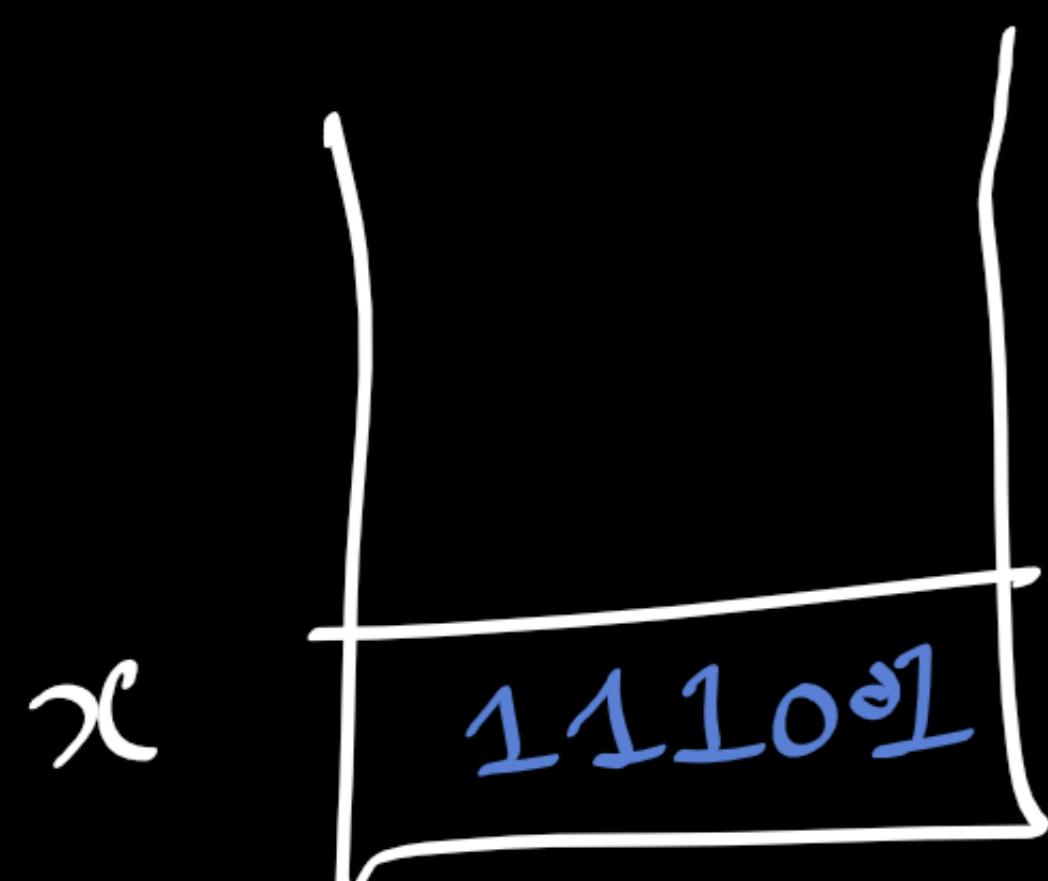


Basics of Bit Manipulation

Let us say we have a variable $x = 57$.

In machine, it will be stored as a binary number



$\text{SysTo}(x) \rightarrow 57$

$$\begin{array}{r} 2 | 57 \\ 2 | 28 \quad 1 \\ \hline 2 | 14 \quad 0 \\ 2 | 7 \quad 0 \\ \hline 2 | 3 \quad 1 \\ 2 | 1 \quad 1 \\ \hline & 0 \quad 1 \end{array}$$

$$(111001)_2$$

- * Store in binary
- * Return / Print in decimal

byte	short	int	long
8 bits (2^8)	16 bits (2^{16})	32 bits (2^{32})	64 bits (2^{64})

$\overbrace{0, 1, 0, 1, 0, 1, 0, 1}^8$ # if we have 4 bits, 2^4 distinct nos are poss.
So, if we have N bits, 2^N distinct nos are possible

Let us assume that there is a datatype called nibble whose size is 4 bits.

① 1000
No is -ve

for calculating value

2's comp of
 1000 is

1000

↓
value is 8

so ans is -8

0	0 0 0 → 0
0	0 0 1 → 1
0	0 1 0 → 2
0	0 1 1 → 3
0	1 0 0 → 4
0	1 0 1 → 5
0	1 1 0 → 6
0	1 1 1 → 7
-8	0 0 0 → 8 -0
-7	0 0 1 → 9 -1
-6	0 1 0 → 10 -2
-5	0 1 1 → 11 -3
-4	1 0 0 → 12 -4
-3	1 0 1 → 13 -5
-2	1 1 0 → 14 -6
-1	1 1 1 → 15 -7

• App $\rightarrow 1$ takes nos from 0 to 2^N-1 (all +ve)

• App $\rightarrow 2$ use MSB for sign & all other bits for value.

• Two 0s, +0 & -0

• After 0111, 1000 has +1 val in bin however -0 & -1 are -1 val diff.

App $\rightarrow 3$ use MSB for sign & complete nos 2's comp as value.

Range: -2^{N-1} to $2^{N-1} - 1$

byte (1 byte = 8 bits) = -2^7 to $2^7 - 1$

Short (2 bytes = 16 bits) = -2^{15} to $2^{15} - 1$

`int (4bytes = 32 bits) = -231 to 231-1`

`long` (8 bytes = 64 bits) = -2^{63} to $2^{63} - 1$

0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7
1	0	0	0	-8
1	0	0	1	-7
1	0	1	0	-6
1	0	1	1	-5
1	1	0	0	-4
1	1	0	1	-3
1	1	1	0	-2
1	1	1	1	-1

Let us say that we want to store 12
in nibble

$$\begin{array}{r} 12 \\ \hline 2 | 12 \\ \hline 2 \quad 6 \quad 0 \\ \hline 2 \quad 3 \quad 0 \\ \hline 2 \quad 1 \quad 1 \\ \hline 0 \quad 1 \end{array} \quad (1100)_2$$

If we try to store 16 i.e 10000, only last 4 bits will be stored.

Convert Bin to Decimal

$$MSB = 0$$

- MCB = {
 - 2's comp
 - Convert to dec
 - -ve Sign

Convert Dec to Bin

- + we
 - convert to Bin
 - fit in bits
- we
 - leave the sign?
 - convert to Bin
 - fit in bits
 - store ZS comp.

!	&	\wedge	$<<$	$>>$
or	AND	XOR	left shift	right shift

$>>>$	\sim	$\neg x$
trns	1's complement	2's complement

Left Shift

$$x = \text{00101011}$$

$$x \ll 3 = 01011\boxed{000}$$

Right Shift

$$y = 10100\boxed{110}$$

$y >> 3 \Rightarrow$ drops last 3 bits
 \Rightarrow Brings 1st 3 bits equal to current MSB & shifts

$$y = \underbrace{111}_{\substack{\rightarrow \text{prev MSB} \\ 3 \text{ bits eq to MSB}}} 10100$$

Triple RS

$$y = 10100\boxed{110}$$

$y >>> 3 \Rightarrow$ Drops 3 LSBs
 \Rightarrow Brings 3 0s at MSBs always.

$$y = \underline{000} 10100$$

\Rightarrow ON, OFF, CHECK/GET & TOGGLE are already discussed in Tries Notes.

Basics Of Bit Manipulation

Easy

◀ Prev

▶ Next

1. You are given a number n.
2. Print the number produced on setting its i-th bit.
3. Print the number produced on unsetting its j-th bit.
4. Print the number produced on toggling its k-th bit.
5. Also, Check if its m-th bit is on or off. Print 'true' if it is on, otherwise print 'false'.

```
public static void main(String[] args){  
    Scanner scn = new Scanner(System.in);  
    int n = scn.nextInt();  
    int i = scn.nextInt();  
    int j = scn.nextInt();  
    int k = scn.nextInt();  
    int m = scn.nextInt();  
  
    //write your code here  
  
    //setting the ith bit  
    int maskForSetting = (1 << i); //or with the number  
    int maskForOff = ~(1 << j); //and with the number  
    int maskForToggle = (1 << k); //XOR with the number;  
    int maskForChecking = (1 << m); //and with number  
  
    int setBitNo = (n | maskForSetting);  
    int unsetBitNo = (n & maskForOff);  
    int toggledNo = (n ^ maskForToggle);  
  
    System.out.println(setBitNo);  
    System.out.println(unsetBitNo);  
    System.out.println(toggledNo);  
  
    if((n & maskForChecking) != 0) {  
        System.out.println(true); //bit was set  
    } else {  
        System.out.println(false); //bit was off  
    }  
}
```

⇒ We have to use brackets () because the precedence of bitwise operators is even lower than assignment operator ("=").

$x = 1 \ll i$

without bracket

$x = (1 \ll i)$

with brackets.

Binary Representation - Gfg

Binary representation

School Accuracy: 50.04% Submissions: 3382 Points: 0

Write a program to print Binary representation of a given number N.

Example 1:

Input:

N = 2

Output:

00000000000000000000000000000010

```
class Solution {  
    static String getBinaryRep(int N){  
        // code here  
        int temp = N;  
        String res = "";  
        while(temp > 0) {  
            int rem = temp % 2;  
            temp = temp / 2;  
            res = "" + rem + res;  
        }  
  
        int len = res.length();  
        int limit = 30-len;  
  
        for(int i=1;i<=limit;i++) {  
            res = "0" + res;  
        }  
  
        return res;  
    }  
}
```

Right Most Set Bit (RMSB Mask)

$$x = 76$$

2	76	
2	38	0
2	19	0
2	9	1
2	4	1
2	2	0
2	1	0
	0	1

$$76 = (1001100)_2$$

↑
Right Most
Set Bit

$$\text{Right Most Set Bit Mask} = 0000\textcircled{1}000$$

All 0s except RMSB

formula to calculate RMSB mask of a no x in $O(1)$ time is $= x \& x''$
i.e $x \&$ 2's complement of x .

Two's comp of x can be calculated as $\sim x + 1$

$$x = A 1's \text{ and } 0's \quad 1 \quad B 0's \quad \textcircled{1}$$

$$\sim x = A 0's \text{ and } 1's \quad 0 \quad B 1's$$

$$x'' = \sim x + 1 = A 0's \text{ and } 1's \quad 1 \quad B 0's \quad \textcircled{2}$$

$$\text{RMSB Mask} = A 0's \quad \textcircled{1} \quad B 0's \quad \textcircled{1} \& \textcircled{2}$$

$$\text{So, RMSB} = x \& (\sim x + 1)$$

2's comp can also be calculated as $-x$

$$\text{So, RMSB Mask} = \underline{x \& -x}$$

```

import java.io.*;
import java.util.*;

public class Main {
    public static void main(String[] args){
        Scanner scn = new Scanner(System.in);
        int n = scn.nextInt();

        //write your code here
        int res = (n & -n);
        System.out.println(Integer.toBinaryString(res));
    }
}

```

} code for
RMSB Mask

Find first set bit

Easy Accuracy: 52.49% Submissions: 42460 Points: 2

Given an integer n . The task is to return the position of **first set bit found from the right side** in the binary representation of the number.

Note: If there is no set bit in the integer n , then return 0 from the function.

Example 1:

Input: $n = 18$
Output: 2
Explanation: Binary representation of 18 is 010010, the first set bit from the right side is at position 2.

⇒ calculate RMSB Mask.
 ⇒ convert it to Binary String
 → The length of the string is our answer.

```

class Solution
{
    //Function to find position of first set bit in the given number.
    public static int getFirstSetBit(int n){

        // Your code here
        int rmsbm = (n & -n);
        String res = Integer.toBinaryString(rmsbm);

        if(res.equals("0") == true) return 0;

        return res.length();
    }
}

```

Find position of set bit

Basic Accuracy: 50.02% Submissions: 27212 Points: 1

Given a number N having only one '1' and all other '0's in its binary representation, find position of the only set bit. If there are 0 or more than 1 set bit the answer should be -1. Position of set bit '1' should be counted starting with 1 from LSB side in binary representation of the number.

1 1 0 1 0 1 0 0
 → There are multiple 1's
 → Hence the answer should be -1
 Also, ans is -1 when there are more than one 1's.

Kernighan's Algorithm

This algorithm is used to count the no of set bits in a Number, without going through all the bits of that number.

Algorithm is as follows

while ($n \neq 0$) {

 calculate RMSB Mask

$N = N - \text{RMSB Mask}$

$\Sigma \text{ count}++;$

$x = 100100100$

① $x = 100100100$

$- \text{rmsbm} = 0000000100$

New $x = 100100000$

counter = 1

② $x = 100100000$

$- \text{rmsbm} = 0001000000$

$x = 100000000$

counter = 2

③ $x = 100000000$

$- \text{rmsbm} = 100000000$

$x = 000000000$

counter = 3

```
import java.io.*;
import java.util.*;

public class Main {

    public static void main(String[] args){
        Scanner scn = new Scanner(System.in);
        int n = scn.nextInt();

        //write your code here
        int counter = 0;
        while(n != 0) {
            int rmsbm = (n & -n);
            n -= rmsbm;
            counter++;
        }

        System.out.println(counter);
    }
}
```

① Use Kernighan's algo to count no of set bits.

② Use rmsbm mask to find 1st set bit pos.

Find position of set bit

Basic Accuracy: 50.02% Submissions: 27212 Points: 1

Given a number N having only one '1' and all other '0's in its binary representation, find position of the only set bit. If there are 0 or more than 1 set bit the answer should be -1. Position of set bit '1' should be counted starting with 1 from LSB side in binary representation of the number.

```
class Solution {  
    static int findPosition(int N) {  
        // code here  
        int counter = 0;  
        int temp = N;  
        //kernighan's algorithm  
        while(temp != 0) {  
            int rmsbm = (temp & -temp);  
            temp -= rmsbm;  
            counter++;  
        }  
  
        if(counter == 0 || counter > 1) return -1;  
  
        int rmsbm = (N & -N);  
        String res = Integer.toBinaryString(rmsbm);  
        return res.length();  
    }  
}
```

Number of 1 Bits

Easy Accuracy: 64.8% Submissions: 38238 Points: 2

Given a positive integer N , print count of set bits in it.

```
// User function Template for Java  
class Solution {  
    static int setBits(int N) {  
        // code here  
        int counter = 0;  
        while(N != 0) {  
            int rmsbm = (N & -N);  
            N -= rmsbm;  
            counter++;  
        }  
  
        return counter;  
    }  
}
```

Kernighan's

algo.

(same code)

No of set bits is also known as Hamming Weight.

191. Number of 1 Bits

Easy 2998 792 Add to List Share

Write a function that takes an unsigned integer and returns the number of '1' bits it has (also known as the Hamming weight).

Note:

- Note that in some languages, such as Java, there is no unsigned integer type. In this case, the input will be given as a signed integer type. It should not affect your implementation, as the integer's internal binary representation is the same, whether it is signed or unsigned.
- In Java, the compiler represents the signed integers using 2's complement notation. Therefore, in Example 3, the input represents the signed integer. -3.

```
public class Solution {  
    // you need to treat n as an unsigned value  
    public int hammingWeight(int n) {  
        int counter = 0;  
        while(n != 0) {  
            int rmsbm = (n & -n);  
            n -= rmsbm;  
            counter++;  
        }  
  
        return counter;  
    }  
}
```

Same code

461. Hamming Distance

Easy 3032 197 Add to List Share

The Hamming distance between two integers is the number of positions at which the corresponding bits are different.

Given two integers x and y , return the **Hamming distance** between them.

$\Rightarrow x \oplus y$ will give 1s at all those bits that are diff. Then count set bits in $x \oplus y$.

```
class Solution {  
    public int hammingDistance(int x, int y) {  
        int xor = (x ^ y);  
  
        int hDist = 0;  
        while(xor != 0) {  
            int rmsbm = (xor & -xor);  
            xor -= rmsbm;  
            hDist++;  
        }  
  
        return hDist;  
    }  
}
```

477. Total Hamming Distance

Medium 1626 80 Add to List Share

The Hamming distance between two integers is the number of positions at which the corresponding bits are different.

Given an integer array `nums`, return the sum of Hamming distances between all the pairs of the integers in `nums`.

$O(N^2)$ approach is generating all pairs and finding hamming distance. \Rightarrow TLE

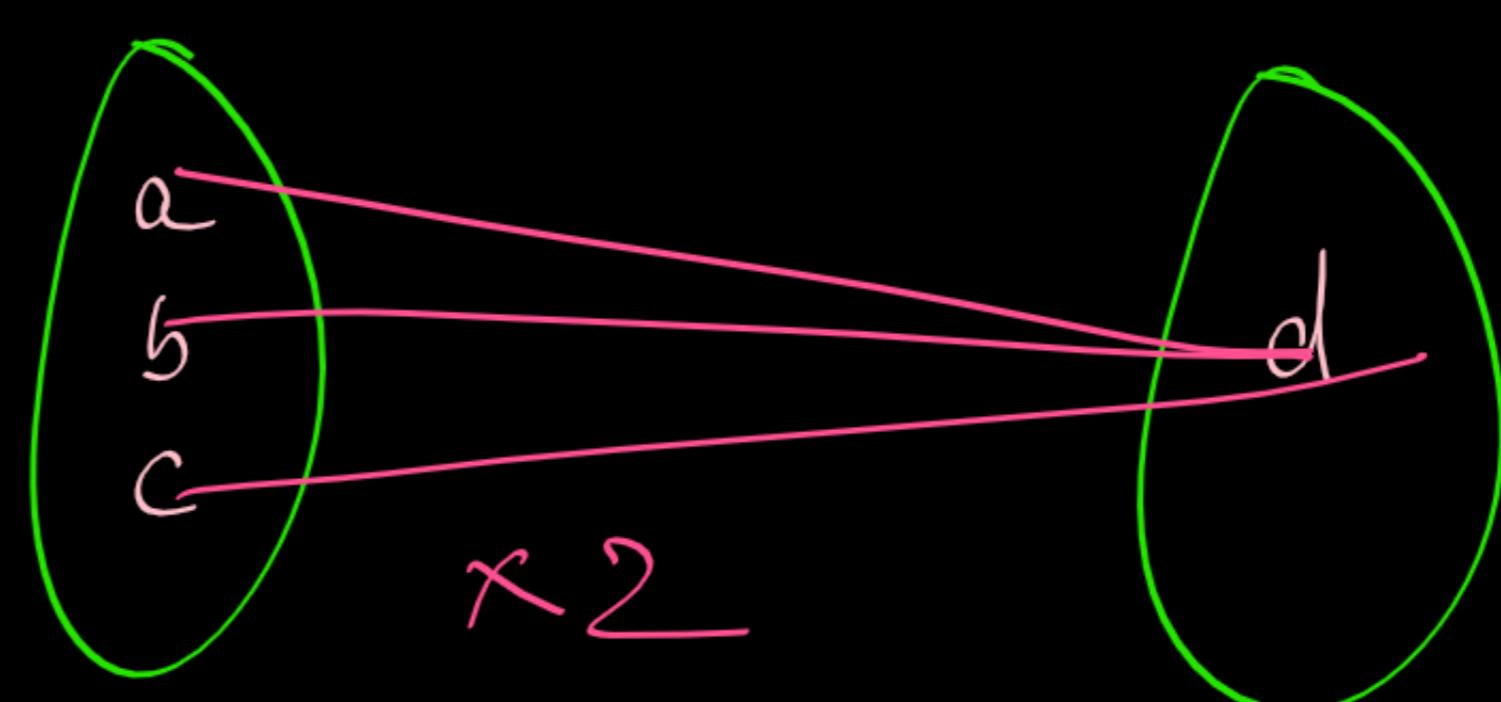
Optimized Approach

We will consider bit by bit all the pairs-

	4 th	3 rd	2 nd	1 st	0 th
a	1	0	1	1	1
b	1	1	0	0	1
c	1	0	1	0	1
d	1	0	0	1	0

Consider 0th bit

ON OFF



ad da } all the pairs
bd db }
cd dc }

Do this for all 32 bits. Hence the TC is $O(32 * N)$

*2 is for Reproducing ques. On Leetcode, consider a bit don't consider bits.

```
class Solution {
    public int totalHammingDistance(int[] nums) {
        long res = 0;

        for(int i=0;i<32;i++) {
            long counton = 0;

            for(int val : nums) {
                int bit = ((val & (1 << i)) == 0) ? 0 : 1;
                if(bit == 1) {
                    counton++;
                }
            }

            long countoff = nums.length - counton;
            long curr = counton * countoff;
            res += curr;
        }

        return (int)res;
    }
}
```

Code for Total Hamming Distance - Leetcod.

338. Counting Bits

Easy 6833 318 Add to List Share

Given an integer n , return an array ans of length $n + 1$ such that for each i ($0 \leq i \leq n$), $\text{ans}[i]$ is the **number of 1's** in the binary representation of i .

```

class Solution {
    public int kerninghans(int n) {
        int counter = 0;
        while(n != 0) {
            int rmsbm = (n & -n);
            n -= rmsbm;
            counter++;
        }
        return counter;
    }

    public int[] countBits(int n) {
        int[] res = new int[n+1];
        for(int i=0;i<=n;i++) {
            res[i] = kerninghans(i);
        }
        return res;
    }
}

```

Bleak Numbers

Bleak Numbers

Medium Accuracy: 55.02% Submissions: 2886 Points: 4

Output: 0
Explanation: 3 is a Bleak number as
 $2 + \text{countSetBit}(2) = 3$.

Your Task:

You don't need to read or print anything. Your task is to complete the function `is_bleak()` which takes n as input parameter and returns 1 if n is **not** a Bleak number otherwise returns 0.

Expected Time Complexity: $O(\log(n) * \log(n))$

Expected Space Complexity: $O(1)$

```

class Solution
{
    public int kerninghans(int n) {
        int counter = 0;
        while(n != 0) {
            int rmsbm = (n & -n);
            n -= rmsbm;
            counter++;
        }
        return counter;
    }

    public int is_bleak(int n)
    {
        // Code here
        for(int i=1;i<=n;i++) {
            int setBitCount = kerninghans(i);
            int res = i + setBitCount;
            if(res == n) return 0;
        }
        return 1;
    }
}

```

for every no less than or equal to N ,
check if the sum of that No & its set bits is equal to N
or Not.

small optimization

Auxiliary Space: $O(1)$

Method 2 (Efficient)

The idea is based on the fact that the largest count of set bits in any number smaller than n cannot exceed ceiling of $\log_2 n$. So we need to check only numbers from range $n - \text{ceilingLog2}(n)$ to n .

Same Bits (Shreyansh & his Bits - GFG)

Shreyansh and his bits

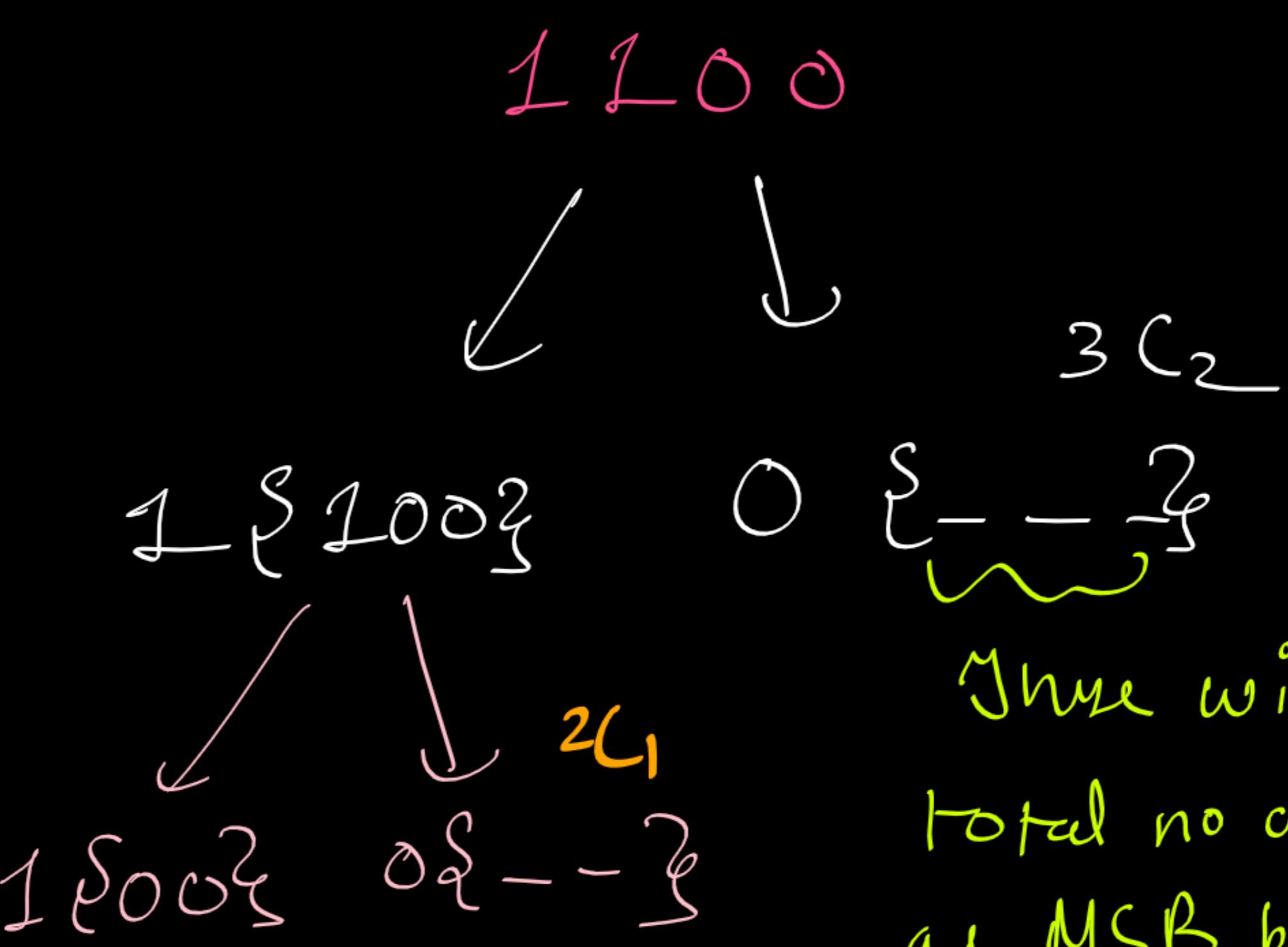
Medium Accuracy: 38.73% Submissions: 395 Points: 4

Shreyansh has an integer N. He is really curious about the binary representation of integers. He sees that any given integer has a number of set bits. Now he wants to find out that how many positive integers, strictly less than N, have the **same number of set bits as N**.

He is a little weak in maths. Help him find the number of integers.

Note: Since N takes large values, brute force won't work.

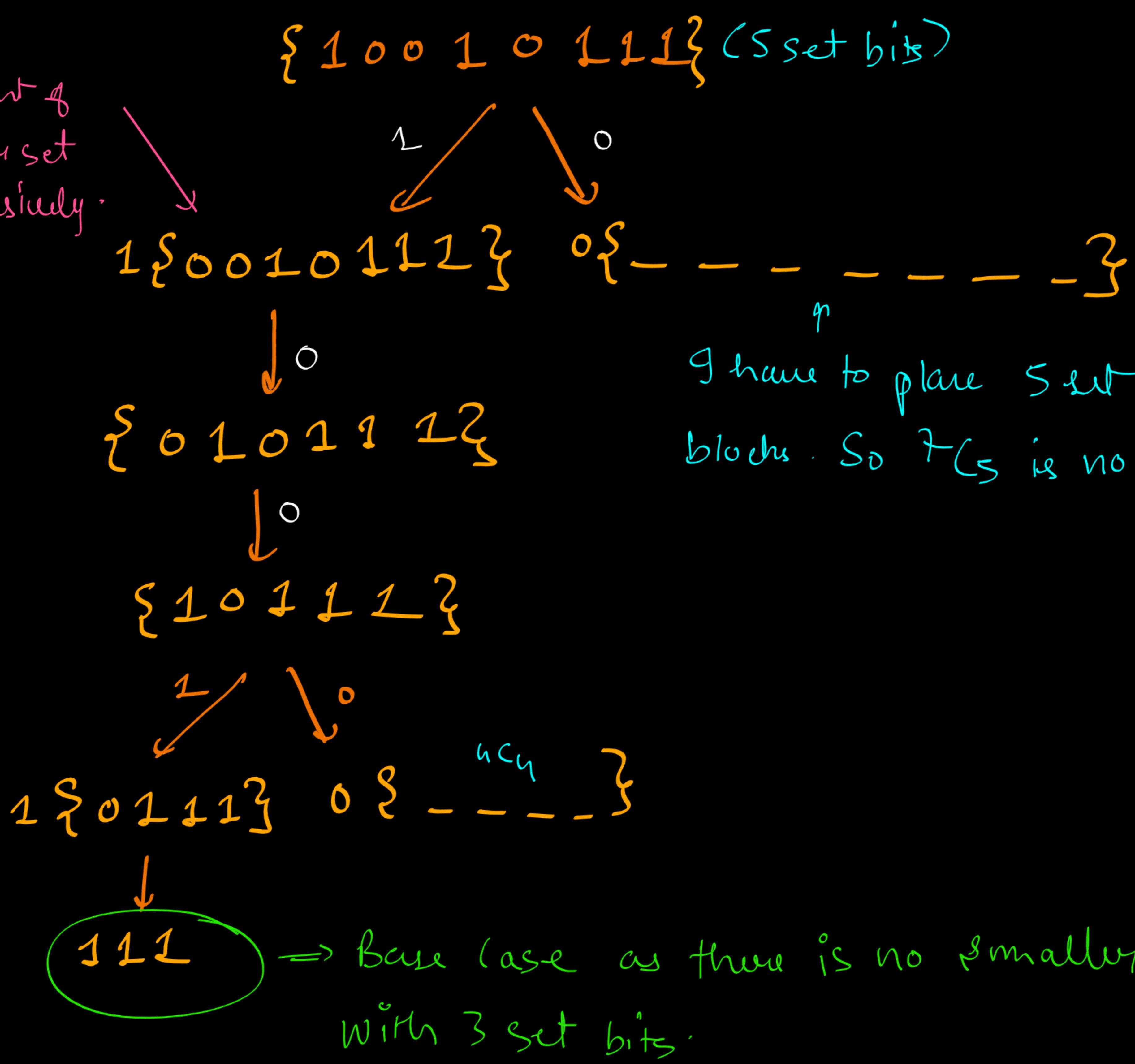
Say, input is 12 \rightarrow 1100



These will be the total no of combs as MSB became 0 i.e. it has contributed in producing a small no.

$f(10010111)$

find count of nos with 4 set bits recursively.



I have to place 5 set bits in 7 blocks. So $7C_5$ is no of ways to do it.

Power of Two

231. Power of Two

Easy 3178 288 Add to List Share

Given an integer n , return `true` if it is a power of two. Otherwise, return `false`.

An integer n is a power of two, if there exists an integer x such that $n == 2^x$.

if a no is a power of 2 then

$$n \& (n-1) = 0$$

```
class Solution {
    public boolean isPowerOfTwo(int n) {
        if(n < 1) return false;
        if((n & (n-1)) == 0) return true;
        return false;
    }
}
```

Highest Power of 2 Less than or Equal to Given No.

keep on left shifting and generating the nos. Stop when you are at a no which is greater than N

```
import java.util.*;
public class Main {
    Run | Debug
    public static void main(String[] args) throws Exception {
        Scanner scn = new Scanner(System.in);
        int n = scn.nextInt();
        System.out.println(highestPowerOf2(n));
        scn.close();
    }

    public static int highestPowerOf2(int n) {
        String bin = Integer.toBinaryString(n);

        int res = 0;
        for(int i=1;i<=bin.length();i++) {
            int curr = (1 << i);
            if(curr > n) break;
            res = curr;
        }

        return res;
    }
}
```

Gray Code { 1 bit-diff }

1 bit

0
1

2 bit

00

01
10 }
11 } Not valid gray code.

3 bit

000
001
011
010

110
111
101
100

Valid 3-bit gray code

00 } Valid
01
11 } 2 bit
10 } gray code.

$$3 \text{ bit Gray Code} = 0 [2 \text{ bit Gray Code}] + 1 [\text{Reverse order of 2 bit Gray Code}]$$

$$N \text{ bit Gray Code} = 0 [N-1 \text{ bit Gray Code}] + 1 [\text{Reverse of } N-1 \text{ bit Gray Code}]$$

[000, 001, 011, 010, 110, 111, 101, 100]

$$\begin{aligned} g_c(3) &\xrightarrow{0[00, 01, 11, 10]} [000, 001, 011, 010] \quad \textcircled{⑥} \\ &\xrightarrow{1[10, 11, 01, 00]} [110, 111, 101, 100] \quad \textcircled{⑦} \\ g_c(2) &\xrightarrow{0[0, 1]} [00, 01] \quad \textcircled{①} \\ &\xrightarrow{1[1, 0]} [11, 10] \quad \textcircled{②} \\ g_c(1) &[0, 1] \end{aligned}$$

This is when we have List<Integer> as Strings

Not that imp as we will have to deal with nos only

Unique Number - 1

136. Single Number

Easy 9687 348 Add to List Share

Given a **non-empty** array of integers `nums`, every element appears twice except for one. Find that single one.

You must implement a solution with a linear runtime complexity and use only constant extra space.

```
class Solution {
    public int singleNumber(int[] nums) {
        int xor = nums[0];
        for(int i=1;i<nums.length;i++) {
            xor = (xor ^ nums[i]);
        }
        return xor;
    }
}
```

Unique Number - 2

137. Single Number II

Medium 4057 477 Add to List Share

Given an integer array `nums` where every element appears **three times** except for one, which appears **exactly once**. Find the single element and return it.

You must implement a solution with a linear runtime complexity and use only constant extra space.

5th 4th 3rd 2nd 1st 0th

Set bits $\rightarrow 10\ 9\ 6\ 4\ 7\ 9$

```
class Solution {
    public int singleNumber(int[] nums) {
        int[] res = new int[32];
        for(int i=0;i<=31;i++) {
            int count1 = 0;
            for(int val : nums) {
                int mask = (1 << i);
                int bit = (val & mask);
                if(bit != 0) count1++;
            }
            res[i] = count1;
        }
        int ans = 0;
        for(int i=0;i<32;i++) {
            if(res[i] % 3 != 0) {
                ans += (1 << i);
            }
        }
        return ans;
    }
}
```

TC = $O(32 \times N)$

5th 4th 3rd 2nd 1st 0th
 $10 - 9 - 6 - 4 - 7 - 9$
 $(3n+1) 3n \quad 3n (3n+1) (3n+1) 3n$

Where there are $3n$ bits, put 0
Where there are $3n+1$ bits, put 1.

5th 4th 3rd 2nd 1st 0th
 $10 - 9 - 6 - 4 - 7 - 9$
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $1 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0$ = 38

↳ This is our unique number.

This solution works because if there was no unique no & all nos repeated thrice, the set bits at all positions of all nos would have been $3n$. But $3n+1$ bit is contributed by one extra number.

	$3n$	$3n+1$	$3n+2$
54	3 4 3 2 1 0 th		
51	1 1 0 0 1 1		
57	2 1 1 0 0 1		
51	1 1 0 0 1 1		
	3 3 1 0 2 3	1 1 0 0 0 1	0 0 + 0 0 0
57	1 1 1 0 0 1		0 0 0 0 1 0
63	1 1 1 1 1 1		
38	1 0 0 1 1 0		
57	1 1 1 0 0 1		
63	1 1 1 1 1 1		
63	1 1 1 1 1 1		
51	1 1 0 0 1 1		

Basically, at any point of time, we have to try to maintain 3 terms $3n$, $3np1$ & $3np2$ such that if a bit position is set $3n$ times, it is set in $3n^2$ diff in $3np1$ & $3np2$ and so is the case for $3np1$ & $3np2$ as well.

If we are able to do this somehow, at last The ans will be $3np1$ term.

So, let us now try to achieve this in $O(N)$ TC without using any extra space.

	$3n$	$3n+1$	$3n+2$
51	110011	000000	000000
57	111001	110011	000000
51	110011	001000	110001
	331023	10101	001000
57	111001		
63	111111		
38	100110		
57	111001		
63	111111		
63	111111		
51	110011		

Initially $3n \Rightarrow 111111 \Rightarrow$ we haven't checked anything.
 $3np1 \Rightarrow 000000$ no. of 1's is at every place is 0. Hence
 $0 \leq 3n$

$3np1 \Rightarrow 000000$ since we haven't checked
 $3np2 \Rightarrow 000000$ any bit so, all are 0s &
 0s are all $3n$ not $3np1$ &
 $3np2$.

Let us take the 1st No

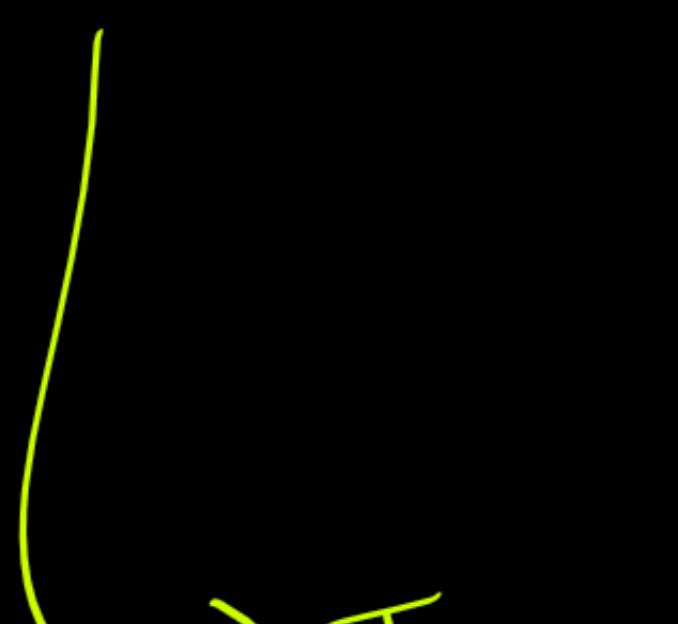
	$3n$	$3np1$	$3np2$
	110011	110011	110011
$3n$	$\underline{111111}$	$\underline{000000}$	$\underline{000000}$
$sbc3n$	$\underline{\underline{110011}}$	$\underline{\underline{000000}}$	$\underline{\underline{000000}}$

All those set bits that are common in $3n$ will now be off in $3n$.
 Because now they have become $3n+1$. (See table)

Let us do it for second no.

$$\begin{array}{r} 3^n \\ \begin{array}{r} 111001 \\ 001100 \\ \hline 001000 \end{array} \end{array}$$

$CSb3n$



Turn this bit ON
for 3^{n+1} & off for 3^n .

$$\begin{array}{r} 3^{n+1} \\ \begin{array}{r} 111001 \\ 110011 \\ \hline 110001 \end{array} \end{array}$$

$(Sb3np1)$

Turn these
bits ON for
 3^{n+2} & off
for 3^{n+1} .

$$\begin{array}{r} 3^{n+2} \\ \begin{array}{r} 111001 \\ 000000 \\ \hline 000000 \end{array} \end{array}$$

$(Sb3np2)$

No bits will
be turned
on for 3^n .
& off for 3^{n+2} .

$$3^n \quad \underline{0} \underline{0} \underline{0} \underline{1} \underline{0} \underline{0}$$

3^{n+1}

$$\underline{0} \underline{0} \underline{1} \underline{0} \underline{1} \underline{0}$$

3^{n+2}

$$\underline{1} \underline{1} \underline{0} \underline{0} \underline{1} \underline{1}$$

new values:

for 3rd value in Array ($idx=2$)

$$\begin{array}{r} 51 \quad 110011 \\ 3^n \quad \begin{array}{r} 000100 \\ \hline 000000 \end{array} \end{array}$$

$Sb<3n$

Turn ON
1s in 3^{n+1} &
off in 3^n

$$\begin{array}{r} 110011 \\ 3^{n+1} \quad \begin{array}{r} 001010 \\ \hline 000010 \end{array} \end{array}$$

$Sb<3np1$

Turn ON 1s
in 3^{n+2} & off
in 3^{n+1} .

$$\begin{array}{r} 110011 \\ 3^{n+2} \quad \begin{array}{r} 110001 \\ \hline 110001 \end{array} \end{array}$$

$Sb<3np2$

Turn ON 1s in
 3^n & off in 3^{n+2}

$$3^n \quad \underline{1} \underline{1} \underline{0} \underline{1} \underline{0} \underline{1}$$

3^{n+1}

$$\underline{0} \underline{0} \underline{1} \underline{0} \underline{0} \underline{0}$$

3^{n+2}

$$\underline{0} \underline{0} \underline{0} \underline{0} \underline{1} \underline{0}$$

new values:

How to do this? Say x is the current value.

$$Sbc3n = (x \& 3n)$$

$$Sbc3np1 = (x \& 3np1)$$

$$Sbc3np2 = (x \& 3np2)$$

Setting bits ON in next values.

$$3n = (3n \& Sbc3np2)$$

$$3np1 = (3np1 \& Sbc3n)$$

$$3np2 = (3np2 \& Sbc3np1)$$

Turning off bits in current values.

$$3n = (3n \& \sim Sbc3n)$$

$$3np1 = (3np1 \& \sim Sbc3np1)$$

$$3np2 = (3np2 \& \sim Sbc3np2)$$

```
class Solution {  
    public int singleNumber(int[] arr) {  
        int tn = -1, tnp1 = 0, tnp2 = 0;  
  
        for(int i=0;i<arr.length;i++) {  
            int sbcw3n = (arr[i] & tn);  
            int sbcw3np1 = (arr[i] & tnp1);  
            int sbcw3np2 = (arr[i] & tnp2);  
  
            tn = (tn & (~sbcw3n));  
            tnp1 = (tnp1 | sbcw3n);  
  
            tnp1 = (tnp1 & (~sbcw3np1));  
            tnp2 = (tnp2 | sbcw3np1);  
  
            tnp2 = (tnp2 & (~sbcw3np2));  
            tn = (tn | sbcw3np2);  
        }  
  
        return tnp1;  
    }  
}
```

Optimized Approach
(Space Optimized)
 $T.C = O(1)$

Unique Number - III

260. Single Number III

Medium 3844 182 Add to List Share

Given an integer array `nums`, in which exactly two elements appear only once and all the other elements appear exactly twice. Find the two elements that appear only once. You can return the answer in [any order](#).

You must write an algorithm that runs in linear runtime complexity and uses only constant extra space.

```
class Solution {
    public int[] singleNumber(int[] nums) {
        int xor = 0;
        for(int val : nums) {
            xor = xor ^ val;
        }

        int rmsbm = (xor & -xor);
        int x = 0;
        int y = 0;

        for(int val : nums) {
            if((val & rmsbm) != 0) {
                x = x ^ val;
            } else {
                y = y ^ val;
            }
        }

        return new int[]{x,y};
    }
}
```

36 - 1 0 0 1 0 0

50 - 1 1 0 0 1 0

24 - 0 1 1 0 0 0

56 - 1 1 1 0 0 0

36 - 1 0 0 1 0 0

24 - 0 1 1 0 0 0

42 - 1 0 1 0 1 0

50 - 1 1 0 0 1 0

0 1 0 0 1 0

↳ This no is a xor of 56 & 42
and also the entire array.

0 1 0 0 1 0

↓

2 meanings for this 1

→ Bits of 2 nos whose this is a

xor are diff

→ With respect to array, odd no
of elements have a set bit at
this position.

36 156 129 158 1
36 129 142 150
= 56 142 =

18

One set will be of all those elements whose bit corresponding

to the RMSB of XOR will be 1 & other set of those nos

whose bit will be 0.

Set 1 will have 1 of the 2 nos & set 2 will have the other.
Rest will be the repeating elec in both the sets. So, XOR(Set1) is
the first unique no & XOR(Set2) is the other unique no.

Calculate Square of Number w/o using *, -, / or pow()

```
square(n) = 0 if n == 0  
if n is even  
    square(n) = 4*square(n/2)  
if n is odd  
    square(n) = 4*square(floor(n/2)) + 4*floor(n/2) + 1
```

if no 'c' x

if ($x \text{ mod } 2 == 0$) {
 return $4 * \text{square}(x/2)$;

else
 return $4 * \text{square}(x/2) + 4 * (x/2) + 1$

// Proof

If n is even $n = 2 * x \rightarrow x = n/2$

$$n^2 \in 4 * x^2 \\ \Rightarrow n^2 = 4 * \left(\frac{n}{2}\right)^2$$

If n is odd

$$n = 2 * x + 1$$

$$n^2 = (2 * x + 1)^2 = 4 * x^2 + 4 * x + 1$$

```
public static int square(int n) {  
    if(n == 0) return 0;  
  
    if(n < 0) n = -n;  
  
    //check if the number is even i.e. divisible by 2  
    //if a number is divisible by 2, its LSB is 0  
  
    int lsb = (n & 1);  
    int x = (n >> 1); //floor(n/2)  
    if(lsb == 1) {  
        //number is odd  
        //  $4 * x^2 + 4 * x + 1$  here x = floor(n/2)  
        int nby2square = square(x);  
        return ((nby2square << 2) + (x << 2) + 1);  
    }  
  
    // if the number is even  
    //  $4 * x^2$   
  
    int nby2square = square(x);  
    return (nby2square << 2);  
}
```

Calculate $7n/8$ without using *, /, + operators.

```
import java.io.*;
import java.util.*;

public class Main {

    public static void main(String[] args){
        Scanner scn = new Scanner(System.in);
        int n = scn.nextInt();
        //write your code here

        int en = (n << 3);
        int num = en - n;
        int res = (num >> 3);
        System.out.println(res);
    }
}
```

$$\frac{7n}{8} = \underline{\underline{(8n - n)}} \quad 8$$

$$8 * n \Leftrightarrow (n \ll 3)$$

$$n/8 \Leftrightarrow (n \gg 3)$$

Divide 2 Integers

29. Divide Two Integers

Medium 2702 9455 Add to List Share

Given two integers dividend and divisor, divide two integers **without** using multiplication, division, and mod operator.

The integer division should truncate toward zero, which means losing its fractional part. For example, 8.345 would be truncated to 8, and -2.7335 would be truncated to -2.

Return the **quotient** after dividing dividend by divisor.

Note: Assume we are dealing with an environment that could only store integers within the **32-bit** signed integer range: $[-2^{31}, 2^{31} - 1]$. For this problem, if the quotient is **strictly greater than** $2^{31} - 1$, then return $2^{31} - 1$, and if the quotient is **strictly less than** -2^{31} , then return -2^{31} .

```
class Solution {
    public int divide(int dividend, int divisor) {
        if(dividend == 1 << 31 && divisor == -1) return Integer.MAX_VALUE;

        boolean sign = (dividend >= 0) == (divisor >= 0) ? true : false;
        dividend = Math.abs(dividend);
        divisor = Math.abs(divisor);

        int res = 0;
        while(dividend - divisor >= 0) {
            int count = 0;
            while(dividend - (divisor << 1 << count) >= 0) {
                count++;
            }
            res += 1 << count;
            dividend -= divisor << count;
        }

        return sign ? res : -res;
    }
}
```

$$\text{dividend} = 10$$

$$\text{divisor} = 3$$

$$\text{dividend} = \text{divisor} * \text{quotient} + \text{remainder}$$

Naive Approach

keep subtracting divisor from dividend and calculate quotient by incrementing counter.

① Dividend = 10
Divisor = 3
 $10 - 3 = 7 \checkmark$
Count = 1

② Dividend = 7
Divisor = 3
 $7 - 3 = 4 \checkmark$
Count = 2

③ Dividend = 4 (i) Dividend = 1
Divisor = 3
 $4 - 3 = 1 \checkmark$
Count = 3
Divisor = 3
 $1 - 3 = -2 \times$

```
class Solution {
    public int divide(int dividend, int divisor) {
        if(dividend == 1 << 31 && divisor == -1) return Integer.MAX_VALUE;
        if(divisor == 1) return dividend;

        boolean sign = (dividend >= 0) == (divisor >= 0) ? true : false;
        dividend = Math.abs(dividend);
        divisor = Math.abs(divisor);

        int res = 0;
        while(dividend - divisor >= 0) {
            res++;
            dividend -= divisor;
        }

        return sign ? res: -res;
    }
}
```

Basic Approach

Submission Detail

992 / 992 test cases passed, but took too long.

Status: Time Limit Exceeded

Submitted: 0 minutes ago

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