

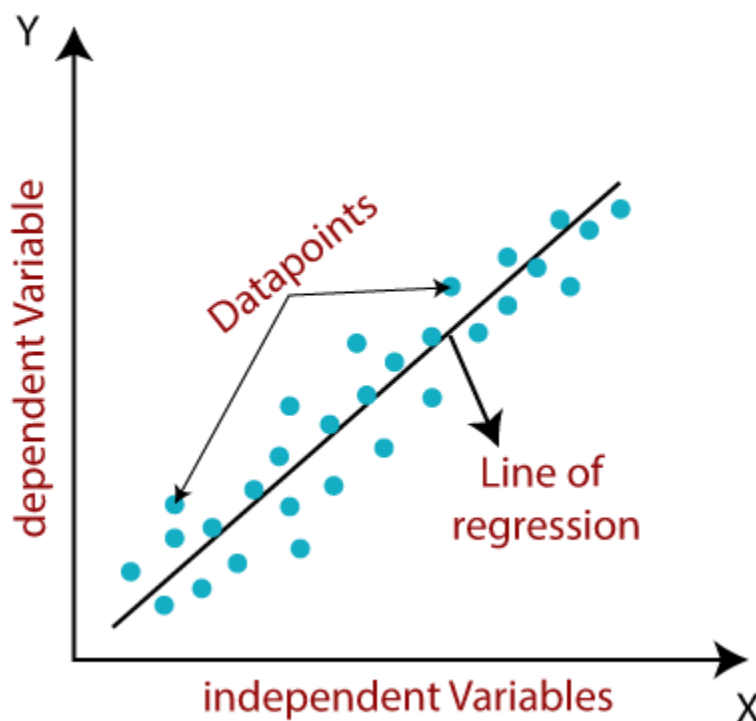
## General Subjective Questions

### 1. Explain the linear regression algorithm in detail.

It is a statistical method that is used for predictive analysis. Linear regression makes predictions for continuous/real or numeric variables such as **sales, salary, age, product price**, etc.

Linear regression algorithm shows a linear relationship between a dependent (y) and one or more independent (x) variables, hence called as linear regression. Since linear regression shows the linear relationship, which means it finds how the value of the dependent variable is changing according to the value of the independent variable.

The linear regression model provides a sloped straight line representing the relationship between the variables. Consider the below image:



$$y = a_0 + a_1x + \varepsilon$$

$Y$  = Dependent Variable (Target Variable)  
 $X$  = Independent Variable (predictor Variable)  
 $a_0$  = intercept of the line (Gives an additional degree of freedom)  
 $a_1$  = Linear regression coefficient (scale factor to each input value).  
 $\varepsilon$  = random error

The values for  $x$  and  $y$  variables are training datasets for Linear Regression model representation.

## Types of Linear Regression

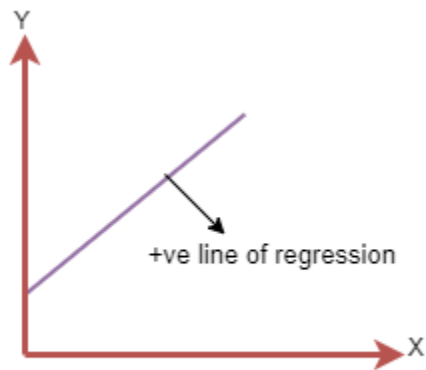
Linear regression can be further divided into two types of the algorithm:

- **Simple Linear Regression:**  
If a single independent variable is used to predict the value of a numerical dependent variable, then such a Linear Regression algorithm is called Simple Linear Regression.
- **Multiple Linear regression:**  
If more than one independent variable is used to predict the value of a numerical dependent variable, then such a Linear Regression algorithm is called Multiple Linear Regression.

## Linear Regression Line

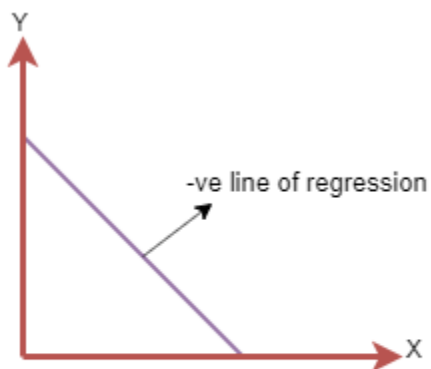
A linear line showing the relationship between the dependent and independent variables is called a **regression line**. A regression line can show two types of relationship:

- **Positive Linear Relationship:**  
If the dependent variable increases on the Y-axis and independent variable increases on X-axis, then such a relationship is termed as a Positive linear relationship.



The line equation will be:  $Y = a_0 + a_1X$

- Negative Linear Relationship:**  
 If the dependent variable decreases on the Y-axis and independent variable increases on the X-axis, then such a relationship is called a negative linear relationship.



The line of equation will be:  $Y = -a_0 + a_1X$

## Finding the best fit line:

When working with linear regression, our main goal is to find the best fit line that means the error between predicted values and actual values should be minimized. The best fit line will have the least error.

The different values for weights or the coefficient of lines ( $a_0, a_1$ ) gives a different line of regression, so we need to calculate the best values for  $a_0$  and  $a_1$  to find the best fit line, so to calculate this we use cost function.

## Cost function-

- The different values for weights or coefficient of lines ( $a_0, a_1$ ) gives the different line of regression, and the cost function is used to estimate the values of the coefficient for the best fit line.
- Cost function optimizes the regression coefficients or weights. It measures how a linear regression model is performing.
- We can use the cost function to find the accuracy of the **mapping function**, which maps the input variable to the output variable. This mapping function is also known as **Hypothesis function**.

For Linear Regression, we use the **Mean Squared Error (MSE)** cost function, which is the average of squared error occurred between the predicted values and actual values. It can be written as:

For the above linear equation, MSE can be calculated as:

$$MSE = \frac{1}{N} \sum_{i=1}^n (y_i - (a_1 x_i + a_0))^2$$

**Where,**

$N$  = Total number of observation  
 $Y_i$  = Actual value  
 $(a_1 x_i + a_0)$  = Predicted value.

**Residuals:** The distance between the actual value and predicted values is called residual. If the observed points are far from the regression line, then the residual will be high, and so cost function will high. If the scatter points are close to the regression line, then the residual will be small and hence the cost function.

## Model Performance:

The Goodness of fit determines how the line of regression fits the set of observations. The process of finding the best model out of various models is called **optimization**. It can be achieved by below method:

### 1. R-squared method:

- R-squared is a statistical method that determines the goodness of fit.
- It measures the strength of the relationship between the dependent and independent variables on a scale of 0-100%.
- The high value of R-square determines the less difference between the predicted values and actual values and hence represents a good model.
- It is also called a **coefficient of determination**, or **coefficient of multiple determination** for multiple regression.
- It can be calculated from the below formula:

$$\text{R-squared} = \frac{\text{Explained variation}}{\text{Total Variation}}$$

## Assumptions of Linear Regression

Below are some important assumptions of Linear Regression. These are some formal checks while building a Linear Regression model, which ensures to get the best possible result from the given dataset.

- **Linear relationship between the features and target:**  
Linear regression assumes the linear relationship between the dependent and independent variables.
- **Small or no multicollinearity between the features:**  
Multicollinearity means high-correlation between the independent variables. Due to multicollinearity, it may difficult to find the true relationship between the predictors and target variables. Or we can say, it is difficult to determine which predictor variable is affecting the target variable and which is not. So, the model assumes either little or no multicollinearity between the features or independent variables.

- **Homoscedasticity** **Assumption:**  
Homoscedasticity is a situation when the error term is the same for all the values of independent variables. With homoscedasticity, there should be no clear pattern distribution of data in the scatter plot.
- **Normal distribution of error terms:**  
Linear regression assumes that the error term should follow the normal distribution pattern. If error terms are not normally distributed, then confidence intervals will become either too wide or too narrow, which may cause difficulties in finding coefficients.  
It can be checked using the **q-q plot**. If the plot shows a straight line without any deviation, which means the error is normally distributed.
- **No autocorrelations:**  
The linear regression model assumes no autocorrelation in error terms. If there will be any correlation in the error term, then it will drastically reduce the accuracy of the model. Autocorrelation usually occurs if there is a dependency between residual errors.

## 2. Explain the Anscombe's quartet in detail.

**Anscombe's Quartet** can be defined as a group of four data sets which are **nearly identical in simple descriptive statistics**, but there are some peculiarities in the dataset that **fools the regression model** if built. They have very different distributions and **appear differently** when plotted on scatter plots.

It illustrates the **importance of plotting the graphs** before analyzing and model building, and the effect of other **observations on statistical properties**. There are these four data set plots which have nearly **same statistical observations**, which provides same statistical information that involves **variance**, and **mean** of all x,y points in all four datasets.

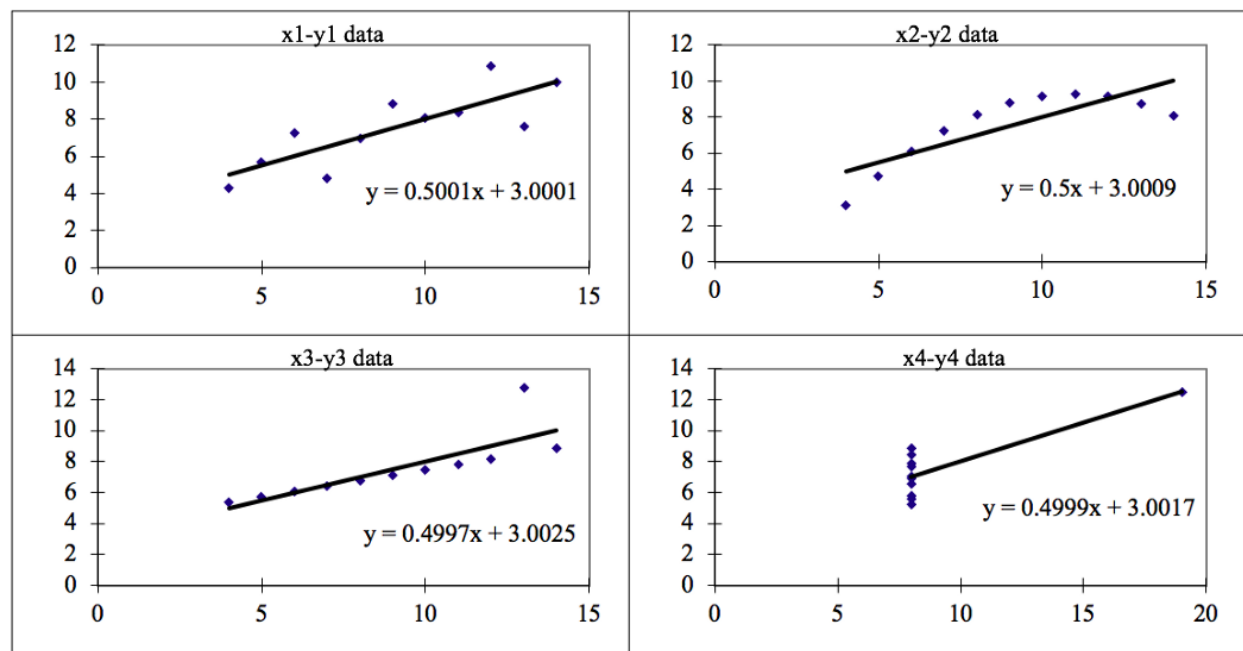
This tells us about the importance of visualising the data before applying various algorithms out there to build models out of them which suggests that the data features must be plotted in order to see the distribution of the samples that can help you identify the various anomalies present in the data like outliers, diversity of the data, linear separability of the data, etc. Also, the Linear Regression can be only be considered a fit for the **data with linear relationships** and is incapable of handling any other kind of datasets. These four plots can be defined as follows:

Anscombe's Data											
Observation	x1	y1		x2	y2		x3	y3		x4	y4
1	10	8.04		10	9.14		10	7.46		8	6.58
2	8	6.95		8	8.14		8	6.77		8	5.76
3	13	7.58		13	8.74		13	12.74		8	7.71
4	9	8.81		9	8.77		9	7.11		8	8.84
5	11	8.33		11	9.26		11	7.81		8	8.47
6	14	9.96		14	8.1		14	8.84		8	7.04
7	6	7.24		6	6.13		6	6.08		8	5.25
8	4	4.26		4	3.1		4	5.39		19	12.5
9	12	10.84		12	9.13		12	8.15		8	5.56
10	7	4.82		7	7.26		7	6.42		8	7.91
11	5	5.68		5	4.74		5	5.73		8	6.89

The statistical information for all these four datasets are approximately similar and can be computed as follows:

Anscombe's Data											
Observation	x1	y1		x2	y2		x3	y3		x4	y4
1	10	8.04		10	9.14		10	7.46		8	6.58
2	8	6.95		8	8.14		8	6.77		8	5.76
3	13	7.58		13	8.74		13	12.74		8	7.71
4	9	8.81		9	8.77		9	7.11		8	8.84
5	11	8.33		11	9.26		11	7.81		8	8.47
6	14	9.96		14	8.1		14	8.84		8	7.04
7	6	7.24		6	6.13		6	6.08		8	5.25
8	4	4.26		4	3.1		4	5.39		19	12.5
9	12	10.84		12	9.13		12	8.15		8	5.56
10	7	4.82		7	7.26		7	6.42		8	7.91
11	5	5.68		5	4.74		5	5.73		8	6.89
				Summary Statistics							
N	11	11		11	11		11	11		11	11
mean	9.00	7.50		9.00	7.500909		9.00	7.50		9.00	7.50
SD	3.16	1.94		3.16	1.94		3.16	1.94		3.16	1.94
r	0.82			0.82			0.82			0.82	

When these models are plotted on a scatter plot, all datasets generates a different kind of plot that is not interpretable by any regression algorithm which is fooled by these peculiarities and can be seen as follows:





The four datasets can be described as:

1. **Dataset 1:** this **fits** the linear regression model pretty well.
2. **Dataset 2:** this **could not fit** linear regression model on the data quite well as the data is non-linear.
3. **Dataset 3:** shows the **outliers** involved in the dataset which **cannot be handled** by linear regression model
4. **Dataset 4:** shows the **outliers** involved in the dataset which **cannot be handled** by linear regression model

Conclusion:

*The four datasets that were intentionally created to describe the importance of data visualisation and how any regression algorithm can be fooled by the same. Hence, all the important features in the dataset must be visualised before implementing any machine learning algorithm on them which will help to make a good fit model.*

What is Pearson's R?

**Correlation is a statistic that measures the relationship between two variables in the finance and investment industries.** It shows the strength of the relationship between the two variables as well as the direction and is represented numerically by the correlation coefficient. The numerical values of the correlation coefficient lies between **-1.0 and +1.0**.

A negative value of the correlation coefficient means that when there is a change in one variable, the other changes in a proportion but in the opposite direction, and if the value of the correlation coefficient is positive, both the variables change in a proportion and the same direction.

**When the value of the correlation coefficient is exactly 1.0, it is said to be a perfect positive correlation.** This situation means that when there is a change in one variable, either negative or positive, the second variable changes in lockstep, in the same direction.

**A perfect negative correlation means that two assets move in opposite directions, while a zero correlation implies no linear relationship at all.** We can determine the strength of the relationship between two variables by finding the absolute value of the correlation coefficient.

## **Pearson's Correlation Coefficient**

In Statistics, the Pearson's Correlation Coefficient is also referred to as **Pearson's  $r$** , **the Pearson product-moment correlation coefficient (PPMCC)**, or **bivariate correlation**. It is a statistic that measures the linear correlation between two variables. Like all correlations, it also has a numerical value that lies between -1.0 and +1.0.

Whenever we discuss correlation in statistics, it is generally Pearson's correlation coefficient. However, it cannot capture nonlinear relationships between two variables and cannot differentiate between dependent and independent variables.

**Pearson's correlation coefficient is the covariance of the two variables divided by the product of their standard deviations.** The form of the definition involves a "product moment", that is, the mean (the first moment about the origin) of the product of the mean-adjusted random variables; hence the modifier product-moment in the name.

Pearson's Correlation Coefficient is named after Karl Pearson. He formulated the correlation coefficient from a related idea by Francis Galton in the 1880s.

### **How is the Correlation coefficient calculated?**

Using the formula proposed by Karl Pearson, we can calculate a **linear relationship** between the two given variables. For example, a child's height increases with his increasing age (different factors affect this biological change). So, we can calculate the relationship between these two variables by obtaining the value of Pearson's Correlation Coefficient  $r$ . There are certain requirements for Pearson's Correlation Coefficient:

- Scale of measurement should be interval or ratio
- Variables should be approximately normally distributed
- The association should be linear
- There should be no outliers in the data

The formula given is:

$$r = \frac{N\sum xy - (\sum x)(\sum y)}{\sqrt{[N\sum x^2 - (\sum x)^2][N\sum y^2 - (\sum y)^2]}}$$

Where,

**N** = the number of pairs of scores

**$\sum xy$**  = the sum of the products of paired scores

**$\sum x$**  = the sum of x scores

**$\sum y$**  = the sum of y scores

**$\sum x^2$**  = the sum of squared x scores

**$\sum y^2$**  = the sum of squared y scores

Some steps are needed to be followed:

**Step 1:** Make a Pearson correlation coefficient table. Make a data chart using the two variables and name them as X and Y. Add three additional columns for the values of XY, X<sup>2</sup>, and Y<sup>2</sup>. Refer to this table.

Person	Age (X)	Income (Y)	XY	X <sup>2</sup>	Y <sup>2</sup>
1					
2					
3					

4					
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**Step 2:** Use basic multiplications to complete the table.

Person	Age (X)	Income (Y)	XY	X^2	Y^2
1	20	1500	30000	400	2250000
2	30	3000	90000	900	9000000
3	40	5000	200000	1600	25000000
4	50	7500	375000	2500	56250000

**Step 3:** Add up all the columns from bottom to top.

Person	Age (X)	Income (Y)	XY	X^2	Y^2
1	20	1500	30000	400	2250000
2	30	3000	90000	900	9000000
3	40	5000	200000	1600	25000000
4	50	7500	375000	2500	56250000

<b>Total</b>	<b>140</b>	<b>17000</b>	695000	<b>5400</b>	<b>92500000</b>
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**Step 4:** Use these values in the formula to obtain the value of r.

$$\begin{aligned}
 r &= [4 * 695000 - 140 * 17000] / \sqrt{\{4 * 5400 - (140)^2\} \{4 * 92500000 - (17000)^2\}} \\
 &= [2780000 - 2380000] / \sqrt{\{21600 - 19600\} \{370000000 - 289000000\}} \\
 &= 400000 / \sqrt{\{2000\} \{81000000\}} \\
 &= 400000 / \sqrt{162000000000} \\
 &= 400000 / 402492.24 \\
 &= 0.99
 \end{aligned}$$

The positive value of Pearson's correlation coefficient implies that if we change either of these variables, there will be a **positive effect** on the other. For example, if we increase the age there will be an increase in the income.

### **Determining the strength of the Pearson product-moment correlation coefficient**

As we have learned from the definition of the Pearson product-moment correlation coefficient, it measures the strength and direction of the linear relationship between two variables.

**The more inclined the value of the Pearson correlation coefficient to -1 and 1, the stronger the association between the two variables.**

## Assumptions

1. For the Pearson  $r$  correlation, both variables should be **normally distributed**. i.e the normal distribution describes how the values of a variable are distributed.
2. There should be **no significant outliers**. Pearson's correlation coefficient,  $r$ , is very sensitive to outliers, which can have a very large effect on the line of best fit and the Pearson correlation coefficient. This means — including outliers in your analysis can lead to misleading results.
3. Each variable should be **continuous** i.e. interval or ratios for example weight, time, height, age etc. If one or both of the variables are ordinal in measurement, then a Spearman correlation could be conducted instead.
4. The two variables have a **linear relationship**.. If the data points have a straight line (and not a curve), then the data satisfies the linearity assumption. If the data you have is not linearly related you might have to run a non-parametric .
5. The observations are **paired observations**. That is, for every observation of the independent variable, there must be a corresponding observation of the dependent variable. For example if

you're calculating the correlation between age and weight. If there are 12 observations of weight, you should have 12 observations of age. **i.e. no blanks.**

**6. Homoscedasticity.** Homoscedasticity describes a situation in which the error term (that is, the “noise” or random disturbance in the relationship between the independent variables and the dependent variable) is the same across all values of the independent variables

4. What is scaling? Why is scaling performed? What is the difference between normalized scaling and standardized scaling?

Feature scaling in machine learning is one of the most critical steps during the pre-processing of data before creating a machine learning model. Scaling can make a difference between a weak machine learning model and a better one.

Machine learning algorithm just sees number — if there is a vast difference in the range say few ranging in thousands and few ranging in the tens, and it makes the underlying assumption that higher ranging numbers have superiority of some sort. So these more significant number starts playing a more decisive role while training the model.

The machine learning algorithm works on numbers and does not know what that number represents. A weight of 10 grams and a price of 10 dollars represents completely two different things



Suppose we have two features of weight and price, as in the below table. The “Weight” cannot have a meaningful comparison with the “Price.” So the assumption algorithm makes that since “Weight” > “Price,” thus “Weight,” is more important than “Price.”

Name	Weight	Price
Orange	15	1
Apple	18	3
Banana	12	2
Grape	10	5

So these more significant number starts playing a more decisive role while training the model. Thus feature scaling is needed to bring every feature in the same footing without any upfront importance. Interestingly, if we convert the weight to “Kg,” then “Price” becomes dominant.

### When to do scaling?

Feature scaling is essential for machine learning algorithms that calculate **distances between data**. If not scale, the feature with a higher value range starts dominating when calculating distances, as explained intuitively in the “why?” section.

The ML algorithm is sensitive to the “**relative scales of features**,” which usually happens when it uses the numeric values of the features rather than say their rank.

**Normalization or Min-Max Scaling** is used to transform features to be on a similar scale. The new point is calculated as:

$$X_{\text{new}} = (X - X_{\text{min}}) / (X_{\text{max}} - X_{\text{min}})$$

This scales the range to [0, 1] or sometimes [-1, 1]. Geometrically speaking, transformation squishes the n-dimensional data into an n-dimensional unit hypercube. Normalization is useful when there are no outliers as it cannot cope up with them. Usually, we would scale age and not incomes because only a few people have high incomes but the age is close to uniform.

**Standardization or Z-Score Normalization** is the transformation of features by subtracting from mean and dividing by standard deviation. This is often called as Z-score.

$$X_{\text{new}} = (X - \text{mean}) / \text{Std}$$

Standardization can be helpful in cases where the data follows a Gaussian distribution. However, this does not have to be necessarily true. Geometrically speaking, it translates the data to the mean vector of original data to the origin and squishes or expands the points if std is 1 respectively. We can see that we are just changing mean and standard deviation to a standard normal distribution which is still normal thus the shape of the distribution is not affected.

Standardization does not get affected by outliers because there is no predefined range of transformed features.

### **Difference between Normalization and Standardization**

S.NO.	Normalization	Standardization
1.	Minimum and maximum value of features are used for scaling	Mean and standard deviation is used for scaling.
2.	It is used when features are of different scales.	It is used when we want to ensure zero mean and unit standard deviation.

S.NO.	Normalization	Standardization
3.	Scales values between [0, 1] or [-1, 1].	It is not bounded to a certain range.
4.	It is really affected by outliers.	It is much less affected by outliers.
5.	Scikit-Learn provides a transformer called <code>MinMaxScaler</code> for Normalization.	Scikit-Learn provides a transformer called <code>StandardScaler</code> for standardization.
6.	This transformation squishes the n-dimensional data into an n-dimensional unit hypercube.	It translates the data to the mean vector of original data to the origin and squishes or expands.
7.	It is useful when we don't know about the distribution	It is useful when the feature distribution is Normal or Gaussian.
8.	It is a often called as Scaling Normalization	It is a often called as Z-Score Normalization.

5. You might have observed that sometimes the value of VIF is infinite. Why does this happen?

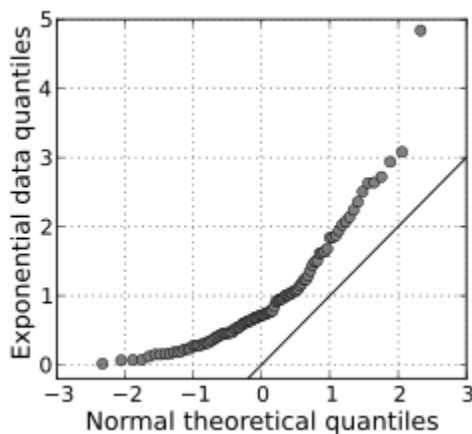
If there is perfect correlation, then  $VIF = \infty$ . This shows a perfect correlation between two independent variables. In the case of perfect correlation, we get  $R^2 = 1$ , which lead to  $1/(1-R^2)$  infinity. To solve this problem we need to drop one of the variables from the dataset which is causing this perfect multicollinearity.

An infinite VIF value indicates that the corresponding variable may be expressed exactly by a linear combination of other variables (which show an infinite VIF as well).

6. What is a Q-Q plot? Explain the use and importance of a Q-Q plot in linear regression.

Q-Q Plots (Quantile-Quantile plots) are plots of two quantiles against each other. A quantile is a fraction where certain values fall below that quantile. For example, the median is a quantile where 50% of the data fall below that point and 50% lie above it. The purpose of Q Q plots is to find out if two sets of data come from the same distribution. A 45 degree angle is plotted on the Q Q plot; if the two data sets come from a common distribution, the points will fall on that reference line.

A Q Q plot showing the 45 degree reference line:



If the two distributions being compared are similar, the points in the Q–Q plot will approximately lie on the line  $y = x$ . If the distributions are linearly related, the points in the Q–Q plot will approximately lie on a line, but not necessarily on the line  $y = x$ . Q–Q plots can also be used as a graphical means of estimating parameters in a location-scale family of distributions.

A Q–Q plot is used to compare the shapes of distributions, providing a graphical view of how properties such as location, scale, and skewness are similar or different in the two distributions.

**Assignment-based Subjective Questions**

## 1. From your analysis of the categorical variables from the dataset, what could you infer about their effect on the dependent variable?

The weather on a particular day effects the count of bike : the least number of bikes are distributed on heavy snow and rain days and most on clear days

Year on Year basis the count of bike distributed is increasing which is a good sign for the business

A working day or not does not have a huge impact , bike distribution is similar

May , June , July and Aug saw a good rise in bike count which also is the reason for more bikes distributed within the same season

The bike distribution is similar on all the days of the week as well

Conclusion : The month , year and type of weather seems to effect the bike count more than other variables.

## 2. Why is it important to use drop\_first=True during dummy variable creation?

If we don't drop the first column then our dummy variables will be correlated .This may affect some models adversely and the effect is stronger when the cardinality is smaller. For example iterative models may have trouble converging and lists of variable importances may be distorted.

If we don't use "drop\_first" you will get a redundant feature, let's see an example.

If we have a feature "Is\_male", you use "get\_dummies" you will get two features "Is\_male\_0" and "Is\_male\_1", but if we look carefully they are redundant actually you just need one of them, the other one will be the exact opposite of the other.

e.g.1:

row	is_male
0	1
1	0

after applying get\_dummies

row	is_male_0	is_male_1
0	0	1
1	1	0

3. Looking at the pair-plot among the numerical variables, which one has the highest correlation with the target variable?

The highest correlation is of the variable temperature.

4. How did you validate the assumptions of Linear Regression after building the model on the training set?

I had checked for the Rsquared values and made a comparison between the train and test values

The error terms were checked and they were normally distributed

A graph was plotted to check if there was a pattern among the residual terms or not.

The F-statistic value was also high , which confirmed it was a good fit model.

5. Based on the final model, which are the top 3 features contributing significantly towards explaining the demand of the shared bikes?

The variables in descending order are :

Temp: +0.4688  
light\_Snow\_Rain\_Thrunderstorm: -0.2843  
Yr : +0.2342

The light\_Snow\_Rain\_Thrunderstorm(LSRT) effects the model negatively :

For an increase of 1 unit in Temp the count of bike increases by .4688

For an increase of 1 unit in LSRT the count of bike decreases by .2843

For an increase of 1 unit in Yr the count of bike increases by .2342