$A, B$ – independent $\Leftrightarrow$ $P(A \cap B) = P(A) \cdot P(B)$

disjoint $\Leftrightarrow$ $A \cap B = \emptyset$

$A, B$ indep. $\Leftrightarrow$ $\begin{cases} P(A|B) = P(A) \\ P(B|A) = P(B) \end{cases}$

$P(A) > 0, P(B) > 0.$

---

2.

$P(A) = 1 - t$

Consider the letters instead, and assign each letter a position of one mailbox.

$\frac{m}{\phantom{x}}$ $\quad \underline{1} \quad \underbrace{\underline{1} \quad \underline{1} \quad \underline{1} \quad \underline{t}}_{\frac{n-m}{\alpha}} = n$

$(N-1)^{(n-m)}$

Total possible cases $= C_n^m \cdot (N-1)^{n-m}$

$\alpha$

1 letter $\Rightarrow$ Cases $\alpha$

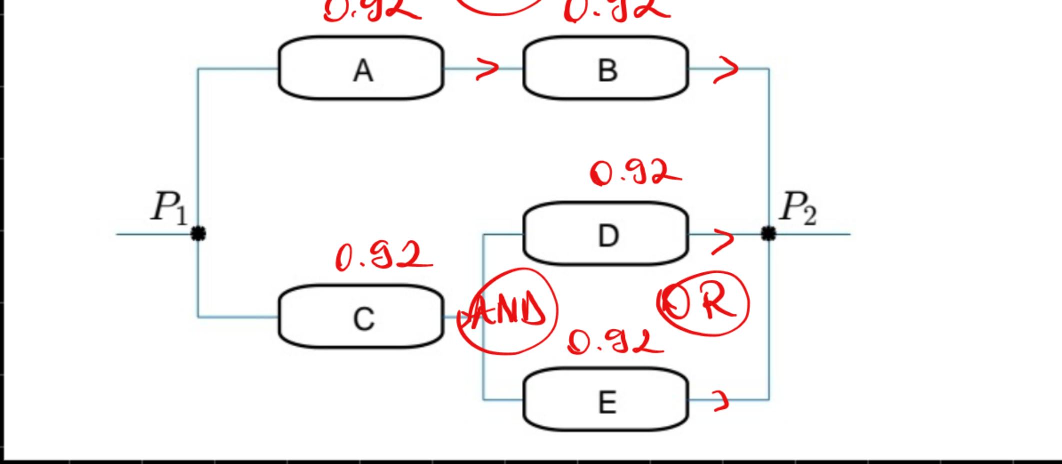
2 letters $\Rightarrow N * N$

$$P = \frac{C_n^m \cdot (N-1)^{n-m}}{N^m}$$

---

4.

AND




$$P(A\cap B) = P(A)\cdot P(B) = 0.92\cdot 0.92$$

↳ using the independence of A and B (definition)

$$P(C\cap(D\cup E)) = P(C)\cdot P(D\cup E) =$$

$$= P(C)\cdot[P(D)+P(E)-P(D\cap E)]$$

$$= P(C)\cdot[P(D)+P(E)-P(D)\cdot P(E)]$$

$$= 0.92\cdot[0.92+0.92-0.92^2]$$

$$\boxed{P((A\cap B)\cup(C\cap(D\cup E))) =}$$

$$P(A\cap B)+P(C\cap(D\cup E)) - P((A\cap B)\cap(C\cap(D\cup E)$$

$$P(A\cap B) + P(C\cap(D\cup E)) - P(A\cap B)\cdot P(C\cap(D\cup E)) =$$

$$0.92^2 + \underbrace{0.92[2\cdot 0.92 - 0.92^2]} - 0.92^2\cdot\underbrace{\qquad} =$$

Translate it into natural language

The Probability of the reliability of the system is the prob. of (A and B to happen) or ((C and (D or B)) to happen.

(+ Redactorle mai ofele)

---

5. 70% .... C
60% .... F
50% .... C & F

$P(C) = 0.7$
$P(F) = 0.6$
$P(C \cap F) = 0.5$

a) $P(\overline{F})$
b) $P(\overline{C} \cap \overline{F})$

c) $P(C \cap \overline{F})$
d) C and F indep.

---

e) $P(C \mid F)$
f) $P(\overline{F} \mid C)$

---

a) $P(\overline{F}) = 1 - P(F) = 0.4$

b) $P(\overline{C} \cap \overline{F}) = P(\overline{C \cup F}) = 1 - P(C \cup F)$

$= 1 - \left( P(C) + P(F) - P(C \cap F) \right)$

$= 1 - (0.7 + 0.6 - 0.5)$

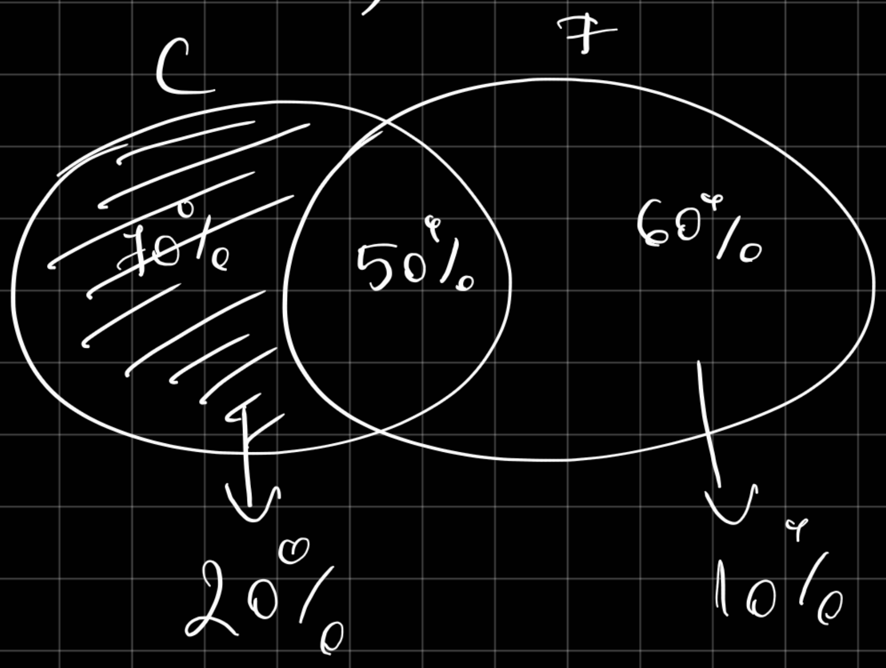
$= 1 - 0.8 = \boxed{0.2}$

d) $P(C \cap F) = 0.5$

$P(C) \cdot P(F) = 0.7 \cdot 0.6 = 0.42 \neq 0.5$

$\Rightarrow$ C and F are not <u>independent</u>.

c) $P(C \cap \overline{F}) = 0.7$ ? $= P(C \setminus F) = 0.2$




$= P(C) - P(C \cap F) = 0.7 - 0.5 = \boxed{0.2}$

e) $P(C|F) = \dfrac{P(C \cap F)}{P(F)} = \dfrac{0.5}{0.6} = \boxed{\dfrac{5}{6}}$

f) $P(\overline{F}|C) = \dfrac{P(\overline{F} \cap C)}{P(C)} = \dfrac{0.2}{0.7} = \boxed{\dfrac{2}{7}}$

---

Bonus :

8. 15 people, 5 cars

$P_1 \quad P_2 \quad - - - - \quad P_{15}$

$\downarrow \quad \downarrow \quad \quad \downarrow$

$(1-5) \quad (1-5) \quad \quad (1-5)$

$\Rightarrow$ Nr possible cases: $\boxed{5^{15}}$

$\Rightarrow$ Nr favorable cases:

$C_1 \quad C_2 \quad C_3 \quad C_4 \quad C_5$

$P_1 \quad P_2 \quad P_3 \quad P_4 \quad P_5$

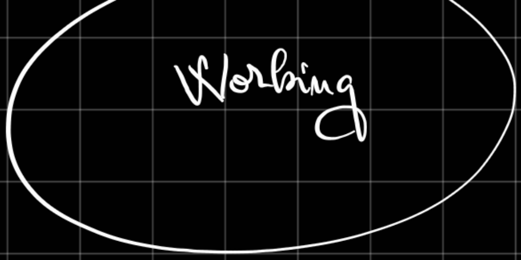
$(P_6 - P_{15}) \rightarrow$ distribute randomly

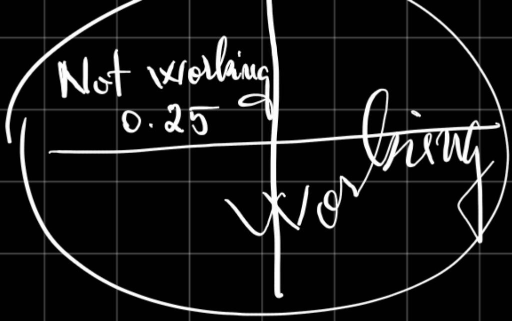
$$\Rightarrow \boxed{C_{15}^{5} \cdot 5^{10}}$$

$$\Rightarrow P = \frac{N_f}{N_p} = \frac{C_{15}^{5} \cdot 5^{10}}{5^{15}} = \boxed{\frac{C_{15}^{5}}{5^{15}}}$$

9. $P_1$ $P_2$

5.




$P(W) = 1$ $\qquad\qquad$ $P(W) = 0.75$

Let $AW$ : choose part from $P_1$, part is working

$BW$ : — " — from $P_2$, working

$$P(AW \cup BW) = P(AW) + P(BW) - P(AW \cap BW)$$

---

6. $S_i$ : shooter "$i$" hits target. (independent)

$P(S_1) = 0.4$, $\quad P(S_2) = 0.5$, $\quad P(S_3) = 0.7$.

$$P\Big(\overbrace{(S_1 \cap \overline{S_2} \cap \overline{S_3})}^{A_1} \cup \overbrace{(\overline{S_1} \cap S_2 \cap \overline{S_3})}^{A_2} \cup$$

$$(S_1 \cap \bar{S}_2 \cap \bar{S}_3))) =$$

Using independence

$$P(S_1) \cdot P(\bar{S}_2 \cap \bar{S}_3) = P(S_1) \cdot P(\bar{S}_2) \cdot P(\bar{S}_3)$$

$A_i$ : Only Shooter $i$ hits the target.

$\Rightarrow$ disjoint

$$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2 \cup A_3) -$$

$$P(A_1 \cap (A_2 \cup A_3)) = P(A_1) + P(A_2) + P(A_3) -$$

$$P(A_2 \cap A_3) - P((A_1 \cap A_2) \cup (A_1 \cap A_3))$$

$$= P(A_1) + P(A_2) + P(A_3) - \underbrace{P(A_2 \cap A_3)}_{\Rightarrow 0} -$$

$$[\underbrace{P(A_1 \cap A_2)}_{\Rightarrow 0} + \underbrace{P(A_1 \cap A_3)}_{\Rightarrow 0} - \underbrace{P(A_1 \cap A_2 \cap A_1 \cap A_3)}_{\Rightarrow 0}]$$

$$= \boxed{P(A_1) + P(A_2) + P(A_3)}$$

$$\Rightarrow P(S_1) \cdot P(\bar{S}_2) \cdot P(\bar{S}_3) + P(\bar{S}_1) \cdot P(S_2) \cdot P(\bar{S}_3)$$

$$+ P(\bar{S}_1) \cdot P(\bar{S}_2) \cdot P(S_3) =$$

$$0.4 \cdot (0.5) \cdot (0.3) + 0.6 \cdot 0.5 \cdot 0.3 +$$

$$0.6 \cdot 0.5 \cdot 0.7 =$$

$$0.060 + 0.09 + 0.2 = \ldots$$

# Theory Review:

$$P(A) = \frac{N_f}{N_p} = \frac{f}{p}$$

A and B – mutually disjoint $\Leftrightarrow$ $A \cap B = \emptyset$

$P(A \cap B) = 0.$

$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}}$$

A, B indep. $\Leftrightarrow$

$$\boxed{\begin{array}{l} P(A|B) = P(A) \\ \text{or} \\ P(A \cap B) = P(A) \cdot P(B). \end{array}}$$

$\cup A_i = S.$

Total Probability Rule: $P(E) = \sum P(A_i) \cdot P(E|A_i)$

Multiplication Rule: $P\left(\bigcap_{i=1}^{n} A_i\right) = P(A_1) \cdot P(A_2|A_1) \cdot P(A_3|A_1 \cap A_2) \cdot \ldots P\left(A_n \middle| \bigcap_{i=1}^{n-1} A_i\right).$

---

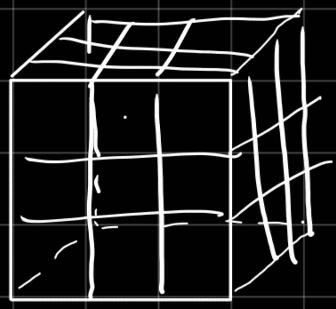
1. 1000 smaller cubes. $= 10^3$

a) 3 colored faces = corner, 4 corners of the cube

$P(X = \text{corner}) = \frac{8}{P} = \frac{\quad}{1000}$

b) $P(X = \text{edge}) = \frac{96}{1000}$

Rubic

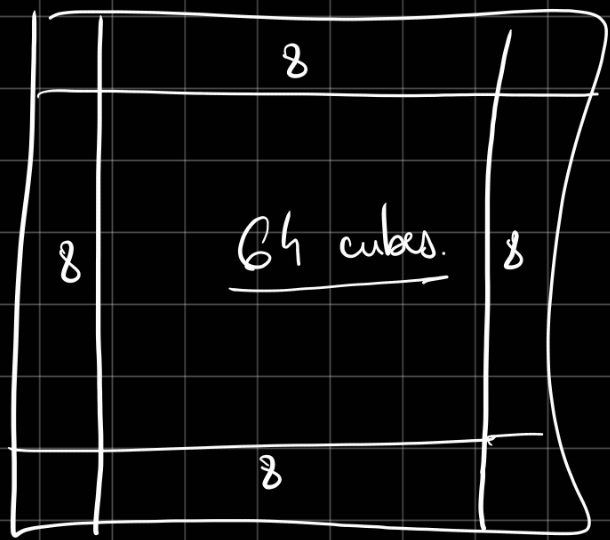


$3^3 = 27$ cubes

$10^3 = 10 \times 10 \times 10$ cube

$\Rightarrow$ Edges: $8 \cdot 4 \cdot 3 = 12 \cdot 8 = 96$

c)




$64^2 \cdot 6 = 384$

$P = \frac{384}{1000}$

d) $\boxed{\frac{1000 - 384 - 96 - 4}{1000}}$

---

③ 8 chars, letters (lower + upper) + digits.

1 million passw/sec.

a) $P = (26+26+10)^8 = 62^8$

$$\frac{62^8/2}{10^6} =$$

b) $$\frac{62^8}{1mil \cdot 60 \cdot 60 \cdot 24 \cdot 7} =$$

7. $P(T \mid GW) = 80\%$

$P(T \mid \overline{GW}) = 50\%$

$P(GW) = 60\%$

$$\frac{P(T \cap GW)}{P(GW)} = 80\%$$

$$\frac{P(T \cap \overline{GW})}{P(\overline{GW})} = 50\%$$

$$P(T \cap GW) = \frac{80}{100} \cdot \frac{60}{100} = \frac{4800}{10000} = 0.48\%$$

$$P(T \cap \overline{GW}) = \frac{50 \cdot 40}{10000} = 0.2\%$$

$S = \{GW, \overline{GW}\}$

---

$$P(T) = P(GW) \cdot P(T \mid GW) + P(\overline{GW}) \cdot P(T \mid \overline{GW})$$

$$P(T) = 0.6 \cdot 0.8 + 0.4 \cdot 0.5$$

$$P(T) = 0.42 + 0.2 = \boxed{0.62}$$