Question

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The solution to the recurrence equation $T(2^k) = 3T(2^{k-1}) + 1$, T(1) = 1 is:

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NVS TGT Mathematic 2019 Shift 1 Official Paper

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1. $2^{\log_3 k}$
2. 2^k
3. $\frac{3^{k+1}-1}{2}$
4. $3^{\log_2 k}$

Answer (Detailed Solution Below)

Option 3 : $\frac{3^{k+1}-1}{2}$

Detailed Solution

Concept:

A **recurrence relation** is an equation that recursively defines a sequence or multidimensional array of values, once one or more initial terms of the same function are given; each further term of the sequence or array is defined as a function of the preceding terms of the same function.

Calculation:

We have

$T(2^k) = 3T(2^{k-1}) + 1$

$= 3^2T(2^{k-2}) + 1 + 3$

$= 3^3T(2^{k-3}) + 1 + 3 + 9$

.... k steps of recursion

$= 3^kT(2^{k-k}) + (1 + 3 + 9 + 27 + ... + 3^{k-1})$

$= 3^k + (\frac{3^k - 1}{2})$

$= (\frac{3^{k+1} - 1}{2})$

∴ The solution to the recurrence equation $T(2^k) = 3T(2^{k-1}) + 1$ is $(\frac{3^{k+1} - 1}{2})$

Hence, the correct answer is option **3)**