# Contents


0.1 To-do list . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1
0.2 Electron plasma oscillations in unmagnetized plasma . . . . . . . . 1
  0.2.1 Physical mechanism . . . . . . . . . . . . . . . . . . . . . . 1
  0.2.2 Test Case . . . . . . . . . . . . . . . . . . . . . . . . . . 2

## 0.1 To-do list

## 0.2 Electron plasma oscillations in unmagnetized plasma

### 0.2.1 Physical mechanism

Here we will consider an electron fluid under local thermal equilibrium, LTE, experiencing small perturbations from an equilibrium state of an homogeneous electron density, $n_0$, and pressure, $p_0$, and vanishing flow, $\mathbf{u}_0 = 0$, and electric field, $\mathbf{E}_0 = 0$. See Goldston and Rutherford (1995) for a more thorough overview.

The small perturbation of the electron density will cause an electric field electric field working to restore the equilibrium. When the electrons reach the equilibrium position they have a kinetic energy and will overshoot causing a new perturbation away from the equilibrium.

The fluid is governed by the following equations:

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{u}_e) = 0 \tag{1a}$$

$$m_e n_e \left( \frac{\partial}{\partial t} + \mathbf{u}_e \cdot \nabla \right) \mathbf{u}_e = e n_e \nabla \phi - \nabla p_e \tag{1b}$$

$$\left( \frac{\partial}{\partial t} + \mathbf{u}_e \cdot \nabla \right) p_e + \frac{5}{3} p_e \nabla \cdot \mathbf{u}_e = 0 \tag{1c}$$

$$\epsilon_0 \nabla^2 \phi = e \left( n_e - n_0 \right) \tag{1d}$$

Assuming the small perturbation to the equilibrium.

$$\text{Perturbation} \rightarrow \begin{cases} n_e = n_0 + \tilde{n}_e \\ p_e = p_0 + \tilde{p}_e \\ \mathbf{u}_e = \tilde{\mathbf{u}}_e \\ \phi = \tilde{\phi} \end{cases}$$

Inserting the perturbation and linearizing the equations we get:

$$\frac{\partial \tilde{n}_e}{\partial t} + \nabla \cdot (n_0 \tilde{\mathbf{u}}_e) = 0 \tag{2a}$$

$$m_e \frac{\partial \tilde{\mathbf{u}}_e}{\partial t} = e \nabla \tilde{\phi} - \frac{\nabla \tilde{p}_e}{n_0} \tag{2b}$$

$$\frac{\partial \tilde{p}}{\partial t} + \frac{5}{3} p_0 \nabla \cdot \tilde{\mathbf{u}}_e = 0 \tag{2c}$$

$$\epsilon_0 \nabla^2 \tilde{\phi} = e \tilde{n}_e \tag{2d}$$

Then we combine the continuity and energy equations, eq. (2a) and eq. (2c).

$$\frac{\partial}{\partial t}\left(\frac{\tilde{p}_e}{p_0}+\frac{5}{3}\frac{\tilde{n}_e}{n_0}\right)=0 \tag{3}$$

The perturbed pressure and density is proportional, $\nabla\tilde{p}_e = (5p_0/3n_0)\,\nabla\tilde{n}_e$. Assuming plane wave solutions along the x-axis, so the differential operators become $\nabla \to ik$ and $\frac{\partial}{\partial t} \to -i\omega$, we can solve for the dispersion relation.

$$\epsilon(\omega,k) = 1+\frac{5}{3}\lambda_{se}^2 k^2 - \frac{\omega^2}{\omega_{pe}^2} \tag{4}$$

Here we have subsituted in the electron debye length $\lambda_{se} = (\epsilon_0 p_0)/(e^2 n_0^2)$, and the electron plasma frequency $\omega_{pe} = e^2 n_0$

### 0.2.2 Test Case

As a test of the validity of our PiC program, we can use the langmuir wave oscillation. This test is inspired by a case set up in Birdsall and Langdon (2004), and modified to fit with our normalization and discretizations.