 4 : 3. If Vijay got 100 papers more than Deepak, then how many papers did Kartik and Ajay together get?
- A. 760
B. 750
C. 650
D. 700
14. A handbag contains coins of 25 paise, 50 paise, and Re. 1 in the ratio of 5 : 6 : 4. If Rs. 33 is in the handbag, then how many 1 rupee coins are there in the bag?
- A. 16
B. 12
C. 10
D. 18
15. A person distributed Rs. 770 among Ravi, Deepak, and Amit. If Ravi has received two-ninth of what Deepak and Amit together receive, then how much did Ravi receive?
- A. 160
B. 120
C. 140
D. 180
16. Chocolates are distributed among children A, B, and C. For every 10 chocolates that A gets, B gets 6 chocolates and for every 4 chocolates that A gets, C gets 3 chocolates. If a total number of 141 chocolates are distributed, then how many chocolates does B get?
- A. 36
B. 32
C. 30
D. 38
17. An office consists of 70 employees. The ratio of women to men in the office is 2 : 3. How many more men should join the office so that the number of women is half the number of men in the office?
- A. 16
B. 12
C. 10
D. 14
18. A sum of Rs. 1248 was divided among A, B, and C in the ratio of $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$, respectively. What is the total share of B?
- A. 360
B. 372
C. 380
D. 384



19. A person has distributed a sum of money to Arpit, Bhim, and Chetan in such a manner that for each rupee Arpit gets, Bhim gets 65 paise and Chetan gets 35 paise. If Chetan's share is Rs. 560, then what is the total sum?
- A. 3600
B. 3200
C. 3100
D. 3800
20. The ratio of the number of male employees and female employees in a company of 720 employees is 7 : 5. How many more female employees should join to make the ratio 1 : 1?
- A. 160
B. 120
C. 130
D. 180
21. The prices of a laptop and its accessories are in the ratio 9 : 5. If the laptop costs Rs. 6800 more than its accessories, then what is the price of the laptop?
- A. 16000
B. 15200
C. 15300
D. 15800
22. After the addition of 7 in both the numerator and denominator, a fraction becomes $\frac{3}{4}$. What is the original fraction?
- A. $\frac{5}{12}$
B. $\frac{7}{9}$
C. $\frac{2}{5}$
D. $\frac{8}{9}$
23. If the salaries of three friends A, B, and C are in the ratio of 5 : 7 : 8. If there is an increase in the salary of all of them by 40%, 50%, and 75%, respectively, what will be the ratio of increased salaries?
- A. 1 : 2 : 3
B. 2 : 1 : 3
C. 3 : 2 : 1
D. 2 : 3 : 4
24. The ratio of pens and pencils in a box is 7 : 8. If the percentage increases in pens and pencils by 20% and 10% respectively, then what will be the new ratio?
- A. 21 : 22
B. 12 : 13
C. 20 : 21
D. 22 : 21
25. In the ratio of 5 : 8, the antecedent is 40, then the consequent is-
- A. 86
B. 62
C. 64
D. 60
26. The population of two cities A and B are in the ratio 2 : 3. If the population is increased by 2000, then the new ratio of the population would become 40 : 57. What is B's population?
- A. 16000
B. 12000
C. 19000
D. 14000
27. If the weights of A, B, and C are in the ratio 2 : 3 : 5. If there is an increase in weight by 15%, 10%, and 20%, then what will be the ratio of their weights?
- A. 23 : 33 : 60
B. 12 : 13 : 14
C. 22 : 21 : 40
D. 33 : 23 : 60
28. Ramesh and Vijay invested in a joint venture in the ratio of 3 : 2. If 5% of the total gains or profits are donated to the trust, and Ramesh's share is Rs. 1710, what is the total gain?
- A. 2500
B. 3000
C. 2000
D. 2800



SOLUTIONS

1. **(C)** If $A : B$ and $B : C$, then C is the third proportion.

As per the given options, 32 is A , hence cannot be the third proportion.

48 is B , hence cannot be the third proportion.

Therefore, option C , i.e., 72 can be the third proportion.

2. **(D)** The data given in this question are not sufficient to answer the same.
Therefore, option D is correct.

3. **(A)** Total sum of their ages before 8 years =
 $96 - (3 \times 8) = 72$

Let the age in ratio be x , therefore, eight years ago, ages of Abhay, Bhanu and Chetan be x , $2x$ and $3x$

So, $x + 2x + 3x = 72$ years

$6x = 72$, $x = 12$ years

Age of Chetan eight years ago = $3x = 3(12) = 36$ years

Present age of Chetan = $36 + 8 = 44$ years

The present age of Chetan is 44 years.

Therefore, option A is correct.

4. **(B)** If $A : B = C : D$, then D is the fourth proportion.

Options A , C and D are the quantities already present in the ratio proportion; thus, they cannot be the fourth proportion.

Therefore, only option B being the different quantity, can be the fourth proportion.

5. **(C)** The ratio among the children A , B , C , and $D = \frac{1}{6} : \frac{1}{8} : \frac{1}{10} : \frac{1}{12}$

On rearranging the ratio = $\frac{120}{6}, \frac{120}{8}, \frac{120}{10}$,

$$\frac{120}{12} = 20 : 15 : 12 : 10$$

Hence, the minimum number of chocolates can be when the common ratio is 1.

Thus, minimum number of chocolates = $20 + 15 + 12 + 10 = 57$.

Therefore, option C is correct.

6. **(A)** The factors of 120 are the integers that divide 120 without leaving any remainder. Therefore, such factors of 120 are, 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, and 120. Hence, there are a total 16 factors of 120. Therefore, option A is correct.

7. **(D)** $6x = 8y$ (1)

$$9z = 5y \quad \text{.....(2)}$$

Dividing (2) by (1), we get

$$\frac{9z}{6x} = \frac{5y}{8y}$$

$$\frac{z}{x} = \frac{5}{12} = \frac{20}{48}$$

Therefore, option D is correct.

8. **(A)** Let $a = 3x$ and $b = 5x$

$$\frac{2a + 3b}{4a - b} = \frac{2(3x) + 3(5x)}{4(3x) - 5x}$$

$$= \frac{21}{7} = 3 : 1$$

Therefore, option A is correct.

9. **(C)** Let the angles in the triangle be $3x$, $4x$, and $5x$

Sum of angles in a triangle = 180

$$\text{Smallest angle} = \frac{3}{12} \times 180 = 45^\circ$$

$$\text{Largest angle} = \frac{5}{12} \times 180 = 75^\circ$$

Difference between smallest and largest = $75 - 45 = 30^\circ$.

Therefore, option C is correct.

10. **(B)** Given that C has Rs. 900



$$B = 900 \times \frac{80}{100} = \text{Rs. } 720$$

$$C = 720 \times \frac{90}{100} = \text{Rs. } 648$$

Total amount A, B, and C have together
 $= 900 + 720 + 648 = \text{Rs. } 2268$
 Therefore, option B is correct.

- 11. (A)** Let the shares be $6x$, $5x$ and $4x$
 So, $6x + 5x + 4x = 9600$
 $15x = 9600$, $x = 640$
 Share of Suresh $= 6x = 6(640) = 3840$
 Share of Mahesh $= 4x = 4(640) = 2560$
 Difference $= 3840 - 2560 = \text{Rs. } 1280$
 Therefore, option A is correct.
- 12. (C)** Let Vineet and Karan's earnings be $5x$ and $6x$ then

$$= \frac{5x + 100}{6x + 100} = \frac{7}{8}$$

 $= 40x + 800 = 42x + 700$
 $x = 50$
 Vineet's earnings $= 5(50) = \text{Rs. } 250$.
 Therefore, option C is correct.
- 13. (D)** Let the shares of papers among Kartik, Ajay, Vijay, and Deepak be $5x$, $2x$, $4x$, $3x$ respectively
 Therefore, $4x - 3x = 100$
 $x = 100$
 Kartik and Ajay share $= 5(100) + 2(100) = 700$
 Therefore, option D is correct.
- 14. (A)** Let the number of coins of 25 paise, 50 paise and Re. 1 be $5x$, $6x$, and $4x$, respectively.
 Then the net value $= 0.25(5x) + 0.50(6x) + 1(4x) = 33$
 $1.25x + 3x + 4x = 33$
 $8.25x = 33$, $x = 4$
 Number of Re. 1 coins $= 4 \times 4 = 16$.
 Therefore, option A is correct.
- 15. (C)** Let the amount received by Ravi, Deepak and Amit be x , y , and z .
 $x + y + z = 770$

$$\frac{2(z + y)}{9} + y + z = 770$$

 $11x + 11y = 6930$

$x + y = 630$, amount received by Deepak and Amit $= \text{Rs. } 630$
 Amount received by Ravi $= 770 - 630 = \text{Rs. } 140$

Therefore, option C is correct.

- 16. (A)** $A : B = 10 : 6 = 5 : 3$ (1)
 $A : C = 4 : 3$ (2)
 Multiplying (1) by 4 and (2) by 5 to get a common value of A
 $A : B = 20 : 12$; $A : C = 20 : 15$
 $A : B : C = 20 : 12 : 15$
 Share of B $= \frac{12}{47} \times 141 = 36$.
 Therefore, option A is correct.
- 17. (D)** Number of women $= \frac{2}{5} \times 70 = 28$
 Number of men $= \frac{3}{5} \times 70 = 42$
 Let x be the more men should join so that

$$\frac{28}{42 + x} = \frac{1}{2}$$

 $56 = 42 + x$, $x = 14$.
 Therefore, option D is correct.
- 18. (D)** Share of $A : B : C = \frac{1}{2} : \frac{1}{3} : \frac{1}{4}$
 $= \frac{6}{12} : \frac{4}{12} : \frac{3}{12} = 6 : 4 : 3$
 Share of B $= \frac{4}{13} \times 1248 = \text{Rs. } 384$.
 Therefore, option D is correct.
- 19. (B)** Share of Arpit $= \frac{560}{35} \times 100 = \text{Rs. } 1600$
 (Rs. 1 = 100 paise)
 Share of Bhim $= \frac{540}{35} \times 65 = \text{Rs. } 1040$
 Total sum $= 1600 + 1040 + 560 = \text{Rs. } 3200$.
 Therefore, option B is correct.
- 20. (B)** Let the number of male employees and female employees be $7x$ and $5x$, respectively.
 $7x + 5x = 720$
 $12x = 720$, $x = 60$
 Number of male employees $= 420$, number of female employees $= 300$



To make the ratio 1 : 1, $420 - 300 = 120$ more female employees should join the company.

Therefore, option B is correct.

- 21. (C)** Let the price of the laptop and its accessories be $9x$ and $5x$, respectively.
Given that $9x - 5x = \text{Rs. } 6800$
 $4x = \text{Rs. } 6800, x = 1700$
Price of laptop = $9(1700) = \text{Rs. } 15300$.
Therefore, option C is correct.

22. (C) Option A $\frac{5+7}{12+7} = \frac{12}{19}$

Option B $\frac{7+7}{9+7} = \frac{14}{16} = \frac{7}{8}$

Option C $\frac{2+7}{5+7} = \frac{9}{12} = \frac{3}{4}$

Therefore, option C is correct.

- 23. (D)** Let the salaries of A, B and C be $5x$, $7x$ and $8x$ respectively
Increment in salary (140% of $5x$), (150% of $7x$), (175% of $8x$)
 $(\frac{140}{100} \times 5x), (\frac{150}{100} \times 7x), (\frac{175}{100} \times 8x)$
 $7x, \frac{21x}{2}$ and $14x$
Thus, the required ratio will be $14x : 21x : 14x$
 $= 2 : 3 : 4$.
Therefore, option D is correct.

- 24. (A)** Let the pens and pencils in the box be $7x$ and $8x$ respectively.
Increased number is (120% of $7x$) and (110% of $8x$)
 $(\frac{120}{100} \times 7x)$ and $(\frac{110}{100} \times 8x)$
So, $\frac{42x}{5}$ and $\frac{44x}{5}$
Thus, the required ratio would be $42 : 44$
 $= 21 : 22$.

Therefore, option A is correct.

- 25. (C)** Let the antecedent and consequent be $5x$ and $8x$, respectively.
 $5x = 40, x = 8$
Consequent = $8(8) = 64$
Therefore, option C is correct.

- 26. (C)** Let $2x$ and $3x$ be the original population of A and B, respectively.
Then, $\frac{2x+2000}{3x+2000} = \frac{40}{57}$
 $57(2x+2000) = 40(3x+2000)$
 $6x = 34000$
 $3x = 17000$
B's present population = $17000 + 2000 = 19000$
Therefore, option C is correct.

- 27. (A)** Let the weights of A, B and C be $2x$, $3x$ and $5x$ respectively.
A's weight = $\frac{115}{100} \times 2x = \frac{23x}{10}$
B's weight = $\frac{110}{100} \times 3x = \frac{33x}{10}$
C's weight = $\frac{120}{100} \times 5x = \frac{6x}{10}$
New required ratio = $23 : 33 : 60$
Therefore, option A is correct.

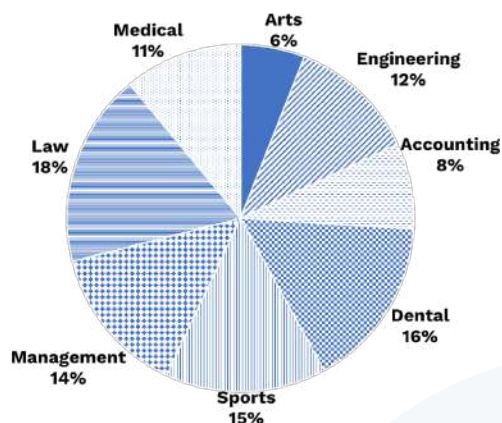
- 28. (B)** Let the profit be Rs. 100
After donating to the trust, Ramesh's share is Rs. $(95 \times \frac{3}{5}) = \text{Rs. } 57$
If Ramesh's share is Rs. 57, total profit = Rs. 100
If Ramesh's share is Rs. 1710, the total profit = Rs. $(\frac{1710 \times 100}{57}) = \text{Rs. } 3000$
Therefore, option B is correct.



PRACTICE QUESTIONS

Questions 1–5: Analyse the given data and answer the following questions:

Total number of students = 2500



The ratio of boys to girls:

DEPARTMENT	BOYS	GIRLS
Engineering	7	5
Accounting	2	3
Dental	5	3
Sports	7	8
Management	3	4
Law	5	4
Medical	6	5
Arts	2	1

- What is the ratio between boys and girls students in engineering and arts respectively?
A. 7 : 4
B. 4 : 7
C. 3 : 4
D. 2 : 3
- What is the difference between the total number of students in management and law?
A. 80
B. 150

- C. 75
D. 100

- What is the ratio between the total number of boys in Dental and Sports and the total number of girls in these two departments, respectively?
A. 13 : 17
B. 17 : 14
C. 13 : 14
D. 14 : 13
- How many girls are pursuing accounts?
A. 130
B. 120
C. 125
D. 110
- What is the difference between boys and girls in the medical department?
A. 25
B. 75
C. 100
D. 40

Questions 6–10: Analyse the given data and answer the following questions:

Total citizens of Konoha are 8000 and there are 3 regions, i.e., P, Q and R in it. Ratio of citizens of these 3 regions (P : Q : R) is 8 : 5 : 3. Number of males in region P is 1900 more than that of females in region R. The ratio of number of females in region P to that of males in region R is 5 : 3. The number of females in region Q is 30% of the number of males in region P.

- What is the average of the number of males in region P and Q together is how much more than the number of females in region P?
A. 650
B. 625
C. 675
D. 610
- 60% and 88% of females of region P and Q, respectively, are working, then find non-



working females of region P is how much per cent less than that working females of region Q?

- A. $\frac{58}{11}$ %
- B. $\frac{67}{11}$ %
- C. $\frac{100}{11}$ %
- D. $\frac{89}{11}$ %

8. Total female citizens of region P and R together is what per cent of total citizens of these two regions together?

- A. 28.18%
- B. 30.18%
- C. 32.18%
- D. 38.18%

9. What is the ratio of the total number of females in all regions to that of the total number of males in all regions?

- A. $\frac{42}{89}$
- B. $\frac{47}{139}$
- C. $\frac{39}{55}$
- D. $\frac{57}{103}$

10. The average of male citizens of region Q and R is how much more than the average of female citizens of these two regions?

- A. 650
- B. 600
- C. 550
- D. 500

SOLUTIONS

1. **(A)** Boys in engineering = $\frac{7}{12}$ of 12% of total students

Boys in arts = $\frac{2}{3}$ of 6% of total students

$$\begin{aligned} \text{Ratio} &= \frac{\frac{7}{12} \text{ of } 12\% \times \text{total students}}{\frac{2}{3} \times 6\% \times \text{total students}} \\ &= \frac{14}{12} \times \frac{3}{2} = 7 : 4 \end{aligned}$$

Therefore, option A is correct.

2. **(D)** Required difference = $2500 \times (18-14)\%$
 $= \frac{2500 \times 4}{100} = 100$

Therefore, D is correct.

3. **(B)** Total students in dental = 16% of total students

$$= \frac{16}{100} \times 2500 = 400 \text{ students in dental}$$

$$\text{Girls in dental} = 400 - 250 = 150$$

$$\text{Total students in sports} = \frac{7}{15} \times 375 = 175$$

$$\text{Girls in sports} = 375 - 175 = 200$$

$$\text{Required ratio} = 425 : 350 = 17 : 14$$

Therefore, option B is correct.

4. **(B)** Total girls = $\frac{3}{5}$ of 8% of total students

$$= \frac{3}{5} \times \frac{8}{100} \times 2500 = 120$$

Therefore, option B is correct.

5. **(A)** Difference = $\frac{6}{11} \times \frac{11}{100} \times 2500 - \frac{5}{11} \times \frac{11}{100} \times$
 2500

$$= 150 - 125 = 25$$

Therefore, option A is correct.

Solutions 6–10:

Let the number of males in region R be $3x$, the number of females in region P will be $5x$.

$$\begin{aligned} \text{Ratio of the citizens in three regions P : Q : R} \\ = 8 : 5 : 3 \end{aligned}$$



$$\text{Citizens in P} = 8 \times \frac{8000}{16} = 4000$$

$$\text{Citizens in Q} = 5 \times \frac{8000}{16} = 2500$$

$$\text{Citizens in R} = 3 \times \frac{8000}{16} = 1500$$

Number of males in region P is 1900 more than that of females in region R.

The given information can be tabulated as:

P(4000)		Q(2500)		R(1500)	
MALES	FEMALES	MALES	FEMALES	MALES	FEMALES
$1500 - 3x + 1900$	$5x$		$30 \frac{\{1500 - 3x + 1900\}}{100}$	$3x$	$1500 - 3x$

The total citizens in region P = 4000

$$(1500 - 3x + 1900) + 5x = 4000$$

$$3400 + 2x = 4000$$

$$x = 300$$

Now, the table will be

P(4000)		Q(2500)		R(1500)	
MALES	FEMALES	MALES	FEMALES	MALES	FEMALES
2500	1500	1750	750	900	600

6. (B) Males in P = 2500, males in Q = 1750

$$\text{Average} = \frac{2500 + 1750}{2} = 2125$$

Females in P = 1500

$$\text{Required difference} = 2125 - 1500 = 625$$

Therefore, option B is correct.

7. (C) Females in P = 1500, working females in P = 60% of 1500, non-working females in P = 40% of 1500 = 600

Females in Q = 750, working females in Q = 88% of 750 = 660

$$\text{Required \%} = (660 - 600) \times \frac{100}{660} = \frac{100}{11} \%$$

Therefore, option C is correct.

8. (D) Females in P and R = 1500 + 600 = 2100
Total citizens of P and R = 4000 + 1500 = 5500

$$\text{Required \%} = 2100 \times \frac{100}{5500} = 38.18\%$$

Therefore, option D is correct.

9. (D) Total number of females in all regions = 1500 + 750 + 600 = 2850
Total number of males in all regions = 2500 + 1750 + 900 = 5150

$$\text{Required ratio} = \frac{2850}{5150} = \frac{57}{103}$$

Therefore, option D is correct.

10. (A) Males in Q = 1750

Males in R = 900

$$\text{Average} = \frac{1750 + 900}{2} = 1325$$

Females in Q = 750

Females in R = 600

$$\text{Average} = \frac{750 + 600}{2} = 675$$

$$\text{Required difference} = 1325 - 675 = 650$$

Therefore, option A is correct.



SYNOPSIS

- Mixtures
- Mean price
- Alligation
- Rule of alligation
- Criss-cross method
- Common types of questions asked
- Summary

MIXTURE

A mixture is the combination of two or more quantities.

Mean price

The cost price of the unit quantity of a mixture is called the mean price.

When a person buys x kg of the first kind of an item and y kg of the second kind of an item, the total amount he spends is $Rs.(px + qy)$ and the total amount of mixture is $(x + y)$ kg. But the ratio in which these two ingredients at the given price must be mixed to produce a mixture of a different desired price is calculated using the rule of alligation.

ALLIGATION

Rule of alligation is used to find the ratio in which two or more ingredients must be mixed at a given price to produce the mixture at a desired price.

Rule of alligation

If two items are mixed, then

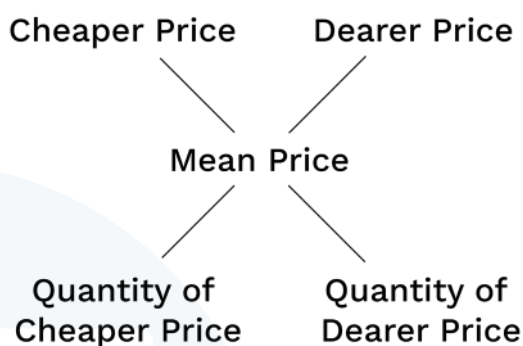
$$\left(\frac{\text{quantity of cheaper item}}{\text{quantity of dearer item}} \right) = \frac{(\text{CP of dearer}) - (\text{mean price})}{(\text{mean price}) - (\text{CP of cheaper})}$$

Or,

$$\text{cheaper quantity} : \text{dearer quantity} = (d - m) : (m - c)$$

Where, d , m , and c are dear price, mean price, and cheap price respectively.

Criss-cross method



The difference between mean price and cheaper price in the place of 'quantity of dearer price' and the difference between mean price and dearer price in the place of 'quantity of cheaper price.'

Example:

In what ratio were two different kinds of cereals costing Rs. 45 per kg and 60 per kg, mixed to get a mixture costing Rs. 50 per kg.

Solution:

Applying the Criss-Cross method:

$$\begin{aligned} \text{Ratio} &= \frac{(60 - 40)}{50 - 45} \\ &= \frac{10}{5} \end{aligned}$$

Therefore, the ratio of the two types of cereals = 2 : 1.

COMMON TYPES OF QUESTIONS ASKED

1. The ratio of items required

In this type of questions, the price and quantity of the mixture along with the price of items are provided, and the students are required to calculate the ratio in which the items are



to be mixed to get the required amount of mixture, using the given price and quantity of the mixture along with the price of items.

Example:

In what proportion must a shopkeeper mix rice type 1 at Rs. 4.08 per kg and rice type 2 at Rs. 5.76 per kg to make a mixture worth Rs. 5.04 per kg?

Solution:

Cheap price = Rs. 4.08

Mean price = Rs. 5.04

Dear price = Rs. 5.76

Thus, according to the alligation formula,
cheap quantity : dear quantity = $(d - m) : (m - c)$

$$\text{Required proportion} = (5.76 - 5.04) : (5.04 - 4.08) \\ = 0.72 :: 0.96 = 3 : 4$$

2. The final quantity of the original item

In these types of questions, the original quantity of a liquid is provided along with the amount of liquid that is being taken out and replaced by some other liquid two or three times, and the students have to calculate the final quantity of the original liquid left.

Example:

A container has 20 L milk. 4 L of milk is replaced with an equal quantity of water. What will be the final quantity of milk left in the container if this process is repeated once again?

Solution:

Original quantity of milk = 20 L

After taking out 4 L and replacing it with water, milk left = 16 L

When repeating the process, after taking out 4 L of this new mixture

$$\text{Quantity of milk taken out} = 4 \times \frac{16}{20} = 3.2 \text{ L}$$

$$\text{Thus, the required quantity of milk left} \\ = 32 - 3.2 \text{ L} = 28.8 \text{ L}$$

3. The ratio of two liquids in the third container

In these types of questions, the ratio of two liquids in two different containers is given, and the students are asked to calculate the new ratio of these two liquids if both the containers are poured out in a third container together.

Example:

Two jars have oil and milk in the ratio 4 : 1 and 5 : 1, respectively. If both jars are emptied into a third jar, then what will be the ratio of oil and water in this third jar?

Solution:

Ratio of oil and milk in 1st jar = 4 : 1

$$\text{Quantity of oil in this 1st jar} = \frac{4}{5}$$

$$\text{Quantity of milk in this 1st jar} = \frac{1}{5}$$

The ratio of oil and milk in 2nd jar = 5 : 1

$$\text{Quantity of oil in this 2nd jar} = \frac{5}{6}$$

$$\text{Quantity of milk in this 2nd jar} = \frac{1}{6}$$

$$\text{In the third jar, the quantity of milk} = \frac{4}{5} + \frac{5}{6} = \frac{49}{30}$$

$$\text{Quantity of water} = \frac{1}{5} + \frac{1}{6} = \frac{11}{30}$$

$$\text{Therefore, required ratio} = \frac{49}{30} : \frac{11}{30} = 49 : 11$$

Note

- If not given, assume the cost of water to be Rs. 0.



Chapter Summary

- Rule of alligation: of two items are mixed, then

$$\left(\frac{\text{quantity of cheaper item}}{\text{quantity of dearer item}} \right) = \frac{(\text{CP of dearer}) - (\text{mean price})}{(\text{mean price}) - (\text{CP of cheaper})}$$

Or,

$$\text{cheaper quantity} : \text{dearer quantity} = (d - m) : (m - c)$$

- Where, d , m , and c are dearer price, mean price, and cheap price, respectively.

PRACTICE QUESTIONS

1. If 50 L of oil and water mixture comprises of 50% oil, then how much more oil should be mixed to make it 70% in the mixture?
A. 33.33 L
B. 28.14 L
C. 16.66 L
D. 11.11 L
2. A mixture of x mL milk and 8 mL water costs Rs. 90 per mL. If the cost price of milk is Rs. 108 mL, then how much milk is there in the mixture?
A. 40 mL
B. 32 mL
C. 36 mL
D. 44 mL
3. In a container, liquid A and liquid B are in their ratio of 7 : 5. If 45 L more of liquid B is mixed in the mixture, the ratio of liquid B and A becomes 8 : 7. Find the amount of liquid B in the old mixture.
A. 65 L
B. 70 L
C. 75 L
D. 80 L
4. A mixture of Rooh Afza and 16 L of water costs rupees 95 per litre. If pure Rooh Afza costs Rs.114 per litre, then what is the amount of Rooh Afza in the original mixture?
A. 65 L
B. 70 L
C. 75 L
D. 80 L
5. A shopkeeper sells a mixture of 45 L milk and 9 L water at Rs. 66 per kg, what is the cost price of pure milk?
A. 86.4 L
B. 76.5 L
C. 79.2 L
D. 84.7 L
6. In a mixture of 15 mL, ratio of two liquids is 4 : 2. What amount of liquid 2 should be mixed in this to make the ratio of the two liquids 1 : 2.
A. 5 mL
B. 8 mL
C. 15 mL
D. 18 mL
7. 150 mg of sugar solution has 20% sugar in it. What quantity of sugar should be added in the sugar water solution to make sugar 25% in the solution?
A. 7 mg
B. 10 mg
C. 15 mg
D. 12 mg
8. 5 kg of alloy X is mixed with 25 kg of alloy Y. If the amount of copper and zinc in alloy



- X is in the ratio 1 : 2, and the amount of copper and zinc in alloy Y is in the ratio of 2 : 3, then what is the ratio of copper to tin in the resulting mixture?
- A. 1 : 3
B. 3 : 5
C. 5 : 7
D. 7 : 11
9. A deodorant contains butane and benzyl alcohol in the ratio 5 : 3. If 8 L of the mixture is replaced with the same quantity of benzyl alcohol, the ratio becomes 3 : 5. What is the total quantity of the container?
- A. 5 L
B. 10 L
C. 15 L
D. 20 L
10. 27 litres of milk and water mixture in the ratio of 4 : 5 is added to a bottle containing x litre pure milk. If the new mixture formed contains 70% milk, what was the original quantity of pure milk in the bottle?
- A. 21 L
B. 23 L
C. 25 L
D. 27 L
11. A mixture of 10 g of wheat and maida contains 20% maida. How much maida should be added to this mixture of wheat and maida to get 33.33% maida?
- A. 6 g
B. 2 g
C. 4 g
D. 5 g
12. A jewellery shop has two qualities of gold, 24 carats, and 22 carats purity gold. In what proportion should he mix both qualities of gold to make an ornament having 20 carats purity?
- A. 1 : 2
B. 5 : 3
C. 3 : 4
D. 4 : 5
13. If selling price of milk is Rs. 100. In what ratio should a salesman mix water in that milk, to sell the water-milk mixture at Rs. 80, and obtain a profit of 50%?
- A. 7 : 8
B. 8 : 9
C. 9 : 11
D. 11 : 13
14. How much amount of butter worth 25 per kg should be blended with 30 kg butter that costs 30 per kg to gain 10% and sell the mixture at 30 per kg?
- A. 36 kg
B. 35 kg
C. 40 kg
D. 38 kg
15. There are two different types of plastic available in market, one having the ratio of components A and B as 2 : 11 and other having same, 5 : 21. In what proportion should a manufacturer combine both the components to get A and B in 7 : 32 ratio?
- A. 1 : 2
B. 3 : 4
C. 2 : 3
D. 5 : 6
16. When 70% of the juice and 30% of the water were removed from a full vessel, it gets emptied by 55%, leaving only 160 L of mixture. What is the remaining amount of juice and water in the mixture?
- A. Water = 60 L, juice = 100 L
B. Water = 90 L, juice = 70 L
C. Water = 80 L, juice = 80 L
D. Water = 100 L, juice = 60 L
17. The ratio of acid to ethanol in a 6 mL liquid is 2 : 1. If the acid-to-ethanol ratio has to be 1 : 2, the amount of ethanol to be added to the mixture is
- A. 5.5 mL
B. 6 mL



- C. 6.5 mL
D. 5 mL
18. 4 mL of liquid from an oil bottle is replaced with water. And then this action is repeated three times more, until the ratio of oil to water in the container equals 16 : 65. How much oil was in the container at the start?
A. 16 mL
B. 12 mL
C. 15 mL
D. 13 mL
19. A container is filled with pulses, 6 parts of which are type 1 and 10 part type 2. How much of the mixture must be replaced with type 2 so that the mixture contains half type 1 and half type 2?
A. $\frac{2}{3}$
B. $\frac{3}{7}$
C. $\frac{1}{5}$
D. $\frac{1}{8}$
20. A mixture of 48 mL contains H_2SO_4 and HCl in the ratio of 3 : 5. How much HCl must be added to this mixture to reverse the ratio?
A. 10 mL
B. 14 mL
C. 8 mL
D. 16 mL
21. How many litres of refined oil costing Rs. 9 per L must be mixed with 27 L of refined oil costing Rs. 7 per L to get a gain of 10% by selling the mixture at Rs. 9.24 L.
A. 61 L
B. 58 L
C. 65 L
D. 63 L
22. Two jars B and A have water and buttermilk mixed in the ratio 2 : 5 and 6 : 7. If a new mixture in vessel C must have water and buttermilk in the ratio 5 : 8, calculate the ratio in which mixture from these two jars must be mixed.
A. 3 : 5
B. 3 : 4
C. 1 : 5
D. 7 : 9
23. A jar full of soda water contains 40% soda. A part of this soda water is replaced by another mixture containing 19% soda, and now the jar contains 26% soda. The quantity of soda water replaced is:
A. $\frac{2}{5}$
B. $\frac{1}{5}$
C. $\frac{2}{3}$
D. $\frac{3}{5}$
24. A juice shop has two big jars of juice. First one contains 25% water and 75% fruit pulp. Second one contains 50% water and 50% fruit pulp. How much juice should the shop mix from each of the containers to get 12 litres of fruit pulp, and ratio of water and pulp being 3 : 5?
A. 2 L
B. 4 L
C. 6 L
D. 8 L
25. The cost of onion A is Rs. 15 per kg and onion B is Rs. 20 per kg. If both types A and B are mixed in the ratio of 2 : 3, then the price per kg of the mixed variety of onion is:
A. 13 per kg
B. 18 kg
C. 16 kg
D. 12 kg
26. In what proportion water must be added to ethanol to gain 20% by selling resulting mixture at the cost price?



- A. 4 : 3
B. 3 : 4
C. 5 : 6
D. 1 : 5
- 27.** In a pot, a mixture of curd and water is in the ratio of 4 : 5. When it is added an additional 8 g of curd, the pot gets full, and the ratio of curd and water becomes 6 : 5. What is the capacity of the pot?
A. 11 g
B. 22 g
C. 33 g
D. 44 g
- 28.** The diluted hydrochloric acid contains 8 litres of HCl and rest water. How many litres of the mixture should be replaced with pure hydrochloric acid to get mixture of 30% concentration if there were initially 32 litres of water in the mixture?
A. 2 L
B. 3 L
C. 4 L
D. 5 L
- 29.** In a mixture of sand and water, the proportion of sand by weight was 75%. If in 60 grams of mixture, 15 gram sand was added, what would be the percentage of sand?
A. 83%
- B. 70%
C. 75%
D. 62%
- 30.** The ratio of ethanol and isopropanol in two different sanitisers is 2 : 3 and 4 : 5. In what ratio are we required to mix the mixtures of two sanitizers in order to get the new mixture in which the ratio of ethanol and isopropanol is 7 : 5?
A. 11 : 7
B. 7 : 5
C. 5 : 3
D. 3 : 1
- 31.** A mixture contains diesel and oil in a ratio of 2 : 3. How much oil should be added to the 60 L mixture to make the ratio of diesel and mixture as 1 : 3?
A. 16
B. 12
C. 10
D. 18
- 32.** The ratio of water and dye is 5 : 2 in the mixture of 28 litres. If 2 litres of dye are added to the mixture, then what will be the ratio of water and dye in the new solution?
A. 1 : 3
B. 1 : 2
C. 3 : 1
D. 2 : 5

SOLUTIONS

- 1. (A)** Initial quantity of oil = 25 L
Oil to be added = x L
Since water remains same,
 30% of $(50 + x) = 25$ L
 $150 + 3x = 250$ L
 $x = 33.33$ L
Therefore, option A is correct.
- 2. (A)** Water : Milk = $(108 - 90) : (90 - 0)$
= $18 : 90 = 1 : 5$

Thus, if the water in mixture is 8 mL, milk quantity will be $8 \times 5 = 40$ mL
Therefore, option A is correct.

- 3. (C)** Liquid A: liquid B = 7 : 5
Liquid B initially = $5x$
Liquid B after addition = $5x + 45$
New ratio = $7 : 8 = 7x : 5x + 45$
 $56x = 35x + 315$
 $21x = 315$



$$x = 15$$

Thus, original liquid B = 75 L

Therefore, option C is correct.

4. (D) Water: Rooh Afza = $(114 - 95) : (95 - 0)$
 $= 19 : 95 = 1 : 5$

Thus, if water in mixture is 16 L, Rooh Afza quantity will be $16 \times 5 = 80$ L

Therefore, option D is correct.

5. (C) $\frac{\text{Quantity of water}}{\text{quantity of milk}} = \frac{x - 66}{66 - 0}$
 $\Rightarrow \frac{45}{9} = \frac{66}{x - 66}$
 $\Rightarrow 5x - 330 = 66$
 $\Rightarrow x = \frac{396}{5} = 79.2$

Therefore, option C is correct.

6. (C) The initial quantity of liquid 1 = 10 mL
 The initial quantity of liquid 2 = 5 mL
 Since liquid 1 remains same,

$$\frac{1}{3} \times (15 + x) = 10 \text{ mL}$$

$$X = 15 \text{ mL}$$

To get the mixture in ratio 1 : 2, (i.e., liquid 2 double the amount of liquid 1)

Liquid 2 to be added = 15 mL

Therefore, option C is correct.

7. (B) Initial quantity of sugar = 30 g
 Since water remains same,
 75% of $(150 + x)$ g = 120 g
 $x = 10$ g
 Sugar to be added = 10 g
 Therefore, option B is correct.

8. (C) Amount of copper in alloy $x = \left(\frac{1}{3}\right) \times 5 = \frac{5}{3}$

$$\text{Amount of zinc in alloy } x = \left(\frac{2}{3}\right) \times 5 = \frac{10}{3}$$

$$\text{Amount of copper in alloy } y = \left(\frac{2}{5}\right) \times 25 = 10 \text{ kg}$$

$$\text{Amount of zinc in alloy } y = \left(\frac{3}{5}\right) \times 25 = 15 \text{ kg}$$

When alloys x and y are mixed, then

$$\text{Amount of copper} = \frac{5}{3} + 10 = \frac{35}{3}$$

$$\text{Amount of zinc} = \frac{10}{3} + 15 = \frac{55}{3}$$

The ratio of copper and zinc in new mixture
 $= 7 : 11$

Therefore, option C is correct.

9. (D) Quantity of butane removed
 $= \frac{5}{5+3} \times 8 = 5 \text{ L}$

Quantity of benzyl alcohol removed

$$= \frac{3}{5+3} \times 8 = 3 \text{ L}$$

New ratio,

$$3 : 5 = \frac{5x - 5}{3x - 3 + 8}$$

$$9x + 15 = 25x - 25$$

$$x = 2.5$$

Total quantity of container = 20 L

Therefore, option D is correct.

10. (B) Quantity of milk in 27 L mixture
 $= \frac{4}{4+5} \times 27 = 12 \text{ L}$

Quantity of water = 15 L

In new mixture,

$$\Rightarrow \frac{7}{3} = \frac{x + 12}{15}$$

$$3x + 36 = 15 \times 7 = 105$$

$$x = 23 \text{ L}$$

Therefore, option C is correct.

11. (B) In the mixture of 10 litres, maida is 20%,
 i.e., 2 kg.

Let x kg of maida be added.

According to the question,

$$\Rightarrow \frac{10 + x}{2 + x} = \frac{100}{33.33} = \frac{3}{1}$$

$$x = 2$$

Therefore, option B is correct.

12. (A) Proportion = $\frac{22 - 20}{20 - 24} = 1 : 2$

Therefore, option A is correct.

13. (A) The total price of milk is now
 $80 \times (1 + x)$

But, he gets a profit of 50%.

$$\text{So, } 80(1 + x) = 150$$



$$1 + x = \frac{15}{8}$$

$$x = \frac{7}{8}$$

Therefore, option A is correct.

$$\begin{aligned} 14. \text{ (A)} \quad (25x + 30 \times 30) \times \frac{110}{100} &= 30(30 + x) \\ (275x + 9900) &= (9000 + 300x) \\ \Rightarrow 300x - 275x &= 900 = x = \frac{900}{25} = 36 \end{aligned}$$

Therefore, option A is correct.

15. (A) Quantity of component A in the first kind of plastic = 213
Quantity of component B in second kind plastic = 526
Required ratio = 178 : 139 = 1 : 2
Therefore, option A is correct.

16. (A) Juice : water = 5 : 3
Quantity of juice in the remaining mixture
= $\frac{5}{8} \times 160 = 100$ L
Similarly, quantity of water
= $\frac{3}{8} \times 160 = 60$ L

Therefore, option A is correct.

17. (B) Quantity of acid in the mixture = $\frac{2}{3} \times 6$
= 4 mL
Quantity of ethanol in the mixture = $\frac{1}{3} \times 6$
= 2 mL
Let the required quantity of ethanol be x L.
 $\Rightarrow \frac{4}{2} + x = \frac{1}{3}$
 $8 = 2 + x$
 $x = 6$ mL

Therefore, option B is correct.

18. (B) Quantity of oil in container = x mL.
Quantity of water left in cask = $[x(1 - 4x)4]$ mL
Thus, $\left[x \left(1 - \frac{4}{x} \right) 4x \right] = 1681$
 $\Rightarrow (1 - 4x)4 = (23)4$

$$\Rightarrow (x - 4x) = \frac{23}{3x} - 12$$

$$x = 12$$

Therefore, option B is correct.

19. (C) Let the container initially contain 16 kg of pulses.
Let a kg of pulses be compressing type 2 pulse.

$$\begin{aligned} \text{Quantity of type 1 pulse in the new mixture} \\ = \left(6 + \frac{6a}{16} + a \right) \end{aligned}$$

$$\begin{aligned} \text{Quantity of type 2 pulse in the new mixture} \\ = \left(10 - \frac{10a}{16} \right) \end{aligned}$$

$$\Rightarrow 6 - \frac{6a}{16} + a = 10 - \frac{10a}{16}$$

$$\Rightarrow 96 - 6a + 16a = 160 - 10a$$

$$\Rightarrow 96 + 10a = 160 - 10a$$

$$\Rightarrow 20a = 64$$

$$\Rightarrow a = \frac{64}{20} = \frac{16}{5}$$

$$\text{Part of mixture replaced} = \left(\frac{1}{16} \right) \times \left(\frac{16}{5} \right) = \frac{1}{5}$$

Therefore, option C is correct.

20. (C) Initial quantity of HCl = $\left(\frac{3}{8} \right) \times 48$ litres
= 18 mL

To get the ratio 5 : 3, new quantity of HCl

$$= \frac{5}{8} = \frac{18 + x}{48 + x}$$

$$= 120 + 5x = 144 + 8x$$

$$3x = 24$$

$$x = 8 \text{ mL}$$

Therefore, option C is correct.

21. (D) S = Rs. 9.24

Gain of 10%

$$C = \text{Rs.} \left(\frac{110}{100} \times 9.24 \right) = \text{Rs. } 8.40$$

According to alligation,

The ratio of 1st and 2nd kinds = 14 : 6 = 7 : 3

Let x kg of butter of 1st be mixed with 27 L of 2nd kind

$$7 : 3 = x : 27$$

$$\Rightarrow x = 7 \times \frac{27}{3} = 63$$

Therefore, option D is correct.

- 22. (D)** Buttermilk in 1-litre mix of A = $\frac{5}{7}$ litre

$$\text{Buttermilk in 1-litre mix of B} = \frac{7}{13} \text{ litre}$$

$$\text{Buttermilk in 1-litre mix of C} = \frac{8}{13} \text{ litre}$$

By rule of the alligation,

$$\text{Required ratio} = \frac{13}{1} : \frac{9}{19}$$

$$= 7 : 9$$

Therefore, option D is correct.

- 23. (C)** Ratio of 1st and 2nd quantities = 7 : 14 = 1 : 2

$$\text{Quantity of soda water replaced} = \frac{2}{3}$$

Therefore, option C is correct.

- 24. (C)** Juice from the first container = x
 Juice from the second container = $(12 - x)$
 Juice pulp in x litres of the first container = $0.75x$
 Water in x litres of the first container = $0.25x$
 Juice pulp in $(12 - x)$ litres of the second container = $0.5(12 - x)$
 Water in $(12 - x)$ litres of the second container = $0.5(12 - x)$
 Ratio of water to juice pulp
 $= [0.25x + 0.5(12 - x)] : [0.75x + 0.5(12 - x)]$
 $= 3 : 5$

$$\Rightarrow \frac{(0.25x + 6 - 0.5x)}{(0.75x + 6 - 0.5x)} = \frac{3}{5}$$

$$\Rightarrow \frac{6 - 0.25x}{6 + 0.25x} = \frac{3}{5} \quad (6 - 0.25x)(0.25x + 6) = 35$$

$$\Rightarrow 30 - 1.25x = 0.75x + 18$$

$$\Rightarrow 2x = 12$$

$$\therefore x = 6$$

Therefore, option C is correct.

- 25. (B)** Price of the mixed variety = x per kg
 According to alligation,

$$\therefore \frac{x - 15}{20 - x} = \frac{3}{2}$$

$$\Rightarrow 60 - 3x = 2x - 30$$

$$\Rightarrow 5x = 9$$

$$\Rightarrow x = 18$$

So, the price of the mixture is Rs. 18 per kg.

Therefore, option B is correct.

$$\text{26. (D)} \quad \frac{\text{Quantity of water}}{\text{Quantity of ethanol}} = \frac{\frac{5}{3}}{\frac{25}{3}} = \frac{1}{5}$$

The ratio of water and ethanol = 1 : 5

Therefore, option D is correct.

- 27. (D)** Let the capacity of the pot be x g.
 Quantity of curd in the mixture before adding more curd = $\left(\frac{4}{9}\right) \times (x - 8)$

After adding curd, the quantity of curd in the mixture = $\frac{6}{11}x$.

$$\Rightarrow \frac{6x}{11} - 8 = \left(\frac{4}{9}\right) \times (x - 8)$$

$$\Rightarrow 10x = 792 - 352 = x = 44.$$

Therefore, option D is correct.

- 28. (D)** Original quantity of water = 32 L
 Original quantity of hydrochloric acid = 8 L
 Initial hydrochloric acid to water ratio = 1 : 4
 Required hydrochloric acid to water ratio = 3 : 7

$$\text{Concentration ratio, } \frac{8 + x}{40 + x} = \frac{3}{10}$$

$$80 + 8x = 120$$

$$x = 5 \text{ L}$$

Therefore, option D is correct.

- 29. (A)** Sand in 60 gram mixture = $60 \times \frac{75}{100} =$

45 gram and water = 15 gram.

After adding 15 gram of sand in mixture, total sand = $45 + 15 = 60$ gram and weight of a mixture = $60 + 15 = 75$ gram.

$$\text{So \% of sand} = 100 \times \frac{60}{75} = 80\%$$

Therefore, option A is correct.

- 30. (C)** Let x be the quantity taken from the first container

$$\text{In } x \text{ litres, the volume of ethanol} = \frac{2x}{5}$$



In x litres, the volume of isopropanol = $\frac{3x}{5}$

Let y be the quantity taken from the second container

In y litres, the volume of ethanol = $\frac{4y}{9}$

In y litres, the volume of isopropanol = $\frac{5y}{9}$

When x litres from the first container and y litre from the second container is mixed, quantity of ethanol in the new mixture

$$= \frac{2x}{5} + \frac{4y}{9}$$

Quantity of isopropanol in the new mixture

$$= \frac{3x}{5} + \frac{5y}{9}$$

According to the question, the ratio of isopropanol to ethanol in the new mixture should be 7 : 5

$$\Rightarrow \frac{3x}{5} + \frac{5y}{9} : \frac{2x}{5} + \frac{4y}{9} = 7 : 5$$

$$\Rightarrow 5 \left(\frac{3x}{5} + \frac{5y}{9} \right) = 7 \left(\frac{2x}{5} + \frac{4y}{9} \right)$$

$$\Rightarrow \frac{5(27x + 25y)}{45} = \frac{7(18x + 20y)}{45}$$

$$\Rightarrow 135x + 125y = 126x + 140y$$

$$\Rightarrow 9x = 15y$$

$$\Rightarrow \frac{x}{y} = \frac{15}{9}$$

$$\Rightarrow \frac{x}{y} = \frac{5}{3}$$

$$\Rightarrow x : y = 5 : 3$$

Therefore, option C is correct.

31. (B) Let the ratio in the mixture be $2x, 3x$

$$2x + 3x = 60,$$

$$5x = 60, x = 12$$

Diesel in mixture = 24 L

To make the ratio 1 : 3, let the mixture be $3x$

$$\text{Thus, } 3 \times 24 = 72$$

$$\text{Oil to be added in the mixture} = 72 - 60 = 12 \text{ L}$$

Therefore, option B is correct.

32. (C) Let the water and dye be $5x$ and $2x$, respectively

$$\text{So, } 5x + 2x = 28$$

$$7x = 28, x = 4$$

$$\text{Dye in the new solution} = 2(4) + 2 = 10 \text{ litres}$$

$$\text{Then the new ratio will be } 30 : 10 = 3 : 1$$

Therefore, option C is correct.

PRACTICE QUESTIONS

Questions 1–5: A dairy house has 5000 L of pure milk. It sells 40% of the total milk to six different milk sellers and utilises remaining 60% of total milk in the dairy itself. Also, each seller, A, B, C, D, E, and F mixes water in their milk. The following table shows the sales of milk to six different sellers by dairy as a percentage of total sales and also the concentration of water in each seller's milk.

PERSON	MILK SOLD BY EACH PERSON	WATER ADDED BY EACH
A	24%	24%
B	10%	10%
C	12%	18%
D	7%	15%
E	28%	25%
F	19%	12%



1. Find the difference between total quantity of water added by seller A and C.
A. 105 litres
B. 91 litres
C. 99 litres
D. 95 litres
 2. Calculate the respective ratio of quantity of water added by seller C and E.
A. 81 : 287
B. 3 : 11
C. 11 : 3
D. 287 : 81
 3. Which seller added the least quantity of water (in litres)?
A. A
B. C
C. B
D. D
 4. If sellers A, B, and C add their diluted milk in one can then what will be the concentration of pure milk in that can?
A. 70.75%
B. 80.25%
C. 90.50%
D. 60.25%
 5. How many litres of water were added by all the sellers in total? (approximately)
A. 500 L
B. 450 L
C. 440 L
D. 490 L
- Questions 6–10:** For his photographic studies, a scientist buys a 50-litre mixture of two chemicals, H_2SO_4 and HCl , on a weekly basis at the rates shown in the line graph and the percentage composition shown in the pie chart. After thoroughly combining them, he sells them to a manufacturing company. Solve the questions below.
6. The average per litre cost of the mixture for the 1st three weeks is:
A. $\frac{2863}{75}$
B. $\frac{2863}{50}$
C. $\frac{2863}{100}$
D. $\frac{2863}{25}$
 7. What is the scientist's profit or loss percentage in the fourth week if he sells that week's mixture to the company for 3297?
A. 75% loss
B. 75% profit
C. 50% profit
D. 50% loss
 8. During the third week, 20% of the mixture from container P was replaced in container Q. Calculate the ratio of the sum of H_2SO_4 in P and HCl in Q and the sum of H_2SO_4 in P and HCl in Q.
A. $\frac{29}{52}$
B. $\frac{73}{52}$
C. $\frac{113}{52}$
D. $\frac{155}{52}$
 9. If the scientist had sold both H_2SO_4 and HCl separately for Rs. 35 and Rs. X per litre in the second week, he would have lost 7%, but if he had sold them at Rs. 28 and Rs. 35 per litre, he would have lost Y per cent. Then what is the product of X and Y ?
A. $\frac{4536}{17}$
B. $\frac{4536}{13}$
C. $\frac{4536}{15}$
D. $\frac{4536}{11}$
 10. If the scientist secretly converted some percentage of HCl to H_2SO_4 and then mixed them, the cost price difference of the mixture for the same volume increased by



Rs. $(\frac{248}{50})$ in the first week. What was the percentage of HCl that was transformed to H_2SO_4 ?

A. $\frac{557}{14}$

B. $\frac{575}{14}$

C. $\frac{577}{14}$

D. $\frac{775}{14}$

SOLUTIONS

1. **(C)** The total quantity of solution = $480 + a$ litre, if seller A adds 'a' litre of water
water = 24%

Thus, 24% of $(480 + a) = a$

$$100a = 24 \times 480 + 24a$$

$$76a = 24 \times 480$$

$$A = \frac{2880}{19} \text{ litres} = \text{approximately } 151.58$$

litres = Quantity of water added by Seller A
Similarly, let the seller C adds c litres of water then

$$8\% \text{ of } (240 + c) = c$$

$$82c = 240 \times 18$$

$$c = \frac{2160}{41} \text{ litres} = 52.68 \text{ litres approximately}$$

= quantity of water added by seller C

$$\text{required difference} = 151.58 - 52.68 = 99$$

litres approximately
Therefore, option C is correct.

2. **(A)** $\frac{2160}{41}$ litres = quantity of water added

by C

Let the seller E adds e litres of water

Then, water = 25% of $(560 + e)$

$$75e = 560 \times 25$$

$$E = \frac{560}{3}$$

$$\text{Required ratio} = \frac{2160}{41} : \frac{560}{3} = 81 : 287$$

Therefore, option A is correct.

3. **(C)** $\frac{2380}{19}$ litres = quantity of water added

by seller A

$\frac{2160}{41}$ litres = quantity of water added by seller C

$\frac{560}{3}$ litres = quantity of water added by seller E

Let the seller B adds b litres of water then,

$$10\% \text{ of } (200 + b) = b$$

$$\frac{200}{9} \text{ litres} = \text{approximately } 22.22 \text{ litres}$$

Let the seller D adds d litres of water

$$15\% \text{ of } (140 + d) = d$$

$$D = \frac{140 \times 15}{85} = 24.70 \text{ litres}$$

= quantity of water added by seller D

Let the seller F add f litres of water then,

$$12\% \text{ of } (380 + f) = f$$

$$88f = 380 \times 12$$

$$f = 51.82 \text{ litres}$$

= quantity of water added by seller F

B added the least quantity (in litres) of water

Therefore, option C is correct.

4. **(B)** The total quantity of new solution = $920 + 226.36 = 1146.36$ litres

$$\text{Quantity of milk in the new can} = 480 + 200 + 240 = 920 \text{ litres}$$

The required concentration of milk in the new solution

$$= \frac{920 \times 100}{1146.36} = 80.25\% \text{ approximately}$$

Therefore, option B is correct.

5. **(D)** Total quantity of water added by all the sellers together = $151.58 + 22.22 + 52.68$



+ 24.70 + 186.67 + 51.82 = approximately
489.67 litres = approximately 490 litres
Therefore, option D is correct.

6. (A) Litres of HCl = 64% of 50 litres

$$= \left(\frac{64}{100} \right) \times 50 = \frac{64}{2}$$

Litres of HCl = 32 litres

Litres of H_2SO_4 = 50 - 32 = 18 litres

$$\text{Average CP of } H_2SO_4 = \frac{49 + 28 + 42}{3} = \frac{\text{Rs. } 119}{3}$$

$$\text{Average CP of HCl} = \frac{35 + 42 + 35}{3} = \text{Rs. } \frac{112}{3}$$

Let CP of per litre mixture be Rs. x .

$$\text{Then, } \left(\frac{119}{3} - x \right) 18 \text{ litres} = \left(x - \frac{112}{3} \right) 32 \text{ litres}$$

$$\Rightarrow \frac{119 \times 9}{3} + \frac{112 \times 16}{3} = 16x + 9x = 25$$

$$\Rightarrow \frac{1071 + 1792}{3} = 25x$$

$$x = \text{Rs. } \frac{2863}{75}$$

Therefore, option A is correct.

7. (C) CP for 4th week = Rs. x .

Then, $(49 - x) 32 \text{ litres} = (x - 35) 18 \text{ litres}$

$$x = \text{Rs. } \frac{2198}{50}$$

CP for 50 litres = Rs. 2198

Profit = Rs. 3297 - 2198 = Rs. 1099

$$\text{Profit \%} = \left(\frac{1099}{2198} \right) \times 100 = 50\% \text{ profit}$$

Therefore, option C is correct.

8. (B) 20% of mixture = $\left(\frac{20}{100} \right) \times 50 \text{ litres}$

= 10 litres (moved from P to Q)

H_2SO_4 in container P = 36% of 10 litres = 3.6 litres

HCl in container P = 64% of 10 litres = 6.4 litres

H_2SO_4 in container Q = 36% of 40 litres = 14.4 litres

HCl in container Q = 64% of 40 litres = 25.6 litres

$$\text{Ratio} = \frac{H_2SO_4 \text{ in P} + HCl \text{ in Q}}{H_2SO_4 \text{ in P} + HCl \text{ in Q}}$$

$$= \frac{3.6 + 25.6}{6.4 + 14.4}$$

$$= \frac{29.2}{20.8} = \frac{73}{52}$$

Therefore, option B is correct.

9. (D) In 2nd week, C P of H_2SO_4 = Rs. 28 and SP = Rs. 35

C P of HCl = Rs. 42 and SP = Rs. X

Total C P = Rs. $(28 \times 18 + 42 \times 32)$ = Rs. (504 + 1344) = Rs. 1848 ... (1)

Total S P = Rs. $(35 \times 18 + X \times 32)$ = Rs. (630 + 32X)

Loss = Rs. $[1848 - (630 + 32X)]$ = Rs. (1218 - 32X)

$$\text{Loss\%} = \left[\frac{1218 - 32X}{1848} \right] \times 100 = 7$$

$$(1218 - 32X) = \frac{7 \times 1848}{100}$$

$$32X = \frac{121800}{100} - \frac{12936}{100}$$

$$X = \frac{3402}{100}$$

Also,

Total SP = Rs. $(28 \times 18 + 35 \times 32)$ = Rs. (504 + 1120) = Rs. 1624

Loss = Rs. $[1848 - 1624]$ = Rs. 224

$$\text{Loss\%} = Y = \left(\frac{224}{1848} \right) \times 100 = \frac{400}{33}$$

$$\text{Then, } X \times Y = \left(\frac{3402}{100} \right) \times \left(\frac{400}{33} \right) = \frac{3402 \times 4}{33}$$

$$= \frac{4536}{11}$$

Therefore, option D is correct.

10. (D) New volume of HCl = 32 - s , if s litres of HCl be converted.

New volume of H_2SO_4 = 18 + s

CP of mixture per litre for the 1st week = Rs. x .

Then, $[49 - x] (18 \text{ litres}) = [x - 35] (32 \text{ litres})$

$$882 - 18x = 32x - 1120$$

$$50x = 1120 + 882 = 2002$$

$$x = \text{Rs. } \frac{2002}{50}$$



Then cost price of mixture will be

$$\frac{2002}{50} + \frac{248}{50} = \frac{2250}{50} = 45$$

$$(18 + s)(49 - 45) = (45 - 35)(32 - s)$$

$$4(18 + s) = 10(32 - s)$$

$$72 + 4s = 320 - 10s$$

$$14s = 320 - 72 = 248$$

$$S = \frac{248}{14} = \frac{124}{7}$$

$$\% \text{ of HCl} = \left[\frac{\frac{124}{7}}{32} \right] \times 100 = \frac{775}{14}$$

Therefore, option D is correct.

