Chapter 1

Verification

1.1 Langmuir Oscillations

In a we consider a homogenous and isotropic plasma, in stable equilibrium, and let the electrons be pushed, causing a slight perturbation of the equilibrium. The slightly uneven distribution will cause an electric field pushing against the perturbation and try to restore the equilibrium. When the electrons reach the equilibrium position they have a velocitykinetic energy and will overshoot causing an equal opposite perturbation. Then it repeats and we have a simple oscillation of the electron density.

Certain assumptions are necessary to derive the oscillation mathematically. First the plasma needs to be in a homogenous and isotropic equilibrium state so the spatial and temporal derivatives is zero. The magnetic field strength also needs to be small enough to be safely ignored. We then consider movements on a timescale so that the inertial effects of the electrons are important, while the ions are considered stationary.

We start from the electron fluid motion equation,

$$mn_e \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) \mathbf{u} = -en_e\mathbf{E} - \nabla p_e \tag{1.1}$$

and we consider a small perturbation to the equilibrium state so the quantities becomes:

$$\mathbf{u} \approx \mathbf{u}_0 + \tilde{\mathbf{u}}; \qquad \mathbf{E} \approx \tilde{\mathbf{E}}; \qquad n \approx n_0 + \tilde{n}; \qquad p \approx p_0 + \tilde{p} \tag{1.2}$$

Here the subscript for the electron is dropped, the subscript 0 is the equilibrium state and the tilde is the perturbation. Then we apply linearization to the equation, so that all the second order terms of the perturbation goes away.

$$mn_0 \frac{\partial}{\partial t} \tilde{\mathbf{u}} = -en_0\tilde{\mathbf{E}} - \nabla \tilde{p}_e \tag{1.3}$$

For simplicities sake the perturbation is a plane wave in the x-direction, $exp[i(kx - i\omega)]$, as well as what we will program for verification use. Then the differential operators become $\nabla \rightarrow ik$ and $\frac{\partial}{\partial t} \rightarrow -i\omega$, using the relation $\tilde{p} = 3T\tilde{n}$, see cite-Goldston INtro to plasma 1995. Then the x-component of the electron motion equation becomes:

$$i\omega m n_0 \tilde{u} = e n_0 \tilde{E} + i3kt\tilde{n} \tag{1.4}$$

Using the same procedure the electron continuity equation, $\frac{\partial n}{\partial t} + \nabla \cdot (\mathbf{u}n) = 0$, becomes

$$-i\omega\tilde{n} = ikn_0\tilde{u} \tag{1.5}$$

Next we look poisson's equation, $\epsilon_0 \nabla \cdot \mathbf{E} = e(n_i - n_e)$