UiO : Faculty of Mathematics and Natural Sciences
University of Oslo

Department of Physics

A Multigrid Poisson Solver for PINC

by

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Overview

Aim of the Thesis

- Develop a new Parallel Multigrid solver for the new Particle-in-Cell model PINC

- Appears in various important areas that affect us
- The Sun, Upper parts of the Earth's Atmosphere
- Important industrial applications
- Plasma cutters, light tubes/bulbs, fusion

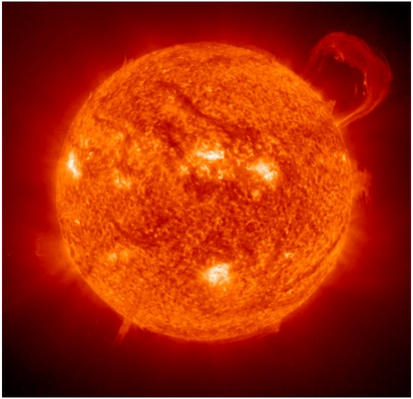


Figure : Picture of the sun, taken with SOHO's Extreme ultraviolet Imaging Telescope (*The Sun imaged by EIT at 304 Å* 2017)

"A plasma is a quasineutral gas of charged and neutral particles which exhibits collective behaviour."

Francis F. Chen (Chen, 1984)

- Neutral and charged particles
- Electromagnetic forces due to the charge

Equations of motion

$$
\begin{aligned}
m\frac{\mathrm{d}\mathbf{v}(t)}{\mathrm{d}t} &= q[\mathbf{E}(\mathbf{r}(t),t) + \mathbf{v}(t) \times \mathbf{B}(\mathbf{r}(t),t)] \\
\frac{\mathrm{d}\mathbf{r}(t)}{\mathrm{d}t} &= \mathbf{v}(t)
\end{aligned}
$$

Maxwell's equations

$$
\begin{aligned}
\nabla \cdot \mathbf{E} &= \frac{\rho}{\varepsilon_0} & \nabla \cdot \mathbf{B} &= 0 \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} & \nabla \times \mathbf{B} &= \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}
\end{aligned}
$$

- Particle based - Computes the Equations of Motion for each particle
- Each particle is interacting with a field, instead of all the other particles

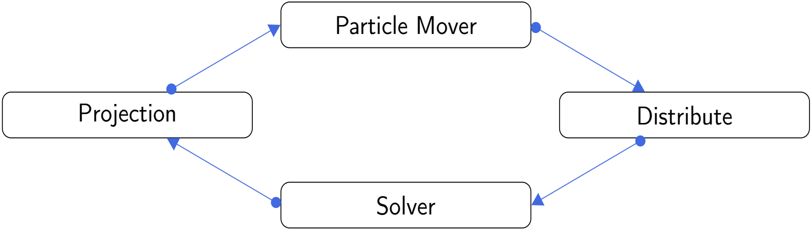


Figure : Schematic overview of the electrostatic PIC cycle. The mover moves all the particles and updates their velocities. Next the particle charges are distributed to a charge density grid. The solver then obtains the electric field on the grid (and magnetic field in a full electromagnetic model when also the currents are weigthed to the grid). Lastly the field values are projected onto the particles.

- Mover - Moves all the particles
- Distribute - Distributes the charges from the particles onto the grid
- Solver - Solves the equation on the grid -> obtains electric potential
- Projection - Field values projected onto the particles

- Solves the Poisson equation, $\epsilon_0 \nabla \cdot \vec{E} = e(n_i - n_e)$.
- Charge distribution (ρ) $\longrightarrow$ Potential (Φ)
- Solves the problem on different grid resolutions

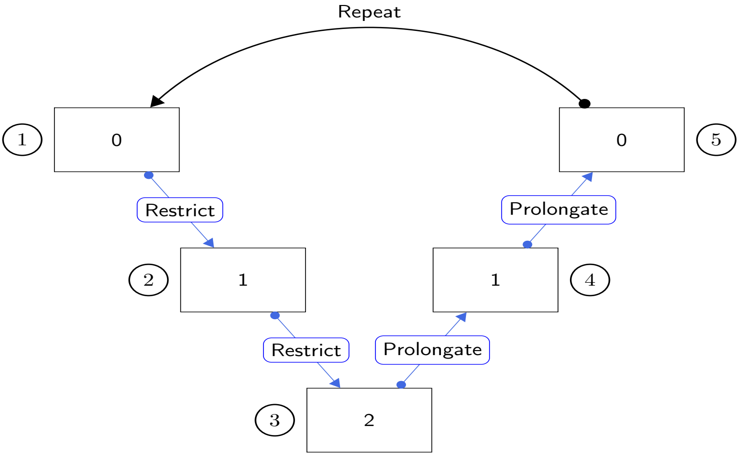


Figure : Schematic overview of the Multigrid cycle. In a three level MG V implementation, there is 5 main steps in a cycle that needs to be considered.

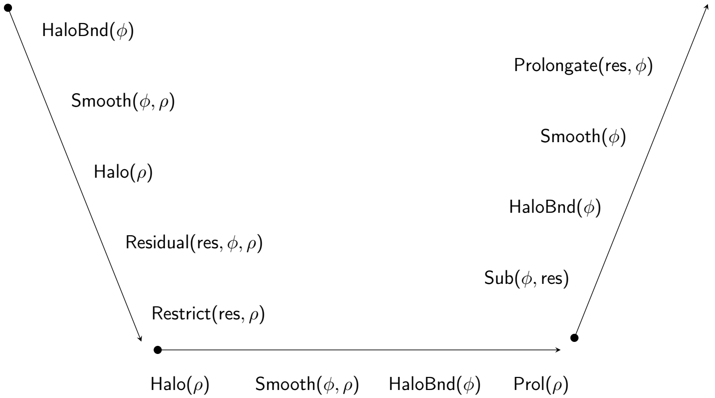


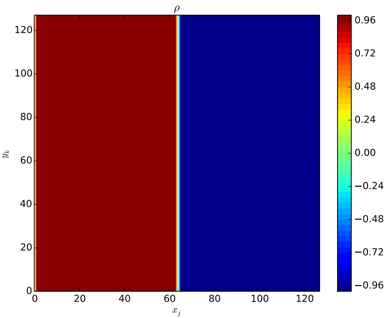
Figure : The algorithmic steps in a 2-level Multigrid algorithm

- HaloBnd - Fill Ghost cells according to boundary condition
- Smooth - Solve with Gauss-Seidel RB
- Halo - Compute Residual
- Restrict - Go to coarser grid
- Retrieve improvement to solution from coarser grid
- Smooth - Improve solution

- Verification Multigrid module
- Verification PINC
- Scaling

Tested on:

- Analytical solutions of sinusoidal and Heaviside charge distributions
- Random charge distributions
- 2 different algorithms (ND and 3D) was tested against each other
- Scaling of the error due to discretization
- Performed well on all the test



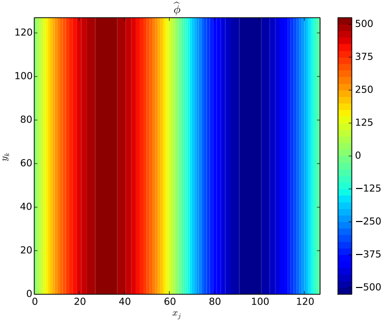


Figure : Slice of the charge distribution (left) and the numerical solution (right) for the resulting potential. The resulting potential has the expected 2nd degree polynomial shape. The residual was on the order 10^{-5}.

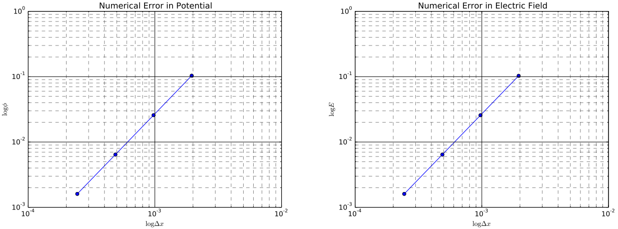


Figure : Logarithmic plot of the 2-norm of the error of the potential ϕ (left) and the x-component of the electric field. The solver was run on a scaled sinus-shaped charge distribution. Both of the plots show a straight line of the error, on the logarithmic plots, with a slope of 2.00. This corresponds to the error scaling with order -2 as a function of the stepsize as expected. All the units are in PINC normalized units.

- PINC was tested by reproducing a Langmuir Plasma Oscillation

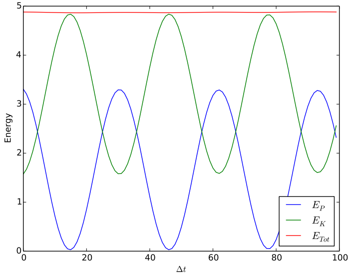


Figure : This shows the time-evolution of the energy in an perturbed plasma. The energies are in normalized units and $\Delta x = 0.1\lambda_{De}$. The total energy has a maximum variation of 0.22%. In the timespan of $10\omega_{pe}$ the plasma oscillates over 1.6 times.

Parallel Scaling of the Multigrid Solver

- Tested by increasing the problem (# grid points), with a corresponding increase in processors
- Timing how long it took to solve the problem satisfactorily
- Scaling was tested on the Abel supercomputer at UiO

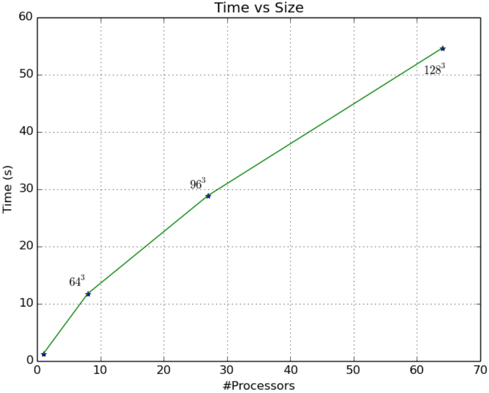


Figure : A Langmuir Oscillation were performed for 10 timesteps with a $(32, 32, 32)$ grid on each processor. This was repeated with increasing amount of processors, $(1, 8, 32, 64)$, to see how the multigrid solver scales.

What was found.

- The multigrid solver was shown to work correctly
- The PINC was shown to work correctly
- Scalability was not shown to work satisfactorily

Next step

- More testing on PINC
- Solve the scaling issue shown
- Add more possibilities to PINC (already underway)
- Different Boundary Conditions are not well tested

Chen, F. F. (1984). *Introduction to Plasma Physics and Controlled Fusion*. en. Boston, MA: Springer US. ISBN: 978-1-4419-3201-3 978-1-4757-5595-4. URL: http://link.springer.com/10.1007/978-1-4757-5595-4 (visited on 07/12/2016).

The Sun imaged by EIT at 304 Å (2017). URL: http://sci.esa.int/soho/43990-the-sun-imaged-by-eit-at-304-aring/ (visited on 01/18/2017).