Poisson Model

- independent trials
- 2 outcomes (S/F)
- $s: p_1$ $s: p_2$ ... $s: p_n$

  $f: q_1$ $f: q_2$ ... $f: q_n$

  1 2 ... n

$$\boxed{P(n,k) = \text{coef. of } x^k \text{ in: } (p_1 x + q_1) \cdot \ldots (p_n x + q_n)}$$

Binomial Model

$$\boxed{P(n,k) = C_n^k \cdot p^k \cdot q^{n-k}}$$

$p \to$ prob of success, $q -$ prob of failure

Hypergeometric Model

- dependent trials
- $n_1 =$ nr of • balls, $N - n_1 =$ nr of o balls.
- $$\boxed{P(n,k) = \frac{C_{n_1}^k \cdot C_{N-n_1}^{n-k}}{C_N^n}}$$

Geometric Model

- independent, infinitely many times
- STOP on 1^{st} success

$$\frac{f}{1}\quad\frac{f}{2}\quad\frac{f}{3}\quad\dots\quad\frac{S}{k}$$

$$\boxed{P_k = p\cdot q^k}$$

Pascal Model

- STOP on n^{th} success

$$\underbrace{\frac{s/f}{1}\quad\frac{s/f}{2}\quad\dots\quad\frac{s/f}{k+n-1}}_{k\times\text{"}f\text{"}+(n-1)\times\text{"}s\text{"}}\quad\frac{\text{"}n^{th}\ s\text{"}}{k+n}$$

$$\boxed{P_K = C_{n+k-1}^{k}\cdot q^k\cdot p^n}$$

Problems

1. 5% def., S - 16 parts.

$p = 0.05$, $q = 0.95$

a) Binomial

b) $P(4) + P(5) + \dots + P(16) =$

$$1 - \left(P(1) + P(2) + P(3)\right)$$

c) $P(\text{at least } 1) = 1 - (0.95)^{k}$

d) $P(<3 \text{ def}) = \sum_{0}^{2} P(16, k)$

(2.) 200 seats = 10 press, 190 others

- 150 people n't rand.

P("all press seats occupied") = ?

Hypergeometric Model:

- we know the nr of seats - 200
- the trials are dependant

$$P(n, k) = \frac{C_{n_1}^{k} \cdot C_{N-n_1}^{n-k}}{C_N^{n}},$$

where n - 10 press seats to choose from.

k - 10 press seats need to be occupied

N - 200 possible seats

$$P(150,10) = \frac{C_{10}^{10} \cdot C_{190}^{140}}{C_{200}^{150}}$$

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3. 10 - computers

3 bad, 7 good, 5 out of them

a). $P(A) = \frac{C_7^3 C_3^2}{C_{10}^5}$.

n_1 - nr of

b)

B - at least 2

A - ex 2 def

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \boxed{\frac{P(A)}{P(B)}}$$

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4. 5 indep. tests.

$t_k = \frac{k}{10}$, $k \in \overline{1,5}$.

a) at least one

b) more than 2

c) all error.

<u>Poisson model:</u>

$P(n,k) =$ coef of x^k in

$(p_1 x + q_1) \cdot (p_2 x + q_2) \cdot \ldots \cdot,$

a) $1 - P(5,0) =$

We exclude the case when no error occurs.

$$P(5,0): \left(\frac{1}{10}x + \frac{9}{10}\right)\left(\frac{2}{10}x + \frac{8}{10}\right) \ldots \Big)$$

$$= \frac{(x+9)(2x+8)(3x+7)(4x+6)(5x+5)}{10^5}$$

Coeff $x^k = x^0 = 1$

$$= \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{10^5}$$

$$\Rightarrow P(5,0) = \frac{5 \cdot 6 \cdot 7 \cdot 8 \cdot 9}{10^5}$$

$$\boxed{A = 1 - \frac{9!}{4!\,10^5}}$$

b) $P(5,3) + P(5,4) + P(5,5) =$

Based on induction $\Rightarrow$

$P(5,3) = \text{coeff } x^3 =$ group pairs of 3 x coeff

$6 \cdot 5 \cdot 6$ $12 \cdot 40$

$$P(5,3) = \frac{6+12+20+24+40+60}{10^5}$$

(* also multiply with constant coeff)

$$P(5,4) = \text{group by } 4 = \frac{24+60+120}{10^5}$$

$$P(5,5) = \frac{5!}{10^5}$$

$$B = \sum_{k=3}^{5} P(5,k)$$

c) $P(5,5) = 5!$

10^5

(5.) cloning properly failure
$P = 0.9$, $q = 0.1$

Geometric Model

$P_2 = $ (2* failure before 1st success)

$$\boxed{P_2 = 0.9 \cdot (0.1)^2}$$

p q

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(3) $P(g) = 0.7$, $P(\bar{g}) = 0.3$

a)

_ _ _ _ _

$$\frac{C_3^2 \cdot C_7^3}{C_{10}^5} \Rightarrow$$ Hypergeometric model?

b) A - at least 2, B = exactly 2

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(B)}{P(A)} =$$

$$P(B) = \frac{C_3^2 \cdot C_7^3}{C_{10}^5} \qquad P(A) = \frac{C_3^2 \cdot C_7^3}{C_{10}^5} + \frac{C_3^3 \cdot C_7^2}{C_{10}^5}$$

4\. a) $A = 1 - P(5,0)$

Poisson model. coeff. x^k in

$$(0.1x + 0.9)(0.2x + 0.8)(0.3x + 0.7)(0.4x + 0.6)(0.5x + 0.5) =$$

$$P(5,0) = 0.5 \cdot 0.6 \cdot 0.7 \cdot 0.8 \cdot 0.9 = \boxed{\frac{5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10}{10^5}}$$

b) $P(B) = P(5,3) + P(5,4) + P(5,5)$

c) $P(5,5) = \frac{24 \cdot 5}{10^5} = \boxed{\frac{120}{10^5}}$

5\. $P(W) = 0.9$, $P(\overline{W}) = 0.1$.

$\Rightarrow$ Geometric Model

$$\boxed{P_2 = p \cdot q^2 = 0.9 \cdot 0.1^2}$$

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6. $p(D) = 0.05$, independent trials.

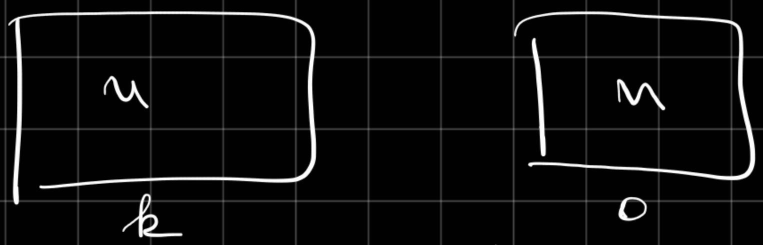
Binomial

a) $P(20,3) = C_{20}^{3} \cdot (0.05)^3 \cdot (0.95)^{17}$

b) Pascal Model, stops on 2^{nd} success after 3 failures

$$P(2,3) = C_{n+k-1}^{k} \cdot p^2 q^3 = C_4^3 \cdot (0.05)^2 \cdot (0.95)^3$$

7.



2n pills $\quad \dfrac{1}{1} \quad \dfrac{2}{2} \quad \dfrac{1}{3} \quad - - - \quad \dfrac{1}{2n}$

$\rightarrow$ box nr

Each pill has a 0.5 chance of being in box 1 or box 2. A - pill from Box 1.

$P(A) = 0.5$, $P(\bar{A}) = 0.5$

We consider a Pascal model where we stop after n successes, and have (n-k) fails. * we stop at the next step after n-th succes, *

$$P(n, (n-k+1)) = \boxed{C_{2n-k}^{n} (0.5)^{n} \cdot (0.5)^{n-k}}$$

* when we see the empty pocket.

$$= \boxed{C_{2n-k}^{n} \left(\frac{1}{2}\right)^{2n-k}}$$

b) $S_n = C_{2n}^{n} + 2 \cdot C_{2n-1}^{n} + \dots + 2^{n} C_{n}^{n}$

$$C_{2n-k}^{n} = 2^{2n-k} \cdot P(n, n-k+1)$$

$$\sum 2^{k} C_{2n-k}^{n} = \sum \cancel{2^{k}} \cdot 2^{2n-\cancel{k}} \cdot P(n, n-k+1)$$

$$= \boxed{2^{2n} \sum P(n, n-k+1)}$$

$$= 2^{2n} \Big(P(n, n+1) + P(n, n) + P(n, n-1) \dots + P(n, 1) \Big)$$

Bonus Problems:

(8.) 4 white, 5 black $\to$ 4 balls
2nd 3rd - black

w b b w

$$\frac{C_4^2 \cdot C_5^2}{C_9^4} = A \text{ (2 balls are black)}$$

div 3 $\to$ 3, 6 $\to$ 2 options

$$\underset{B}{P(4 \text{ dice}, 2w)} = \boxed{\frac{2^2 \cdot 6^2}{6^4}}$$

$$\underset{C}{P(3 \text{ dice}, 2w)} = \boxed{\frac{2^2 \cdot 6^1}{6^3}}$$

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(\overline{A} \cap C) = P(\overline{A}) \cdot P(C)$$

(9.) 3 students - 10 questions.

α

0.5, 0.8, 0.6.

3 Separate Binomial Models.

A – first stud. passes $= P(10,5) + P(10,6) + \dots P(10,10)$

$= 1 - \left(P(10,1) + P(10,2) + P(10,3) + P(10,4)\right)$

B – second stud passes.

C – failing $P_F = P(10,1) + P(10,2) + P(10,3) + P(10,4)$

$$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C) =$$

$$= a \cdot a \cdot (1-a)$$

$$= \boxed{a^2(1-a)}$$
