Notations

A or B = $A \cup B$

A and B = $A \cap B$

$\overline{A}$ = not A

Ω = the set of all events.

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a) 15 chairs ← 10 stud. A_{15}^{10}

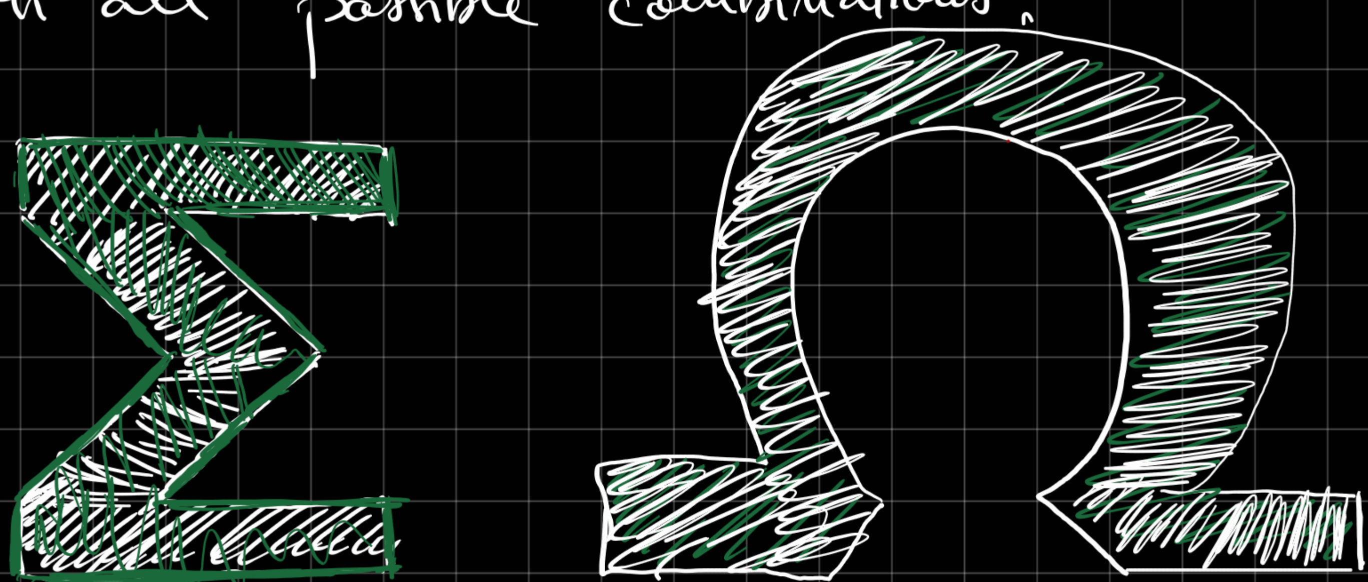
b) 10 ch P_{10}

c) 3 dice

$6 \cdot 6 \cdot 6 = 6^3$

* solved with the fundamental principle of counting / product rule

* explanation: you can draw a graph with all possible combinations.



Gen. randomly 2 letters, 3 digits

$\Rightarrow 26^2 \cdot 10^3$ ↳ unordered

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a) possible 2 let, 3 digits → Ordered

_ _ _ _ _ letters first digits first

$\Rightarrow \boxed{C_5^2 \cdot 26^2 \cdot 10^3}$ or $\boxed{C_5^3 \cdot 26^2 \cdot 10^3}$

- choose pos. for letters
- then fill the rest with numbers

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b) possible dist 2 let, dist 3 digits.

$\Rightarrow C_5^2 \cdot C_{26}^2 \cdot C_{10}^3$

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(5) A_i: "shooter i hits the target". $i = 1,2,3$

a) $A = A_1 \cup A_2 \cup A_3$

b) $B = \overline{A_1} \cap \overline{A_2} \cap \overline{A_3} = \overline{A}$

c) $C = A_1 \cap A_2 \cap A_3$

d) $D = (A_1 \cap \overline{A_2} \cap \overline{A_3}) \cup (\overline{A_1} \cap A_2 \cap \overline{A_3}) \cup (\overline{A_1} \cap \overline{A_2} \cup A_3$

e) $E = (\overline{A_1} \cap A_2 \cap A_3) \cup (A_1 \cap \overline{A_2} \cap A_3) \cup$

$(A_1 \cap A_2 \cap \overline{A_3})$

$$(A_1 \cap A_2 \cap A_3)$$

$$\boxed{E = (A \setminus C) \setminus D}$$

(4.) A_i : " ticket i winning ", $i = \overline{1,n}$

a) $A = A_1 \cap A_2 \cap A_3 \dots \cap A_n$

b) $B = \overline{A_1} \cap \overline{A_2} \cap \overline{A_3} \dots \cap \overline{A_n}$

c) $C = A_1 \cup A_2 \cup A_3 \cup \dots A_n$

d) $D = \bigcup_{i=1}^{n} \left(A_i \cap \overline{A_1} \cap \overline{A_2} \cap \dots \cap \overline{A_{i-1}} \cap \overline{A_{i+1}} \cap \dots \overline{A_n} \right)$

$$D = \bigcup_{i=1}^{n} \left(A_i \bigcap_{\substack{k=1 \\ k \neq i}}^{n} \overline{A_k} \right)$$

e) $E = \bigcup_{i,j=(1,1)}^{(n,n)} \left(A_i \cap A_j \bigcap_{\substack{k=1 \\ k \neq i,j}}^{n} \overline{A_k} \right)$

f) $F = \left((A_1 \cap A_2) \cup A_3 \cup A_4 \cup \dots A_n \right) \cup \dots$

$$F = \bigcup_{i,j=(1,1)}^{(n,n)} \left((A_i \cap A_j) \cancel{\bigcup_{\substack{k=1 \\ k \neq i,j}}^{n} A_k} \right)$$

not needed.

g) $G = \bigcup_{i,j=(1,1)}^{(n,n)} \left((A_i \cup A_j) \bigcap_{\substack{k=1 \\ k \neq i,j}}^{n} \overline{A_k} \right)$

$F = C \setminus D$

$G = \overline{F} \cup E = B \cup D \cup E$

* interpret in natural language also, to find combined solutions

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Extra

(3) 10 free out of 25 total, 5 games, 3 antivirus.

a) C_{25}^{10} because the 10 packages' order doesn't matter.

b) $C_5^3 \cdot C_{20}^7$

c) $C_5^3 \cdot C_3^2 \cdot C_{17}^5$

Euler's Gamma Function: $\Gamma : (0,\infty) \to (0,\infty)$,

$$\boxed{\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x}\, dx.}$$

1. $\Gamma(1) = 1$

2. $\Gamma(a+1) = a \cdot \Gamma(a)$, $(\forall)\ a > 0$.

3. $\Gamma(n+1) = n!\,,\ (\forall)\ n \in \mathbb{N};$

4. $\Gamma\left(\frac{1}{2}\right) = \sqrt{2}\int_0^\infty e^{-\frac{t^2}{2}}\,dt = \int_{\mathbb{R}} e^{-t^2}\,dt = \sqrt{\pi}.$

Euler's Beta Functions: $\beta : (0,\infty)\times(0,\infty)\to(0,\infty)$,

$$\boxed{\beta(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1}\,dx.}$$

1. $\beta(a,1) = \frac{1}{a}\,,\ (\forall)\ a>0$
2. $\beta(a,b) = \beta(b,a)\,,\ (\forall)\ a,b>0$
3. $\beta(a,b) = \frac{a-1}{b}\beta(a-1,b+1)\,,\ a>1, b>0.$
4. $\beta(a,b) = \frac{b-1}{a+b-1}\beta(a,b-1) = \frac{a-1}{a+b-1}\beta(a-1,b)$
5. $\beta(a,b) = \frac{\Gamma(a)\cdot\Gamma(b)}{\Gamma(a+b)}$