

Chapter 4 – Vectors:

- Vector Properties & Representations

$$\mathbf{A} = A\hat{\mathbf{a}} \quad \text{Diagram: } \begin{array}{c} \text{A} \\ | \\ \text{---} \\ \text{---} \end{array} \xrightarrow{\mathbf{v}_1}$$

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$$

$$\mathbf{D} = \mathbf{A} - \mathbf{B}$$

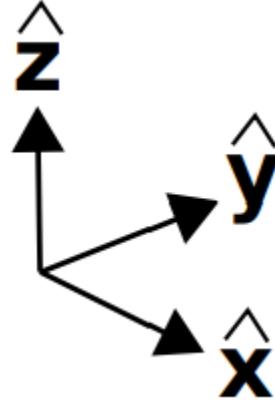
$$= (\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z) - (\hat{\mathbf{x}}B_x + \hat{\mathbf{y}}B_y + \hat{\mathbf{z}}B_z) \\ = \hat{\mathbf{x}}(A_x - B_x) + \hat{\mathbf{y}}(A_y - B_y) + \hat{\mathbf{z}}(A_z - B_z)$$

$$\mathbf{C} = \mathbf{A} + \mathbf{B}$$

$$= (\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z) + (\hat{\mathbf{x}}B_x + \hat{\mathbf{y}}B_y + \hat{\mathbf{z}}B_z) \\ = \hat{\mathbf{x}}(A_x + B_x) + \hat{\mathbf{y}}(A_y + B_y) + \hat{\mathbf{z}}(A_z + B_z)$$

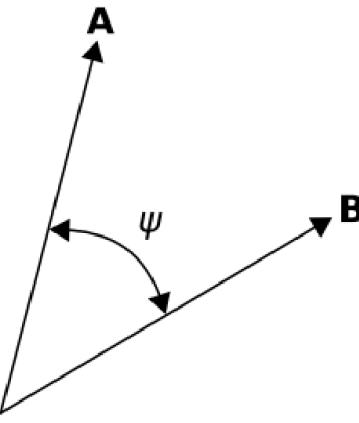
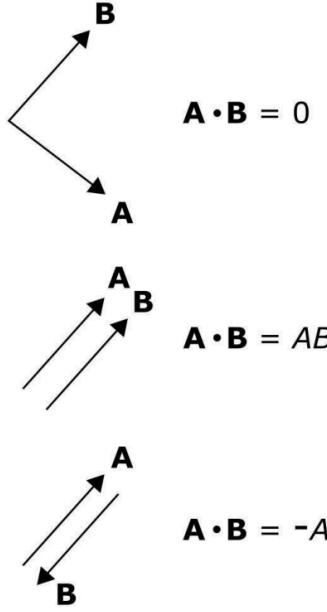
Vector representation $\mathbf{A} =$	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\phi}A_\phi + \hat{\theta}A_\theta$	$\hat{\mathbf{R}}A_R + \hat{\Theta}A_\theta + \hat{\Phi}A_\phi$
○ Magnitude			
Magnitude of \mathbf{A} $ \mathbf{A} =$	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_\theta^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
○ Unit Vector			

$$\begin{aligned}\hat{\mathbf{a}} &= \frac{\mathbf{A}}{|\mathbf{A}|} = \frac{\mathbf{A}}{\sqrt{A_x^2 + A_y^2 + A_z^2}} \\ &= \hat{\mathbf{x}}A_x (A_x^2 + A_y^2 + A_z^2)^{-1/2} \\ &\quad + \hat{\mathbf{y}}A_y (A_x^2 + A_y^2 + A_z^2)^{-1/2} \\ &\quad + \hat{\mathbf{z}}A_z (A_x^2 + A_y^2 + A_z^2)^{-1/2}\end{aligned}$$



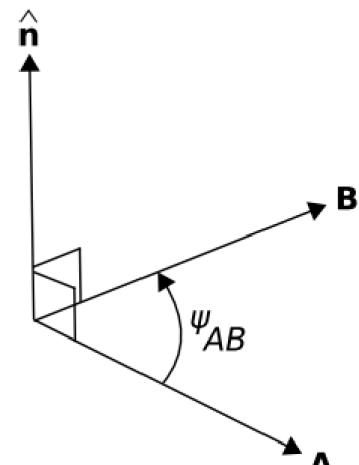
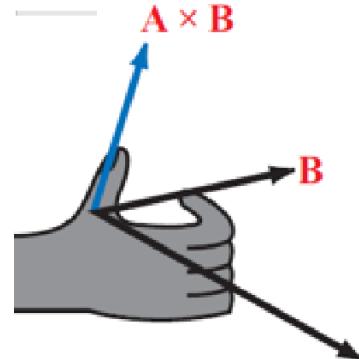
- Dot Product & Associated Properties

$\mathbf{A} \cdot \mathbf{B} = AB \cos \psi$		$\hat{\mathbf{x}} \cdot \hat{\mathbf{x}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{x}} \cdot \hat{\mathbf{z}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{z}} = 0$
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	$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$ $\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C}$	
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Dot product $\mathbf{A} \cdot \mathbf{B} = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = A_r B_r + A_\theta B_\theta + A_\phi B_\phi$

- Cross Product & Associated Properties

$\mathbf{A} \times \mathbf{B} = \hat{\mathbf{n}} AB \sin \psi_{AB}$	$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$ (anticommutative) $\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{C}$ (distributive) $\mathbf{A} \times \mathbf{A} = 0$	
	$\mathbf{A} \times \mathbf{B} = \hat{\mathbf{x}}(A_y B_z - A_z B_y) + \hat{\mathbf{y}}(A_z B_x - A_x B_z) + \hat{\mathbf{z}}(A_x B_y - A_y B_x)$ $\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	

Cross product $\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} \hat{\mathbf{r}} & \hat{\mathbf{\theta}} & \hat{\mathbf{\phi}} \\ A_r & A_\theta & A_\phi \\ B_r & B_\theta & B_\phi \end{vmatrix}$

- Cartesian Coordinate System
 - Length, Surface, and Volume Integration & Differential Forms
- Cylindrical Coordinate System

ECE 2214 Exam 2

- Conversion from Cartesian to Cylindrical
- Length, Surface, and Volume Integration & Differential Forms
- Spherical Coordinate System
 - Conversion from Cartesian to Spherical
 - Length, Surface, and Volume Integration & Differential Forms

Differential length $d\mathbf{l} =$	$\hat{\mathbf{x}} dx + \hat{\mathbf{y}} dy + \hat{\mathbf{z}} dz$	$\hat{\mathbf{r}} dr + \hat{\theta} r d\phi + \hat{\mathbf{z}} dz$	$\hat{\mathbf{R}} dR + \hat{\theta} R d\theta + \hat{\phi} R \sin\theta d\phi$
Differential surface areas	$ds_x = \hat{\mathbf{x}} dy dz$ $ds_y = \hat{\mathbf{y}} dx dz$ $ds_z = \hat{\mathbf{z}} dx dy$	$ds_r = \hat{\mathbf{r}} r d\phi dz$ $ds_\phi = \hat{\theta} dr dz$ $ds_z = \hat{\mathbf{z}} r dr d\phi$	$ds_R = \hat{\mathbf{R}} R^2 \sin\theta d\theta d\phi$ $ds_\theta = \hat{\theta} R \sin\theta dR d\phi$ $ds_\phi = \hat{\phi} R dR d\theta$
Differential volume $dV =$	$dx dy dz$	$r dr d\phi dz$	$R^2 \sin\theta dR d\theta d\phi$