Finite Automata

$\delta : Q \times \Sigma \longrightarrow \mathcal{P}(Q)$

state — Q; elem of alphabet — Σ

→ transition function (can be represented as table)

Explicit representation or Table representation

or Graphical representation (as a graph)

Configuration: (q, x), $q \in Q$, $x \in \Sigma^*$

initial config. (q_0, w), $q_0 \in Q$

final config (q_f, ε), $q_f \in F$

$(p, a, x) \vdash (q, x) \iff q \in \delta(p, a)$

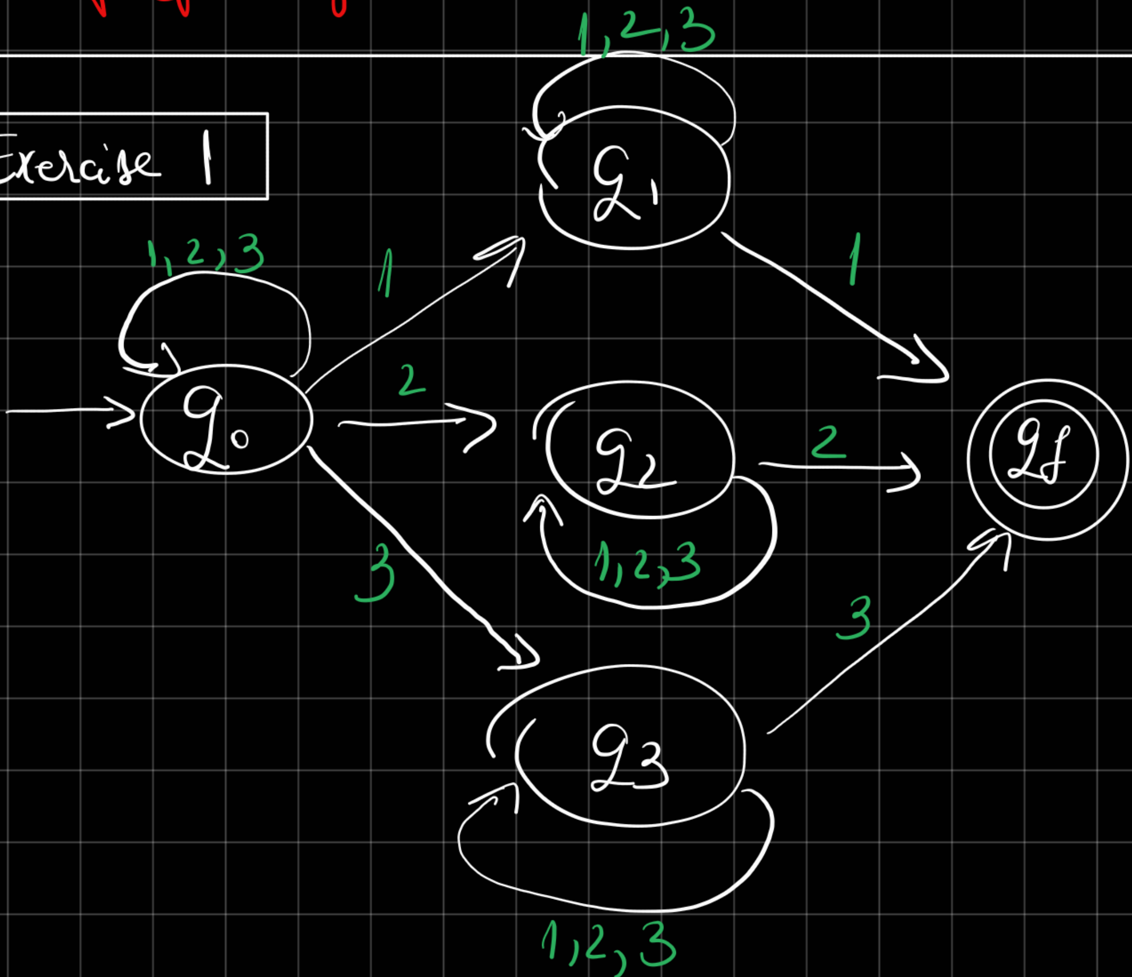
(direct transition)

$p \xrightarrow{a} q$

$$\boxed{L(M) = \{ w \in \Sigma \mid (q_0, w) \vdash^{*} (q_f, \varepsilon), q_f \in F \}}$$

Language definition.

Exercise 1

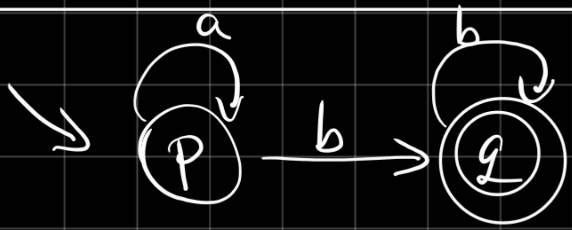


Prove that $w = 12321 \in L(M)$

$$(q_0, 12321) \vdash (q_1, 2321) \vdash^{3} (q_1, 1) \vdash (q_f, \varepsilon)$$

$$\Rightarrow (q_0, w) \vdash^{*} L(M) \Rightarrow \boxed{w \in L(M)}$$

(2)



$L = \{ a^n b^m \mid n \in \mathbb{N}, m \in \mathbb{N}^* \}$

? $L(M) \subseteq L$ ✓

? $L \subseteq L(M)$

? $(\forall)\ n, m \in \mathbb{N},\ m \neq 0,\ \boxed{a^n b^m \in L(M)}$

Let $n, m \in \mathbb{N},\ m \neq 0$ be fixed

$$(p, a^n b^m) \overset{n}{\underset{i}{\vdash}} (p, b^m) \vdash (q, b^{m-1}) \overset{m-1}{\underset{ii)}{\vdash}} (q, \varepsilon)$$

$$\Rightarrow (p, a^n b^m) \vdash^{*} (q, \varepsilon) \Rightarrow \boxed{a^n b^m \in L(M)}$$

$$P(n) : (p, a^n) \vdash^{n} (p, \varepsilon)$$

$\underline{I}$ $\quad n = 0 \Rightarrow (p, \varepsilon) \vdash^{0} (p, \varepsilon) \Rightarrow P(0)$ true

$\underline{II}$. $P(k) \Rightarrow P(k+1)$

$$(p, a^k) \vdash^{k} (p, \varepsilon) \quad \text{true}$$

$$(p, a^{k+1}) \vdash^{k} (p, a) \vdash (p, \varepsilon) \Rightarrow P(k+1) \text{ true}$$

$\Rightarrow$ i) true

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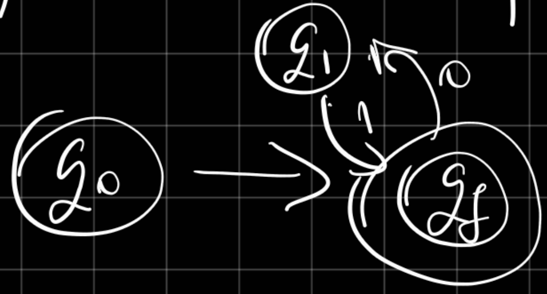
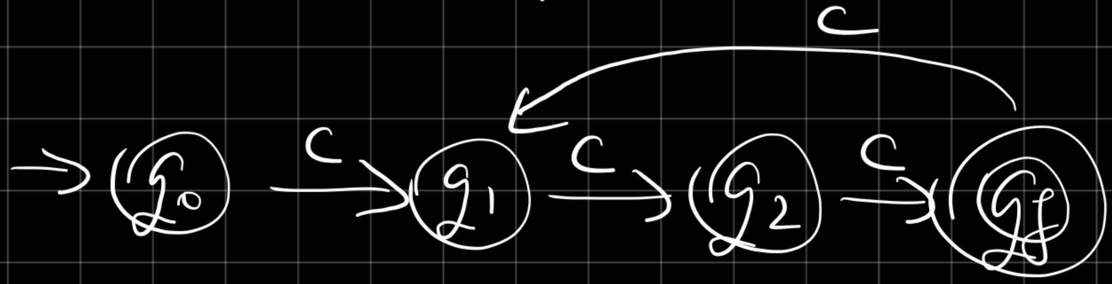
d) $L = \{ 0(01)^n \mid n \in \mathbb{N} \}$



e) $L = \{c^{3n} \mid n \in \mathbb{N}^*\}$



<u>for $n \in \mathbb{N}$ $\Rightarrow$</u> q_0 is also a final state.

f)