Unit-1

Em-Theory÷

Physical quantity

i) Scalar ÷ Only magnitude ∵(Example ÷ Mass, speed, Electric current).

ii) Vector ÷ Both magnitude and direction. ∵(Example ÷ velocity, force).

Types ÷

→ Zero vector ⇒ It has zero magnitude like a dot. or we can say $\overrightarrow{AB}$ is zero vector means $\vec{A}$ & $\vec{B}$ is coincides.

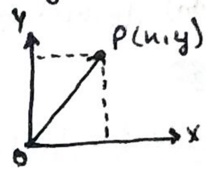
→ Unit vector ⇒ It is represented by $\vec{A}$, $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$

→ Like & Unlike vector ⇒ If they are parallel or more in the same direction then they are like other.

→ Co-initial vector ⇒ It is a vector starting from Single point.

→ Co-planer vector ⇒ no. of vector lie on the same plane.

→ Equal vector ⇒ If $\vec{A}$ and $\vec{B}$ are given then they have equal magnitude then they are called Equal vector.



→ Position vector ⇒ we can say that $\overrightarrow{OP}$ is a position vector.

$\overrightarrow{OP} = (x-0)\hat{i} + (y-0)\hat{j} = x\hat{i} + y\hat{j}$

→ Displacement vector ⇒ If in the plane XY two point A and B lie about A Displace toward B So we can say that $\overrightarrow{AB}$ is a displacement vector

$\overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$

* Product Rule ÷

i) Dot product or Scalar

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$

$= AB \cos\theta$

ii) vetor / cross product.

$\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n}$

$\hat{n}$ = unit vector.

Ques → find $\vec{A} \times \vec{B}$, if $\vec{A} = 2\hat{i} + \hat{k}$, $\vec{B} = \hat{i} - \hat{j} + \hat{k}$

Sol ⇒ $\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \Rightarrow i(A_yB_z - A_zB_y) - \hat{j}(A_xB_z - A_zB_x) + \hat{k}(A_xB_y - A_yB_x)$.

$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & -1 & 1 \end{vmatrix} \Rightarrow \hat{i}(0+1) - \hat{j}(2-1) + \hat{k}(-2-0)$

$\Rightarrow \hat{i} - \hat{j} - 2\hat{k}$ or $(1, -1, -2)$.

Ques → find $\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_y$.

$= 2 \cdot 1 + (0)(-1) + 1 \cdot 1 = 2 + 1 = 3$.

* Same equation = $\vec{A} \perp \vec{B}$ or $(\vec{A} \parallel \vec{B})$

$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| \cdot |\vec{B}|}$

Note ÷ $\boxed{\vec{A} \cdot \vec{B} = 0} = AB \cos 90^\circ$

If $\vec{A} \times \vec{B} = 0 = AB \sin 0^\circ$

* **field** → It is the region of space in which at each point on Scalar point function or vector have value.

* **Scalar field** → It is defined as region of space in which each point is associated with a Scalar point function has a value.
Example : ① Electric potential ② Temp distributed in a rod ③ pressure distributed in a liquid.

* **Vector field** → It is defined as region of space in which each point is associated with a vector point function has a value
Example : ① Electric field ② Gravitational field.

* **Ordinary differential equation**

$$\text{Slope} = \frac{df(x)}{dx}$$

$$df = \frac{df}{dx}\cdot dx$$

Total derivative

* **Gradient** : In the rectangle coordinate, the Gradient of a Scalar function $F(x,y,z)$

$$\nabla F(x,y,z) = \vec{\nabla} F = \hat{i}\frac{\partial F}{\partial x} + \hat{j}\frac{\partial F}{\partial y} + \hat{k}\frac{\partial F}{\partial z}$$

$$\vec{\nabla} F = \frac{\partial F}{\partial s}\hat{s} + \frac{\partial F}{\partial \phi}\hat{\phi}\frac{1}{s} + \frac{\partial F}{\partial z}\hat{z} \quad \text{Cylindrical Coordinate system.}$$

$$\vec{\nabla} = \frac{\partial F}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial F}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial F}{\partial \phi}\hat{\phi} \quad \text{Spherical coordinate System.}$$

* **Gradient ; directional derivative.**

$$dF = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy + \frac{\partial F}{\partial z}dz$$

dF is variation in F for a small change x, y and z.

And is nothing but the dot product of ∇F with $\vec{dl} = \hat{i}dx + \hat{j}dy + \hat{k}dz$.

i.e .... $dF = \vec{\nabla} F\cdot\vec{dl} = |\nabla F||dl|\cos\theta$.

So maximum when $\theta = 0$ i.e when spatial change is in the direction of the vector ∇F.

Ques : If $f(x,y,z) = x^2yz$ find grad at $(1,-1,1)$

$$\Rightarrow \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)f \cdot \vec{dr}$$

$$\Rightarrow \hat{i}\frac{\partial}{\partial x}(x^2yz) + \hat{j}\frac{\partial}{\partial y}(x^2yz) + \hat{k}\frac{\partial}{\partial z}(x^2yz)$$

$$\Rightarrow \nabla f = \hat{i}\,2xyz + \hat{j}\,x^2z + \hat{k}\,x^2y$$

$$\Rightarrow \nabla f = \hat{i}\,2\cdot1\cdot(-1) + \hat{j}(1^2)\cdot1 + \hat{k}(1^2)\cdot(-1).$$

$$= -2\hat{i} + \hat{j} - \hat{k}$$

* **Directional derivative**

If $\hat{n}$ is the unit vector, then $\hat{n}.\nabla f$ is called as directional derivative of function $f(x,y)$ in the direction.

Ques ÷ find the directional derivative of $f(x,y,z) = x^2yz$ in the derivate $4\hat{i}-3\hat{k}=\vec{A}$ at $(1,-1,1)$.

Sol ÷ $\hat{n} = \frac{\vec{A}}{|\vec{A}|} = \frac{1}{5}(4\hat{i}-3\hat{k})$

$\nabla f = \left(\hat{i}\frac{d}{dx} + \hat{j}\frac{d}{dy} + \hat{k}\frac{\partial}{dz}\right)(x^2yz)$

$\nabla f = \hat{i}\,2xyz + \hat{j}\,x^2z + \hat{k}\,x^2y$.

$f(x,y,z) = f(1,-1,1) = \hat{i}.2.1(-1).1 + \hat{j}(1^2)(1) + \hat{k}(1^2)-1$

$= -2\hat{i}+\hat{j}-k$

So, directional derivative

$\hat{n}.\nabla f|_{(1,-1,1)} = \frac{1}{5}(4\hat{i}-3\hat{k})(-2\hat{j}+\hat{j}-\hat{k}). = \frac{1}{5}(-8+3) = -1.$

Ques ÷ find grad$f$ if $f=r$ where $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$. $\boxed{\because \nabla f = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}}$

Sol ÷ Similarly ÷

$f = r = \sqrt{x^2+y^2+z^2}$

$\frac{\partial f}{\partial x} = \frac{\partial(\sqrt{x^2+y^2+z^2})}{\partial x} = \frac{1\,\cancel{2}x}{\cancel{2}\sqrt{x^2+y^2+z^2}} = \frac{x}{\sqrt{x^2+y^2+z^2}}$

Similarly for y and z :-

$\frac{df}{\partial y} \Rightarrow \frac{y}{\sqrt{x^2+y^2+z^2}}, \quad \frac{\partial f}{\partial z} \Rightarrow \frac{z}{\sqrt{x^2+y^2+z^2}}$

So $\nabla f = \hat{i}\frac{x}{\sqrt{x^2+y^2+z^2}} + \hat{j}\frac{y}{\sqrt{x^2+y^2+z^2}} + \hat{k}\frac{z}{\sqrt{x^2+y^2+z^2}} = \frac{x\hat{i}+y\hat{j}+z\hat{k}}{r} = \frac{\vec{r}}{r} = \hat{r}$

* **Divergence**

$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$, $\boxed{\vec{\nabla}.\vec{F} = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}}$ → formula of divergen it can be 0, +ve, -ve.

Ques ÷ $\vec{F} = x^2y\hat{i} + y^2z\hat{j} + z^2x\hat{k}$ Find Divergence.

$\vec{\nabla}.\vec{F} = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z} \Rightarrow \frac{\partial x^2y}{\partial x} + \frac{\partial}{\partial y}y^2z + \frac{\partial}{\partial z}z^2x.$

$\Rightarrow 2xy + 2yz + 2zx.$

$\vec{\nabla}.\vec{F}|_{(1,0,-1)} = 2.1.0 + 2.0.(-1) + 2.(-1).1 = -2$

Note ÷ div $\vec{F}$ = outflow − inflow.

* ## Curl of a Vector ($\vec{\nabla} \times \vec{A}$)

If $\vec{A}(x,y,z) = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ be defined and differentiable at each point (x,y,z) in a region of space then the curl of $\vec{A}$ is cross product of $\vec{\nabla}$ and $\vec{A}$.

i.e. curl $\vec{A} = \vec{\nabla}\times\vec{A} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \times (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})$

$$\vec{\nabla}\times\vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{i} - \left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right)\hat{j} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{k}$$

$$\vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$\therefore$ Curl of a vector is vector quantity.

Ques → If $\vec{F}(x,y,z) = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ find $\vec{\nabla}\times\vec{F}$ at $(1,0,-1)$.

Sol → $$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$\therefore$ Note If $\vec{\nabla}\times\vec{F} = 0$
Then $\vec{F}$ is irrotational field.

Given: $\vec{F} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$

$$\text{Div } \vec{F} = \nabla\cdot\vec{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

$$= \frac{\partial (x^2)}{\partial x} + \frac{\partial (y^2)}{\partial y} + \frac{\partial (z^2)}{\partial z} = 2x + 2y + 2z.$$

$$\text{Curl } \vec{F} = \nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & y^2 & z^2 \end{vmatrix} = \vec{0}$$

* ## Gauss divergence theorem :- ~~The Statement~~

→ The surface integral of the normal component of a vector ($\vec{A}$) taken over closed surface (S) is equal to the volumn integral of the divergence of same vector ($\vec{A}$) taken over closed volume (V) enclosed by same Surface (S).

→ It is also defined as the flux of vector field over any closed surface 'S' is equal to the volume integral of the divergence of the vector field. $\vec{A}$ over a volume 'V' enclosed by surface 'S'.

i.e $$\iint_S \vec{A}\, ds = \iiint_V (\text{div } \vec{A})\, dv.$$

Ques:- If S is ~~en~~closed surface enclosing a volume 'V', vector point function equallent to $\vec{F} = x\hat{i} + 2y\hat{j} + 3z\hat{k}$ Show that $\oiint \vec{F}\cdot ds = 6V$

Sol - Give that, $\vec{F} = x\hat{i} + 2y\hat{j} + 3z\hat{k}$

We Know that $\oiint \vec{F}\cdot ds = \iiint (\vec{\nabla}\cdot\vec{F})\, dv$

So, $\nabla \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = \frac{\partial (x)}{\partial x} + \frac{\partial (2y)}{\partial y} + \frac{\partial (3z)}{\partial z} = 1+2+3 = 6$

$\rightarrow \oiint \vec{F}\cdot ds = \iiint (\vec{\nabla}\cdot\vec{F}\, dv$

$= \iiint 6\cdot dv = 6\iiint dv = 6V.$

Ques → Show that $\oiint x^2 dy\,dz + y^2 dz\,dx + 2z(xy-x-y)\,dx\,dy$. where S is a closed surface of the Cube $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$

Sol- $\oiint \vec{F}\cdot\vec{ds} = \iiint (\vec{\nabla}\cdot\vec{F})\, dv.$

$\Rightarrow F_x\, dy\, dz + F_y\, dz\, dx + F_z\, dx\, dy.$

$\Rightarrow (\hat{i} F_x + \hat{j} F_y + \hat{k} F_z).(\hat{i}\, dy dz + \hat{j}\, dz dx + \hat{k}\, dx dy).$

So in equation, $x^2 dy\, dz + y^2 dz dx + 2z(xy-x-y)\, dx dy.$

$\Rightarrow [\hat{i}x^2 + \hat{j}y^2 + \hat{k}\, 2z(xy-x-y)].[\hat{i} dy dz + \hat{j} dz dx + \hat{k}\, dx dy]$

So $\vec{F} = x^2\hat{i} + y^2\hat{j} + 2z(xy-x-y)\hat{k}$

$\vec{\nabla}\cdot\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$

$= \frac{\partial (x^2)}{\partial x} + \frac{\partial (y^2)}{\partial y} + \frac{\partial\, 2z(xy-x-y)}{\partial z}$

$\vec{\nabla}\cdot F \Rightarrow 2x + 2y + 2xy - 2x - 2y = 2xy.$

Hence, $\int_{x=0}^{x=1}\int_{y=0}^{y=1}\int_{z=0}^{z=1} 2xy\, dx\, dy\, dz = 2\int_{x=0}^{x=1} x dy \cdot \int_{y=0}^{y=1} y dx \cdot \int_{z=0}^{z=1} dz = 2\left[\frac{x^2}{2}\right]_0^1 \cdot \left[\frac{y^2}{2}\right]_0^1$

$= \cancel{2}\cdot\frac{1}{\cancel{2}}\cdot\frac{1}{2}\cdot 1 = \frac{1}{2}$

* Stokes theorum :-

The line integral of a vector $(\vec{A})$ taken around a closed curve (c) is equal to the surface integral of the curl of the same vector $(\vec{A})$ taken over surface (S) bounded by same closed path (c).

It is also defined as the flux of the curl of a vector field of $\vec{A}$ over any Surface 's' is equal to the integral of the vector field '$\vec{A}$' on a closed path 'c'.

$$\oint_c \vec{A}\, dr = \iint_S \text{curl}\, \vec{A}\, ds \quad \text{or} \quad \iint_S (\vec{\nabla}\times\vec{F})\, \vec{ds}$$

Ques:- Evaluate by stoke's theorm $\oint_c e^x dx + 2y dy - dz$ where 'c' is curve

$x^2+y^2=4, z=2$

Sol:- $\int \vec{F}\cdot\vec{dl} = \iint (\vec{\nabla}\times\vec{F})\cdot\hat{n}\,ds$.

$e^x dx + 2y dy - dz = (\hat{i}e^x + \hat{j}2y - \hat{k})(\hat{i}dx + \hat{j}dy + \hat{k}dz)$

$= \vec{F}\cdot\vec{dl}$

$\vec{F} = \hat{i}e^x + \hat{j}2y - \hat{k}$

So, $\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_x & f_y & f_z \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^x & 2y & -1 \end{vmatrix}$

$\Rightarrow \hat{i}\left(\frac{\partial f_z}{\partial y} - \frac{\partial f_y}{\partial z}\right) - \hat{j}\left(\frac{\partial f_z}{\partial x} - \frac{\partial f_x}{\partial z}\right) + \hat{k}\left(\frac{\partial f_y}{\partial x} - \frac{\partial f_x}{\partial y}\right)$

$\Rightarrow \hat{i}\left(\frac{\partial}{\partial y}(-1) - \frac{\partial}{\partial z}2y\right) - \hat{j}\left(\frac{\partial}{\partial x}(-1) - \frac{\partial}{\partial z}e^x\right) + \hat{k}\left(\frac{\partial}{\partial x}2y - \frac{\partial}{\partial y}e^x\right)$

$\Rightarrow 0.$

Ques:- $\oint_c \sin z\,dx - \cos x\,dy + \sin y\,dz$ where 'c' is the boundary of the rectangle where $0 \leq x \leq \pi$, $0 \leq y \leq 1$, $z=3$.

Sol:- $\oint \vec{F}\cdot\vec{dl} = \iint (\vec{\nabla}\cdot\vec{F})\cdot\vec{ds} = \iint (\vec{\nabla}\times\vec{F})\cdot\hat{n}\,ds$.

$\sin z - \cos x + \sin y = (\hat{i}\sin z - \hat{j}\cos x + \hat{k}\sin y)\cdot(\hat{i}dx + \hat{j}dy + \hat{k}dz)$.

$= \vec{F}\cdot\vec{dl}$

So, $\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \sin z & -\cos x & \sin y \end{vmatrix}$ $\Rightarrow \hat{i}(\cos y - 0) - \hat{j}(0 - \cos z) + \hat{k}(\sin x - 0)$

$\Rightarrow \hat{i}\cos y + \hat{j}\cos z + \hat{k}\sin x.$

★ **Poission & laplace equation**:- By Gauss's law in electrostatic

$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$, where ρ = volum charge density, $\rho = q/V$ = charge/Volum

→ Again the Relation b/w Electric field & Electic potential.

$\vec{E} = -\text{grad}\,V = -\nabla V$

$\Rightarrow \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$, $\Rightarrow \vec{\nabla}(-\nabla\vec{V}) = \frac{\rho}{\varepsilon_0}$ $\Rightarrow -\nabla^2 V = \frac{\rho}{\varepsilon_0}$

$\Rightarrow$ $\boxed{\nabla^2 V = -\frac{\rho}{\varepsilon_0}}$ Poisson's Equation.

→ In the charge free region, $q = 0 \Rightarrow \rho = 0$

$$\boxed{\nabla^2 V = 0} \longrightarrow \text{Laplace equation.}$$

Ques - verify the laplace equation the potential if function $V(x,y,z) = x^2y^2z$.

Sol ÷ $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

$$\nabla^2 V = \frac{\partial^2(x^2y^2z)}{\partial x^2} + \frac{\partial^2(x^2y^2z)}{\partial y^2} + \frac{\partial^2(x^2y^2z)}{\partial z^2}$$

$$= 2y^2z + 2x^2z + 0$$

$$= 2y^2z + 2x^2z \neq 0 \quad \text{Ans.}$$

Ques - check the field $\vec{F} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ is solenoide / irrotational field.

Sol - $\bar{\nabla} \cdot \bar{F} = 0 \longrightarrow$ Solenoidal field.

$\bar{\nabla} \times \bar{F} = 0 \longrightarrow$ irrotational field. or field can be conservative.

$$\bar{\nabla} \cdot \bar{f} = \frac{\partial x^2}{\partial x} + \frac{\partial y^2}{\partial y} + \frac{\partial z^2}{\partial z} = 2x + 2y + 2z \neq 0$$

$$\bar{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_x & f_y & f_z \end{vmatrix} = 0$$

Ques ÷ check $\vec{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ irrotational & solenoidal.

Sol - $$\vec{\nabla} \times \vec{F} = \hat{i}\left|\frac{\partial}{\partial y}xy - \frac{\partial}{\partial z}zx\right| - \hat{j}\left|\frac{\partial}{\partial x}xy - \frac{\partial}{\partial z}yz\right| + \hat{k}\left|\frac{\partial}{\partial x}zx - \frac{\partial}{\partial y}yz\right|$$

$$= \hat{i}|x - x| - \hat{j}|y - y| + \hat{k}|z - z| \Rightarrow \hat{i}(0) + \hat{j}(0) + \hat{k}(0) = 0$$

So it is irrotational.

→ Maxwell's Equations. (Integral form).

(i) $\oint_s E \cdot \vec{ds} = \frac{q_{enc}}{\varepsilon_0}$ (Gaus's Law in Electricity)

(ii) $\oint_s \vec{B} \cdot \vec{ds} = 0$ (Gaus's Law in Magnetism)

(iii) $\oint \vec{E} \cdot \vec{dl} = -\frac{d\phi_B}{dt}$ (Faraday's Law of Induction).

(iv). $\oint_l \vec{B} \cdot \vec{dl} = \mu_0 i_{enc} + \boxed{\mu_0 \varepsilon_0 \frac{d\phi_E}{dt}}$

(Modified Ampere's Circulital law).

Maxwell's Equations (Differential form).

(i) $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \Rightarrow \vec{\nabla} \cdot \vec{D} = \rho$.

(ii) $\vec{\nabla} \cdot \vec{B} = 0 \Rightarrow \vec{\nabla} \cdot \vec{B} = 0$

(iii) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

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(iv) $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \Rightarrow$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Note :-

$$\vec{D} = \varepsilon_0 \vec{E} \text{ and } \vec{B} = \mu_0 \vec{H}$$

Guass law for electric field :- The total electric flux passing through any closed

$$\rightarrow \oiint \vec{E} \cdot \vec{ds} = \frac{q}{\varepsilon_0}$$

But, $q = \iiint \rho . dv$

So,

$$\oiint \vec{E} \cdot \vec{ds} = \frac{1}{\varepsilon_0} \iiint \rho . dv .$$

By Gauss - div. theorem.

$$\oiint \vec{E} \cdot \vec{ds} = \iiint (\vec{\nabla} \cdot \vec{E}) dv .$$

$$\rightarrow \iiint (\vec{\nabla} \cdot \vec{E}) dv = \frac{1}{\varepsilon_0} \iiint \rho . dv$$

$$\rightarrow \vec{\nabla} \cdot \vec{E} = \frac{1}{\varepsilon_0} \rho$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \Rightarrow \vec{\nabla} \cdot (\varepsilon_0 \vec{E}) = \rho$$

$$\Rightarrow \vec{\nabla} \cdot \vec{D} = \rho$$

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(2) $\oiint \vec{B} \cdot \vec{ds} = 0$. — (i)

By Gauss - div. - Theorem.

$$\oiint \vec{B} \cdot \vec{ds} = \iiint (\overline{\nabla} \cdot \overline{B}) dv.$$

So from eq (i).

$$\iiint (\overline{\nabla} \cdot \overline{B}) dv = 0.$$

$\Rightarrow$ $\overline{\nabla} \cdot \overline{B} = 0$ Ans.

Proved.

(4) $\oint \vec{B} \cdot \vec{dl} = \mu_0 \cdot I$.

But $I = \oiint \vec{J} \cdot \vec{ds}$

So, $\oint \vec{B} \cdot \vec{dl} = \mu_0 \iint \vec{J} \cdot \vec{ds}$

$\Rightarrow$ $\oint \overline{H} \cdot \overline{dl} = \iint \overline{J} \cdot \vec{ds}$ — (1)

Stoke's theorem.

$$\oint \vec{H} \cdot \vec{dl} = \iint (\overline{\nabla} \times \overline{H}) \cdot \vec{ds} \quad — (2)$$

from (1) and (2)

$$\iint (\overline{\nabla} \times \overline{H}) \cdot \vec{ds} = \iint \vec{J} \cdot \vec{ds}$$

$\Rightarrow$ $\overline{\nabla} \times \overline{H} = \vec{J}$

$\Rightarrow$ div curl $\cdot \vec{H}$ = div $\vec{J}$

$\Rightarrow$ $\boxed{0 = \overline{\nabla} \cdot \vec{J}}$

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→ Maxwell modified that total current density $= J + J_d$

So, $\text{curl } H = J + J_d$

→ $\text{Div} \cdot \text{curl } H = \text{Div}(J + J_d)$

→ $\text{Div } J + \text{Div } J_d = 0$

→ $-\dfrac{\partial \rho}{\partial t} + \text{Div } J_d = 0$

→ $-\dfrac{\partial (\text{Div } D)}{dt} + \text{Div } J_d = 0$

→ $-\text{div } \dfrac{\partial D}{\partial t} + \text{Div } J_d = 0$

→ $J_d = \dfrac{\partial D}{\partial t}$