④ $G = (\{S, A, B\}, \{a, b\}, P, S)$

P: $S \to aA$

$A \to aA \mid bB \mid b$

$B \to bB \mid b$

? $\Leftrightarrow$ RE

$$x = ax + b$$

$x = a^*b$ is a solution

$$ax + b = aa^*b + b = a^+b + b = (a^+ + \varepsilon)b$$

$$= \boxed{a^*b}.$$

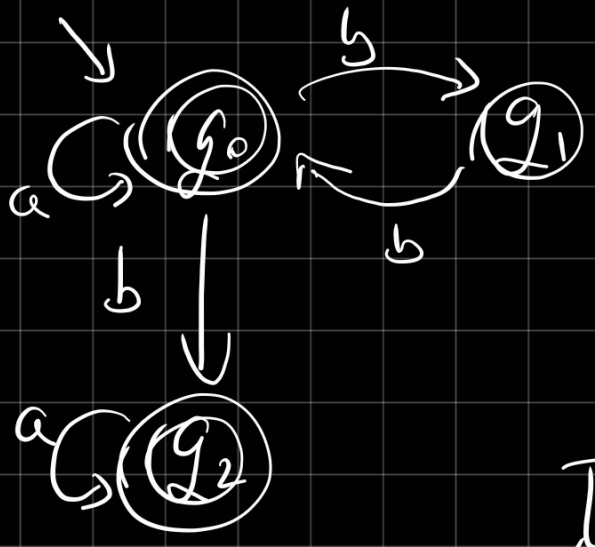
$$\begin{cases} S = aA \\ A = aA + bB + b \\ B = bB + b \end{cases}$$

$\Leftrightarrow$ $B = b^*b = b^+$

$\Leftrightarrow$ $A = a^*b^+$

$\Leftrightarrow \quad S = a^+ b^+$

III. RE $\Leftrightarrow$ FA



$$X = Xa + b$$

$$X = ba^*$$

OR

appears just for the initial state

$$\begin{cases} q_0 = q_1 b + q_0 a + \varepsilon_{(\text{no step})} \\ q_1 = q_0 b \\ q_2 = q_0 b + q_2 a \end{cases}$$

$$q_0 = q_0 b^2 + q_0 a + \varepsilon = q_0 (b^2 + a) + \varepsilon$$

$$\Rightarrow q_0 = \varepsilon \cdot (b^2 + a)^* = \boxed{(b^2 + a)^*}$$

$$\Rightarrow q_2 = (b^2 + a)^* b + q_2 a$$

$$q_2 = (b^2+a)^* b\, a^*$$

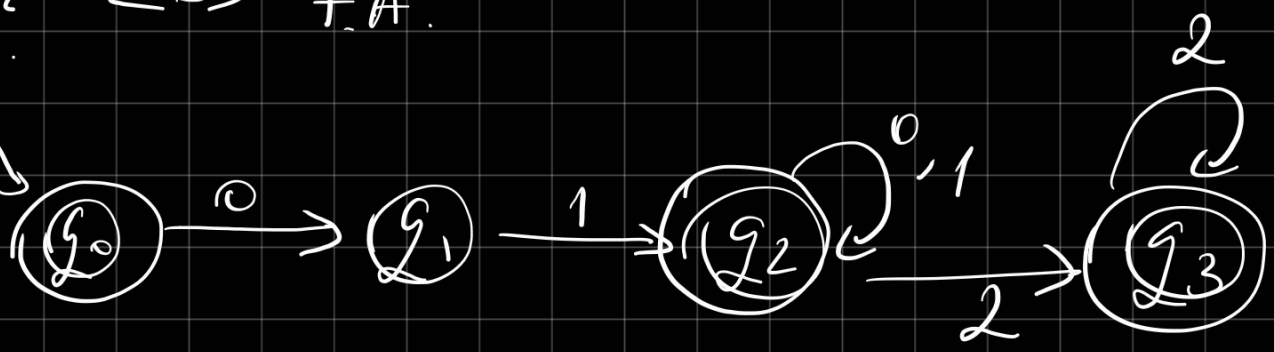
$$q_0 + q_2 = (b^2+a)^* + (b^2+a)^* b\, a^*$$

$$\Rightarrow \boxed{q_0 + q_2 = (b^2+a)^* (\varepsilon + b\, a^*)}$$

---

6. $0\,1\,(0+1)^*\,2^*$

? $\Leftrightarrow$ F.A.



HW Take any REGEX $\Rightarrow$ FA.