Assignment 2

AI1110: Probability and Random Variables
Indian Institute of Techonology, Hyderabad

Gunethra Bommineni*

10.15.1.12 A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 (see Fig. 4), and these are equally likely outcomes. What is the probability that it will point at:

(i) 8?
(ii) an odd number?
(iii) a number greater than 2?
(iv) a number less than 9?

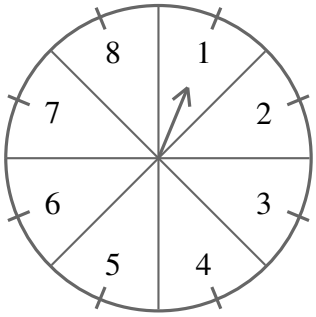

Fig. 4. Spinner

Solution: Let X be a random variable defined as the value given by the pointer. The distribution is unform since all the outcomes are equally likely.

$$\therefore \Pr(X = i) = \frac{1}{8} \tag{1}$$

Let $F_X(i)$ be the Cumulative distribution function(CDF) such that;

$$F_X(i) = P(X \le i) \tag{2}$$

$$= \begin{cases} 0, & i \le 0 \\ \frac{i}{8} & 1 \le i \le 8 \\ 1, & i \ge 9 \end{cases} \tag{3}$$

See python code for PMF and CDF plots: [1]

*The student is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: ee22btech11205@iith.ac.in.

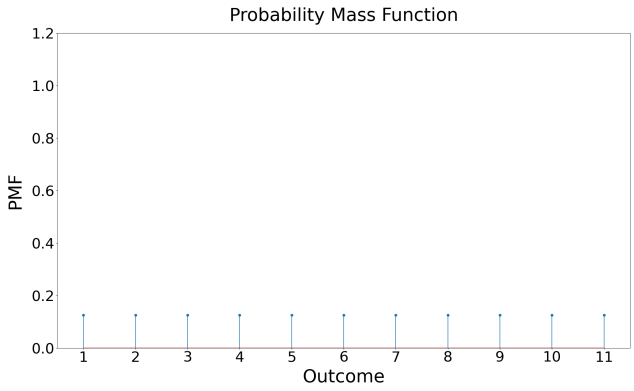

Fig. 4. Plot of Probability Mass Function

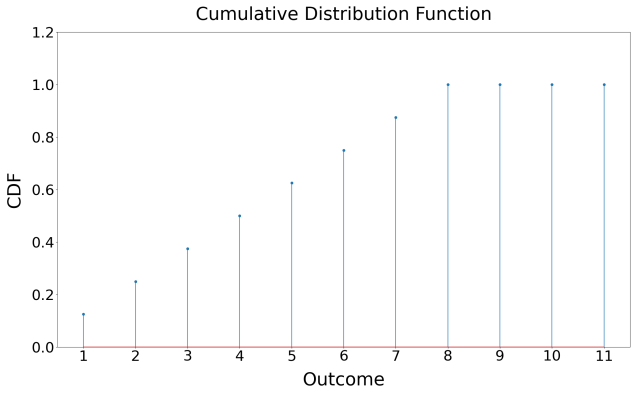

Fig. 4. Plot of Cumulative Distribution Function

(i) For i = 8, required probability is equivalent to;

$$\Pr(X = 8) = F_X(8) - F_X(7) \tag{4}$$

$$= \frac{1}{8} = 0.125 \tag{5}$$

(ii) For i being odd, required probability is equiv-

alent to;

$$\begin{aligned}\Pr\left(X=\{1,3,5,7\}\right)&=F_X\left(7\right)-F_X\left(6\right)+F_X\left(5\right)-F_X\left(4\right)\\&\quad+F_X\left(3\right)-F_X\left(2\right)+F_X\left(1\right)-F_X\left(0\right)\\&=\frac{4}{8}=0.5\end{aligned}\tag{6}$$

(iii) For i greater than 2, required probability is equivalent to;

$$\Pr\left(X>2\right)=1-\Pr\left(X\leq 2\right)\tag{7}$$

$$=1-\left(F_X\left(2\right)-F_X\left(0\right)\right)\tag{8}$$

$$=\frac{6}{8}=0.75\tag{9}$$

(iv) For i less than 9, required probability is equivalent to;

$$\Pr\left(1\leq X<9\right)=F_X\left(8\right)-F_X\left(0\right)\tag{10}$$

$$=\frac{8}{8}=1\tag{11}$$

See simulation using python: [2]

References

[1] https://github.com/Gunethra/AI1110_2023/tree/master/Assignment_2/code/pmf.py
https://github.com/Gunethra/AI1110_2023/tree/master/Assignment_2/code/cdf.py.

[2] https://github.com/Gunethra/AI1110_2023/tree/master/Assignment_2/code/code.py.