Theory of zero-width band gap effect in photonic crystals made of metamaterials

S. Zouhdi,[1] A. V. Dorofeenko,[2] A. M. Merzlikin,[2] and A. P. Vinogradov[1,2,*]
[1]*Laboratoire de Génie Electrique de Paris Supélec, Plateau de Moulon, 91192, Gif sur Yvette Cedex, France*
[2]*Institute of Theoretical and Applied Electromagnetism, OIVT, Russian Academy of Sciences, 125412, Moscow, Izhorskaya, 13/19, Russia*

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It is shown that the band structure of a one-dimensional (1D) photonic crystal (PC) made of successive layers of single-epsilon-negative (ENG) and single-mu-negative (MNG) media can exhibit a photonic band gap (PBG) of zero width. Although this band gap consists of band edges only, the normal component of group velocity does not vanish. It is shown that the condition of existence of a zero width band gap, coincides with the Alù-Engheta conditions [A. Alù and N. Engheta, IEEE Tr. AP **51**, 2558 (2003)] for the restoration of the amplitude and phase of an electromagnetic wave by means of ENG-MNG bilayers. At the conjugate Alù-Engheta conditions the band gap of zero width transforms into a zero-φ_{eff} band gap [H. Jiang, H. Chen, H. Li, *et al.*, Phys. Rev. E **69**, 066607 (2004)] that allows using this PC for perfect imaging. The origin and robustness of these effects is studied.

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I. INTRODUCTION

In 1968 Veselago[1] predicted that a slab of left-handed material with relative permittivity and permeability $\varepsilon=\mu=-1$ works as a lens focusing a point source. In the literature, these media are called double-negative media (DNG-media). In 2000, Pendry[2] noted that such a lens has the ability to restore the whole wave spectrum of the source, including its evanescent modes. Thus, this lens produces a perfect image of the source. Since, at this stage, materials with $\varepsilon=\mu=-1$ do not exist for optical frequencies, Pendry proposed to use noble metals with $\varepsilon<0,\mu=1$ as an alternative to DNG media.[2] The drawback of employing SNG media instead of DNG-media is that (i) the image is formed only for TM-polarized light in the case of media with negative permittivity and positive permeability (ENG) or TE-polarized light in the case of media with negative permeability and positive permittivity (MNG), (ii) the image is not perfect: details much smaller than the wavelength will be restored whereas larger ones will be lost.

To overcome these difficulties, a bilayer superlens was suggested in Ref. 3 by Alù and Engheta. This lens consists of two layers one of which is made of ENG material and the other of MNG material. If the impedance values of the layers are of opposite signs ($\zeta_{ENG}=-\zeta_{MNG}$), that is

$$k_{z1}/\varepsilon_1 = -k_{z2}/\varepsilon_2 \text{ (TM polarization)},$$

$$\mu_1/k_{z1} = -\mu_2/k_{z2} \text{ (TE polarization)}, \tag{1a}$$

and the values of the optical thickness of the layers are identical

$$k_{z1}d_1 = k_{z2}d_2, \tag{1b}$$

then the bilayer system can transmit an incident plane wave with no change of its amplitude and phase.[3] Here k_{zj} is the normal component of the wave vector and d_j is the thickness of the jth layer.

As one easily sees, the transfer matrix of a bilayer is $\hat{T}=S_{01}J_1S_{12}J_2S_{21}S_{10}$, where indexes 0, 1, 2 indicate the host medium, first and second layers respectively, where

$$S_{ij} = \begin{pmatrix} \dfrac{\zeta_j+\zeta_i}{2\zeta_j} & \dfrac{\zeta_j-\zeta_i}{2\zeta_j} \\ \dfrac{\zeta_j-\zeta_i}{2\zeta_j} & \dfrac{\zeta_j+\zeta_i}{2\zeta_j} \end{pmatrix} \text{ and } J_i = \begin{pmatrix} \exp(ik_{zi}d_i) & 0 \\ 0 & \exp(-ik_{zi}d_i) \end{pmatrix} \tag{2}$$

are the transfer matrices of the boundary and of the layer, ζ_j is the wave resistance in the jth layer ($\zeta_j=k_z/\varepsilon_j$ for TM polarization and $\zeta_j=\mu_j/k_z$ for TE polarization). Under fulfillment of the Alu-Engheta conditions (1) $S_{12}=S_{21}=\left(\begin{smallmatrix}0&1\\1&0\end{smallmatrix}\right)$; and it is easy to see that $J_1S_{12}J_2S_{21}$ is a unitary matrix. As $S_{01}=S_{10}^{-1}$, the whole $\hat{T}$ matrix is also equal to the unit matrix. Therefore the wave is transmitted without any change in amplitude and phase. It is worth emphasizing that due to the single-negative nature of each layer the electromagnetic waves inside them are of evanescent type. Thus, the transfer of phase and amplitude is realized by evanescent waves.

Equation (1a) is the condition for the surface plasmon or magnon excitation. In the layers adjacent to the surface, excitations decay with scales $i/k_{z1}, i/k_{z2}$, respectively. Equation (1b) means that amplitude of fields on the surfaces adjacent to the given one are identical. In other words, the growth of the evanescent wave achieved in the first layer is compensated by its decay in the second one (Fig. 1). In the general case, the wave in each layer is a mixture of fields of two surface plasmon or magnon. Equation (1b) means that the fields of surface plasmon or magnon excited on the next nearest surfaces are continuous across the intermediate interface.

For further analysis it is convenient to rewrite Eq. (1) in the following form:

$$\varepsilon_1 d_1 = -\varepsilon_2 d_2 \text{ (for TM\_polarization)}, \tag{3a}$$

$$\mu_1 d_1 = -\mu_2 d_2 \text{ (for TE\_polarization)}, \tag{3b}$$

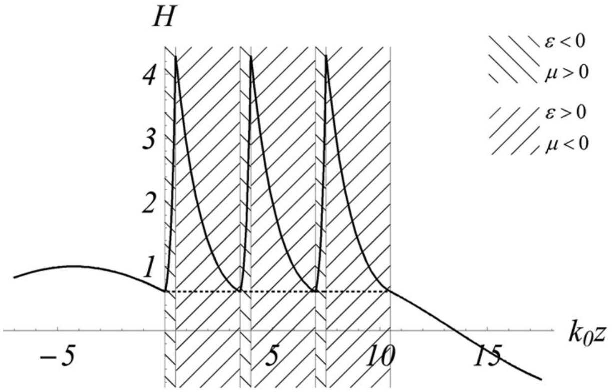

FIG. 1. A scheme of a 1D ENG-MNG periodic crystal, satisfying the condition (3a) ($\varepsilon_1=-6$, $\mu_1=11$, $\varepsilon_2=1$, $\mu_2=-1$, $\varepsilon_2 d_2 = -\varepsilon_1 d_1$) and a field distribution of a TM-polarized wave, meeting the condition (3c) with $k_0(d_1+d_2)=2$.

$$\frac{k_x}{k_0} = \pm \sqrt{\frac{\mu_1/\varepsilon_1 - \mu_2/\varepsilon_2}{1/\varepsilon_1^2 - 1/\varepsilon_2^2}} \text{ (TM\_polarization)}, \tag{3c}$$

$$\frac{k_x}{k_0} = \pm \sqrt{\frac{\varepsilon_1/\mu_1 - \varepsilon_2/\mu_2}{1/\mu_1^2 - 1/\mu_2^2}} \text{ (TE\_polarization)}, \tag{3d}$$

where k_x is a tangential wave number and $k_0=\omega/c$. At first sight the conditions (3a) and (3b) concern the crystal properties, not the wave ones. But it is the fulfillment of one of them that determines the polarization of the harmonic that shows a unitary transmission. The transversal wave number k_x of the harmonic is correspondingly determined by (3c) or (3d). The simultaneous fulfillment of both (3a) and (3b) results in the trivial case $k_x=0$ that corresponds to polarization degeneration. Indeed, in this case (3) reduces to

$$k_x^2/k_0^2 = [d_2^2/d_1^2]k_x^2/k_0^2. \tag{3e}$$

If a single ENG-MNG bilayer transmits a given incident wave with unchanged amplitude and phase, then a chain of such bilayers will produce the same effect (Fig. 1).

In accordance with (3e) the case $d_1=d_2$, referred to in Ref. 3 as "conjugate matching" of layers, plays a special role. In this case we have

$$\varepsilon_1 = -\varepsilon_2, \quad \mu_1 = -\mu_2, \quad d_1 = d_2. \tag{4}$$

Equations (3) are fulfilled by all the Fourier-components simultaneously, which means the reproduction of the whole spectrum for both TE and TM polarizations. The Alù-Engheta bilayer works like a system with canalization:[4] in both cases the source and image should be situated on opposite sides of the lens whereas for a DNG and ENG single layer lens, the distances d_{sl} between source and lens, d_{il} between image and lens are related with lens thickness d_l by the well known equality $d_{sl}+d_{il}=d_l$.

The analysis of the ENG/MNG bilayer was performed in Ref. 3 using the transmission line theory. Below we propose to analyze the corresponding multilayer structure by means of the photonic crystals band theory.

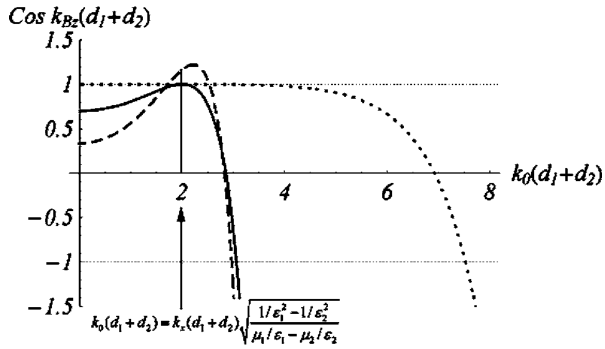

FIG. 2. Dependence of $\cos k_{Bz}(d_1+d_2)$ on $k_0(d_1+d_2)$. The solid line's parameters in the point, shown by the arrow, coincide with ones in Fig. 1. The dashed line indicates a typical dependence with band gap (the parameters are the same but $\varepsilon_2 d_2 = -0.8\varepsilon_1 d_1$). The dotted line illustrates the case with the parameters being closer to the ideal case (4) (it differs from the solid line by $\varepsilon_1=-1.1$, $\mu_1 = 1.263$).

II. ZERO-WIDTH BAND GAP AND IMAGING

The formal band theory of one-dimensional (1D) photonic crystal is well developed.[5,6] In the 1D PC the z dependence of the magnetic field (the z axis is normal to the layers) for TM Bloch wave can be expressed as $H_y=f(z)\exp(ik_{Bz}z)$, where $f(z)$ is a crystal's periodic function and k_{Bz} is the z component of the Bloch wave number. In a PC, made up of two alternating homogeneous layers, k_{Bz} is the solution of the dispersion equation[6,7]

$$\cos[k_{Bz}(d_1+d_2)] = \cos(k_{z1}d_1)\cos(k_{z2}d_2) - \frac{1}{2}\left(\frac{\zeta_1}{\zeta_2}+\frac{\zeta_2}{\zeta_1}\right)\sin(k_{z1}d_1)\sin(k_{z2}d_2), \tag{5}$$

where k_{zj}, d_j, ζ_j, are respectively the normal component of the wave vector, the thickness of the jth layer and its impedance.

Keeping in mind the employment of the PC as a superlens we are interested in the near field Bloch waves with tangential wave number $k_x \neq 0$. The term "near field wave" implies that although the Bloch wave may travel in the z direction, the field in each layer is always the sum of two evanescent (in the z direction) waves.[8]

A typical dependence of $\cos k_{Bz}(d_1+d_2)$ on k_0 is presented in Fig. 2 by the dashed line. In those regions where $|\cos k_{Bz}(d_1+d_2)|<1$ the value of k_{Bz} is real, which corresponds to the pass band. A band gap exists in the domains $|\cos k_{Bz}(d_1+d_2)|>1$ where k_{Bz} is complex. The condition $\cos k_{Bz}(d_1+d_2)=\pm 1$ corresponds to the band edges.

In terms of band theory, a wave transmission without modification in phase and amplitude through a PC slab can be achieved if the z directed component of the Bloch vector is equal to zero or, what is just the same, $\cos k_{Bz}(d_1+d_2)=1$. In the general case, the latter condition corresponds to the band edge. This means that, first, the corresponding group velocity should be equal to zero and there may be no energy transfer in the infinite system. Second, in a PC slab of finite thickness, due to degeneration, there appears a linear solution in z that prevents the fields to remain constant within the slab. However, as shown below, if the Alù-Engheta conditions are fulfilled, the linear solution is absent and both (linearly independent) solutions are Bloch waves with $k_{Bz}=0$. In this case the condition $\cos k_{Bz}(d_1+d_2)=1$ corresponds to the local maximum of $\cos k_{Bz}(d_1+d_2)$ (the solid line in Fig. 2). We treat this point as a band gap of zero width.

What does occur in zero-width band gap in reality? Taking into account that, for example, for TM polarization $H_y=f(z)\exp(ik_{Bz}z)$,

$$E_x(z) = -(i/\varepsilon k_0)\partial H_y/\partial z = -(i/\varepsilon k_0)[f'(z) + ik_{Bz}f(z)]\exp(ik_{Bz}z)$$

with $f(z)=|f(z)|\exp[i\varphi(z)]$ and $f'(z)=[|f(z)|' + i|f(z)|\varphi'(z)]\exp[i\varphi(z)]$, we can calculate the normal component of the Poynting vector

$$S_z = \frac{c}{8\pi}\mathrm{Re}(E_xH_y^*) = \frac{c}{8\pi\varepsilon(z)k_0}\mathrm{Re}[-if'(z)f^*(z) + k_{Bz}f(z)f^*(z)] = \frac{c}{8\pi\varepsilon(z)k_0}|f(z)|^2[\varphi'(z) + \mathrm{Re}\,k_{Bz}].$$

We can see that S_z does not reduce to the Poynting vector S_z^{plane} of the plane wave traveling with wave number $k_z=k_{Bz}$ and with a constant amplitude $C=|f(z_0)|=\mathrm{const}$: $S_z^{plane}(z)=\frac{c}{8\pi\varepsilon k_0}|C|^2\mathrm{Re}\,k_{Bz}$. Opposite to S_z^{plane}, the S_z does not equal zero even for $k_{Bz}=0$. Since the function $f(z)$ is crystal periodic, its phase $\varphi(z)$ can change by the value of $2\pi n$ by passing over a period. Indeed, adding the reciprocal lattice vectors to the Bloch vector, we can arrive at the situation with $n=0$. Then $\varphi(z)$ is periodic and $\varphi'(z)$ changes the sign on the crystal period. It turns out to be positive in one layer and negative in the other. At that time the sign of $\varepsilon(z)$ also alternates so that the energy flow does not change direction and absolute value. Note, that in the case of Alu-Engetha primitive cell, at $n=0$ we have $k_{Bz}=0$. Thus, the energy flow is connected with the phase gradient only.

In spite of the fact that both solutions to the Maxwell equations have the same wave number $k_{Bz}=0$, there is no degeneracy because these solutions are independent. Indeed, due to real values of the constitutive parameters the solutions are connected by the following relationship: $H_{y1}(z)=H_{y2}^*(z)$. As a consequence, $\varphi_1(z)=-\varphi_2(z)$ and $\varphi_1'(z)=-\varphi_2'(z)$, which creates fluxes of opposite signs.

The violation of the Alu-Engheta condition (for example, an impedance mismatch) results in an opening of the real band gap, where there is no energy flow. The reason is that there appears a reflection of the Bloch wave. The solution is now a mixture of two evanescent Bloch waves dying in opposite directions. The width of the band gap as well as of the interval where the group velocity tends to zero is proportional to the impedance mismatch.

In the general case, the band gap of zero width is formed due to the crossing of two isofrequencies (solid line in Fig. 3) or two dispersion curves (solid line in Fig. 4). Thus, there is no degeneracy, the phase velocity ω/k_{Bz} is equal to infinity

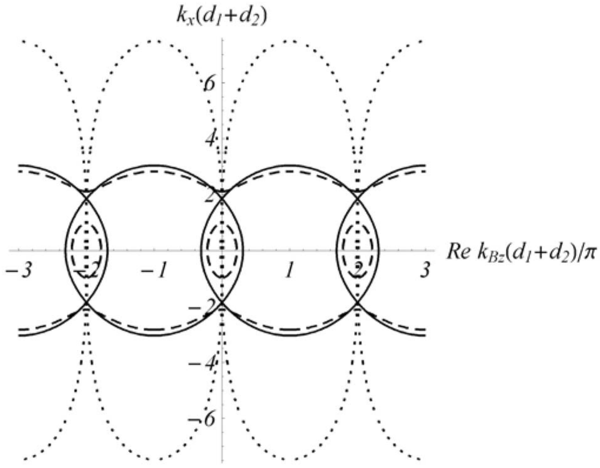

FIG. 3. Isofrequencies representation. The solid line's parameters in the points $k_{Bz}(d_1+d_2)/\pi=2n$, $n\in\mathbb{Z}$, coincide with ones in Fig. 1. The dashed line indicates a typical dependence with band gap (the parameters are the same but $\varepsilon_2d_2=-0.8\varepsilon_1d_1$). The dotted line illustrates the case with the parameters being closer to the ideal case (4) (it differs from the solid line by $\varepsilon_1=-1.1$, $\mu_1=1.263$).

($k_{Bz}=0$) and the group velocity is *not* equal to zero because of nonzero slope of dispersion curves. The crossing of isofrequencies or dispersion curves is for sure an unusual phenomenon. Indeed, usually it is accompanied with the coupling of the corresponding modes which results in mode hybridization and opening of a band gap[5] (the dashed lines in Figs. 3 and 4).

The formation of a band gap of zero width in the ENG-MNG PC (see Figs. 3 and 4) is also a consequence of the decoupling of modes (the reflection coefficient from the Alù-Engheta bilayer is equal to zero), which appears at fulfillment of the conditions (3). In the general case we have two pass bands and a band gap in k_0 values (the dashed curve in Fig. 4). There is the zero-ϕ_{eff} band gap,[9] separating two bands.[10] Under conditions (3) this zero-ϕ_{eff} band gap shrinks to the band gap of zero width, the curves reconnect such that the group velocities become non-zero (the solid curves in Fig. 4).

As we tend to the fulfillment of (4), the curves are pressed toward the axis (dotted curve in Figs. 3 and 4). When (4) is

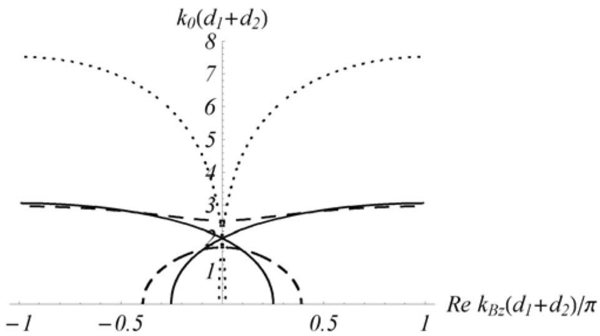

FIG. 4. Dispersion curves corresponding to the curves in Fig. 2.

fulfilled, the Bloch wave number is equal to zero at each value of k_x [see Eq. (5)] and the curves coincide with the axis. When the parameters approach the conditions (4), the value of $\cos k_{Bz}(d_1+d_2)$ tends to unity for all k_x (the dotted line in Fig. 2). The zero width band gap extends to zero-ϕ_{eff} band gap with $k_{Bz}=0$ at any k_x and any frequency [actually, due to the frequency dispersion, the condition (4) may be fulfilled at unique frequency]. As a consequence a wave with any value of k_x can be transmitted across the ENG-MNG PC without change in phase and amplitude. Though the mechanism is connected with plasmon or magnon resonance such as in a DNG lens opposite to DNG lens the perfect image can be observed if and only if the object is immediately placed on the surface of the lens. Moreover the image is formed on the opposite surface of the system. In this sense the situation reminds one of the case of canalization.[4]

III. DECOUPLING

In the 1D system the magnetic field H of the TM wave is determined by the equation

$$H''(z) + (\varepsilon(z)\mu(z)k_0^2 - k_x^2)H(z) = \frac{\varepsilon'(z)}{\varepsilon(z)}H'(z). \tag{6a}$$

Operating the variable change $\{\varepsilon,\mu\}$ to $\{k_z,\zeta\}$ $[k_z^2 = \varepsilon(z)\mu(z)k_0^2 - k_x^2,\ \zeta = k_z/\varepsilon]$ one can rewrite Eq. (6a) as

$$H''(z) - \frac{k_z'(z)}{k_z(z)}H'(z) + k_z^2(z)H(z) = -\frac{\zeta'(z)}{\zeta(z)}H'(z). \tag{6b}$$

If the impedance has the same value throughout the system, the waves exhibit no reflection. In other words waves coming towards each other do not couple. Thus the phenomena that are connected with reflection, such as the opening of the band gaps, do not appear. Indeed, if $\zeta'(z)=0$, Eq. (6b) has two uncoupled, independent solutions

$$H(z) = \exp\left(\pm i\int_0^z k_z(\tilde{z})d\tilde{z}\right). \tag{6c}$$

and

$$H(z) = \exp\left(\pm i\int_0^z [k_z(\tilde{z}) - \langle k_z\rangle]d\tilde{z}\right)\exp(\pm i\langle k_z\rangle z), \tag{6d}$$

where $\langle\cdots\rangle$ means averaging over the period. The first function $\exp(\pm i\int_0^z[k_z(\tilde{z})-\langle k_z\rangle]d\tilde{z})$ is a lattice function. The second function can be treated as a Bloch exponent with $k_{Bz} = \pm\langle k_z\rangle = \pm\int k_z(z)dz\big/\int dz$.

In the case of different impedance values, there appears a reflected wave that at $k_{Bz}=\{0,\pm\pi/d\}$ results in an opening of the band gaps. At edges of the band gap the group velocity vanishes. The vanishing is connected with equalization of amplitudes of two opposite traveling harmonics constituting the Bloch wave. If there is no coupling then there is no reflected wave and the fulfillment of the Bragg condition does not result in the appearance of opposite traveling harmonic in the Bloch wave.

In the ENG-MNG PC the k_z and, consequently, $\langle k_z\rangle$ are purely imaginary quantities and in the case of constant impedance we observe an all-round band gap. If the sign of the impedance value changes, one can expect coupling and opening of bands separated with band gaps (see Ref. 8 for details). This concerns both $k_{Bz}(k_0)$ at fixed k_x and $k_{Bz}(k_x)$ at fixed k_0.

Under the Alù-Engheta conditions (3) the band gap in the $k_{Bz}(k_x)$ dependence is about to open. As the reflection from a primitive cell is equal to zero we encounter the situation of quasiuncoupled waves. Indeed, since the dependence of $\zeta(z)=k_z(z)/\varepsilon(z)$ consists only in the change of sign, it is convenient to introduce a function $\chi(z)$ that is equal to $+1$ in one layer and to -1 in the other. By employing the following transformation

$$\frac{\varepsilon'(z)}{\varepsilon(z)} = \frac{d}{dz}\ln\varepsilon(z) = \frac{d}{dz}\ln\frac{k_z(z)}{\zeta(z)} = \frac{d}{dz}\ln\frac{\chi(z)\cdot k_z(z)}{\chi(z)\cdot\zeta(z)}$$
$$= \frac{[\chi(z)k_z(z)]'}{\chi(z)k_z(z)} - \frac{[\chi(z)\zeta(z)]'}{\chi(z)\zeta(z)},$$

one can rewrite Eq. (6a) as

$$H''(z) - \frac{[\chi(z)k_z(z)]'}{\chi(z)k_z(z)}H'(z) + [k_z\chi(z)]^2H(z)$$
$$= -\frac{[\chi(z)\zeta(z)]'}{\chi(z)\zeta(z)}H'(z). \tag{6e}$$

It is very important that under the Alù-Engheta conditions, $\chi(z)\zeta(z)=\text{const}$. The right-hand side in (6e) vanishes and Eq. (6e) reduces to the case considered above. The given above formulas for the solution and the Bloch wave number hold after the substitution

$$k_z(z) \to k_z(z)\chi(z):\ H(z) = \exp\left(\pm i\int_0^z \chi(\tilde{z})k_z(\tilde{z})d\tilde{z}\right),$$

$$k_{Bz} = \pm\langle\chi(z)k_z(z)\rangle$$

as if the coupling was absent in the system. Note that for $\zeta_1=-\zeta_2$ the latter formula coincides with the result obtained from the dispersion equation (5), which gives $k_{Bz}=k_{z1}d_1 - k_{z2}d_2\big/d_1+d_2=\langle\chi(z)k_z(z)\rangle$. In the Alù-Engheta system $\int_0^{d_1+d_2}\chi(z)k_z(z)dz=0$ [Eq. (1b)], which results in $k_{Bz}=0$. The latter equality assures that the wave is restored after passing through the crystal.

IV. ROBUSTNESS ESTIMATION

In the previous part of the paper we explored the Alù-Engheta system, meeting the "matching condition" (3a) and (3c) or (3b) and (3e). Now we will consider the "conjugate matched" case (4), when decoupling at any k_x appears and the perfect imaging is possible. In terms of the band gap theory we deal with zero-ϕ_{eff} band gap with $k_{Bz}=0$. Let us estimate the robustness of such a system against fluctuations of the permittivity/permeability, losses and layer thickness.

Below we consider the robustness of the perfect Alù-Engheta superlens ($\varepsilon_1=-1$, $\mu_1=1$, $\varepsilon_2=1$, $\mu_2=-1$). It turns out that deviation of real or imaginary parts of permittivity

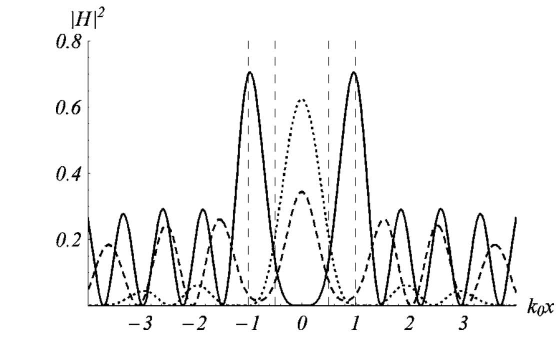

FIG. 5. The image of two slits (the slits, shown by vertical dashed lines, have the widths $k_0\delta=0.5$ and are placed at a distance $k_0\Delta=1$) formed by an imperfect conjugate matched lossless system [$\varepsilon_1=-0.99$, $\mu_1=1$, $\varepsilon_2=1$, $\mu_2=-1.02$, $k_0(d_1+d_2)=2$, $\varepsilon_2 d_2=-\varepsilon_1 d_1$] including one bilayer (solid line), five bilayers (dashed line), and ten bilayers (dotted line).

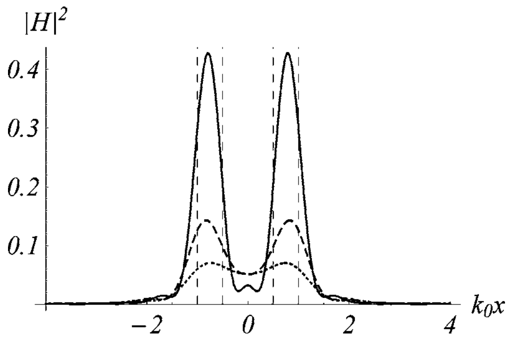

FIG. 7. The image of two slits ($k_0\delta=0.5$, $k_0\Delta=1$) formed by an imperfect conjugate matched system with losses [$\varepsilon_1=-1+0.01i$, $\mu_1=1$, $\varepsilon_2=1$, $\mu_2=-1+0.02i$, $k_0(d_1+d_2)=2$, $d_1=d_2$] including one bilayer cell (solid line), five cells (dashed line), and ten cells (dotted line). The presence of both amplitude and phase change in transfer function leads to the corruption of the image.

and/or permeability from $\varepsilon_1=-1$, $\mu_1=1$, $\varepsilon_2=1$, $\mu_2=-1$ decreases the resolution of the Alù-Engheta lens.

If losses are not taken into account, in the nonperfect system the transmission coefficient $T(k_x)$ as a function of k_x has a system of singularities, corresponding to eigenstates.[11] In spite of the fact that phases of the transfer functions are constant, the nonconstancy of the absolute values of the transfer functions leads to destruction of the image (see Fig. 5). Depending on the type of disturbance, instead of zero-ϕ_{eff} band gap with $k_{Bz}=0$ at any k_x one can observe a band gap of zero width [the relation (3a) or (3b) is still valid, Fig. 6, inset 1, Fig. 3 the dotted curve] or a real band gap [both relations (4) and (3) are violated, Fig. 6, inset 2].

The appearance of losses violates the "conjugate matching" condition (4) and as a consequence spoils the image (see Fig. 7) even if the real parts satisfy relations (4) because the impedances of different layers could not be identical and instead of zero-ϕ_{eff} band gap a true band gap should form. In fact, it is a pseudoband gap, because the Lyapunov factor differs from zero at any frequency due to losses (the case of optical gain[13] is not considered here). The losses change both the real and imaginary part of k_{zB} (see also Ref. 14). Thus, instead of $\mathrm{Re}\, k_{zB}=0$ for any k_x the dependence of the $\mathrm{Re}\, k_{zB}$ on k_x in the pseudoband gap has the form as in the zero-width band gap (see Fig. 8 and insets therein).

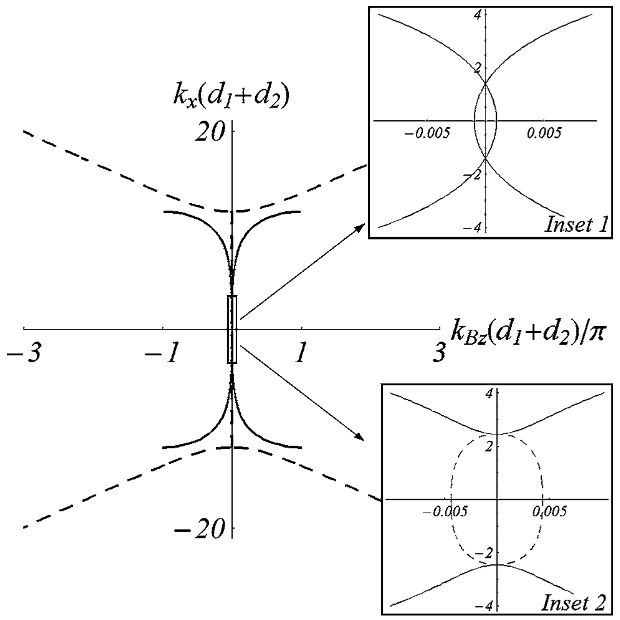

FIG. 6. The dependence of $\mathrm{Re}\, k_{Bz}(d_1+d_2)/\pi$ (solid line) and $\mathrm{Im}\, k_{Bz}(d_1+d_2)/\pi$ (dashed line) on k_x. The first inset corresponds to the formation of the zero-width band gap (the parameters are the same as in Fig. 5), the second one appears at violation of the condition (3a): $\varepsilon_2 d_2=-\varepsilon_1 d_1\cdot 1.01$.

V. CONCLUSIONS

The Alù-Engheta conditions have found a clear physical sense in terms of the band theory of photonic crystals,

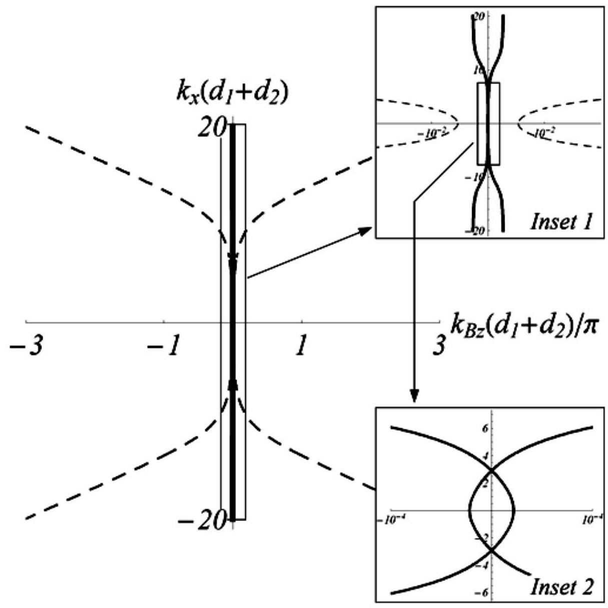

FIG. 8. The dependence of $\mathrm{Re}\, k_{Bz}(d_1+d_2)/\pi$ (solid line) and $\mathrm{Im}\, k_{Bz}(d_1+d_2)/\pi$ (dashed line) on k_x. The parameters are the same as in Fig. 7. The insets show the details of the plot.

namely, we found that they correspond to the conditions of appearance of a band gap of zero width. At the very frequency of this band gap the phase velocity is equal to infinity, whereas the group velocity differs from zero. It is this property, which appears to be due to the phenomena of quasidecoupling of opposite traveling waves, that permits using such a structure as a superlens. Violation of quasidecoupling results in destruction of the image whether this violation be due to the real or imaginary parts of permittivity and permeability. At the same frequency the Veselago and Alù-Engheta lenses may still be working, which concerns our parameters. Thus, it seems that the Alù-Engheta lens is preferable because it produces almost the same results as the Veselago lens does but its manufacturing requires single-negative materials only.

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*E-mail: a-vinogr@yandex.ru

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[10] It is worth emphasizing that in PC containing single-negative materials (negative PC) the resonant scattering is responsible for bands of transmittance whereas in common (positive) PC the resonance (Bragg) scattering is responsible for band gaps (Ref. 4). Indeed, disregarding scattering phenomena we arrive at the band gap in the first case and at the pass band in the second. Thus, the nature of band gaps in PC with single-negative materials and in common PC is different. As a consequence the band gaps exhibit different scaling behavior (Ref. 9).

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