Seminar 3

1. Given the grammar $G = (N, \Sigma, P, S)$

$N = \{S, C\}$

$\Sigma = \{a, b\}$

$$P: \quad S \to \overset{1}{ab} \mid \overset{2}{aCSb}$$
$$C \to \underset{3}{S} \mid \underset{4}{\underline{bSb}}$$
$$CS \to b$$

prove that $w = ab(ab^2)^2 \in L(G)$

obs: $(ab)^2 = abab \neq a^2b^2 = aabb$.

Start with S, end up with "w" did 2 steps in 1.

$$S \underset{2}{\Rightarrow} a\boxed{C}Sb \underset{4}{\Rightarrow} \underline{ab}SbSb \underset{1}{\overset{2}{\Rightarrow}} ab(ab^2)^2 = w$$

$$S \overset{4}{\Rightarrow} w \Rightarrow \boxed{w \in L(G)}$$

2. Given the grammar $G = (N, \Sigma, P, S)$

$N = \{S\}$

$\Sigma = \{a, b, c\}$

$P: S \to a^2 S \mid bc$

find $L(G)$.

$$L(G) = \{bc, a^2bc, a^4bc \dots a^{2n}bc\}$$

$$L = \{a^{2n}bc\}$$

Prove:

$$L = L(G) \iff \boxed{\underbrace{L \subset L(G)}_{\text{Proof 1}} \ \&\& \ \underbrace{L(G) \subset L}_{\text{Proof 2}}}$$

$a^{2k}bc \in L(G)$

Proof 1

$P(k): a^{2k}bc \in L(G), \ k \in \mathbb{N}$

i) ? for $k = 0$, $bc \in L(G)$

$S \Rightarrow bc \in L(G) \Rightarrow \boxed{P(0) \text{ true}}$

ii) assume $P(k)$ true $\Rightarrow P(k+1)$ true

$S \overset{2}{\underset{1}{\Rightarrow}} a^2 S \underset{P(k)}{\Rightarrow} a^2 a^{2k} bc \Rightarrow a^{2(k+1)}bc$

$\Rightarrow P(k+1)$

i), ii) $\Rightarrow P(k)$ true $(\forall)$ $k \in \mathbb{N} \Rightarrow$ I true

Proof 2

$L(G) \subseteq L$

$$S \underset{2}{\Rightarrow} bc = a^0bc \in L$$
$$\Downarrow_1$$
$$a^2S \underset{2}{\Rightarrow} a^2bc \in L$$
$$\Downarrow_1$$
$$a^4S \underset{2}{\Rightarrow} a^4bc \in L$$
$$\Downarrow_1$$
$$a^6S \underset{2}{\Rightarrow} \dots$$
$$\Downarrow_1 \dots$$

Find a grammar that generates

$$L = \{\underbrace{0^n 1^n 2^m}_{\text{sequence}} \mid n, m \in \mathbb{N}^*\}$$

$N = \{S\}$

$\Sigma = \{0, 1, 2\}$

P :

$$S = AB$$
$$A = 0A1 \mid 01$$

$$B = 2B \mid 2$$

I. $L \subseteq L(G)$

$(\forall)\, n, m \in \mathbb{N}^* \quad 0^n 1^n 2^m \in L(G)$

Let $n, m \in \mathbb{N}^*$ be fixed.

$$S \underset{1}{\Rightarrow} AB \overset{n}{\underset{\boxed{a)}}{\Rightarrow}} 0^n 1^n B \overset{m}{\underset{\boxed{b)}}{\Rightarrow}} 0^n 1^n 2^m \quad (\forall)\, n, m \in \mathbb{N}^*$$

$$\boxed{\begin{array}{l} a)\ A \overset{n}{\underset{2}{\Rightarrow}} 0^n 1^n \ (\forall)\, n \in \mathbb{N}^* \\ b)\ B \overset{m}{\underset{1}{\Rightarrow}} 2^m \ (\forall)\, m \in \mathbb{N}^* \end{array}}$$

Proove a, b with <u>induction</u>.

Proof a)

Let $P(k)$: $A \overset{k}{\Rightarrow} 0^k 1^k$, $k \in \mathbb{N}^*$

i) $P(1)$: $A \Rightarrow 01$ true based on pr 2

ii) Assume $P(k)$ true $\Rightarrow$ $P(k+1)$ true

$$P(k+1) = A \overset{k+1}{\Rightarrow} 0^{k+1} 1^{k+1}$$

$$P(k+1) = A \underset{2}{\Rightarrow} 0A1 \overset{k}{\Rightarrow} 0^k 1^k \Rightarrow 0 \cdot 0^k 1^k \cdot 1$$

$k+1$

$A \Rightarrow 0^{k+1} 1^{k+1} \Rightarrow \boxed{P(k+1) \text{ true}}$

$i, ii \Rightarrow$ (a) True

$L(G) \subseteq L$

Make the tree for A and B.

$A \underset{2}{\Rightarrow} 01$

$A \underset{1}{\Rightarrow} 0A1 \underset{2}{\Rightarrow} 0^2 1^2$

$0A1 \underset{1}{\Rightarrow} 0^2 A 1^2 \underset{2}{\Rightarrow} 0^3 1^3$

$0^2 A 1^2 \underset{1}{\Rightarrow} 0^3 A 1^3 \ldots$

$\Rightarrow \boxed{A \text{ generates only } 0^n 1^n} \ \forall\, n \in \mathbb{N}^*.$

$B \underset{2}{\Rightarrow} 2$

$B \underset{1}{\Rightarrow} 2B \underset{2}{\Rightarrow} 2^2$

$2B \underset{1}{\Rightarrow} 2^2 B \underset{2}{\Rightarrow} 2^3$

$2^2 B \underset{1}{\Rightarrow} 2^3 B \ldots$

B only generates $2^m \ \forall\, m \in \mathbb{N}^*$

B only generates $2^m \ \forall m \in \mathbb{N}$

$\Rightarrow$ S generates only concatenations of AB

$\Rightarrow \boxed{S \Rightarrow 0^n 1^n 2^m}$