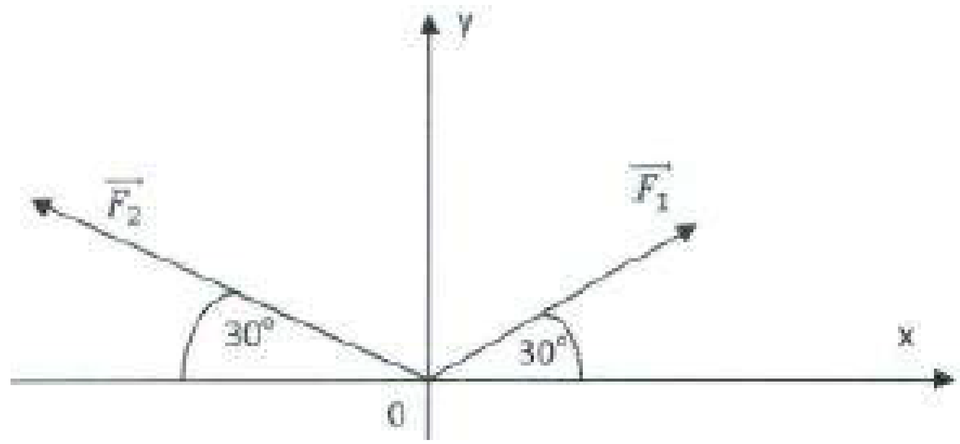
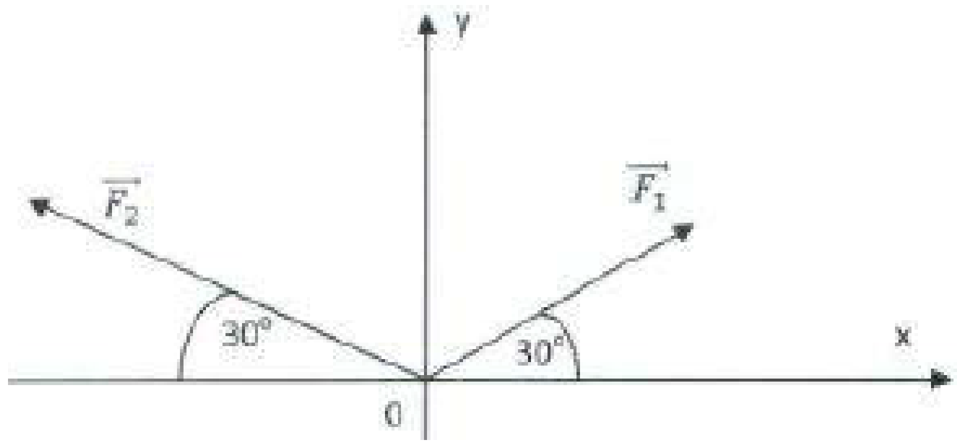
1. Two vectors, F_1 = 20 N and F_2 = 30 N, have direction as shown in the figure below. Determine the resultant of components of vectors in x-axis and y-axis.

A. 5√3 N and -25 N
B. -5√3 N and 25 N
C. 25 N and 5√3 N
D. 30 N and 25√3 N



1. Two vectors, F_1 = 20 N and F_2 = 30 N, have direction as shown in the figure below. Determine the resultant of components of vectors in x-axis and y-axis.

A. $5\sqrt{3}$ N and -25 N
B. $-5\sqrt{3}$ N and 25 N
C. 25 N and $5\sqrt{3}$ N
D. 30 N and $25\sqrt{3}$ N



2. **Calculate the cross product between a=(3,−3,1) and b=(4,9,2).**

A. $15\hat{\imath} + 2\hat{\jmath} + 30\widehat{K}$

B. $5\hat{\imath} + 20\hat{\jmath} + 10\widehat{K}$

C. $-15\hat{\imath} - 2\hat{\jmath} + 39\widehat{K}$

D. $15\hat{\imath} + 2\hat{\jmath} + 39\widehat{K}$

2. Calculate the cross product between a=(3,−3,1) and b=(4,9,2).

A. $15\hat{\imath} + 2\hat{\jmath} + 30\widehat{K}$
B. $5\hat{\imath} + 20\hat{\jmath} + 10\widehat{K}$
C. $-15\hat{\imath} - 2\hat{\jmath} + 39\widehat{K}$
D. $15\hat{\imath} + 2\hat{\jmath} + 39\widehat{K}$

3. Calculate the vector $a \times b$ when $a = 2i + j - k$ and $b = 3i - 6j + 2k$.

A. $-4i - 7j - 15k$
B. $-3i - 4j - 10k$
C. $4i + 7j + 15k$
D. $3i + 4j + 10k$

3. Calculate the vector *a* × *b* when *a* = 2*i* + *j* - *k* and *b* = 3*i* - 6*j* + 2*k* .

A.−4i − 7j − 15k

B.−3i − 4j − 10k

C.4i +7j + 15k

D.3i +4j + 10k

Here, we have $\underline{a} = 2\underline{i} + \underline{j} - \underline{k}$ and $\underline{b} = 3\underline{i} - 6\underline{j} + 2\underline{k}$

$$\begin{aligned}
\underline{a} \times \underline{b} &= \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & 1 & -1 \\ 3 & -6 & 2 \end{vmatrix} \\
&= \underline{i}\begin{vmatrix} 1 & -1 \\ -6 & 2 \end{vmatrix} - \underline{j}\begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} + \underline{k}\begin{vmatrix} 2 & 1 \\ 3 & -6 \end{vmatrix} \\
&= \underline{i}[(1)(2)-(-1)(-6)] - \underline{j}[(2)(2)-(-1)(3)] + \underline{k}[(2)(-6)-(1)(3)] \\
\underline{a} \times \underline{b} &= \underline{i}[2-6] - \underline{j}[4+3] + \underline{k}[-12-3] \\
&= -4\underline{i} - 7\underline{j} - 15\underline{k}.
\end{aligned}$$

4. Calculate the dot product of a and b when $a = 3i - 4j + k$ and $b = 2i + 5j - k$.

A. 15
B. -15
C. 10
D. 5

4. Calculate the dot product of a and b when *a* = 3*i* - 4*j* + *k* and *b* = 2*i* + 5*j* – *k*.

A. 15
B. -15
C. 10
D. 5

5. If $a = 4i+2j - k$ and $b = 2i-6i-3k$ then calculate a vector that is perpendicular to both a and b

A.–10i + 12j – 20k
B.12i -10j – 28k
C.12i + 10j – 20k
D.–12i + 10j – 28k

5. If *a* = 4*i*+2*j* -*k* and *b* = 2*i*-6*i*-3*k* then calculate a vector that is perpendicular to both *a* and *b*

$\underline{a} \times \underline{b}$ is a vector that is perpendicular to both $\underline{a}$ and $\underline{b}$.
Here, we have $\underline{a} = 4\underline{i} + 2\underline{j} - \underline{k}$ and $\underline{b} = 2\underline{i} - 6\underline{j} - 3\underline{k}$.

$$\underline{a} \times \underline{b} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 4 & 2 & -1 \\ 2 & -6 & -3 \end{vmatrix}$$

$$= \underline{i}\begin{vmatrix} 2 & -1 \\ -6 & -3 \end{vmatrix} - \underline{j}\begin{vmatrix} 4 & -1 \\ 2 & -3 \end{vmatrix} + \underline{k}\begin{vmatrix} 4 & 2 \\ 2 & -6 \end{vmatrix}$$

$$= \underline{i}[(2)(-3)-(-1)(-6)] - \underline{j}[(4)(-3)-(-1)(2)] + \underline{k}[(4)(-6)-(2)(2)]$$

$$\underline{a} \times \underline{b} = \underline{i}[-6-6] - \underline{j}[-12+2] + \underline{k}[-24-4]$$

$$= -12\underline{i} + 10\underline{j} - 28\underline{k}.$$

A.–10i + 12j – 20k

B.12i -10j – 28k

C.12i + 10j – 20k

D.–12i + 10j – 28k

6. Find the angle between two vectors $\vec{a}$ = $\boldsymbol{7\hat{\imath} + 1\hat{\jmath}}$ and $\vec{b}$ = $\boldsymbol{5\hat{\imath} + 5\hat{\jmath}}$

A.40.46 Degrees
B.46.76 Degrees
C.36.86 Degrees
D.32.24 Degrees

6. Find the angle between two vectors $\vec{a} = 7\hat{\imath} + 1\hat{\jmath}$ and $\vec{b} = 5\hat{\imath} + 5\hat{\jmath}$

A.40.46 Degrees
B.46.76 Degrees
C.36.86 Degrees
D.32.24 Degrees

$\bar{a} \cdot \bar{b} = 5 \cdot 7 + 1 \cdot 5 = 35 + 5 = 40.$

Calculate vectors magnitude:

$|\bar{a}| = \sqrt{7^2 + 1^2} = \sqrt{49 + 1} = \sqrt{50} = 5\sqrt{2}$
$|\bar{b}| = \sqrt{5^2 + 5^2} = \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}$

Calculate the angle between vectors:

$$\cos \alpha = \frac{\bar{a} \cdot \bar{b}}{|\bar{a}| \cdot |\bar{b}|} = \frac{40}{5\sqrt{2} \cdot 5\sqrt{2}} = \frac{40}{50} = \frac{4}{5} = 0.8$$

7. Find the angle between two vectors $\vec{a} = 3\hat{\imath} + 4\hat{\jmath} + 0\widehat{k}$ and $\vec{b} = 4\hat{\imath} + 4\hat{\jmath} + 2\widehat{k}$.

A.21.09 degrees
B.25.09 degrees
C.18.09 degrees
D.20.09 degrees

7. Find the angle between two vectors $\vec{a} = 3\hat{\imath} + 4\hat{\jmath} + 0\hat{k}$ and $\vec{b} = 4\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$.

A.21.09 degrees

B.25.09 degrees

C.18.09 degrees

D.20.09 degrees

$\overline{a} \cdot \overline{b} = 3 \cdot 4 + 4 \cdot 4 + 0 \cdot 2 = 12 + 16 + 0 = 28.$

Calculate vectors magnitude:

$|\overline{a}| = \sqrt{3^2 + 4^2 + 0^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

$|\overline{b}| = \sqrt{4^2 + 4^2 + 2^2} = \sqrt{16 + 16 + 4} = \sqrt{36} = 6$

Calculate the angle between vectors:

$$\cos \alpha = \frac{\overline{a} \cdot \overline{b}}{|\overline{a}| \cdot |\overline{b}|} = \frac{28}{5 \cdot 6} = \frac{14}{15}$$

8. If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $|\vec{r}| = (x^2 + y^2 + z^2)^{1/2}$, What would be the value of $\nabla(|\vec{r}|)^n$?

Hint: $(|\vec{r}|)^n$ = $(x^2 + y^2 + z^2)^{n/2}$

A.n$(|\vec{r}|)^{n+2}\vec{r}$

B.n$(|\vec{r}|)^{n-2}\vec{r}$

C.(n-1)$(|\vec{r}|)^{n-2}\vec{r}$

D.(n$-1)(|\vec{r}|)^{n+2}\vec{r}$

8. If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $|\vec{r}| = (x^2 + y^2 + z^2)^{1/2}$, What would be the value of $\nabla(|\vec{r}|)^n$?

Hint: $(|\vec{r}|)^n$ = $(x^2 + y^2 + z^2)^{n/2}$

A.n$(|\vec{r}|)^{n+2}\vec{r}$

B.n$(|\vec{r}|)^{n-2}\vec{r}$

C.(n-1)$(|\vec{r}|)^{n-2}\vec{r}$

D.(n-1)$(|\vec{r}|)^{n+2}\vec{r}$

Solution next slide

1. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $\nabla(|\vec{r}|)^n = ?$

$|\vec{r}| = (x^2+y^2+z^2)^{1/2}$, $|\vec{r}|^n = (x^2+y^2+z^2)^{n/2}$

$\therefore \nabla(|\vec{r}|)^n = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(x^2+y^2+z^2)^{n/2}$

Let us consider only the differential of x-variable first,

$\therefore \hat{i}\frac{\partial}{\partial x}(x^2+y^2+z^2)^{\frac{n}{2}} = \hat{i}\frac{n}{2}(x^2+y^2+z^2)^{\frac{n}{2}-1} \cdot 2x$

$= \hat{i}nx\,(x^2+y^2+z^2)^{\frac{n-2}{2}}$

Similarly, $\hat{j}\frac{\partial}{\partial y}(x^2+y^2+z^2)^{\frac{n}{2}} = \hat{j}ny\,(x^2+y^2+z^2)^{\frac{n-2}{2}}$

and $\hat{k}\frac{\partial}{\partial z}(x^2+y^2+z^2)^{\frac{n}{2}} = \hat{k}nz\,(x^2+y^2+z^2)^{\frac{n-2}{2}}$

$\therefore \nabla(|\vec{r}|)^n = \hat{i}\frac{\partial}{\partial x}(x^2+y^2+z^2)^{\frac{n}{2}} + \hat{j}\frac{\partial}{\partial y}(x^2+y^2+z^2)^{\frac{n}{2}} + \hat{k}\frac{\partial}{\partial z}(x^2+y^2+z^2)^{\frac{n}{2}}$

$= \hat{i}nx(x^2+y^2+z^2)^{\frac{n-2}{2}} + \hat{j}ny(x^2+y^2+z^2)^{\frac{n-2}{2}} + \hat{k}nz(x^2+y^2+z^2)^{\frac{n-2}{2}}$

$= n(x^2+y^2+z^2)^{\frac{n-2}{2}}(\hat{i}x + \hat{j}y + \hat{k}z)$

$= n\left\{(x^2+y^2+z^2)^{1/2}\right\}^{(n-2)} \underbrace{(x\hat{i} + y\hat{j} + z\hat{k})}_{\vec{r}}$

$= n\,|\vec{r}|$

$\boxed{= n\,(|\vec{r}|)^{n-2}\,\vec{r}}$

9. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $|\vec{r}| = (x^2 + y^2 + z^2)^{1/2}$, What would be the value of $\nabla(\frac{1}{|\vec{r}|})^n$?

Hint: $(|\vec{r}|)^n = (x^2 + y^2 + z^2)^{n/2}$; $(\frac{1}{|\vec{r}|})^n = (x^2 + y^2 + z^2)^{-n/2}$

A. $\frac{-n}{(|\vec{r}|)^{n+2}} \vec{r}$

B. $\frac{n}{(|\vec{r}|)^{n+2}} \vec{r}$

C. $\frac{-n}{(|\vec{r}|)^{n-2}} \vec{r}$

D. $\frac{n}{(|\vec{r}|)^{n-2}} \vec{r}$

9. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $|\vec{r}| = (x^2 + y^2 + z^2)^{1/2}$, What would be the value of $\nabla(\frac{1}{|\vec{r}|})^n$?

Hint: $(|\vec{r}|)^n = (x^2 + y^2 + z^2)^{n/2}$; $(\frac{1}{|\vec{r}|})^n = (x^2 + y^2 + z^2)^{-n/2}$

A. $\frac{-n}{(|\vec{r}|)^{n+2}}\ \vec{r}$

B. $\frac{n}{(|\vec{r}|)^{n+2}}\ \vec{r}$

C. $\frac{-n}{(|\vec{r}|)^{n-2}}\ \vec{r}$

D. $\frac{n}{(|\vec{r}|)^{n-2}}\ \vec{r}$

Solution next slide

2. $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $\nabla\left(\frac{1}{|\vec{r}|}\right)^n = ?$

$|\vec{r}| = (x^2+y^2+z^2)^{\frac{1}{2}} \Rightarrow \left(\frac{1}{|\vec{r}|}\right)^n = (x^2+y^2+z^2)^{-\frac{n}{2}}$

$\therefore \nabla\left(\frac{1}{|\vec{r}|}\right)^n = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(x^2+y^2+z^2)^{-n/2}$.

First let us consider the x-differential only —

$\therefore \hat{i}\frac{\partial}{\partial x}(x^2+y^2+z^2)^{-\frac{n}{2}} = \hat{i}\left(-\frac{n}{2}\right)(x^2+y^2+z^2)^{-\frac{n}{2}-1} \cdot 2x$

$= -nx(x^2+y^2+z^2)^{-\frac{(n+2)}{2}}\hat{i}$

Similarly, $\hat{j}\frac{\partial}{\partial y}(x^2+y^2+z^2)^{-\frac{n}{2}} = -ny(x^2+y^2+z^2)^{-\frac{(n+2)}{2}}\hat{j}$

$\hat{k}\frac{\partial}{\partial z}(x^2+y^2+z^2)^{-\frac{n}{2}} = -nz(x^2+y^2+z^2)^{-\frac{(n+2)}{2}}\hat{k}$

$\therefore \nabla\left(\frac{1}{|\vec{r}|}\right)^n = -nx(x^2+y^2+z^2)^{-\frac{(n+2)}{2}}\hat{i} - ny(x^2+y^2+z^2)^{-\frac{(n+2)}{2}}\hat{j} - nz(x^2+y^2+z^2)^{-\frac{(n+2)}{2}}\hat{k}$

$= -n(x^2+y^2+z^2)^{-\frac{(n+2)}{2}}(x\hat{i}+y\hat{j}+z\hat{k})$

$= -n\{\underbrace{(x^2+y^2+z^2)^{\frac{1}{2}}}_{|\vec{r}|}\}^{-(n+2)}\underbrace{(x\hat{i}+y\hat{j}+z\hat{k})}_{\vec{r}}$

$= -n(|\vec{r}|)^{-(n+2)}\vec{r}$

$\boxed{= -\frac{n}{(|\vec{r}|)^{n+2}}\vec{r}.}$

10. $\varphi = x^{3/2} + y^{3/2} + z^{3/2}$, Calculate $\nabla(\varphi)$

A. $\frac{3}{2}\left(x^{\frac{3}{2}}\hat{\mathbf{i}} + y^{\frac{3}{2}}\hat{\mathbf{j}} + z^{\frac{3}{2}}\hat{\mathbf{k}}\right)$

B. $\frac{1}{2}\left(x^{\frac{-3}{2}}\hat{\mathbf{i}} + y^{-\frac{3}{2}}\hat{\mathbf{j}} + z^{\frac{-3}{2}}\hat{\mathbf{k}}\right)$

C. $-\frac{1}{2}\left(x^{\frac{-3}{2}}\hat{\mathbf{i}} + y^{-\frac{3}{2}}\hat{\mathbf{j}} + z^{\frac{-3}{2}}\hat{\mathbf{k}}\right)$

D. $\frac{3}{2}\left(x^{\frac{1}{2}}\hat{\mathbf{i}} + y^{\frac{1}{2}}\hat{\mathbf{j}} + z^{\frac{1}{2}}\hat{\mathbf{k}}\right)$

10. $\varphi = \mathbf{x}^{3/2} + \mathbf{y}^{3/2} + \mathbf{z}^{3/2}$, **Calculate** $\nabla(\varphi)$

$$\vec{\nabla}\phi = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(x^{3/2} + y^{3/2} + z^{3/2})$$

$$= \frac{3}{2}(\hat{i}x^{1/2} + \hat{j}y^{1/2} + \hat{k}z^{1/2})$$

A. $\frac{3}{2}\left(\mathbf{x}^{\frac{3}{2}}\hat{\mathbf{i}} + \mathbf{y}^{\frac{3}{2}}\hat{\mathbf{j}} + \mathbf{z}^{\frac{3}{2}}\hat{\mathbf{k}}\right)$

B. $\frac{1}{2}\left(\mathbf{x}^{\frac{-3}{2}}\hat{\mathbf{i}} + \mathbf{y}^{-\frac{3}{2}}\hat{\mathbf{j}} + \mathbf{z}^{\frac{-3}{2}}\hat{\mathbf{k}}\right)$

C. $-\frac{1}{2}\left(\mathbf{x}^{\frac{-3}{2}}\hat{\mathbf{i}} + \mathbf{y}^{-\frac{3}{2}}\hat{\mathbf{j}} + \mathbf{z}^{\frac{-3}{2}}\hat{\mathbf{k}}\right)$

D. $\frac{3}{2}\left(\mathbf{x}^{\frac{1}{2}}\hat{\mathbf{i}} + \mathbf{y}^{\frac{1}{2}}\hat{\mathbf{j}} + \mathbf{z}^{\frac{1}{2}}\hat{\mathbf{k}}\right)$

11. If $\vec{A} = 3y^2z^2\hat{i} + 3x^2z^2\hat{j} + 3x^2y^2$, What would be the value of $\nabla.\vec{A}$?

A.3

B.7

C.0

D.5

11. If $\vec{A} = 3y^2z^2\hat{i} + 3x^2z^2\hat{j} + 3x^2y^2$, What would be the value of $\nabla.\vec{A}$?

$$\vec{A} = 3y^2z^2\hat{i} + 3x^2z^2\hat{j} + 3x^2y^2\hat{k}$$

$$\vec{\nabla}\cdot\vec{A} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot(3y^2z^2\hat{i} + 3x^2z^2\hat{j} + 3x^2y^2\hat{k})$$

$$= \frac{\partial}{\partial x}(3y^2z^2) + \frac{\partial}{\partial y}(3x^2z^2) + \frac{\partial}{\partial z}(3x^2y^2)$$

$$= \mathbf{0}$$

A.3

B.7

C.0 ***(If a vector has zero divergence, it is called solenoidal vector)***

D.5

12. If $\vec{\mathbf{A}} = (\mathbf{x} + 3\mathbf{y})\hat{\mathbf{i}} + (2\mathbf{y} + 3\mathbf{z})\hat{\mathbf{j}} + (\mathbf{x} + \mathbf{a}\mathbf{z})\hat{\mathbf{k}}$ and it is a solenoidal vector. What is the magnitude of $\mathbf{a}$?

A.-3

B.+3

C.+2

D.-2

12. If $\vec{\mathbf{A}} = (\mathbf{x} + 3\mathbf{y})\hat{\mathbf{i}} + (2\mathbf{y} + 3\mathbf{z})\hat{\mathbf{j}} + (\mathbf{x} + \mathbf{az})\hat{\mathbf{k}}$ and it is a solenoidal vector. What is the magnitude of a?

A.-3

B.+3

C.+2

D.-2

$$\vec{A} = (x + 3y)\hat{i} + (2y + 3z)\hat{j} + (x + az)\hat{k}$$

$$\text{div } A = \vec{\nabla} \cdot \vec{A} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z} \right) \cdot [(x + 3y)\hat{i} + (2y + 3z)\hat{j} + (x + az)\hat{k}] = 0$$

$$\frac{\partial}{\partial x}(x + 3y) + \frac{\partial}{\partial y}(2y + 3z) + \frac{\partial}{\partial z}(x + az) = 0$$

$$1 + 2 + a = 0$$

$$a = \mathbf{-3}$$

13. If $\vec{\mathbf{r}} = \mathbf{x}\hat{\mathbf{i}} + \mathbf{y}\hat{\mathbf{j}} + \mathbf{z}\hat{\mathbf{k}}$ and $|\vec{\mathbf{r}}| = (\mathbf{x}^2 + \mathbf{y}^2 + \mathbf{z}^2)^{1/2}$, What would be the value of $\nabla.\left[(|\vec{\mathbf{r}}|)^3\,\vec{\mathbf{r}}\right]$?

Hint: $(|\vec{\mathbf{r}}|)^3$ = $(\mathbf{x}^2 + \mathbf{y}^2 + \mathbf{z}^2)^{3/2}$

A.4 $(|\vec{\mathbf{r}}|)^3$
B.5 $(|\vec{\mathbf{r}}|)^3$
C.6 $(|\vec{\mathbf{r}}|)^3$
D.7 $(|\vec{\mathbf{r}}|)^3$

13. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $|\vec{r}| = (x^2 + y^2 + z^2)^{1/2}$, What would be the value of $\nabla.\left[(|\vec{r}|)^3\,\vec{r}\right]$?

Hint: $(|\vec{r}|)^3$ = $(x^2 + y^2 + z^2)^{3/2}$

A.4 $(|\vec{r}|)^3$

B.5 $(|\vec{r}|)^3$

C.6 $(|\vec{r}|)^3$

D.7 $(|\vec{r}|)^3$

$$\vec{\nabla}\cdot(r^3\vec{r}) = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \frac{\hat{k}\partial}{\partial z}\right)\cdot[(x^2+y^2+z^2)^{3/2}(\hat{i}x + \hat{j}y + \hat{k}z)]$$

$$= \frac{\partial}{\partial x}[x(x^2+y^2+z^2)^{3/2}] + \frac{\partial}{\partial y}[y(x^2+y^2+z^2)^{3/2}] + \frac{\partial}{\partial z}[z(x^2+y^2+z^2)^{3/2}]$$

Now $\frac{\partial}{\partial x}[x(x^2+y^2+z^2)^{3/2}] = (x^2+y^2+z^2)^{3/2} + x\frac{3}{2}2x[x^2+y^2+z^2]^{1/2}$

$$= r^3 + 3x^2r$$

Similarly, $\frac{\partial}{\partial y}[y(x^2+y^2+z^2)^{3/2}] = r^3 + 3y^2r$

and $\frac{\partial}{\partial z}[z(x^2+y^2+z^2)^{3/2}] = r^3 + 3z^2r$

so $\vec{\nabla}(r^3\vec{r}) = 3r^3 + 3(x^2+y^2+z^2)r$

$$= 3r^3 + 3r^2\,r = \mathbf{6r^3}$$

14. $\vec{A} = \frac{-2z^2y}{x^3}\hat{\imath} + \frac{z^2}{x^2}\hat{\jmath} + \frac{2yz}{x^2}\hat{k}$, Calculate $\nabla X \vec{A}$

A.0

B.1

C.2

D.3

E.4

14. $\overrightarrow{\mathbf{A}} = \frac{-2z^2y}{x^3}\hat{\mathbf{i}} + \frac{z^2}{x^2}\hat{\mathbf{j}} + \frac{2yz}{x^2}\hat{\mathbf{k}}$**, Calculate** $\nabla X \overrightarrow{\mathbf{A}}$

A.0

B.1

C.2

D.3

E.4

$$\text{Now } \vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{-2z^2y}{x^3} & \frac{z^2}{x^2} & \frac{2yz}{x^2} \end{vmatrix}$$

$$= \hat{i}\left[\frac{\partial}{\partial y}\left(\frac{2yz}{x^2}\right) - \frac{\partial}{\partial z}\left(\frac{z^2}{x^2}\right)\right] + \hat{j}\left[\frac{\partial}{\partial z}\left(\frac{-2z^2y}{x^3}\right) - \frac{\partial}{\partial x}\left(\frac{2yz}{x^2}\right)\right] + \hat{k}\left[\frac{\partial}{\partial x}\left(\frac{z^2}{x^2}\right) - \frac{\partial}{\partial y}\left(\frac{-2z^2y}{x^3}\right)\right]$$

$$= \hat{i}\left[\frac{2z}{x^2} - \frac{2z}{x^2}\right] + \hat{j}\left[\frac{-4yz}{x^3} + \frac{4yz}{x^3}\right] + \hat{k}\left[\frac{-2z^2}{x^3} + \frac{2z^2}{x^3}\right]$$

$$\vec{\nabla} \times \vec{A} = \mathbf{0}$$

15. If $\vec{\mathbf{A}} = \mathbf{yz\hat{i} + xz\hat{j} + xy\hat{k}}$, calculate $\boldsymbol{\nabla}.\vec{\mathbf{A}}$.

A.4
B.8
C.0
D.7

15. If $\vec{\mathbf{A}} = \mathbf{yz\hat{i} + xz\hat{j} + xy\hat{k}}$, calculate $\nabla.\vec{\mathbf{A}}$.

$$\vec{\nabla}\cdot\vec{A} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot(\hat{i}A_x + \hat{j}A_y + \hat{k}A_z)$$

A.4

B.8

C.0

D.7

$$= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$= \frac{\partial(yz)}{\partial x} + \frac{\partial(xz)}{\partial y} + \frac{\partial(xy)}{\partial z}$$

$$\therefore \qquad \vec{\nabla}\cdot\vec{A} = 0$$

16. If $\overrightarrow{\mathbf{A}} = \mathbf{yz\hat{i}} + \mathbf{xz\hat{j}} + \mathbf{xy\hat{k}}$, calculate $\mathbf{\nabla X \overrightarrow{A}}$.

A.4
B.0
C.15
D.7

16. If $\vec{A} = yz\hat{i} + xz\hat{j} + xy\hat{k}$, calculate $\nabla X \vec{A}$.

A.4

B.0

C.15

D.7

$$\text{curl } \vec{A} = \vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ yz & xz & xy \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial xy}{\partial y} - \frac{\partial xz}{\partial z}\right) + \hat{j}\left(\frac{\partial yz}{\partial z} - \frac{\partial xy}{\partial x}\right) + \hat{k}\left(\frac{\partial xz}{\partial x} - \frac{\partial yz}{\partial y}\right)$$

$$= \hat{i}(x - x) + \hat{j}(y - y) + \hat{k}(z - z)$$

$\therefore$ $\qquad \boldsymbol{\vec{\nabla} \times \vec{A} = 0}$

17. If, $\vec{A} = x^2 y\hat{\imath} + (x - y)\hat{k}$, Calculate $\nabla . \vec{A}$

A.5xy
B.-2xy
C.-5xy
D.2xy

17. If, $\vec{A} = x^2 y\hat{\imath} + (x - y)\hat{k}$, Calculate $\nabla . \vec{A}$

Given $\vec{A} = x^2 y\hat{i} + (x - y)\hat{k}$

A.5xy

B.-2xy

C.-5xy

D.2xy

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$A_x = x^2y,\ A_y = 0 \text{ and } A_z = (x - y)$$

$$\vec{\nabla}\cdot\vec{A} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot(\hat{i}A_x + \hat{j}A_y + \hat{k}A_z)$$

$$= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = \frac{\partial(x^2y)}{\partial x} + \frac{\partial(0)}{\partial y} + \frac{\partial(x-y)}{\partial z}$$

$$\vec{\nabla}\cdot\vec{A} = \mathbf{2xy}$$

18. If, $\vec{A} = x^2 y\hat{\imath} + (x - y)\hat{k}$, Calculate $\nabla X \vec{A}$

A. $\hat{\imath} + \hat{\jmath} + x^2\hat{k}$

B. $-\hat{\imath} - \hat{\jmath} - x^2\hat{k}$

C. $2\hat{\imath} + 4\hat{\jmath} + 2x^2\hat{k}$

D. $-2\hat{\imath} - 4\hat{\jmath} - 2x^2\hat{k}$

18. If, $\vec{A} = x^2 y\hat{\imath} + (x-y)\hat{k}$, Calculate $\nabla X \vec{A}$

A. $\hat{\imath} + \hat{\jmath} + x^2\hat{k}$

B. $-\hat{\imath} - \hat{\jmath} - x^2\hat{k}$

C. $2\hat{\imath} + 4\hat{\jmath} + 2x^2\hat{k}$

D. $-2\hat{\imath} - 4\hat{\jmath} - 2x^2\hat{k}$

$$\text{curl } \vec{A} = \vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 y & 0 & (x-y) \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial(x-y)}{\partial y} - \frac{\partial(0)}{\partial z}\right) + \hat{j}\left(\frac{\partial x^2 y}{\partial z} - \frac{\partial(x-y)}{\partial x}\right) + \hat{k}\left(\frac{\partial(0)}{\partial x} - \frac{\partial x^2 y}{\partial z}\right)$$

$$= \hat{i}(-1) + \hat{j}(-1) - x^2\hat{k}$$

$$\vec{\nabla} \times \vec{A} = -\hat{\boldsymbol{i}} - \hat{\boldsymbol{j}} - \boldsymbol{x}^2\hat{\boldsymbol{k}}$$

19. If 2000 flux lines enter a given volume of space and 4000 lines diverge from it, find the total charge within the volume.

A. 6.32×10^{-10} C

B. 2.71×10^{-5} C

C. 5.69×10^{-13} C

D. 1.77×10^{-8} C

19. If 2000 flux lines enter a given volume of space and 4000 lines diverge from it, find the total charge within the volume.

Given $\phi_1 = 2000\,\text{Vm}$ and $\phi_2 = 4000$ Vm.

According to Gauss's theorem,

$$\phi = \frac{q}{\varepsilon_0} \tag{i}$$

Net flux emerging out of the surface, i.e.,

$$\phi = \phi_2 - \phi_1 = 4000 - 2000 = 2000 \text{ Vm}$$

By using Eq. (i), we get

$$q = \varepsilon_0 \phi = 8.85 \times 10^{-12} \times 2000$$
$$= \mathbf{1.77 \times 10^{-8}\ C}$$

A. 6.32 X 10^{-10} C
B. 2.71 X 10^{-5} C
C. 5.69 X 10^{-13} C
D. 1.77 X 10^{-8} C

20. A point charge of 13.5 Micro Coulomb is enclosed at the center of the cube of side 6.0 cm. Find the electric flux (i) through the whole volume and (ii) through one face of the cube.

13.5 μC=13.5 X $10^{-6} C$, a= 6 cm

A. **1.525 X 10^{6} $N.m^2/C$ and 2.54 X $10^{5}$$N.m^2/C$**
B. **1.525 X 10^{8} $N.m^2/C$ and 2.54 X $10^{7}$$N.m^2/C$**
C. **1.525 X 10^{4} $N.m^2/C$ and 2.54 X $10^{3}$$N.m^2/C$**
D. **1.525 X 10^{10} $N.m^2/C$ and 2.54 X $10^{9}$$N.m^2/C$**
E. **1.525 X 10^{5} $N.m^2/C$ and 2.54 X $10^{5}$$N.m^4/C$**

20. A point charge of 13.5 Micro Coulomb is enclosed at the center of the cube of side 6.0 cm. Find the electric flux (i) through the whole volume and (ii) through one face of the cube. (13.5 μC=13.5 X 10^{-6}C, a= 6 cm)

(i) According to Gauss's theorem, the total flux through the whole volume

$$\phi = \frac{q}{\varepsilon_0}$$

$$= \frac{13.5 \times 10^{-6}}{8.85 \times 10^{-12}}$$

$$= 1.525 \times 10^{6} \text{ Nm}^2/\text{C}$$

Since a cube has 6 faces of equal area, the flux through one face of the cube would be

$$= \frac{1}{6}\frac{q}{\varepsilon_0} = \frac{1.525}{6} \times 10^{6} \text{ Nm}^2/\text{C}$$

$$= \mathbf{2.54 \times 10^{5} \text{ Nm}^2/\text{C}}$$

A. **1.525 X 10^6 N. m^2/C and 2.54 X 10^5N. m^2/C**
B. **1.525 X 10^8 N. m^2/C and 2.54 X 10^7N. m^2/C**
C. **1.525 X 10^4 N. m^2/C and 2.54 X 10^3N. m^2/C**
D. **1.525 X 10^{10} N. m^2/C and 2.54 X 10^9N. m^2/C**
E. **1.525 X 10^5 N. m^2/C and 2.54 X 10^5N. m^4/C**

21. A point charge of 11Coulomb is located at the center of a cube of side 5.0cm. Calculate the electric flux through each surface.

A. **2.07 X** $10^{13}\mathrm{N}.\frac{\mathrm{m}^2}{\mathrm{C}}$

B. **2.07 X** $10^{11}\mathrm{N}.\frac{\mathrm{m}^2}{\mathrm{C}}$

C. **2.07 X** $10^{9}\mathrm{N}.\frac{\mathrm{m}^2}{\mathrm{C}}$

D. **2.07 X** $10^{7}\mathrm{N}.\frac{\mathrm{m}^2}{\mathrm{C}}$

21. A point charge of 11Coulomb is located at the center of a cube of side 5.0cm. Calculate the electric flux through each surface.

A. $2.07 \text{ X } 10^{13} \mathrm{N}.\frac{\mathrm{m}^2}{\mathrm{C}}$

B. $2.07 \text{ X } 10^{11} \mathrm{N}.\frac{\mathrm{m}^2}{\mathrm{C}}$

C. $2.07 \text{ X } 10^{9} \mathrm{N}.\frac{\mathrm{m}^2}{\mathrm{C}}$

D. $2.07 \text{ X } 10^{7} \mathrm{N}.\frac{\mathrm{m}^2}{\mathrm{C}}$

Given $q = 11\,\mathrm{C}$ and $a = 5.0\,\mathrm{cm}$

As a cube has six faces of equal area, so the flux through each surface of the cube is

$$= \frac{1}{6}\frac{q}{\varepsilon_0} = \frac{11}{6 \times 8.85 \times 10^{-12}}$$

$$= \mathbf{2.07 \times 10^{11}\ Nm^2/C}$$

22. A hollow metallic sphere of radius 0.1m has 10^{-8} Coulomb of charge uniformly spread over it. Determine the electric field intensity (i) on the surface of the sphere (ii) at point 7cm away from the center and (iii) at point 0.5m away from the center. ***(USE Gauss's Law and draw Gaussian surface)***

A. $9 \text{ X } \frac{10^5 \text{N}}{\text{C}}, 0 \text{ and } 0.36 \text{ X } 10^5 \text{N/C}$

B. $9 \text{ X } \frac{10^3 \text{N}}{\text{C}}, 9 \text{ X } \frac{10^3 \text{N}}{\text{C}} \text{ and } 0.36 \text{ X } 10^3 \text{N/C}$

C. $9 \text{ X } \frac{10^3 \text{N}}{\text{C}}, 0 \text{ and } 0.36 \text{ X } 10^3 \text{N/C}$

D. $9 \text{ X } \frac{10^3 \text{N}}{\text{C}}, 0.36 \text{ X } \frac{10^3 \text{N}}{\text{C}} \text{ and } 0.36 \text{ X } 10^3 \text{N/C}$

22. A hollow metallic sphere of radius 0.1m has 10^{-8} Coulomb of charge uniformly spread over it. Determine the electric field intensity (i) on the surface of the sphere (ii) at point 7cm away from the center and (iii) at point 0.5m away from the center. ***(USE Gauss's Law and draw Gaussian surface)***

A. $9\,X\,\frac{10^5 N}{C}, 0 \text{ and } 0.36\,X\,10^5 N/C$

B. $9\,X\,\frac{10^3 N}{C}, 9\,X\,\frac{10^3 N}{C} \text{ and } 0.36\,X\,10^3 N/C$

A. $9\,X\,\frac{10^3 N}{C}, 0 \text{ and } 0.36\,X\,10^3 N/C$

B. $9\,X\,\frac{10^3 N}{C}, 0.36\,X\,\frac{10^3 N}{C} \text{ and } 0.36\,X\,10^3 N/C$

Given radius of the hollow sphere (R) = 0.1 m and charge on it $q = 10^{-8}$ C.

Formula used for electric field intensity

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

(i) Intensity on the surface of the sphere ($r = R$) is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$

$$= \frac{1}{4 \times 3.14 \times 8.85 \times 10^{-12}} \times \frac{10^{-8}}{(0.1)^2}$$

$$= 9 \times 10^9 \times 10^{-6}$$

$$= \mathbf{9 \times 10^3\ N/C}$$

(ii) Intensity at distance 7.0 cm away from the centre. This point lies inside the sphere so that inside the sphere electric field will be zero, i.e.,

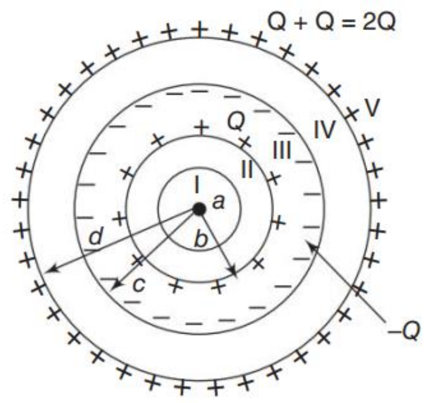
$$E = 0$$

(iii) Intensity at 0.5m away from the centre

$$E = 9 \times 10^9 \times \frac{10^{-8}}{(0.5)^2}$$

$$E = \mathbf{0.36 \times 10^3 N/C}$$

23. Consider two conducting concentric spherical shells of inner and outer radii a, b, c, and d as shown in the figure. Both the shells are given a positive charge Q. Calculate the electric fields at different distances (r) from the Centre of the inner sphere. $r<a$, $a<r<b$, $b<r<c$ and $r>d$



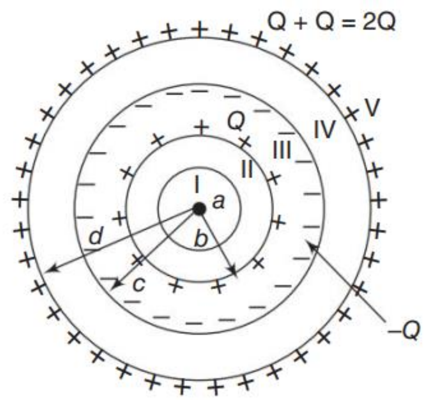
A. $\frac{Q}{4\pi\epsilon_0 r^2}\hat{r}, \frac{-Q}{4\pi\epsilon_0 r^2}\hat{r}, 0, 0$

B. $0, \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}, 0, \frac{Q}{2\pi\epsilon_0 r^2}\hat{r}$

C. $0, \frac{Q}{8\pi\epsilon_0 r^2}\hat{r}, \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}, \frac{-Q}{4\pi\epsilon_0 r^2}\hat{r}, 0$

D. $0, 0, \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}, \frac{Q}{2\pi\epsilon_0 r^2}\hat{r}$

23. **Consider two conducting concentric spherical shells of inner and outer radii a, b, c, and d as shown in the figure. Both the shells are given a positive charge Q. Calculate the electric fields at different distances (r) from the Centre of the inner sphere. $r<a$, $a<r<b$, $b<r<c$ and $r>d$**



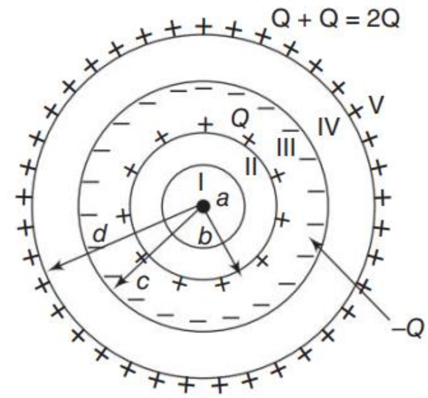
A. $\frac{Q}{4\pi\epsilon_0 r^2}\hat{r}, \frac{-Q}{4\pi\epsilon_0 r^2}\hat{r}, 0, 0$

B. $0, \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}, 0, \frac{Q}{2\pi\epsilon_0 r^2}\hat{r}$

C. $0, \frac{Q}{8\pi\epsilon_0 r^2}\hat{r}, \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}, \frac{-Q}{4\pi\epsilon_0 r^2}\hat{r}, 0$

D. $0, 0, \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}, \frac{Q}{2\pi\epsilon_0 r^2}\hat{r}$

23. Consider two conducting concentric spherical shells of inner and outer radii a, b, c, and d as shown in the figure. Both the shells are given a positive charge Q. Calculate the electric fields at different distances (r) from the Centre of the inner sphere. $r<a$, $a<r<b$, $b<r<c$ and $r>d$



Field in region I ($r < a$)

There is no charge enclosed by the Gaussian surface within this region.

Hence, Gauss's law reads $\oint \vec{E} \cdot d\vec{S} = \frac{q_{\text{enc}}}{\varepsilon_0} = 0$

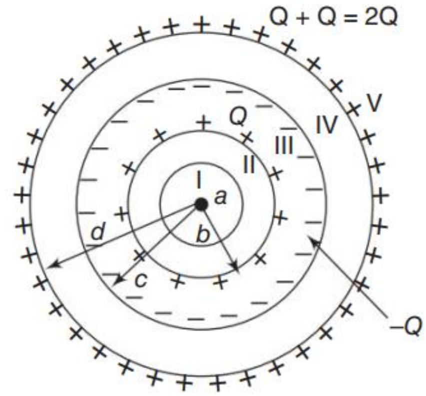
$\Rightarrow \qquad \vec{E} = 0 \quad \text{for} \quad r < a$

Field in region II ($b < r < a$)

In this region also, the charge enclosed by the Gaussian surface would be zero.

$\Rightarrow \qquad \vec{E} = 0 \quad \text{for} \quad a < r < b.$

23. Consider two conducting concentric spherical shells of inner and outer radii a, b, c, and d as shown in the figure. Both the shells are given a positive charge Q. Calculate the electric fields at different distances (r) from the Centre of the inner sphere. r<a, a<r<b, b<r<c and r>d



Field in region III ($b < r < c$)

Here enclosed charged will be as Q.

$\therefore$ Gauss's law reads $\oint \vec{E} \cdot d\vec{S} = Q/\varepsilon_0$

or $E4\pi r^2 = Q/\varepsilon_0 \Rightarrow \vec{E} = \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$

Field in region IV ($c < r < d$)

In this region, the enclosed charge $= -Q + Q = 0$

$\Rightarrow$ The field $\vec{E} = 0$.

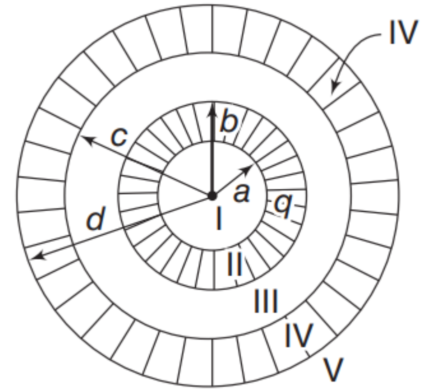
Field in region V ($r > d$)

The charge enclosed by Gaussian surface in this region will be $2Q - Q + Q = 2Q$

$\therefore$ Gauss's law reads $\oint \vec{E} \cdot d\vec{S} = \dfrac{2Q}{\varepsilon_0}$

$$\Rightarrow \qquad \vec{E} = \frac{2Q}{4\pi\varepsilon_0 r^2}\hat{r} = \frac{Q}{2\pi\varepsilon_0 r^2}\hat{r}$$

24. Consider two non-conducting concentric spherical shells of inner and outer radii a, b, c, and d as shown in the figure. Both the shells are given a positive charge Q. Calculate the electric fields at different distances (r) from the Centre of the inner sphere. r<a, a<r<b, b<r<c and r>d



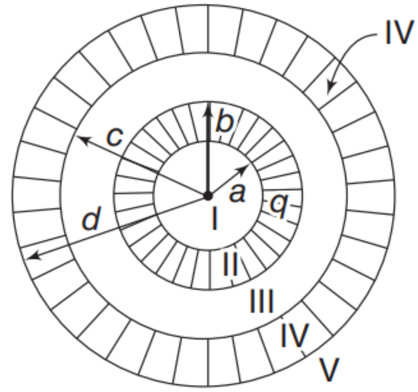
A. $\frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$, $\frac{Q(r^3-a^3)}{4\pi\epsilon_0 r^2(b^3-a^3)}\hat{r}$, 0, $\frac{Q}{2\pi\epsilon_0 r^2}\hat{r}$

B. 0, $\frac{Q(r^3-a^3)}{4\pi\epsilon_0 r^2(b^3-a^3)}\hat{r}$, $\frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$, $\frac{Q(r^3-c^3)}{4\pi\epsilon_0 r^2(d^3-c^3)}\hat{r}$, $\frac{Q}{2\pi\epsilon_0 r^2}\hat{r}$

C. 0, $\frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$, 0, $\frac{Q}{2\pi\epsilon_0 r^2}\hat{r}$

D. 0, $\frac{Q(r^3-a^3)}{4\pi\epsilon_0 r^2(b^3-a^3)}\hat{r}$, $\frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$, $\frac{Q(r^3-c^3)}{4\pi\epsilon_0 r^2(d^3-c^3)}\hat{r}$, $\frac{Q}{16\pi\epsilon_0 r^2}\hat{r}$

24. Consider two non-conducting concentric spherical shells of inner and outer radii a, b, c, and d as shown in the figure. Both the shells are given a positive charge Q. Calculate the electric fields at different distances (r) from the Centre of the inner sphere. $r<a$, $a<r<b$, $b<r<c$ and $r>d$



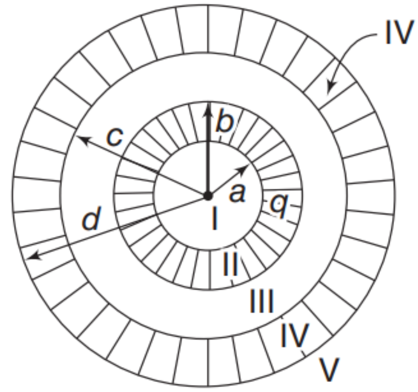
A. $\frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$, $\frac{Q(r^3-a^3)}{4\pi\epsilon_0 r^2(b^3-a^3)}\hat{r}$, 0, $\frac{Q}{2\pi\epsilon_0 r^2}\hat{r}$

B. 0, $\frac{Q(r^3-a^3)}{4\pi\epsilon_0 r^2(b^3-a^3)}\hat{r}$, $\frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$, $\frac{Q(r^3-c^3)}{4\pi\epsilon_0 r^2(d^3-c^3)}\hat{r}$, $\frac{Q}{2\pi\epsilon_0 r^2}\hat{r}$

C. 0, $\frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$, 0, $\frac{Q}{2\pi\epsilon_0 r^2}\hat{r}$

D. 0, $\frac{Q(r^3-a^3)}{4\pi\epsilon_0 r^2(b^3-a^3)}\hat{r}$, $\frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$, $\frac{Q(r^3-c^3)}{4\pi\epsilon_0 r^2(d^3-c^3)}\hat{r}$, $\frac{Q}{16\pi\epsilon_0 r^2}\hat{r}$

24. Consider two non-conducting concentric spherical shells of inner and outer radii a, b, c, and d as shown in the figure. Both the shells are given a positive charge Q. Calculate the electric fields at different distances (r) from the Centre of the inner sphere. r<a, a<r<b, b<r<c and r>d



Charge density of inner spherical shell

$$\rho_1 = \frac{Q}{\frac{4}{3}\pi(b^3 - a^3)}$$

Charge density of outer spherical shell $\rho_2 = \dfrac{Q}{\frac{4}{3}\pi(d^3 - c^3)}$

Region I ($r < a$)

Charge enclosed by Gaussian surface for $r < a$ will be zero.

Hence, Gauss's law $\oint \vec{E} \cdot d\vec{S} = 0 \Rightarrow \vec{E} = 0$.

Region II ($a < r < b$)

In this region, the charge enclosed by Gaussian surface will be $\rho_1 \cdot \frac{4}{3}\pi\,(r^3 - a^3)$

Hence, Gauss's Law reads

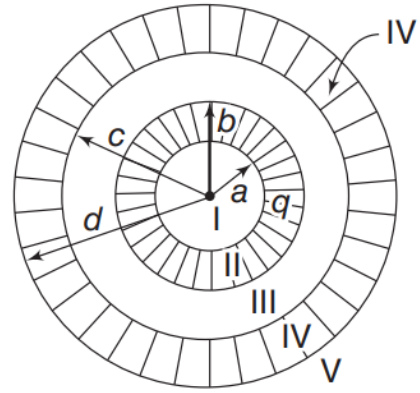
$$\oint \vec{E} \cdot d\vec{S} = \frac{Q}{\varepsilon_0 \frac{4}{3}\pi(b^3 - a^3)} \cdot \frac{4}{3}\pi(r^3 - a^3)$$

$$\Rightarrow \qquad E \cdot 4\pi r^2 = \frac{Q(r^3 - a^3)}{\varepsilon_0(b^3 - a^3)}$$

or

$$\vec{E} = \frac{Q(r^3 - a^3)}{4\pi\varepsilon_0 r^2 (b^3 - a^3)}\hat{r}$$

24. Consider two non-conducting concentric spherical shells of inner and outer radii a, b, c, and d as shown in the figure. Both the shells are given a positive charge Q. Calculate the electric fields at different distances (r) from the Centre of the inner sphere. r<a, a<r<b, b<r<c and r>d



Region III ($b < r < c$)

Charge enclosed by Gaussian surface will be Q.

$$\Rightarrow \qquad \vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

Region $(r > d)$

Charge enclosed by Gaussian surface will be

$$Q + Q = 2Q$$

Hence,
$$\oint \vec{E} \cdot d\vec{S} = \frac{2Q}{\varepsilon_0}$$

or
$$\vec{E} = \frac{2Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

$$= \frac{Q}{2\pi\varepsilon_0 r^2}\hat{r}$$