Goos-Hänchen shift

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An extremely simple derivation of the Goos-Hänchen shift is presented for total internal reflection at a plane interface between two semiinfinite dielectric media, as well as for optical waveguides of plane and circular cross section. The derivation is based on energy considerations, requires knowledge of Fresnel's equation only, and shows explicitly that the shift is due to the flow of energy across the dielectric boundary.

I. Introduction

When a beam of light is totally reflected from a plane interface between two dielectric media, the reflected beam is shifted by an amount s as shown in Fig. 1.

$$s^E = \left(\frac{2}{k_1}\right)\frac{\tan\alpha_c}{(\sin^2\alpha - \sin^2\alpha_c)^{1/2}}, \tag{1a}$$

$$s^H = s^E/\sin^2\alpha_c, \tag{1b}$$

$$k_1 = 2\pi n_1/\lambda, \tag{2}$$

where the superscripts E and H indicate that the electric vector and magnetic vector, respectively, are parallel to the interface. n_1 is the refractive index of the optically denser medium, λ is the wavelength in vacuum, α is the inclination to the normal, and α_c is the critical angle, $\sin\alpha_c = n_2/n_1$, where n_2 is the refractive index of the optically less dense medium. These classical expressions are valid only when $\alpha \cong \alpha_c$ with $\alpha > \alpha_c$ and are discussed in detail in a comprehensive review by Lotsch.[1] The shift is due to the penetration of the beam into the optically less dense medium and its associated trajectory.[1]

Here we present an extremely simple derivation of Eq. (1) requiring knowledge only of Fresnel's equations for reflection of a plane wave from a plane interface. A concise expression for the shift in optical waveguides is also derived. Our derivation applies only when $n_1 \cong n_2$. This approximation allows us to extract the relevant physics with the minimum effort. It is also the situation of specific interest for many optical waveguides.

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II. Derivation of Classical Equations for Goos-Hänchen Shift

Kapany and Burke[2] provide an interesting derivation of Eq. (1) in which they invoke the theory of both dielectric and metallic waveguides. Following their conceptual lead, we consider wave propagation in a slightly absorbing dielectric slab waveguide of thickness d and index of refraction n_1, surrounded by an unbounded, nonabsorbing medium of refractive index n_2. The waveguide is used to circumvent the difficulties associated with finite beam widths.[2,3] Loss allows us to use energy attenuation instead of phase change consideration. The Goos-Hänchen shift (GHS) is then displayed as an accumulated effect along the length of the waveguide instead of a beam displacement on a single reflection.

We begin with a strictly classical geometric optics description of total internal reflection within the dielectric slab of Fig. 2. The ray penetrates into the optically less dense medium 2, reemerging into medium 1, shifted a distance s along the interface. In accordance with classical geometric optics, the energy associated with the ray is confined entirely to the medium through which it propagates. For our analysis, it is unnecessary to know the trajectory of the ray in medium 2.

Our method finds the s that is necessary for the ray attenuation, determined from the above geometric optics model, to agree with that determined by electromagnetic theory. Although this may seem an unorthodox definition of the GHS, our results are consistent with more conventional treatments.

The analysis is facilitated by defining a total extended pathlength l_{tot} in Fig. 2.

$$l_{\text{tot}} = z/\sin\alpha \tag{3a}$$

$$= (z_1 + s)N(s)/\sin\alpha \tag{3b}$$

$$= (l + s/\sin\alpha)N(s), \tag{3c}$$

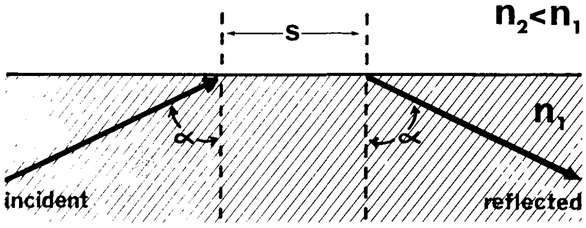

Fig. 1. Total internal reflection ($\alpha > \alpha_c$) from a plane interface exhibiting the Goos-Hänchen shift s. The arrows represent light rays.

where z is the length of the waveguide, l is the pathlength in medium 1 between successive reflections, z_l is the projection of l parallel to the waveguide axis, and $N(s)$ is the number of reflections.

A. Ray Pathlength Determination of Attenuation

Because only medium 1 is absorbing, the power of rays attenuates as $\exp(-\gamma)$ after $N(s)$ reflections.

$$\gamma = l\xi N(s), \tag{4}$$

where ξ is the power attenuation coefficient of a plane wave in medium 1.

$$\xi = 2k_1 n_1{}^i n_1{}^r, \tag{5}$$

where $n_1 = n_1{}^r + in_1{}^i$.

B. Plane Wave Determination of Attenuation

An alternative expression for γ is required to find s. This is provided by an elementary plane wave treatment that approximates the exact electromagnetic mode analysis. In the above derivation of γ we did not consider explicitly the effects of reflections at the core–cladding interface. This was included in the shift s. In the present derivation, the ray is assumed to have the trajectory of Fig. 2 with $s = 0$; but the effects of reflections are explicitly included by using Fresnel's equations.

In addition to the attenuation $l\xi$ due to the physical pathlength of the ray in medium 1, there is also a flow T of evanescent energy into the waveguide. This flow is found from Fresnel's equations. Combining these effects,

$$\gamma = (l\xi + T)N(\mathrm{o}), \tag{6}$$

where $N(\mathrm{o})$ is the number of reflections when $s = \mathrm{o}$. From Eq. (3c),

$$N(\mathrm{o}) = l_{\mathrm{tot}}/l \tag{7a}$$

$$= (1 + s/l\sin\alpha)N(s). \tag{7b}$$

T is the Fresnel power transmission coefficient when medium 1 is absorbing.

$$T = 1 - \frac{\text{power of reflected wave}}{\text{power of incident wave}}. \tag{8}$$

We give T only for $\alpha \cong \alpha_c$ with $\alpha > \alpha_c$, since this is the condition for the validity of Eq. (1). Thus,[4]

$$T^E \cong T^H \sin^2\alpha_c \tag{9a}$$

$$\cong -\frac{2}{k_1}\left(\frac{\sin^2\alpha_c}{\cos\alpha_c}\right)\frac{\xi}{(\sin^2\alpha - \sin^2\alpha_c)^{1/2}}, \tag{9b}$$

provided $\sin^2\alpha - \sin^2\alpha_c \gg n_1{}^i/n_1{}^r$, where $n_1{}^i$ and $n_1{}^r$ are defined by Eq. (5) and the superscripts E and H are the same as in Eq. (1).

C. Determination of s

Equating Eqs. (4) and (6) leads to

$$s = -Tl\sin\alpha/(l\xi + T). \tag{10}$$

Since $l = d/\cos\alpha$, it can be shown from Eq. (9) that $(T/l\xi) \to 0$ when $k_1 d(\sin^2\alpha - \sin^2\alpha_c)^{1/2} \to \infty$. Thus, by letting d be arbitrarily large and $\alpha \neq \alpha_c$, we obtain an expression for the Goos-Hänchen shift at a plane interface

$$s = -T\sin\alpha/\xi, \tag{11}$$

which is identical to Eq. (1) when $\alpha \cong \alpha_c \cong \pi/2$, i.e., when $\sin^2\alpha_c \cong 1$ and is in error by only 3% when $\alpha_c = 80°$.

It should be emphasized that we have not only provided a simple derivation for s, but we have also shown explicitly that *the Goos-Hänchen shift* s *is proportional to the flow of energy* ($-$T/ξ) *from medium 1 to medium 2.* Furthermore, our derivation underscores the fact that the GHS is implicitly contained in Fresnel's classical equations for reflection at a plane interface. In the geometric optics model, s is that additional pathlength associated with the ray trajectory in medium 2.

This concludes our discussion of the GHS at a plane interface between two semiinfinite dielectric media.

III. Goos-Hänchen Shift in Optical Waveguides

Equation (11) gives the GHS on the slab optical waveguide[5] provided $\alpha \cong \alpha_c \cong \pi/2$ and

$$k_1 d(\sin^2\alpha - \sin^2\alpha_c)^{1/2} \gg 1. \tag{12}$$

With only a slight modification, Eq. (11) also holds

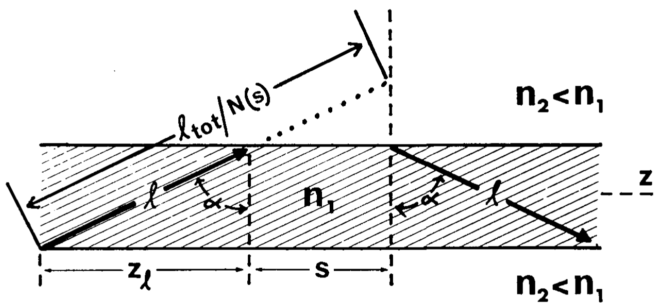

Fig. 2. Model used to illustrate the trajectory of a ray undergoing total internal reflection within a dielectric slab. The dotted line is the geometric continuation of the ray into medium 2 that would exist if $n_1 = n_2$. $N(s)$ is the number of reflections in a specified length.

for the optical waveguide of circular cross section. Then s for skew rays is no longer parallel to the z axis. The projection z_s of s parallel to the z axis is $z_s = s \cos \theta_z/\sin\alpha$, where θ_z is the inclination of the incident ray to the direction of the waveguide axis.

Ray methods are of interest for multimode $k_1 d \gg 1$ waveguides, so Eq. (12) holds except when α is very close to α_c.

We could terminate our discussion of the GHS on optical waveguide here; however, it is of interest to express s in terms of the well-known characteristic waveguide parameter $\eta(\theta_z,\alpha)$, where[6,7]

$$\eta(\theta_z, \alpha) = \frac{\text{power of the mode in the core}}{\text{total guided power of the mode}} \tag{13a}$$

$$\cong 1 - \frac{2 \sin^2\theta_z}{k_1 d \cos^2\alpha_c(\sin^2\alpha - \sin^2\alpha_c)^{1/2}}, \tag{13b}$$

where Eq. (13b) holds when $\theta_z \ll 1$, $n_1 \cong n_2$, and Eq. (12) is satisfied. To do this we replace the attenuation coefficient Eq. (6) found from plane wave concepts with that from electromagnetic theory. Thus[6,7]

$$\gamma \cong z\xi\eta(\theta_z, \alpha), \tag{14}$$

where z is the length of the guide, ξ is given by Eq. (5), $n_1 \cong n_2$, and $\theta_2 \ll 1$. Now we equate Eqs. (4) and (14), noting that for the waveguide of circular cross section $z = l_{\text{tot}} \cos\theta_z$ and $N(s) = l_{\text{tot}}/(l + s/\cos\theta_z)$, leading to

$$s = \left[l\frac{P_{\text{cl}}}{P_{\text{co}}}\right]\sin\alpha, \tag{15}$$

which is valid when $\alpha \cong \alpha_c \cong \pi/2$, with $\theta_z \ll 1$, but need not satisfy, e.g., Eq. (12). The pathlength in the core between two successive reflections is

$$l = d\cos\alpha/\sin^2\theta_z.$$

P_{cl} is the power of the mode within the cladding, and P_{co} is the power of the mode within the core.

$$\frac{P_{\text{cl}}}{P_{\text{co}}} = \frac{1 - \eta(\theta_z, \alpha)}{\eta(\theta_z, \alpha)}. \tag{16}$$

Equation (15) shows explicitly that the GHS is proportional to the ratio of cladding to core modal power. It is valid for both the slab and circular cross section waveguide. Subject to Eq. (12) and $n_1 \cong n_2$, s given by Eq. (15) is identical to Eq. (11). This follows using the asymptotic form of ν in Eq. (13b).

IV. Discussion

We have found expressions for the GHS using the classical geometric optics model of Fig. 2. The simplicity of the derivation is a consequence of our assumption that $\alpha \cong \alpha_c \cong \pi/2$. Then the GHS is most significant, and the energy associated with the ray is approximately confined to the medium through which it propagates.

Finally, we note that by equating Eqs. (6) and (14) we can express the waveguide parameter η in terms of the Fresnel power transmission coefficient T of Eq. (9):

$$\eta = 1 + T/l\xi. \tag{17}$$

This expression is valid only when $n_1 \cong n_2$, $\theta_z \ll 1$, and Eq. (12) is satisfied.

References

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The report, Projections of Science and Engineering Doctorate Supply and Utilization, 1980 and 1985, (NSF 75-301), updates earlier NSF studies conducted in 1969 and 1971 and incorporates new data and improved methodologies. The report contains two sets of models on the supply and utilization of science and engineering doctorates and details on the assumptions used in their development. In the future, NSF plans to reexamine the projection assumptions and methodologies as changes develop in enrollment, funding, and utilization patterns.

The report, Projections of Science and Engineering Doctorate Supply and Utilization, 1980 and 1985, is available from the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402 for $1.30 per copy, stock number 038-000-00212.