Unit-1

Problem 1: You are shopping on Flipkart and have selected two items. As you proceed to the payment page, you see different payment options: **Credit Card, Debit Card, GPay, and Cash on Delivery**.

Which **logical operator** is used in the payment statement that allows you to choose any one of these payment modes?

A) AND
B) OR
C) NOT
D) XOR

👉 **What is your answer and why?**

Problem 2: Suppose that Smartphone A has 256 MB RAM and 32 GB ROM, and the resolution of its camera is 8 MP; Smartphone
B has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4 MP; and Smartphone C has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 MP. Determine the truth value of each of
these propositions.

a) Smartphone B has the most RAM of these three smartphones.

b) Smartphone C has more ROM or a higher resolution camera than Smartphone B.

c) Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.

d) If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.

e) Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A.

Problem 3: A traffic rule states: "If the traffic light is green, you can go; if the light is red, you must stop." Express this rule with logical expression and explain with the truth table.

Problem 4: In a small town, there are two main conditions for residents to be eligible for a special government benefit:

1. Condition A: The resident must be over the age of 65.
2. Condition B: The resident has a disability.
3. Condition C: The resident must be a taxpayer.

The government decides that a resident qualifies for the benefit if all conditions A, B and C are true. Using logic, can we determine if a resident qualifies for the benefit based on the following:

- A resident is over 65 years old or has a disability, and is a taxpayer.
- A resident is not over 65, but has a disability and is a taxpayer.
- A resident is not over 65 iff the resident has no disability.
- If a resident is under 65 then the resident has no disability, and is not a taxpayer.

Problem 5: Let's consider a classroom with students and their participation in class discussions. Define the following predicates:

P(x): "Student x participates in class."

A(x): "Student x is active in class."

Now, consider the following statement:

"All active students participate in class, but some students who are not active still participate in class."

Express this situation using quantifiers and predicates, and determine if the statement is true or false, given the set of students in the class.

Problem 6:(a)

1. A city traffic rule states

"If the traffic light is red, and a vehicle crosses the intersection, the driver will be fined."
Represent the rule using logical connectives (p, q, r) where:

p: The traffic light is red.

q: The vehicle crosses the intersection.

r: The driver will be fined.

Write the negation of the statement and explain its meaning.

Problem 6: (b)

2. A library offers free membership under the following conditions:
"A person will qualify for free membership if they are a student or they are over 60 years old, and they live in the city."

Represent the conditions using logical connectives:

p: The person is a student.

q: The person is over 60 years old.

r: The person lives in the city.

Write the negation of the statement and describe its meaning.

Problem 7:

A smart traffic light system is programmed with the following logic:

A: "The pedestrian button is pressed."

B: "The traffic light is red."

C: "Vehicles are stopped."

The system ensures safety using the condition: $(A \rightarrow B) \wedge (B \rightarrow C)$.

(a) Express this condition in terms of a truth table and analyse if there are any scenarios where pedestrian safety might be compromised.
(b) Is this implication logically equivalent to $A \rightarrow C$? Justify your answer using logical proof techniques.

Problem 8:

Situation:
A bank has an automatic loan approval system that grants loans only if:

- The applicant has a **good credit score (C)**.
- The applicant has a **stable income (I)** or **a strong guarantor (G)**.

However, if the applicant has **bad financial history (B)**, the loan is automatically rejected.

Question:

1. Express the loan approval condition using propositional logic.
2. Determine whether the following applicant qualifies for a loan based on the logical statement:
   - The applicant has a good credit score.
   - The applicant has a strong guarantor but no stable income.
   - The applicant has no bad financial history.
3. Use a truth table to verify the correctness of the loan approval rule.

Problem 9:

(i)Situation*:* A school is organizing a quiz competition with three teams—Team A, Team B, and Team C. The following conditions apply:

- Team A will win if and only if Team B does not win.
- If Team C wins, then Team A does not win.
- Either Team A or Team C wins, but not both.

Question: Using logical reasoning and propositional logic, determine the possible outcomes for the winning teams.

Problem 10:

Suppose a company is conducting a survey on whether employees have completed certain tasks. The statement is, "The employee has completed task 1, and either task 2 or task 3." How can we rewrite this statement equivalently?

Problem 11:

a) It snows whenever the wind blows from the northeast.

Situation-based question: Imagine you're a meteorologist analyzing weather patterns and creating a model for snowfall based on wind direction. You're trying to determine if the statement holds true in different situations.
Question: If the wind blows from the northeast, then it snows. Can you construct a truth table to verify this statement for different wind directions? What are the conditions under which it does not snow even if the wind blows from the northeast?

If the wind blows from the northeast, then it snows. Create a truth table where "p" represents "wind blows from the northeast" and "q" represents "it snows". What happens if p is true and q is false?

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b) The apple trees will bloom if it stays warm for a week.

Situation-based question: As a horticulturist, you're studying the blooming patterns of apple trees. You're testing the hypothesis that warmth for a week causes blooming. You also want to account for extreme cases where other factors might interfere.
Question: If it stays warm for a week, then the apple trees will bloom. Can you construct a truth table to explore all possible conditions? How would the truth of this conditional statement change if it stays warm for less than a week or if the temperature fluctuates? What factors other than temperature might affect blooming?

If it stays warm for a week, then the apple trees will bloom. Consider a truth table where "p" represents "it stays warm for a week" and "q" represents "the apple trees bloom." What happens when "p" is false but "q" is true?

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c) That the Pistons win the championship implies that they beat the Lakers.

Situation-based question: You're analyzing the results of a basketball season, and you're looking at the relationship between the Pistons winning the championship and their victory over the Lakers.
Question: If the Pistons win the championship, then they beat the Lakers. How would you model this situation using a truth table? What happens if the Pistons win the championship but did not beat the Lakers? What logical reasoning can explain the relationship between the two events?

If the Pistons win the championship, then they beat the Lakers. Create a truth table to check the truth values of "p" (Pistons win championship) and "q" (Pistons beat Lakers). What if "p" is true and "q" is false?

d) It is necessary to walk eight miles to get to the top of Long’s Peak.

Situation-based question: As a hiker, you're planning a trip to Long's Peak. You’ve heard it’s necessary to walk eight miles to reach the summit, but you want to verify this claim and understand if there are any alternate paths.
Question: If you want to get to the top of Long’s Peak, then you must walk eight miles. Can you represent this as a necessary condition in logic? Create a truth table with "p" representing "you want to get to the top of Long’s Peak" and "q" representing "you walk eight miles." What happens if "q" is false?

If you want to get to the top of Long’s Peak, then you must walk eight miles. How would you structure the truth table for this statement, and what would be the logical interpretation if "q" (the need to walk eight miles) is false?

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e) To get tenure as a professor, it is sufficient to be world famous.

Situation-based question: As an academic administrator, you are reviewing the criteria for granting tenure. You’ve been told that being world-famous is sufficient to secure tenure, but you want to understand the full implications of this claim.
Question: If you are world famous, then you will get tenure. Can you create a truth table to verify the sufficiency of being world famous for getting tenure? What might happen if you are not world famous but still get tenure due to other factors?

If you are world famous, then you will get tenure. Explore a truth table with "p" representing "you are world famous" and "q" representing "you get tenure." What if "p" is false but "q" is true? What does that imply for the sufficiency condition?

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f) If you drive more than 400 miles, you will need to buy gasoline.

Situation-based question: You are preparing for a road trip and trying to plan your refueling stops. You need to understand if driving more than 400 miles necessitates a fuel stop.
Question: If you drive more than 400 miles, then you will need to buy gasoline. How can you use a truth table to model this statement? What would happen if you drive exactly 400 miles or less than 400 miles?

If you drive more than 400 miles, then you will need to buy gasoline. Create a truth table with "p" representing "you drive more than 400 miles" and "q" representing "you need to buy gasoline." What happens if "p" is false but "q" is true?

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g) Your guarantee is good only if you bought your CD player less than 90 days ago.

Situation-based question: You're reading the terms of a warranty for your CD player and need to determine if it's still valid. You understand that the guarantee depends on how long ago the CD player was purchased.
Question: If you bought your CD player less than 90 days ago, then your guarantee is good. How would you express this as a conditional statement? Create a truth table to explore the relationship between the purchase date and the guarantee status. How would you handle the case where the purchase was made after 90 days?

If you bought your CD player less than 90 days ago, then your guarantee is good. Construct a truth table with "p" representing "CD player bought less than 90 days ago" and "q" representing "guarantee is good." What if "p" is false but "q" is true?

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h) Jan will go swimming unless the water is too cold.

Situation-based question: You're organizing a pool party, and Jan is uncertain about going swimming based on the temperature of the water. You want to predict Jan's decision.
Question: If the water is not too cold, then Jan will go swimming. How would you structure a truth table to model this situation? What happens if the water is too cold but Jan still decides to swim? Does the "unless" statement change the logic, and if so, how?

If the water is not too cold, then Jan will go swimming. Construct a truth table to analyze "p" (water is not too cold) and "q" (Jan swims). How does this differ from a typical "if p, then q" statement?

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i) We will have a future, provided that people believe in science.

Situation-based question: You're discussing the future of society in a class on philosophy and science. You want to explore the role of belief in science in shaping our future.
Question: If people believe in science, then we will have a future. How would you model this conditional statement using a truth table? What happens if people do not believe in science? Is the statement still valid under all conditions?

If people believe in science, then we will have a future. Create a truth table with "p" representing "people believe in science" and "q" representing "we have a future." What happens when "p" is false?

Problem 12:

Scenario:
Consider the following propositions:

- p: "The car is parked."
- q: "The car has fuel."
- r: "The car will start."

The following logical statements are given:

- $p \implies q$: If the car is parked, then it has fuel.
- $\neg q \implies \neg r$: If the car has no fuel, then it will not start.
- $p \implies r$: If the car is parked, then it will start.

Tasks:

1. Express the following propositions using logical symbols:
   - If the car is parked, then it has fuel and will start.
   - If the car does not have fuel, then the car will not start.
2. Simplify the expression:
   - $\neg(p \implies q) \vee (q \implies r)$

Problem 13:

You have an automated security system for a building, and the conditions for the building to be secure are as follows:

Rule1: Building is secure if all doors are locked and all windows are closed

Rule2: If any door is unlocked, the building is not secure

Rule3: If any window is open, the building is not secure

Define D_i: $Door\ i\ is\ locked$, W_j: $Window\ j\ is\ closed$, and let S be the proposition that building is secure.

Then, express the following statement in propositional logic:

1. The building is secure if and only if all doors are locked and all windows are closed
2. For every door i, if the door is unlocked, the building is not secure

Problem 14:

1. A person can obtain a driver's license if they satisfy the following conditions:
   a) **Condition A:** The person must be at least 18 years old.
   b) **Condition B:** The person must pass a driving test.
   c) **Condition C:** The person must not have a medical condition that prevents them from driving.

Determine if a person is eligible for a driver's license based on the following:

- A person is at least 18 or has passed the driving test and does not have a medical condition.
- A person is not at least 18, but has passed the driving test and does not have a medical condition.
- A person is not at least 18 iff they have a medical condition preventing them from driving.
- If a person is under 18, then they have a medical condition and have not passed the driving test.

Problem 15

Problem Statement:

A radar system follows these rules for detecting aircraft:

1. **If an aircraft is detected by the radar (D), then it must be within the radar range (R).**
   - $(D \rightarrow R)$
2. **If an aircraft is not within the radar range (¬R), then it is either flying too low (L) or using stealth technology (S).**
   - $(\neg R \rightarrow (L \vee S))$
3. **A certain aircraft is not within the radar range (¬R) and is not flying too low (¬L).**

Task:

1. Represent the given statements in propositional logic.
2. Construct a **truth table** to determine whether the aircraft must be using stealth technology (S).
3. Use logical reasoning to explain the conclusion.

Unit-2

Problem 1: A company tracks the number of new users joining its platform each month. Let U_n represent the number of new users in the nth month. The number of users follows the recurrence relation:

$$U_n = 5U_{n-1} - 6U_{n-2}$$

with initial conditions $U_0 = 2$ and $U_1 = 5$.

(a) Identify the type of recurrence relation and determine if it is homogeneous or non-homogeneous.

(b) Solve the recurrence relation using the characteristic equation method.

(c) If instead, the recurrence relation was modified to

$$U_n = 5U_{n-1} - 6U_{n-2} + 2^n$$

explain how the Method of Inverse Operator could be used to solve it.

Problem 2: A bus driver pays all tolls, using only nickels and dimes, by throwing one coin at a time into the mechanical toll collector.

a) Find a recurrence relation for the number of different ways the bus driver can pay a toll of n cents (where the order in which the coins are used matters).

b) In how many different ways can the driver pay a toll of 45 cents?

Problem 3: Suppose a company buys a machine, and the value of the machine depreciates each year. The depreciation rate is fixed, meaning that the value of the machine each year is a constant percentage of its value from the previous year.

Problem 4: In a pond, the number of fish at the beginning of year n is given by F_n. The fish population doubles every year, but each year, a fixed number of fish die off. If there are initially 10 fish in the pond, and the pond loses 2 fish each year, model the population of fish in the pond after ‘n’ years using a recurrence relation.

Problem 5: A small business starts with 100 customers in the first month. Each subsequent month, the number of new customers is equal to the number of customers from the previous month, minus 10 customers who stop using the service.

Find the number of customers at the end of the 5th month.

Problem 6:

A person invests money in a bank such that every year the total amount doubles the previous year's interest and adds an initial fixed deposit. Suppose the recurrence relation is given by

$A_n = 3A_{n-1} - 2A_{n-2}$,
where $A_0 = 1000$ and $A_1 = 2000$.

Find the solution to the recurrence relation.

Calculate the amount in the account at the end of 5 years.

Problem 7:

A computer virus spreads in such a way that:

- Initially, one computer is infected.
- Each infected computer spreads the virus to two new computers every day.

(a) Write a recurrence relation to model the number of infected computers on n-th day.
(b) Find how many computers are infected after 6 days.

Problem 8:

Situation:
A start-up company grows by hiring new employees every year.

- The company starts with **5 employees** in its first year.
- Each year, the company hires **twice the number of employees hired in the previous year**.

Question:

1. Form a recurrence relation for the number of employees hired in the n^{th} year.
2. Solve the recurrence relation to find how many employees the company will have in its **fifth year**.
3. Find the **total number of employees** in the company at the end of **five years**.

Problem 9:

Situation*:* A person decides to save money by depositing a fixed amount every month into a savings account. The person saves ₹100 in the first month, ₹150 in the second month, ₹200 in the third month, and so on. The amount saved each month increases by ₹50 compared to the previous month.

Question: Find the total amount saved after n months

Problem 10:

A savings account earns interest annually, and the amount in the account each year is a sum of the previous year's amount and the interest on it. For simplicity, let's assume the interest rate is constant and the interest is compounded annually.

Problem 11:

Q1. You are working as a financial advisor, and you are tasked with modeling the growth of an investment fund that has a special dividend system. At the start of the year, a deposit of $100,000 is made into the account, and at the end of each year, two types of dividends are awarded based on the account balance:

1. The first dividend is 20% of the current balance in the account at the end of the year.
2. The second dividend is 45% of the balance in the account from the previous year.

The client is interested in understanding how their investment will grow over time and wants to know the expected amount in the account after a given number of years, assuming that no withdrawals are made from the account during this time.

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a) Find a recurrence relation for {Pn}, where Pn is the amount in the account at the end of n years if no money is ever withdrawn.

Question: Suppose you are tasked with helping the client understand the mathematical growth of their investment. How can you represent the balance in the account at the end of each year, denoted by P_n, using a recurrence relation?

- You know that the initial deposit is $100,000 at the start (i.e., P_0 = 100,000).
- At the end of each year, two dividends are added:
  - The first dividend is 20% of the amount at the end of the current year.
  - The second dividend is 45% of the amount from the previous year.

Using this information, can you derive a recurrence relation for the sequence P_n that models the account balance after n years? What is the expression for P_{n+1} in terms of P_n?

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b) How much is in the account after n years if no money has been withdrawn?

Question: Now that you have derived the recurrence relation, the client wants to know how much will be in the account after 5 years, assuming no money has been withdrawn during that time. They are interested in seeing how the account balance grows year by year due to the dividends.

- Using the recurrence relation you derived, what is the account balance after nn years, say after 5 years? How would you calculate the balance for specific values of nn?
- Can you create a table showing the account balance at the end of each year for the first 5 years? What would the account balance be at the end of year 1, year 2, and so on, based on the recurrence relation?

Problem 12:

Question 1: Modelling with Recurrence Relations

Scenario:
A small company has a policy for tracking customer growth. The number of customers C_n at the n^{th} month is tracked, and the following recurrence relation holds:

$$C_n = 1.2 \cdot C_{n-1} - 50, for\, n \geq 1, \qquad C_0 = 100$$

Where:

- C_0 is the initial number of customers at the start of month 0, and
- The company expects a 20% monthly growth in customers, with a reduction of 50 customers due to attrition each month.

Tasks:

1. Write down the recurrence relation for C_n in words.
2. Calculate the number of customers in the 3rd month, C_3.

Question 2: Homogeneous Linear Recurrence Relations with Constant Coefficients

Scenario:
A population of rabbits grows according to the following recurrence relation, where the population at each time step depends on the population of the previous two time steps:

$$P_n = 3P_{n-1} - 2P_{n-2}, \qquad \text{for } n \geq 2, \qquad P_0 = 4, \qquad P_1 = 7$$

Where:

- P_n represents the population of rabbits at time n,
- $P_0 = 4$ and $P_1 = 7$ are the initial populations.

Tasks:

1. Solve the recurrence relation for P_n by finding the characteristic equation and solving it.
2. Find P_2, the population at time $n = 2$.

Question 3: Non-Homogeneous Recurrence Relation

Scenario:
A manufacturing process produces a certain number of products each week. The number of

products produced P_n at the n^{th} week is given by the following non-homogeneous recurrence relation:

$$P_n = 4P_{n-1} + 10, \qquad \text{for } n \geq 1, \qquad P_0 = 3$$

Where:

- $P_0 = 3$ is the initial number of products produced in the first week,
- The production rate increases by 10 units each week, but is also influenced by the previous week's production.

Tasks:

1. Solve the recurrence relation using the method of inverse operators for non-homogeneous recurrence relations.
2. Find P_3, the number of products produced in the 3rd week.

Question 4: Solving Recurrence Relation Using Generating Functions

Scenario:
A certain type of bacteria grows in such a way that its population at time n, denoted by B_n, follows the recurrence relation:

$$B_n = 3B_{n-1} + 5, \qquad \text{for } n \geq 1, \quad B_0 = 2$$

Where:

- $B_0 = 2$ is the initial population of bacteria.

The recurrence is non-homogeneous because of the constant term (5).

Tasks:

1. Solve the recurrence relation using generating functions.
2. Find the explicit formula for B_n and calculate B_4, the population of bacteria at time $n = 4$.

Question 5: Modelling and Solving with Recurrence Relations

Scenario:
A software company hires employees in such a way that the number of new hires H_n in the n^{th} month follows the recurrence relation:

$$H_n = 1.5 \cdot H_{n-1} + 20, \qquad \text{for } \; n \geq 1, \qquad H_0 = 30$$

Where:

- $H_0 = 30$ is the initial number of employees hired in month 0, and

- The company hires employees based on the previous month's hiring rate, with a fixed increase of 20 new hires each month.

Tasks:

1. Express the recurrence relation in words and solve it for the general case.
2. Calculate the number of employees hired in month 3, H_3.

Problem 13

An employee joined a company in 1999 with a starting salary of Rs 50000. Every year this employee receives a raise of rs 1000 plus 5% of the salary of the previous year. Find a recurrence relation for the salary of this employee after n months.

Problem 14:

A pair of rabbits is placed on an island. A pair of rabbits does not breed until they are 2 months old. After they are 2 months old, each pair of rabbits produces another pair each month. Find an implicit form of recurrence relation for the number of pairs of rabbits on the island after n months, assuming that no rabbits ever die. Also, solve the recurrence relation and find its explicit form.

Problem 15:

A certain species of tree reproduces such that the number of trees at the beginning of year n is given by T_n. Every year, the number of trees triples, but 7 trees are cut down for lumber. If the forest initially has 30 trees, model the tree population after n years using a recurrence relation.

Problem 16

Problem Statement:

A factory uses a line of **autonomous robots** for assembling products. Each robot can either work independently or rely on instructions from the previous robot in the sequence. The number of functioning robots in the factory on day n follows this recurrence relation:

$$R(n) = R(n-1) + 2R(n-2)$$

where:

- $R(n)$ is the number of functioning robots on day n.
- $R(n-1)$ is the number of functioning robots on the previous day.
- $R(n-2)$ is the number of functioning robots two days ago.
- Initially, there were **3 robots on day 1** and **5 robots on day 2**.

Task:

1. **Calculate** the number of functioning robots on day 3, day 4, and day 5 using the recurrence relation.
2. **Find a general formula** for $R(n)$ if possible.
3. **Analyze** whether the number of functioning robots grows exponentially or linearly.

Unit-3

Problem 1: You are managing a library where **100 students** have borrowed books. Out of these:

- **60 students** borrowed a Math book.
- **50 students** borrowed a Science book.
- **20 students** borrowed both Math and Science books.

Using the **Principle of Inclusion-Exclusion**, how many students borrowed at least one book?

A) 80
B) 90
C) 100
D) 110

👉 **What is your answer and why?**

Problem 2: In how many ways can a photographer at a wedding arrange 6 people in a row from a group of 10 people, where the bride and the groom are among these 10 people, if

a) the bride must be in the picture?

b) both the bride and groom must be in the picture?

c) exactly one of the bride and the groom is in the picture?

Problem 3: You have 10 pairs of socks in a drawer, each pair consisting of one left and one right sock. You randomly pick 11 socks from the drawer. Prove that, by the **Pigeonhole Principle**, you must have at least one complete pair of socks.

Problem 4: In a survey of 100 students, 60 students play football, 50 students play basketball, and 30 students play both football and basketball. How many students play either football or basketball, or both, using the principle of inclusion-exclusion?

Problem 5: In a company, employees can be either managers or subordinates. The relation "is a direct report to" defines a partial order. Given the following relationships:

John is a manager of Sarah.
Sarah is a manager of David.
John is a manager of Emily.
Determine the partial order among the employees.

Problem 6:

A company has 15 employees and 5 offices. The employees are randomly assigned to these offices.

Prove that at least one office has at least 3 employees.

If each office can accommodate a maximum of 4 employees, show that at least one office exceeds its capacity.

Problem 7:

At a university, students must follow prerequisite rules when registering for courses:

- Course A is a prerequisite for Course B
- Course B is a prerequisite for Course C
- Course A is also a prerequisite for Course D, but Course D is independent of Course C.

(a) Represent this prerequisite structure as a directed graph.
(b) Does this relation form a partial order?
(c) Identify any maximal and minimal elements in the order.

Problem 8:

Situation:
A university is conducting an online programming contest.

- 100 students registered for the contest.
- 60 students have experience in **Python**.
- 45 students have experience in **C++**.
- 25 students have experience in **both Python and C++**.

Question:

1. How many students have experience **only in Python**?
2. How many students have experience **only in C++**?
3. Using the **principle of inclusion-exclusion**, find how many students have experience in **at least one** of the two languages.
4. How many students have **no experience** in either Python or C++?

Problem 9:

Situation*:* In a library, books are classified by subject. The subjects are labelled S1, S2, S3, and S4 with the following classification hierarchy:

- Subject S1 covers the basics of subjects S2 and S3.
- Subject S2 covers the basics of subject S4.
- Subject S3 covers the basics of subject S4.

Problem 10:

A company has 10 employees, and each employee is assigned to one of 5 departments. Prove that at least one department must have at least 2 employees.

Problem 11:

Question 1: Principle of Inclusion-Exclusion

Scenario:
In a city, there are 200 students. Among these students:

- 150 students like basketball,
- 120 students like football,
- 100 students like both basketball and football.

Tasks:

1. How many students like either basketball or football? Use the **Principle of Inclusion-Exclusion** to calculate this.
2. How many students do not like either basketball or football?

Question 2: Pigeonhole Principle

Scenario:
You are organizing a tournament with 50 teams. Each team is assigned a color from a set of 8 colors.

According to the **Pigeonhole Principle**, determine how many teams must have the same color if the 50 teams are assigned colors randomly.

Tasks:

1. Using the **basic Pigeonhole Principle**, determine the minimum number of teams that must share the same color.
2. If you had 12 colors to assign to the 50 teams, what is the minimum number of teams that must share the same color? Use the **generalized Pigeonhole Principle**.

Question 3: Equivalence Relations

Scenario:
Consider a set A={1,2,3,4,5}, and a relation R on A where $a\,R\,b$ if and only if the sum $a + b$ is even.

Tasks:

1. Prove that R is an **equivalence relation** on A by checking if it satisfies the reflexive, symmetric, and transitive properties.
2. List the equivalence classes under the relation R.

Question 4: Partial and Total Ordering Relations

Scenario:
Consider the set $B = \{a, b, c, d\}$, and the relation $\leq$ defined as the usual less-than-or-equal relation on numbers, with the additional condition that:

- $a \leq b, b \leq c, \text{and } c \leq d,$
- The relation is **partial** (i.e., not every pair of elements is comparable).

Tasks:

1. Verify if the relation $\leq$ is a **partial order**. Justify your answer based on reflexivity, antisymmetry, and transitivity.
2. If the set is expanded to $B = \{a, b, c, d, e\}$, and the relation is total (i.e., every pair of elements is comparable), would the relation still be a **partial order** or a **total order**? Justify your answer.

Question 5: Lattice and Hasse Diagram

Scenario:
Consider the set $C = \{1,2,3,4,6,12\}$ with the divisibility relation |, i.e., $a \leq b$ if a divides b. We want to represent this set as a **lattice** and draw its **Hasse diagram**.

Tasks:

1. Prove that $(C, |)$ forms a **lattice** by showing that every pair of elements has a greatest lower bound (GLB) and least upper bound (LUB).
2. Draw the **Hasse diagram** for the set $C = \{1,2,3,4,6,12\}$ under the divisibility relation |.

Problem 12:

If 15 students are assigned to 4 discussion group , what is the minimum number of students in one group.

Problem 13:

A library has a collection of 10 different books, and they are to be assigned to 5 distinct shelves. Each shelf can hold any number of books, but no shelf can hold more than 3 books. You need to figure out how many different ways the books can be distributed across the shelves.

Problem 14:

In a language school, 80 students are learning French, 65 students are learning Spanish, and 40 students are learning both French and Spanish. How many students are learning at least one of these two languages using the principle of inclusion-exclusion?

Problem 15:

A railway ticketing system categorizes tickets based on the following priority order:

1. **VIP Ticket (V)** – Highest priority
2. **First Class Ticket (F)** – Higher than economy but lower than VIP
3. **Economy Class Ticket (E)** – Lower than first class but higher than general
4. **General Ticket (G)** – Lowest priority

The relation R is defined as "**is preferred over**" (e.g., VIP is preferred over First Class, First Class is preferred over Economy, etc.).

Tasks:

1. **Draw the Hasse Diagram** representing this preference order.
2. **Find the Maximal, Minimal, Greatest, and Least elements** in the Hasse Diagram.