Project 1 in FYS-3150

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1 Task a: Prepare the problem, implement Runge-Kutta 4

1.1 Preparing simple ODE

For the first part of the project we will set up a simple system system of just the Earth orbiting the Sun, with the Sun acting as a constant anchor unnaffected by the earth since its mass is several orders of magnitude larger than the Earth, $M_S >> M_E$. This simple classical system is governed by Newtons laws. We will use index notation with $\frac{\partial^2 x}{\partial t^2} = \partial^2 x$, with alle derivatives being with respect to time unless spesified otherwise. If an index repeats summation over i is implied, so for $i = 1, 2, 3$, it will mean: $x_i x_i = x_1 x_1 + x_2 x_2 + x_3 x_3$

From Newtons second law we get:

$$M_E \partial^2 x_i = F_i \tag{1}$$

The only force acting on the earth is the gravitational force

$$\partial^2 x_i = \frac{x_i}{(x_j x_j)^{1/2}} \frac{(GM_S M_E)/(x_j x_j)}{M_E} \tag{2}$$

$$\partial^2 x_i = x_i \frac{GM_S}{(x_j x_j)^{3/2}} \tag{3}$$

Now we will divide it into a coupled set of two first order ODE's

$$y_i^{(1)} = x_i \tag{4a}$$

$$y_i^{(2)} = \partial y_i^{(1)} \tag{4b}$$

We then insert our new equationset, (4), into equation (3) and obtain

$$\partial y_i^{(1)} = y_i^{(2)} \tag{5a}$$

$$\partial y_i^{(2)} = y_i^{(1)} \frac{GM_S}{(y_j^{(1)} y_j^{(1)})^{3/2}} \tag{5b}$$

Now we have a set of 2 coupled first order ODE's, with 2 unknowns, and we will now discretize them.

1.2 Runge-Kutta 4 method

The Runge-Kutta 4 method is based on using 4 steps to calculate the next iteration of the function we approximate. Given a that the derivative of a function is given by a general function $f(t, y$

$$y(t) = \int f(t, y) dt \tag{6}$$

Given that y_k is known the next point, y_{k+1} on the function is given by

$$y_{k+1} = y_k + \int_t^{t+1} f(t,y)dt \tag{7}$$

Let h be the stepsize and then a Taylor expand around half the step, $t + 1/2$, results in

$$f(t,y) = f(t_{i+1/2}, y_{i+1/2}) + (t - t_{i+1/2})\frac{\mathrm{d}}{\mathrm{d}t}(f(t_{i+1/2}, y_{i+1/2})) + O(h^3) \tag{8}$$

We then integrate this from $t_i \to t_{i+1}$ using a midpoint rule and get

$$\int_t^{t+1} f(t,y)dt \approx hf(t_{i+1/2}, y_{i+1/2}) \tag{9}$$

Inserting this into equation (7)

$$y_{k+1} = y_k + hf(t_{i+1/2}, y_{i+1/2}) + O(h^3) \tag{10}$$

Then a forward Euler method is used to approximate $y_{i+1/2} \approx y_i + \frac{h}{2}\frac{\mathrm{d}y}{\mathrm{d}t} = y_i + \frac{h}{2}f(t_i, y_i)$

$$s \tag{11}$$