Computer Graphics

Assignment 2: Planet in Space

2019. 5. 6. (Mon)

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1. Introduction

The goal of Assignment 2 is to draw a colored 3D sphere on the screen.

2. Implementation

To implement the colored 3D sphere, vertices and triangles should be defined.

① Vertex

Each vertex has three attributes: **position coordinate**, **normal vector**, and **texture coordinate**.

- Position coordinate

The definition of the sphere is the set of points that has same distance (r) from the center (O). So, using spherical coordinate system is easier to represent the sphere than using cartesian coordinate system. If the center of the sphere is the origin $O(0,0,0)$, it can be represented by the set of points $P(r,\theta,\varphi)$ in spherical coordinate system which can be converted to $\boldsymbol{P(r\sin\theta\cos\varphi, r\sin\theta\sin\varphi, r\cos\theta)}$ in cartesian coordinate system (radius $r \geq 0$, latitude angle $\theta \in [0,\pi]$, and longitude angle $\varphi \in [0,2\pi]$).

- Normal vector

The vector $\overrightarrow{OP}$ can be the normal vector of the vertex at position P, because a plane that contacts with the sphere at only one position P exists. For convenience, let's normalize the vector $\overrightarrow{OP} = (r\sin\theta\cos\varphi, r\sin\theta\sin\varphi, r\cos\theta)$ and denote it as a vector $\vec{n}$. Since the length of the vector $\overrightarrow{OP}$ is $\|\overrightarrow{OP}\| = \sqrt{(r\sin\theta\cos\varphi)^2 + (r\sin\theta\sin\varphi)^2 + (r\cos\theta)^2} = r$, the unit vector of normal vector is $\boldsymbol{\vec{n}} = \frac{1}{\|\overrightarrow{OP}\|}\overrightarrow{OP} = (\boldsymbol{\sin\theta\cos\varphi}, \boldsymbol{\sin\theta\sin\varphi}, \boldsymbol{\cos\theta})$.

- Texture coordinate

On the surface of the sphere, latitude and longitude can be used as 2D coordinates. It means, a coordinate (φ,θ) can be used as a texture coordinate. However, to simplify this, each coordinate value can be normalized between 0 and 1. So, the texture coordinate is $\boldsymbol{T(x,y)} = (\boldsymbol{\varphi}/\boldsymbol{2\pi}, \boldsymbol{1-\theta/\pi})$.

② Triangle

Computers can't express all points of a sphere. Therefore, the surface of the sphere must be approximated through several triangles. One way to do this is to select the vertices on the surface in a lattice form and make triangles with the three adjacent vertices. Since the shape of the grid is square, except near the pole, one grid must be filled with two triangles: **an upward triangle** and **a downward triangle**.

Let $0 \leq \varphi_k < \varphi_{k+1} \leq 2\pi$ and $0 \leq \theta_k < \theta_{k+1} \leq \pi$. And denote the vertex at position $P(r,\theta_k,\varphi_k)$ as V_{θ_k,φ_k}. Then, one grid adjacent can be represented with four vertices V_{θ_k,φ_k}, $V_{\theta_k,\varphi_{k+1}}$, $V_{\theta_{k+1},\varphi_k}$, and $V_{\theta_{k+1},\varphi_{k+1}}$. The upward triangle in a grid can be represented with three vertices V_{θ_k,φ_k}, $V_{\theta_{k+1},\varphi_k}$, and $V_{\theta_{k+1},\varphi_{k+1}}$. Also, the downward triangle can be represented with three vertices V_{θ_k,φ_k}, $V_{\theta_k,\varphi_{k+1}}$, and $V_{\theta_{k+1},\varphi_{k+1}}$.

3. Additional features

① When 'r' key is pressed, the rotation of the sphere is toggled.

4. Discussion

Attributes are interpolated between vertices. Therefore, at the boundary of longitude, the color of texture is interpolated when triangles use a same vertex V as $V_{\theta_k,0}$ and $V_{\theta_k,2\pi}$. Also, at the both poles, it is interpolated when triangles use a same vertex V_1 as $V_{0,\varphi}$ ($\varphi \in [0, 2\pi]$), and a same vertex V_2 as $V_{\pi,\varphi}$ ($\varphi \in [0, 2\pi]$). So, not to interpolate the color and make clear boundaries, it should have duplicate vertices at the points described above. And it causes some waste of performance.