EAS 502

Introduction to Probability Theory for Data Science

Project 2 (Analytical solution)

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Strategy 1:

At each unit of time, the robot will move 1 unit toward the object regardless of the object movement.

Solution:

Given at each unit of time the robot will move 1 unit toward the object.

Let object moving left or right be q and the object staying in position be 1-2q

We know that, E[T] = $\sum P(D)E[T|D]$

Let,

A_1 : object moving towards right

A_2 : object moving towards left

A_3 : object stops moving

For D = 1 i.e., distance between object and robot being 1 unit

E[T|D=1] = $\sum_{i=1}^{3} P(A_i)E[T|D = 1 \cap A_i]$

E[T|D=1] =$P\big((A_1)\big)E[T|D = 1 \cap A_1]$+$P\big((A_2)\big)E[T|D = 1 \cap A_2]$+$P\big((A_3)\big)E[T|D = 1 \cap A_3]$

E[T|D=1] = q * $E[T|D = 1 \cap A_1]$ + q * 1 + (1-2q) * 1

E[T|D=1] = $\frac{1}{1-q}$

For D = 2 i.e., distance between object and robot being 2 units

E[T|D=2] = = $\sum_{i=1}^{3} P(A_i)E[T|D = 2 \cap A_i]$

E[T|D=2] =$P\big((A_1)\big)E[T|D = 2 \cap A_1]$+$P\big((A_2)\big)E[T|D = 2 \cap A_2]$+$P\big((A_3)\big)E[T|D = 2 \cap A_3]$

E[T|D=2] = q*(1+ E[T|D=2])+q* E[T|D=2]+(1-2q)* E[T|D=1]

E[T|D=2] = $\frac{1-2q}{1-q}$ * E[T|D=1] + $\frac{1}{1-q}$

That implies for a distance of k units (where k > 2) between robot and object,

E[T|D=k]=$P\big((A_1)\big)E[T|D = k \cap A_1]$+$P\big((A_2)\big)E[T|D = k \cap A_2]$+$P\big((A_3)\big)E[T|D = k \cap A_3]$

E[T|D=k] = q*(1+ E[T|D=k])+q*(1+ E[T|D=k-2])+(1-2q)* (1+E[T|D=k-1]) [Recursive Formula]

Strategy 2:

At each unit of time, the robot will move 1 unit toward the object if the object moves to either left or right, and the robot will stop if the object stops.

Solution:

Given at each unit of time, the robot will move 1 unit toward the object if the object moves to either left or right, and the robot will stop if the object stops.

Let object moving left or right be q and the object staying in position be 1-2q.

We know E[T] = $\sum P(D)E[T|D]$

Let,

A_1 : object moving towards right and robot moving towards right

A_2 : object moving towards left and robot moving towards right

A_3 : object stops moving and robot stops moving

For D = 1 i.e., distance between object and robot being 1 unit

E[T|D=1] = $\sum_{i=1}^{3} P(A_i)E[T|D = 1 \cap A_i]$

E[T|D=1] =$P\big((A_1)\big)E[T|D = 1 \cap A_1]$+$P\big((A_2)\big)E[T|D = 1 \cap A_2]$+$P\big((A_3)\big)E[T|D = 1 \cap A_3]$

E[T|D=1] = q + q*(1 + E[T|D=1]) + (1-2q)*(1+ E[T|D=1])

E[T|D=1] = $\frac{1}{q}$

For D = 2 i.e., distance between object and robot being 2 units

E[T|D=2] = $\sum_{i=1}^{3} P(A_i)E[T|D = 2 \cap A_i]$

E[T|D=2] =$P\big((A_1)\big)E[T|D = 2 \cap A_1]$+$P\big((A_2)\big)E[T|D = 2 \cap A_2]$+$P\big((A_3)\big)E[T|D = 2 \cap A_3]$

E[T|D=2] = q*(1+ E[T|D=2]) + q + (1-2q)* (1+E[T|D=2])

E[T|D=2] = $\frac{1}{q}$

For D = 3 i.e., distance between object and robot being 3 units

E[T|D=3] = $\sum_{i=1}^{3} P(A_i)E[T|D = 3 \cap A_i]$

E[T|D=3]=$P\big((A_1)\big)E[T|D = 3 \cap A_1]$+$P\big((A_2)\big)E[T|D = 3 \cap A_2]$+$P\big((A_3)\big)E[T|D = 3 \cap A_3]$

E[T|D=3] = q*(1+ E[T|D=3]) + q*(1+ E[T|D=1]) + (1-2q)*(1+ E[T|D=3])

E[T|D=3] = $\frac{1}{q}$ + E[T|D=1]

For D = 4 i.e., distance between object and robot being 4 units

E[T|D=4] = $\sum_{i=1}^{3} P(A_i)E[T|D = 4 \cap A_i]$

E[T|D=4]=$P\big((A_1)\big)E[T|D = 4 \cap A_1]$+$P\big((A_2)\big)E[T|D = 4 \cap A_2]$+$P\big((A_3)\big)E[T|D = 4 \cap A_3]$

$E[T|D=4] = q*(1+ E[T|D=4]) + q*(1+ E[T|D=2]) + (1-2q)*(1+ E[T|D=4])$

$E[T|D=4] = \frac{1}{q} + E[T|D=2]$

That implies for a distance of k units (where k > 2) between robot and object,

$E[T|D=k] = \frac{1}{q} + E[T|D=k-2]$

Table of results:

Strategy	POM = 0.25	POM = 0.4
1	12.2501(1)	12.4021(2)
2	14.2505(3)	12.9048(4)

(1) output of project2_srigunak_dyamarth(0.25,1)

(2) output of project2_srigunak_dyamarth(0.4,1)

(3) output of project2_srigunak_dyamarth(0.25,2)

(4) output of project2_srigunak_dyamarth(0.4,2)