

Assignment 1

AI1110: Probability and Random Variables
Indian Institute of Technology Hyderabad

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Question 13.2.12:

Problem Statement

A die is tossed thrice. Find the probability of getting an odd number at least once.

Solution

Let X be a random variable defined as the number of odd number occurrences in three trials. Possible values of random variable X are;

$$X = \{0, 1, 2, 3\} \quad (1)$$

Required values of X are;

$$X = \{1, 2, 3\} \quad (2)$$

which correspond to at least one odd number. Probability of an observation being odd is;

$$p = \frac{3}{6} = \frac{1}{2} \quad (3)$$

Let $F_X(i)$ be the **Cumulative distribution function**(CDF) such that;

$$F_X(i) = \Pr(X > i) \quad (4)$$

$$\Pr(X = i) = {}^n C_i \times p^i \times (1 - p)^{(n-i)} \quad (5)$$

$$\text{where } n = 3 \text{ and } i \in \{0, 1, 2, 3\} \quad (6)$$

On solving;

$$\Pr(X = 1) = {}^3 C_1 \times \left(\frac{1}{2}\right)^1 \times \left(\frac{1}{2}\right)^{(3-1)} \quad (7)$$

$$\Rightarrow \Pr(X = 1) = \frac{3}{8} \quad (8)$$

$$\Pr(X = 2) = {}^3 C_2 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^{(3-2)} \quad (9)$$

$$\Rightarrow \Pr(X = 2) = \frac{3}{8} \quad (10)$$

$$\Pr(X = 3) = {}^3 C_3 \times \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^{(3-3)} \quad (11)$$

$$\Rightarrow \Pr(X = 3) = \frac{1}{8} \quad (12)$$

Required probability of at least one odd observation is equivalent to $F_X(0)$.

$$F_X(0) = \Pr(X = (i > 0)) \quad (13)$$

$$= \sum_{i=1}^3 \Pr(X = i) \quad (14)$$

$$= \frac{7}{8} \quad (15)$$

Therefore, probability of at least one odd observation is;

$$\therefore \Pr(\text{At least one odd}) = \frac{7}{8} = 0.875 \quad (16)$$

Conclusion

The probability of getting an odd number at least once is 0.875 as also calculated using the python code. [1]

REFERENCES

[1] https://github.com/Gunethra/AI1110_2023/tree/master/code.

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