Contents

0.1 To-do list

Chapter 1

Verification and Performance

1.1 Electron plasma oscillations in unmagnetized plasma

1.1.1 Physical mechanism

Here we will consider an electron fluid under local thermal equilibrium, LTE, experiencing small perturbations from an equilibrium state of an homogeneous electron density, n_0, and pressure, p_0, and vanishing flow, $\mathbf{u}_0 = 0$, and electric field, $\mathbf{E}_0 = 0$. See **goldston˙introduction˙1995** for a more thorough overview.

The small perturbation of the electron density will cause an electric field electric field working to restore the equilibrium. When the electrons reach the equilibrium position they have a kinetic energy and will overshoot causing a new perturbation away from the equilibrium.

The fluid is governed by the following equations:

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{u}_e) = 0 \tag{1.1a}$$

$$m_e n_e \left(\frac{\partial}{\partial t} + \mathbf{u}_e \cdot \nabla \right) \mathbf{u}_e = e n_e \nabla \phi - \nabla p_e \tag{1.1b}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{u}_e \cdot \nabla \right) p_e + \frac{5}{3} p_e \nabla \cdot \mathbf{u}_e = 0 \tag{1.1c}$$

$$\epsilon_0 \nabla^2 \phi = e \left(n_e - n_0 \right) \tag{1.1d}$$

Assuming the small perturbation to the equilibrium.

$$\text{Perturbation} \rightarrow \begin{cases} n_e = n_0 + \tilde{n}_e \\ p_e = p_0 + \tilde{p}_e \\ \mathbf{u}_e = \tilde{\mathbf{u}}_e \\ \phi = \tilde{\phi} \end{cases}$$

Inserting the perturbation and linearizing the equations we get:

$$\frac{\partial \tilde{n}_e}{\partial t} + \nabla \cdot (n_0 \tilde{\mathbf{u}}_e) = 0 \tag{1.2a}$$

$$m_e \frac{\partial \tilde{\mathbf{u}}_e}{\partial t} = e\nabla\tilde{\phi} - \frac{\nabla \tilde{p}_e}{n_0} \tag{1.2b}$$

$$\frac{\partial \tilde{p}}{\partial t} + \frac{5}{3} p_0 \nabla \cdot \tilde{\mathbf{u}}_e = 0 \tag{1.2c}$$

$$\epsilon_0 \nabla^2 \tilde{\phi} = e\tilde{n}_e \tag{1.2d}$$

Then we combine the continuity and energy equations, eq. (1.2a) and eq. (1.2c).

$$\frac{\partial}{\partial t}\left(\frac{\tilde{p}_e}{p_0} + \frac{5}{3}\frac{\tilde{n}_e}{n_0}\right) = 0 \tag{1.3}$$

The perturbed pressure and density is proportional, $\nabla \tilde{p}_e = (5p_0/3n_0)\,\nabla \tilde{n}_e$. Assuming plane wave solutions along the x-axis, so the differential operators become $\nabla \to ik$ and $\frac{\partial}{\partial t} \to -i\omega$, we can solve for the dispersion relation.

$$\epsilon(\omega, k) = 1 + \frac{5}{3}\lambda_{se}^2 k^2 - \frac{\omega^2}{\omega_{pe}^2} \tag{1.4}$$

Here we have subsituted in the electron debye length $\lambda_{se} = (\epsilon_0 p_0)/(e^2 n_0^2)$, and the electron plasma frequency $\omega_{pe} = e^2 n_0$

1.1.2 Test Case

Since the physics behind the electron plasma wave is quite simple, we want to verify that our code works by letting it reproduce the waves.

- Homogenous plasma
- No fields
- Equilibrium? (Lattice structure?) First test this
- No thermal velocity / (Not necessary?)
-

1.2 Notation

While the notation is described in the main text at its first instance, we have also included this small note on the notation used to make it easier to look up. Notation that are only used in locally in smaller sections are not included here.

In the PinC project we have decided to try to keep different indexes tied different objects to help avoid confusion and increase readability. Since the i index is reserved for incrementing particles, the spatial x, y, z-indexes are j, k, l instead of the more usual i, j, k. So to make the transition between this document and the code easier we have also used the j, k, l indexes to denote the spatial area. This is the convention used by Birdsall and Langdon, (cite plasma physics via simulation).

Subscripts are usually used to denote spatial index, and a superscripts are usually reserved for temporal cases. So $\Phi^n_{j,k,l}$ means the potential at the timestep n and position j, k, l. When plasma theory is involved the subscript can also signify the particle species.

Φ	Electric Potential
ρ	Charge Density