Gulin Tudor 913

Homework 4

1) d) $\sum_{n\geq 2} \frac{1}{(\ln n)^{\ln n}}$

$\frac{1}{(\ln n)^{\ln n}} > 0$, decreasing

Condention test: $\sum_{n\geq 1} x_n$ „like" $\sum_{n\geq 0} 2^n x_{2^n}$

$\sum_{n\geq 0} 2^n x_{2^n} = \sum_{n\geq 0} \frac{2^n}{(\ln 2^n)^{\ln 2^n}} = \sum_{n\geq 0} \frac{2^n}{(n\ln 2)^{n\ln 2}}$

Root test

$= \sum \sqrt[n]{\frac{2^n}{(n\ln 2)^{n\ln 2}}} = \frac{2}{(n\ln 2)^{\ln 2}} \to 0 < 1$

$\Rightarrow$ converges $\Rightarrow$

$\Rightarrow \sum 2^n x_{2^n}$ converges $\Rightarrow \sum x_n$ converges

3) 1) $\sum_{n\geq 1} \frac{1\cdot 3\cdot \ldots \cdot(2n-1)}{2\cdot 4\cdot \ldots \cdot 2n} \cdot \frac{1}{n^2}$

Raabe-Duhamel: $n\left(\frac{x_n}{x_{n+1}} - 1\right)$, $x_n = \frac{1\cdot 3\cdot \ldots \cdot(2n-1)}{2\cdot 4\cdot \ldots \cdot 2n} \cdot \frac{1}{n^2}$

$\lim_{n\to\infty} n\cdot\left(\frac{1\cdot 3\cdot \ldots (2n-1)}{2\cdot 4\cdot \ldots \cdot 2n} \cdot \frac{1}{n^2} : \left(\frac{1\cdot 3\cdot \ldots \cdot(2n+1)}{2\cdot 4\cdot \ldots \cdot 2(n+1)} \cdot \frac{1}{(n+1)^2}\right) - 1\right)$

$= \lim_{n\to\infty} n\cdot\left(\frac{2(n+1)}{2n+1}\cdot\frac{(n+1)^2}{n^2} - 1\right) = \lim_{n\to\infty} n\left(\frac{2(n+1)^3}{(2n+1)n^2} - 1\right)$

$= \lim_{n\to\infty} \frac{2n^3 + 6n + 6n^2 + 2 - 2n^3 - n^2}{2n^3 + n^2} \cdot n$

$= \lim_{n\to\infty} \frac{5n^2 + 6n + 2}{2n^2 + n} = \frac{5}{2} > 1 \Rightarrow \sum x_n$ converges

2) d) $\sum_{n\geq 1} \sin(\pi\sqrt{n^2+1}) = \sum_{n\geq 1} \sin\left(\pi(\sqrt{n^2+1} - n) + n\pi\right)$

$= \sum_{n\geq 1} (-1)^n \sin\left(\pi\cdot\frac{(n^2+1-n^2)}{\sqrt{n^2+1}+n}\right) = \sum_{n\geq 1} (-1)^n \sin\frac{\pi}{\sqrt{n^2+1}+n}$

$= \sum_{n\geq 1} (-1)^n s_n$

$s_n \xrightarrow[n\to\infty]{} 0$

$\sin$ - decreasing on $\left(\frac{\pi}{2}, 0\right)$ $\Big|$ $\xRightarrow{\text{Leibniz}} \sum(-1)^n s_n$ converges