



I. Aim:

Determining error correction capability of given (n,k) code using Hamming bound.

II. Software Required:

Matlab;

III. Theory:

Hamming Code: The Hamming code is a single error correction linear block code with $(n, k) = (2^m - 1, 2^m - 1 - m)$
 $2^r m + r + 1$ where, $r = \text{redundant bit}$, $m = \text{data bit}$

where $m = n - k$ is the number of check bits in the codeword.

The simplest non-trivial code is for $m = 3$, which is the (7, 4) Hamming code.

The generator matrix of the Hamming code has dimension $k \times n$ and is of the form

$$G = [I_k \times k \ S]$$

For this Hamming code, S has one row for each possible m -bit string with weight at least 2.

There are exactly $k = 2^m - m - 1$ such strings. Any Hamming code is a 1-error correcting code as the two conditions above are satisfied.

$$k = 2^q - q - 1 \quad q = (n - k)$$
$$2^q - 1$$

Hamming code is an error correction system that can detect and correct errors when data is stored or transmitted. It requires adding additional parity bits with the data. It is commonly used in error correction code (ECC). Hamming code uses a block parity mechanism. The data is divided into blocks, and parity is added to the block. Hamming code can correct single-bit errors and detect the presence of two-bit errors in a data block.

Binary single error correcting perfect codes are called Hamming codes.

Consider (n, k) linear block code that has following parameters

- Block length $n = 2^q - 1$
- Number of message bits $k = 2^q - q - 1$



- Number of parity bits $q = (n-k)$
- Where $q \geq 3$.

This code is called Hamming code.

IV. Matlab Code

By Linear Code Method:

```
clc;
%Taking inputs of codeword length,messgebits,generator and Message matrix
n=input('Enter codeword length\n');
k=input('Enter No of msg bits\n');
G=input('Enter Generator matrix\n');
q=n-k;
if k==2^q-q-1 && n==2^q-1
    for i = 1 : 2^k % Iterate through Vector with Specified Increment
        for j = k : -1 : 1
            if rem(i - 1, 2 ^ (-j + k + 1)) >= 2 ^ (-j + k)
                M(i, j) = 1;
            else
                M(i, j) = 0;
            end
        end
    end
    disp('Message matrix');
    disp(M); %Generating Parity Matrix using generator matrix
    P=G(:,k+1:n);
    disp('Parity matrix');
    disp(P);
    c=mod(M*P, 2);%c=mp
    disp('Code words vector');%displaying Codewords
    disp(c);
    weight = zeros(2^k,1);
    for i=1:2^k
        for j=1:k
            if M(i,j) == 1
                weight(i) = weight(i) + 1;
            end
        end
    end
    for i=1:2^k
        for j=1:n-k
            if c(i,j) == 1
                weight(i) = weight(i) + 1;
            end
        end
    end
    sort(weight);%Finding minimum no of ones
    dmin=weight(2);
    disp('Minimum weight(dmin):');
    disp(dmin);%error Detection,correction
    ed=dmin-1;ec=ed/2;
    disp('Error Detection:');disp(ed);
    disp('Error Correction:');disp(ec);
else
    disp('It is not a Hamming code');
end
```



By Normal Method:

```
clc;
n=input('enter the code word length : n :');
k=input('enter the message bits: k :');
m=n-k ;
z=2^m ;
i=0;
for j=0:1:n
    sum=0;
    for i=0:1:j
        c=factorial(n)/(factorial(n-1)*factorial(i));
        sum=sum+c;
    end
    if sum >= zj=j-1;
        sum=sum-c;
        break;
    end
end
op = sprintf('(%d,%d) can correct all combinations of %d errors and it can also correct✓
%d combinations of %d errors', n,k,j,z-sum,j+1);
disp(op);
```

V. Code Explanation:



VI. Matlab Outputs:

By Linear Code Method:

We Use (7,4)Linear Block Code as Hamming Code

```
Enter codeword length
7
Enter No of msg bits
4
Enter Generator matrix
[1 0 0 0 1 1 1;0 1 0 0 1 1 0;0 0 1 0 1 0 1;0 0 0 1 0 1 1];
Message matrix
  0     0     0     0
  0     0     0     1
  0     0     1     0
  0     0     1     1
  0     1     0     0
  0     1     0     1
  0     1     1     0
  0     1     1     1
  1     0     0     0
  1     0     0     1
  1     0     1     0
  1     0     1     1
  1     1     0     0
  1     1     0     1
  1     1     1     0
  1     1     1     1

Parity matrix
  1     1     1
  1     1     0
  1     0     1
  0     1     1

Code words vector
  0     0     0
  0     1     1
  1     0     1
  1     1     0
  1     1     0
  1     0     1
  0     1     1
  0     0     0
  1     1     1
  1     0     0
  0     1     0
  0     0     1
  0     0     1
  0     1     0
  1     0     0
  1     1     1

Minimum weight (dmin):
3
```



MATLAB Command Window

```
Error Detection:
2

Error Correction:
1

>> HammingCode
Enter codeword length
6
Enter No of msg bits
3
Enter Generator matrix
[1 0 0 0 1 1;0 1 0 1 0 1;0 0 1 1 1 0]
It is not a Hamming code
>>
```

By Normal Method:

We Use Different Hamming Codes With Minimum length of 3.

MATLAB Command Window

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```
enter the code word length : n :3
enter the message bits: k :8
(3,8) can correct all combinations of -1 errors and it can also correct 3.125000e-02✓
combinations of 0 errors
>> Hammingcodeerrorcorrection
enter the code word length : n :4
enter the message bits: k :77
(4,77) can correct all combinations of -1 errors and it can also correct 1.058791e-22✓
combinations of 0 errors
>> Hammingcodeerrorcorrection
enter the code word length : n :5
enter the message bits: k :4
(5,4) can correct all combinations of -1 errors and it can also correct 2 combinations✓
of 0 errors
>> Hammingcodeerrorcorrection
enter the code word length : n :8
enter the message bits: k :2
(8,2) can correct all combinations of 8 errors and it can also correct 4.225377e+01✓
combinations of 9 errors
>>
```

VII. Conclusion:

We Successfully Determined the error correction capability of given (n,k) code using Hamming bound on Matlab