

## Chapter 4 – Vectors:

## ● Vector Properties & Representations

$$\mathbf{A} = A\hat{\mathbf{a}} \quad \text{○} \nearrow \mathbf{v}_1 \quad \mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$$

$$\mathbf{D} = \mathbf{A} - \mathbf{B}$$

$$\begin{aligned} &= (\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z) - (\hat{\mathbf{x}}B_x + \hat{\mathbf{y}}B_y + \hat{\mathbf{z}}B_z) \\ &= \hat{\mathbf{x}}(A_x - B_x) + \hat{\mathbf{y}}(A_y - B_y) + \hat{\mathbf{z}}(A_z - B_z) \end{aligned}$$

$$\mathbf{C} = \mathbf{A} + \mathbf{B}$$

$$\begin{aligned} &= (\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z) + (\hat{\mathbf{x}}B_x + \hat{\mathbf{y}}B_y + \hat{\mathbf{z}}B_z) \\ &= \hat{\mathbf{x}}(A_x + B_x) + \hat{\mathbf{y}}(A_y + B_y) + \hat{\mathbf{z}}(A_z + B_z) \end{aligned}$$

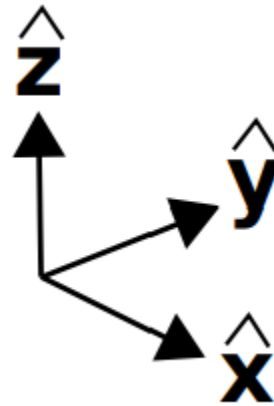
**Vector representation  $\mathbf{A}$  =**  $\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$   $\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$   $\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$

- **Magnitude**

<b>Magnitude of A</b>	$ A  = \sqrt{A_x^2 + A_y^2 + A_z^2}$	$\sqrt{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt{A_R^2 + A_\theta^2 + A_\phi^2}$
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- **Unit Vector**

$$\begin{aligned}\hat{\mathbf{a}} &= \frac{\mathbf{A}}{|\mathbf{A}|} = \frac{\mathbf{A}}{\sqrt{A_x^2 + A_y^2 + A_z^2}} \\ &= \hat{\mathbf{x}}A_x (A_x^2 + A_y^2 + A_z^2)^{-1/2} \\ &\quad + \hat{\mathbf{y}}A_y (A_x^2 + A_y^2 + A_z^2)^{-1/2} \\ &\quad + \hat{\mathbf{z}}A_z (A_x^2 + A_y^2 + A_z^2)^{-1/2}\end{aligned}$$

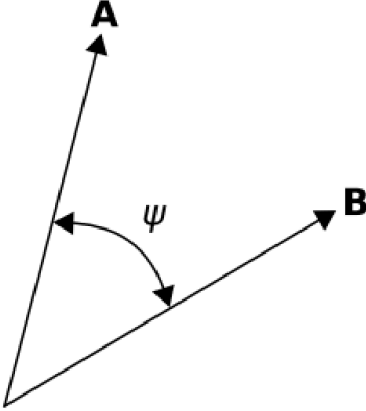
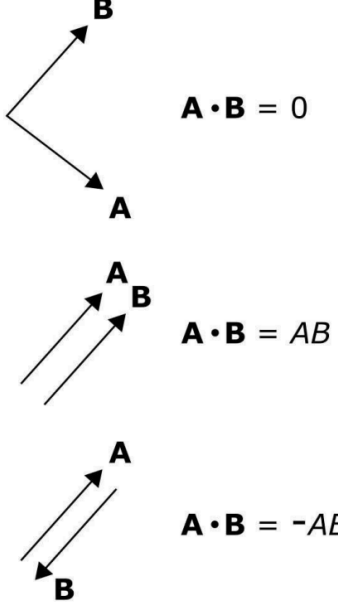


- **Dot Product & Associated Properties**

$$\mathbf{A} \cdot \mathbf{B} = AB \cos \psi$$

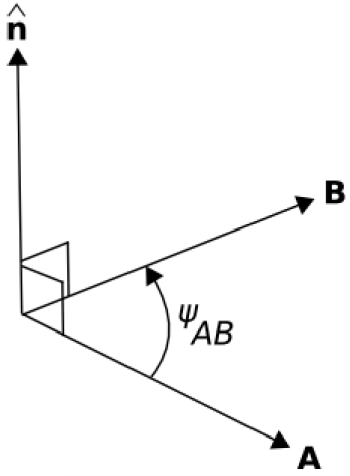
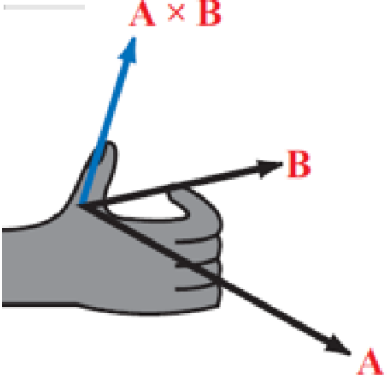
$$\hat{\mathbf{x}} \cdot \hat{\mathbf{x}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{z}} = 1$$

$$\hat{x}.\hat{y} = \hat{x}.\hat{z} = \hat{y}.\hat{z} = 0$$

	$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$ $\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C}$	
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**Dot product**  $\mathbf{A} \cdot \mathbf{B} = \begin{vmatrix} A_x B_x + A_y B_y + A_z B_z & A_r B_r + A_\phi B_\phi + A_z B_z & A_R B_R + A_\theta B_\theta + A_\phi B_\phi \end{vmatrix}$

• **Cross Product & Associated Properties**

$\mathbf{A} \times \mathbf{B} = \hat{\mathbf{n}} AB \sin \psi_{AB}$	$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A} \quad (\text{anticommutative})$ $\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{C} \quad (\text{distributive})$ $\mathbf{A} \times \mathbf{A} = 0$	
	$\mathbf{A} \times \mathbf{B} = \hat{\mathbf{x}}(A_y B_z - A_z B_y) + \hat{\mathbf{y}}(A_z B_x - A_x B_z) + \hat{\mathbf{z}}(A_x B_y - A_y B_x)$ $\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	

**Cross product**  $\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \quad \begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix} \quad \begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$

- Cartesian Coordinate System
  - Length, Surface, and Volume Integration & Differential Forms
- Cylindrical Coordinate System

- Conversion from Cartesian to Cylindrical
- Length, Surface, and Volume Integration & Differential Forms
- Spherical Coordinate System
  - Conversion from Cartesian to Spherical
  - Length, Surface, and Volume Integration & Differential Forms

<b>Differential length</b> $d\mathbf{l} =$	$\hat{\mathbf{x}} dx + \hat{\mathbf{y}} dy + \hat{\mathbf{z}} dz$	$\hat{\mathbf{r}} dr + \hat{\boldsymbol{\phi}} r d\phi + \hat{\mathbf{z}} dz$	$\hat{\mathbf{R}} dR + \hat{\boldsymbol{\theta}} R d\theta + \hat{\boldsymbol{\phi}} R \sin \theta d\phi$
<b>Differential surface areas</b>	$ds_x = \hat{\mathbf{x}} dy dz$ $ds_y = \hat{\mathbf{y}} dx dz$ $ds_z = \hat{\mathbf{z}} dx dy$	$ds_r = \hat{\mathbf{r}} r d\phi dz$ $ds_\phi = \hat{\boldsymbol{\phi}} dr dz$ $ds_z = \hat{\mathbf{z}} r dr d\phi$	$ds_R = \hat{\mathbf{R}} R^2 \sin \theta d\theta d\phi$ $ds_\theta = \hat{\boldsymbol{\theta}} R \sin \theta dR d\phi$ $ds_\phi = \hat{\boldsymbol{\phi}} R dR d\theta$
<b>Differential volume</b> $dV =$	$dx dy dz$	$r dr d\phi dz$	$R^2 \sin \theta dR d\theta d\phi$