Uncorrelated Locality-Sensitive Multi-view Discriminant Analysis (ULMVDA)

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Chapter 1

Introduction

1.1 Multi-View Discriminant Analysis (MvDA)

Multi-View Discriminant Analysis is the specialised form of Multi-View Learning. In the case of Multi-View Learning, we take into consideration the data obtained from multiple views of the data. When we look at the real-world applications, we find that the same object can be observed at different viewpoints or can be described with different descriptors, thus generating the multi-view data. Since more useful information exists in multiple views than in a single one, multi-view learning has attracted a lot of research interests. To further improve the learning curve we started using the Discriminant analysis with the Multi-view Analysis to improve the overall learning.

1.2 Uncorrelated Locality-Sensitive Multi-View Discriminant Analysis (ULSMDA)

Uncorrelated Locality Sensitive Multi-View Discriminant Analysis jointly learns multiple view-specific transformations. For each view, by mapping data into the projected subspace, the nearby samples with the same label are close to each other while the nearby samples with different labels are far apart. Here the learning process depends on the fact that we increase the vector distance between the elements of the different classes and decrease the distance between the elements of the same class. We further combine the distances found using the vector distancing with the elements of the affinity matrix of the in-class Graphs we create, this provides us with multi-view data, which enhances the performance of our learning algorithm. By this way, the discriminant and local geometrical structure information in multiple views can be well preserved. ULSMDA provides a multi-view sample distance term to promote one-to-one data consistency across views. Further, the learning process is marked complete by the sampling of the data set using the PCA transformation and then we try to identify the variable parameters to meet the requirements of our learning algorithm. Once we find the accurate parameters we set the parameters and then train the model to obtain the required accuracy. We design uncorrelated constraints for the multi-view learning process, to reduce the redundancy among transformations learned from different views.

Chapter 2

Mathematical Formulation

2.1 Derivation and Formulation

When developing the formulation for the ULMSDA we first try to understand all the requirements and the parameters which we use in the mathematical formulation of our approach. Let us start by stating the assumptions and understanding the various parameters we use.

We assume that is the training samples and for each X_k (k=1,..., M) is made up of n-dimensional samples which are derived from C classes. For each $x_k{}^p$ denotes the pth sample from the kth view. Also, we use the projection vectors($v_1, v_2, v_3, ..., v_m$) to explore the discriminant and the local geometrical structure so that we can explore multiple views of the information.

Now for each view, we construct two graphs one within the class $(G_w{}^k$) and one between the class($G_b{}^k$). Further, we find the K-nearest neighbors and obtain the neighbour set of the same i.e. $N^k(x_k{}^p) = \{x_k{}^{p1}, x_k{}^{p2}, ...x_k{}^{pk}\}$. We further divide the neighbour sets into two smaller subsets- one which contains elements owing to the same and different class labels with $x_k{}^p$. For simplification, we further define the affinity matrix of both the within-class Graph $(G_w{}^k$), where its elements would be represented as $W_w{}^k$.

$$W^k_{w,pq} = \begin{cases} 1, & \text{if } x^p_k \in N^k_w(x^q_k) \text{ or } x^q_k \in N^k_w(x^p_k) \\ 0, & \text{otherwise} \end{cases}$$

$$W^k_{b,pq} = \begin{cases} 1, & \text{if } x^p_k \in N^k_b(x^q_k) \text{ or } x^q_k \in N^k_b(x^p_k) \\ 0, & \text{otherwise} \end{cases}$$

Each element describes the weight of the edge between $x_k{}^p$ and $x_k{}^q$.

We can further comprehend the within-class weights as values either 0 or 1. After this when we talk about the values we need to minimize the value of the following term:

$$\min_{v_1, v_2, \ldots, v_M} \sum_{k=1}^{M} \left(\sum_{pq} (y_k^p - y_k^q)^2 W_{w,pq}^k - \sum_{pq} (y_k^p - y_k^q)^2 W_{b,pq}^k \right) \quad \ldots\ldots(1)$$

Now we add a one-to-one data consistency across views, we now add a multi-view sample distance term:

$$\sum_{k=1}^{M-1} \sum_{l=k+1}^{M} d(v_k^T X_k, v_l^T X_l) \quad \ldots\ldots(2)$$

When we minimize the cross-view distance of projected samples, we can then find the closeness between the multi-view samples that characterize the same objects. Now, to maintain the cross-view consistency we require to reduce the redundancy as well. For this, we require the projective vectors corresponding to different views to be mutually orthogonal i.e. $v_k^t(v_l) = 0$, $k \neq 1$.

Thus, we reach our objective function which states:

$$\min_{v_1, v_2, \ldots, v_M} \frac{1}{2} \sum_{k=1}^{M} \left(\sum_{pq} (y_k^p - y_k^q)^2 W_{w,pq}^k - \sum_{pq} (y_k^p - y_k^q)^2 W_{b,pq}^k \right) + \sum_{k=1}^{M-1} \sum_{l=k+1}^{M} d(v_k^T X_k, v_l^T X_l)$$
$$\text{s.t.} \quad v_k^T v_l = 0, \; k \neq l \quad \ldots\ldots(3)$$

By further simplifying our equation we can write it as follows:

$$\frac{1}{2}\sum_{pq}(y_k^p - y_k^q)^2 W_{w,pq}^k = \frac{1}{2}\sum_{pq}(v_k^T x_k^p - v_k^T x_k^q)^2 W_{w,pq}^k$$
$$= \sum_p v_k^T x_k^p D_{w,pp}^k x_k^{pT} v_k - \sum_{pq} v_k^T x_k^p W_{w,pq}^k x_k^{qT} v_k$$
$$= v_k^T X_k (D_w^k - W_w^k) X_k^T v_k = v_k^T X_k L_w^k X_k^T v_k$$
………(4)

Here, we have added the constant to simplify the calculations and then we further apply the Laplacian Matrix which can further simplify the equation. And hence we can rewrite equation 3 as follows:

$$\min_{v_1, v_2, \ldots, v_M} \sum_{k=1}^{M} v_k^T X_k (L_w^k - L_b^k) X_k^T v_k$$
$$+ \sum_{k=1}^{M-1} \sum_{l=k+1}^{M} d(v_k^T X_k, v_l^T X_l)$$
$$\text{s.t.} \quad v_k^T v_l = 0, \; k \neq l$$
……(5)

We can finally reduce down our equation by multiplying the diagonal values of the affinity matrix by a constant and then we apply regularization. Thus we obtain the following equation:

$Gv = \lambda H' v$

Then we find the projected features of our training sample set and the query sample set and then fuse the features. Finally, we apply the nearest neighbour classifier with cosine distance to classify the Z^y .

Chapter 3
ULMVDA Algorithm

In this section we will look at the algorithmic flow of how we can implement the UMLVDA algorithm and all the required optimizations in the previous available algorithms.

3.1 ULMVDA Algorithm

Input:

Training data X ;

Number of Views M;

Number of classes C;

Output:

Transformed data Z;

1: Construct the within class and the interclass graphs- $G_w{}^k$, $G_b{}^k$.

2: Find the set of nearest neighbours divided on the ground of within class and inter-class i.e.

$N^k(x_k{}^p) = \{x_k{}^{p1}, x_k{}^{p2}, ...x_k{}^{pk}\}$

3: Create the affinity matrix for both the created graphs.

4: Minimize the obtained term

$$\min_{v_1, v_2, \ldots, v_M} \sum_{k=1}^{M} \left(\sum_{pq} (y_k^p - y_k^q)^2 W_{w,pq}^k - \sum_{pq} (y_k^p - y_k^q)^2 W_{b,pq}^k \right)$$

5: Improve data consistency by introducing the multi-view sample distance term.

$$\sum_{k=1}^{M-1} \sum_{l=k+1}^{M} d\left(v_k^T X_k, v_l^T X_l\right)$$

6: Projective vectors are created for each view such that they are mutually orthogonal.

$$v_k^T v_l = 0, \; k \neq l$$

7: Using Laplacian matrix obtain:

$$\min_{v_1, v_2, \ldots, v_M} \; v^T G v$$
$$\text{s.t.} \quad v^T H v = 0$$

8: Apply regularization on H, to get $Gv = \lambda H' v$.

9: Obtain the projected feature vector ${Z_X}^k$ and fuse Z^X and Z^Y .

10: Apply nearest neighbour classification using cosine distance to classify the Z^y .

11: return Z.

Chapter 4

Learning outcome

Some of the major learning outcomes from the **Uncorrelated Locality-Sensitive Multi-view Discriminant Analysis** are as follows:

1. ULSMDA works on the principle of in-class and inter-class vector distance i.e. minimizing one and maximising the other. Hence adding heavy losses for elements that can be grouped together, which ultimately increases the accuracy of the whole system.

2. ULSMDA improves the performance of multi-view discriminant analysis technique significantly.

3. ULSMDA achieves better classification accuracies when compared with several multi-view discriminant analysis methods.

Appendix A

References

1. https://link.springer.com/article/10.1007/s40009-019-00864-4
2. https://www.researchgate.net/publication/314251895_Multi-view_Learning_Overview_Recent_Progress_and_New_Challenges