

A photograph of a modern concrete staircase with a metal handrail. On each step, there is a small, glowing green rectangular light. The lights are arranged in a zigzag pattern that follows the curve of the stairs, starting from the bottom left and ending at the top right.

Social Network Analysis

CASCADE BEHAVIORS AND NETWORK
EFFECTS

Information Diffusion

- ❑ Diffusion is the net movement of anything from a region of higher concentration to a region of lower concentration
- ❑ Driven by a gradient in concentration
- ❑ Information Diffusion is the process by which information is spread from one place to another through interactions
- ❑ Diffusion process involves three main elements:
 - ❑ Sender: An entity (or a group of entities) responsible for initiating the diffusion process
 - ❑ Receiver: An entity (or a group of entities) receives the diffusion information from the sender(s)
 - ❑ Medium: The channel through which the diffusion information is sent from the sender(s) to the receiver(s).

Cascade Behavior: Real-world Instances

Healthcare

- Disease Propagation
- Epidemic Spreading

Socio-political Cascades

- Arab Spring Movement in 2010 – 2012
- From small protests began in Leipzig to Fall of the Berlin Wall in 1989
- #MeToo movement against sexual abuse and sexual harassment

Financial Market Cascades

- Market Bubble
- A stock becomes overly popular among investors
- Viral marketing

Social network

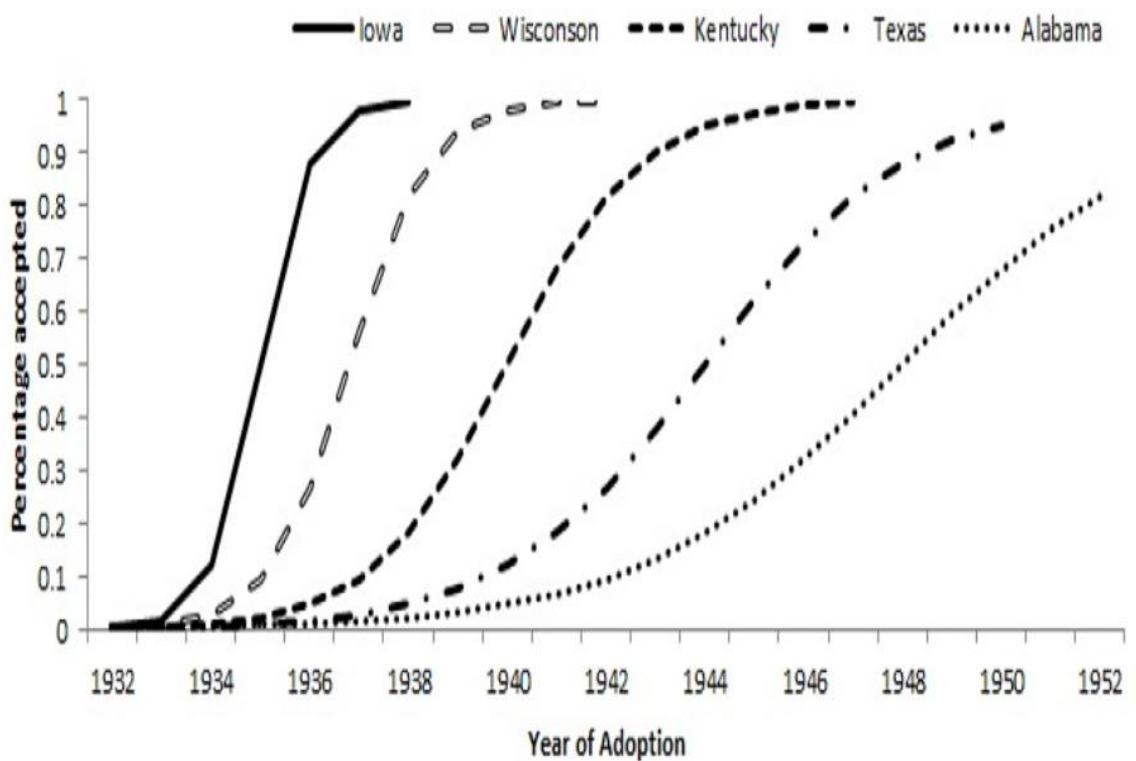
- Rumor spread
- Belief Spread
- Fake News virality

Information Diffusion: Information Cascade



- An **Information cascade** is a phenomenon in which a number of people make the same decision in a sequential fashion.
- The phenomenon is found widely in **behavioral economics** and **network theory**
- Similar to, but not identical to **herd behavior**

Information Diffusion: Diffusion of Innovations

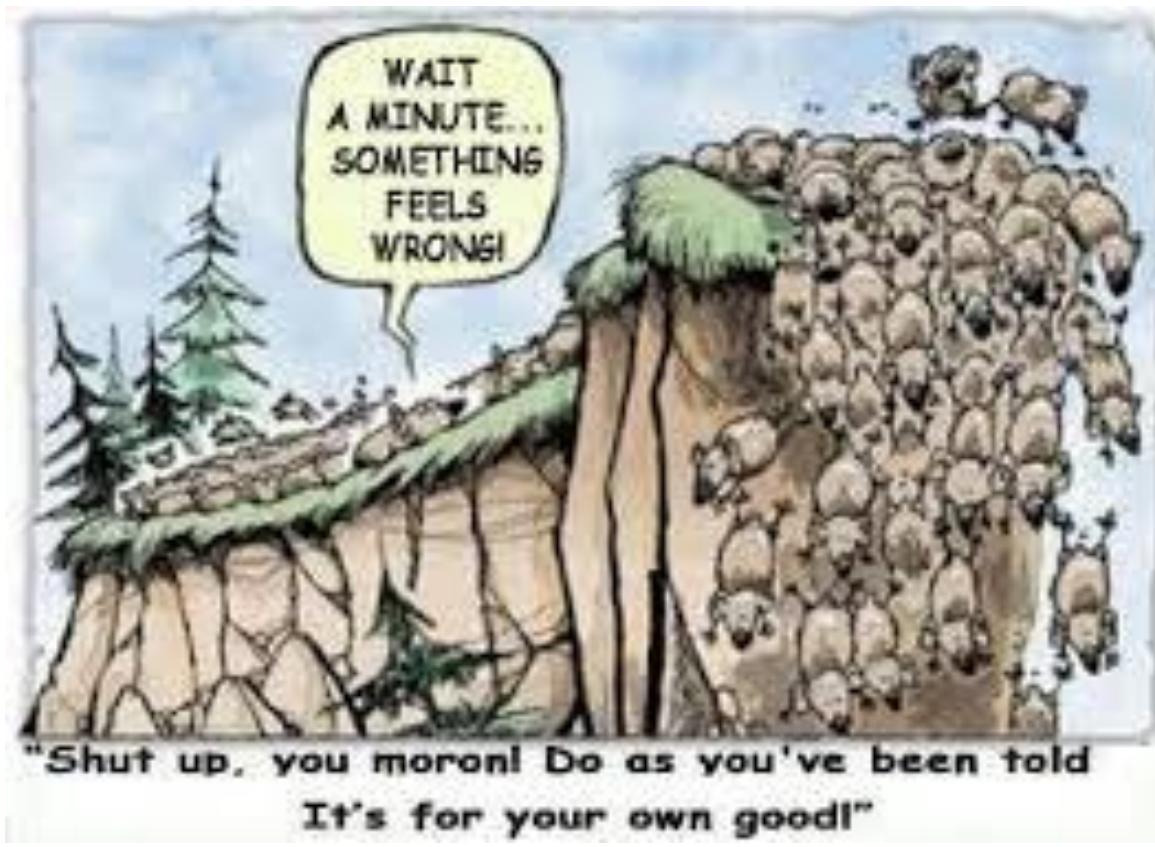


- Success/failure of an innovation is highly guided by the structure of the network formed by the initial adopters
- adoption of hybrid seed corn by farmers
- influenced by their neighbors in the community
- adoption of a new drug by the doctors
- Assurance from social peer connections

Adoption of hybrid seed corn by farmers in USA

<http://homepage.cs.uiowa.edu/~sriram/196/spring12/lectureNotes/Lecture15.pdf>

Information Diffusion: Herd Behavior



- The behavior of individuals in a group acting collectively without centralized direction
- Human based herd behaviour: demonstrations, riots, general strikes, religious gatherings, judgement and opinion-forming, etc.
- Often a useful tool in marketing; if used properly, can lead to increases in sales
- Herding behavior turns violent sometimes, particularly when confronted by an opposing ethnic or racial group

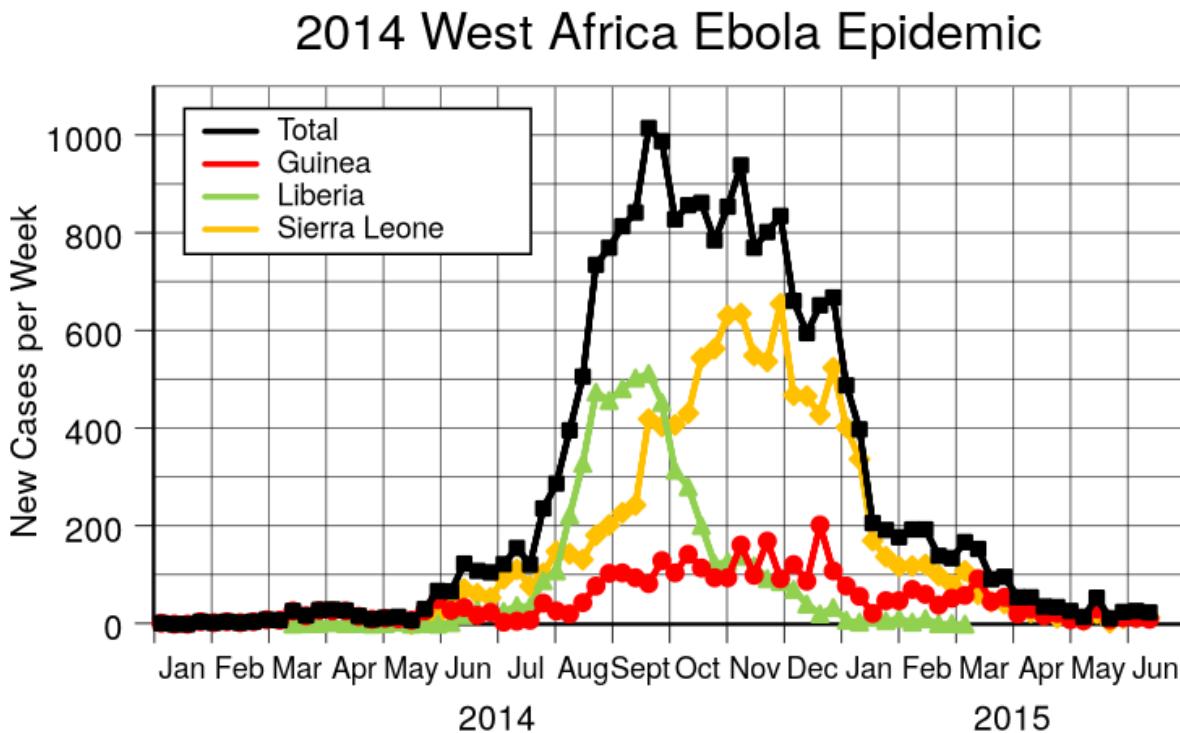
Information Diffusion: Echo Chambers



- ❑ Situations in which beliefs are amplified or reinforced by communication and repetition inside a closed system
- ❑ A harmonious group of people amalgamate and develop tunnel vision
- ❑ Social communities become fragmented by echo chambers
- ❑ Causes powerful reinforcements of rumors and fake news due to the unchallenged trust in the evidence supplied by their peers

<https://theconversation.com/the-problem-of-living-inside-echo-chambers-110486>

Information Diffusion: Epidemics

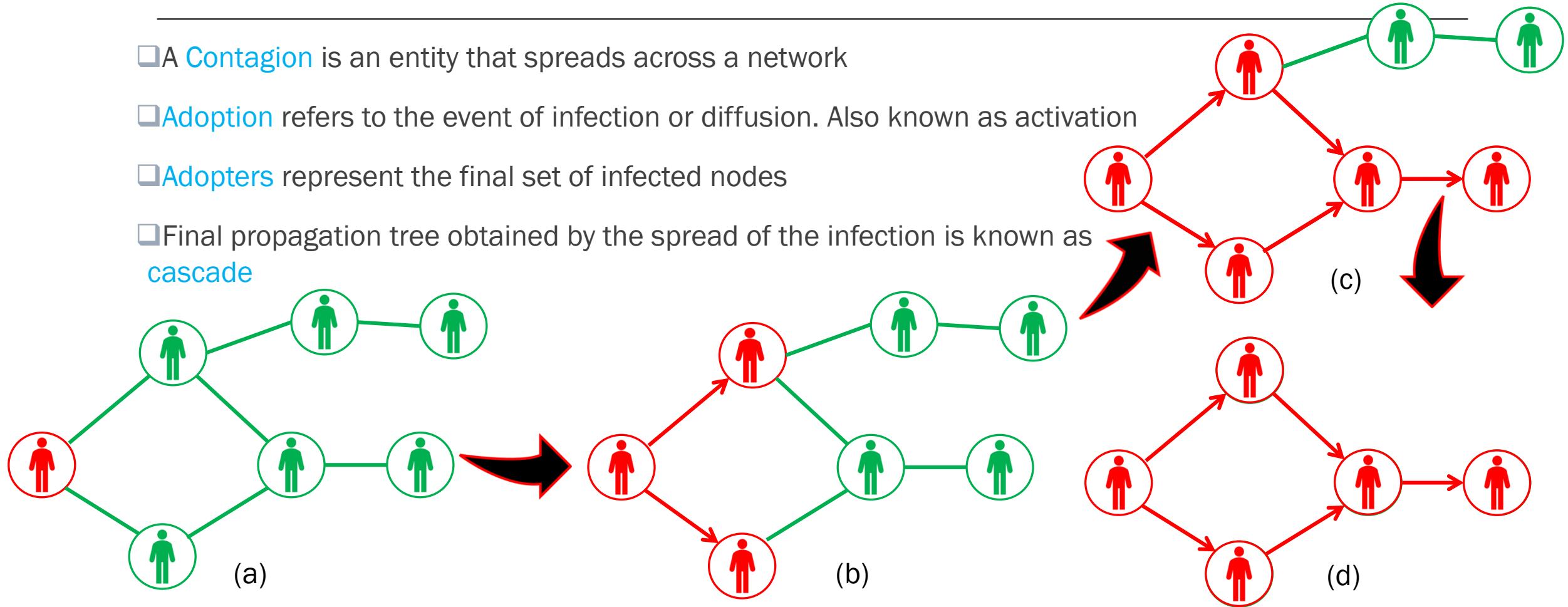


- ❑ Rapid spread of disease to a large number of people in a given population within a short period of time
- ❑ Epidemic models are similar to diffusion of innovations models
- ❑ Only difference is: individuals do not decide whether to become infected or not

https://commons.wikimedia.org/wiki/File:2014_West_Africa_Ebola_Epidemic_-_New_Cases_per_Week.svg

Information Diffusion: Terminologies

- ❑ A **Contagion** is an entity that spreads across a network
- ❑ **Adoption** refers to the event of infection or diffusion. Also known as activation
- ❑ **Adopters** represent the final set of infected nodes
- ❑ Final propagation tree obtained by the spread of the infection is known as **cascade**



Cascade Model: Decision-based Model

- ❑ Given a network, each node has the **freedom to decide** whether to adopt a contagion or not
- ❑ Originated from the idea of **local interaction models** described by Morris in 2000
- ❑ Decision at each node is influenced by the **behavior of nodes in its neighborhood**
- ❑ Nodes decide to adopt a new contagion driven by a **direct benefit** or **payoff**
- ❑ The **payoff** by adopting a contagion is directly proportional to the **number of its neighbors** that have adopted the same contagion
- ❑ Can be explained using a **two-player coordination game**
 - ❑ Given a number of strategies, the end goal of the players is to coordinate on the same strategy to maximize their payoffs

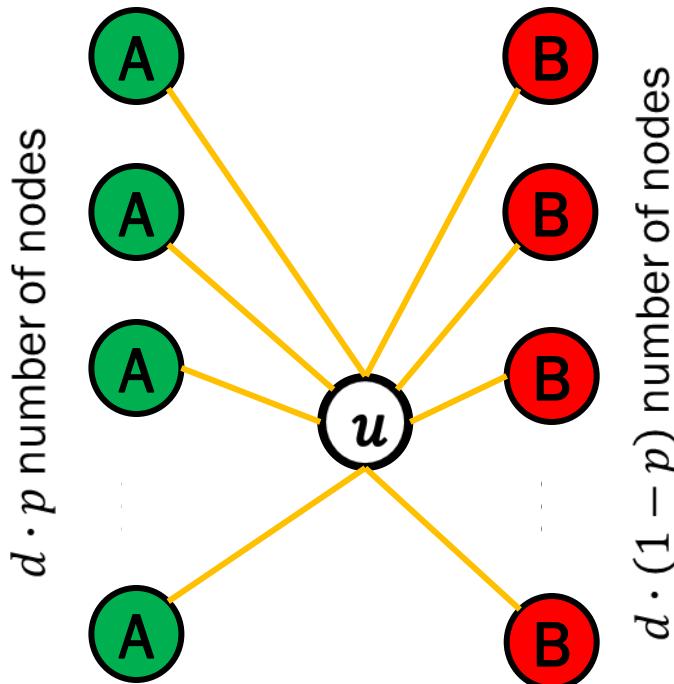
Decision-based Cascade Model: Two-player Coordination Game

| u 's decision | v 's decision | Payoff |
|-----------------|-----------------|--------|
| A | A | a^* |
| B | B | b^* |
| A | B | 0 |
| B | A | 0 |

Payoff distribution for different adoption strategies
* a and b are positive constants

- A and B: two possible strategies that each node in network $G(V, E)$ could adopt
- Each node u will play its own independent game
- Final payoff is the sum of payoffs for all the games
- To calculate the required threshold at which a node u would decide to go with strategy A

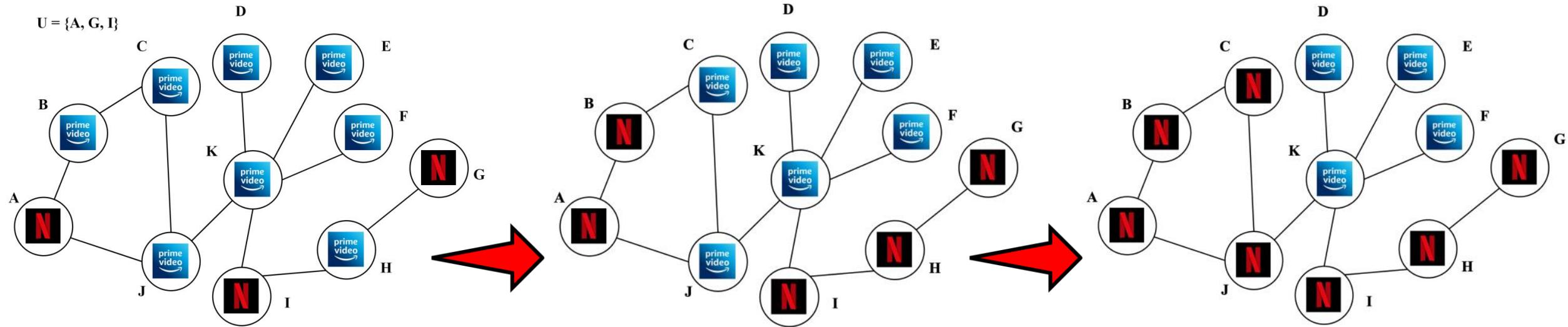
Decision-based Cascade Model: Two-player Coordination Game



- ❑ Node u has d neighbours
 - ❑ p fraction of neighbours adopt strategy A
 - ❑ Rest adopts strategy B
- ❑ Total payoff for node u if it goes with strategy A = $a \cdot d \cdot p$
- ❑ Total payoff for node u if it goes with strategy B = $b \cdot d \cdot (1 - p)$
- ❑ Node u would adopt contagion A if

$$p \geq \frac{b}{a+b}$$

Decision-based Cascade Model: Illustration



The threshold for a switch from Amazon Prime Video to Netflix at a node
is 0.50

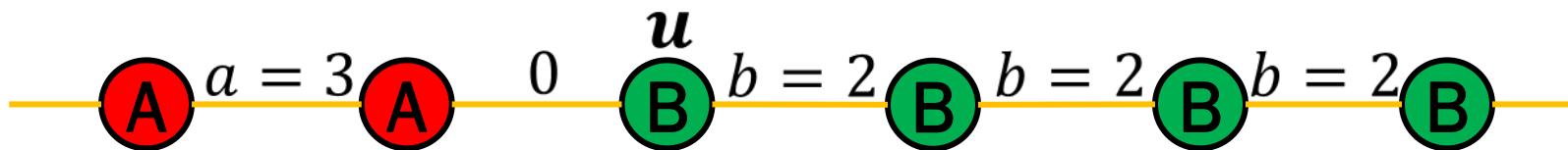
Multiple Choice Decision-based Cascade Model

- Allows a node to adopt more than one strategy/behavior
- In case a node prefers to go with both the strategies A and B, it would incur an additional cost c
- The revised payoff distribution:

| u 's decision | v 's decision | Payoff |
|-----------------|-----------------|--------------|
| AB | A | a^* |
| AB | B | b^* |
| AB | AB | $\max(a, b)$ |

Payoff for a multiple choice decision model
* a and b are positive constants

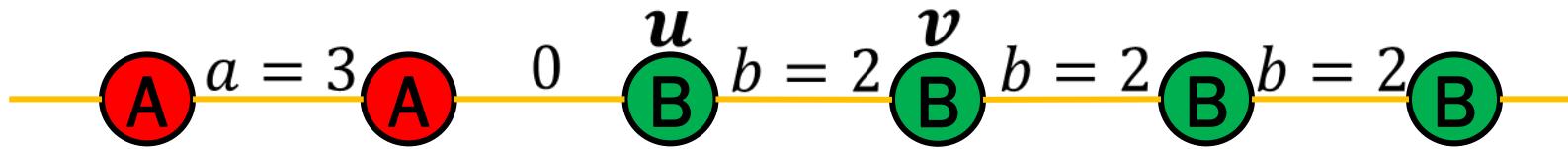
Cascades for Infinite Chain Networks: Single Choice



- ❑ Consider the case: $a = 3, b = 2$
- ❑ Two possible choice for node u
 - ❑ Stick with **strategy B**, total payoff: $0 + 2 = 2$
 - ❑ Switch to **strategy A**, total payoff: $3 + 0 = 3$
- ❑ So, node u would adopt strategy A
- ❑ And the cascade continues...



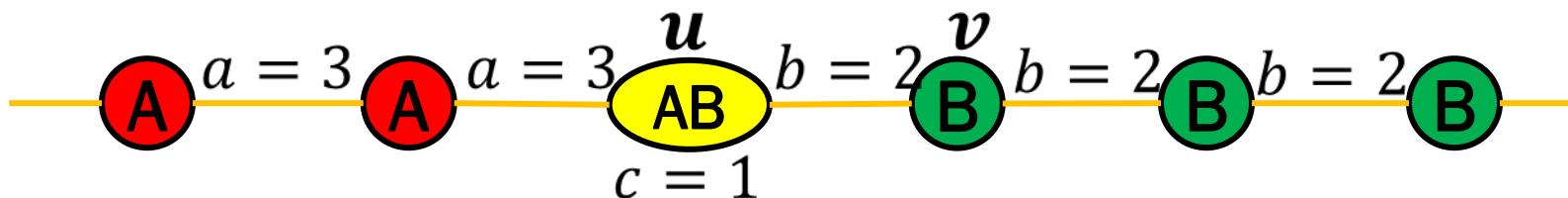
Cascades for Infinite Chain Networks: Multiple Choice: Case I



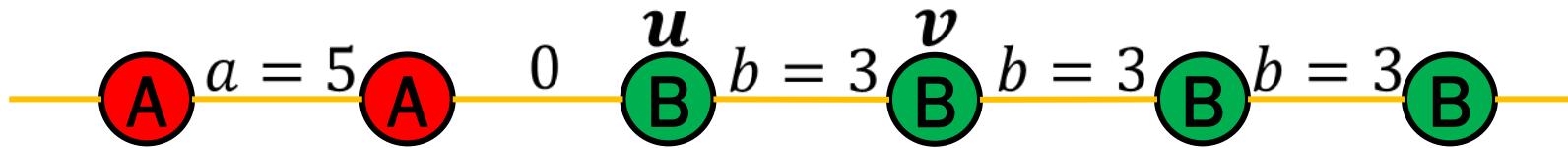
- ❑ Consider the case: $a = 3, b = 2, c = 1$
- ❑ Two possible choice for node u
 - ❑ Stick with **strategy B**, total payoff: $0 + 2 = 2$
 - ❑ Switch to **strategy A**, total payoff: $3 + 0 = 3$
 - ❑ Switch to **strategy AB**, total payoff: $3 + 2 - 1 = 4$

❑ So, node u would adopt strategy AB

❑ And system is stable now!!

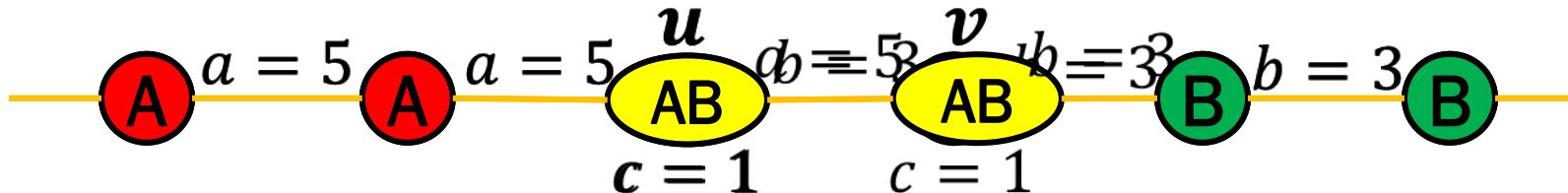


Cascades for Infinite Chain Networks: Multiple Choice: Case II



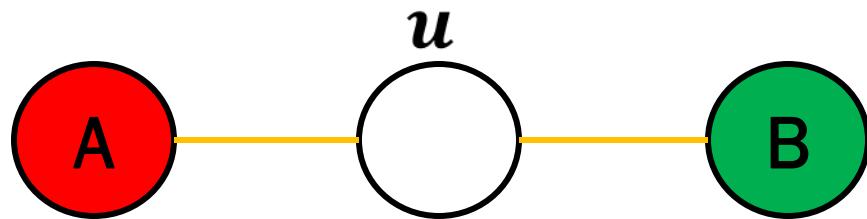
- Consider the case: $a = 5, b = 3, c = 1$
- Two possible choice for node u
 - Stick with **strategy B**, total payoff: $0 + 3 = 3$
 - Switch to **strategy A**, total payoff: $5 + 0 = 5$
 - Switch to **strategy AB**, total payoff: $5 + 3 - 1 = 7$
- So, node u would adopt strategy AB

- Two possible choice for node v
 - Stick with **strategy B**, total payoff: $3 + 3 = 6$
 - Switch to **strategy A**, total payoff: $5 + 0 = 5$
 - Switch to **strategy AB**, total payoff: $5 + 3 - 1 = 7$
- So, node v would adopt strategy AB
- And the cascade continues!!

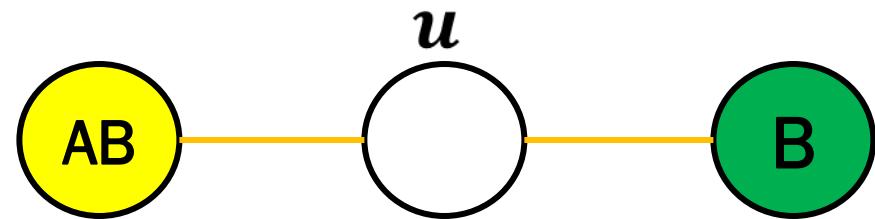


Cascade in Infinite Chain Networks: Generic Model

- Let us consider an infinite chain network with strategy set $\{A, B, AB\}$
- We consider the scenario: $a = a, b = 1, c = c$
- Two possible cases may arise:

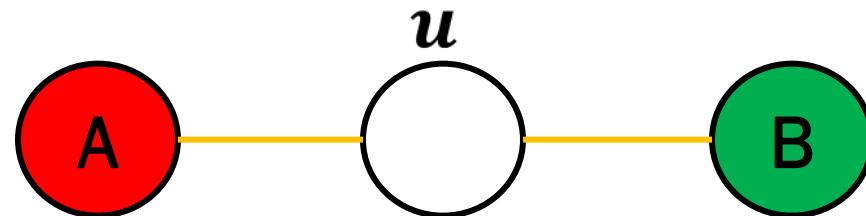


Case A



Case B

Generic Model: Case A



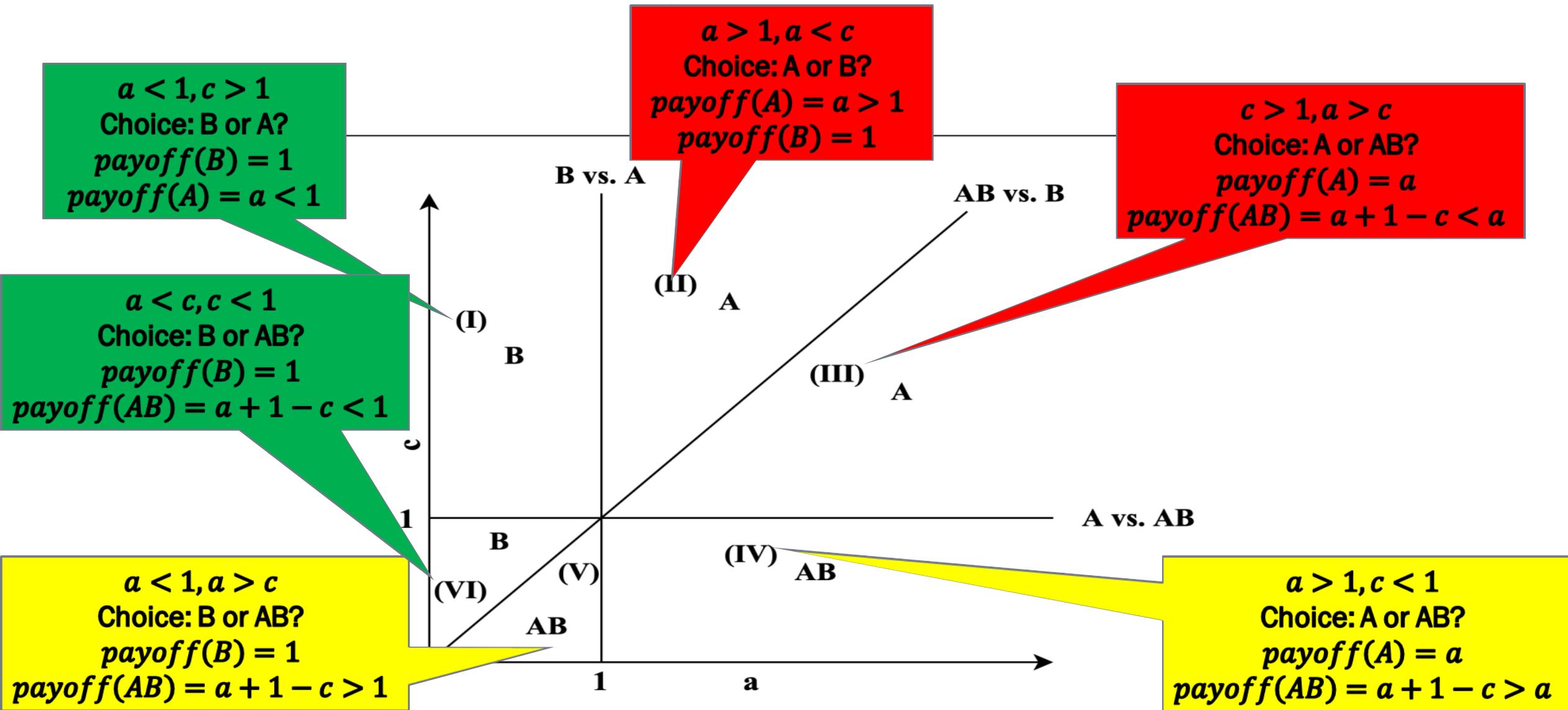
❑ Three possible options for node u

1. Adopt Behavior A; Payoff = $a + 0 = a$
2. Adopt Behavior B; Payoff = $0 + 1 = 1$
3. Adopt Behavior AB; Payoff = $a + 1 - c$

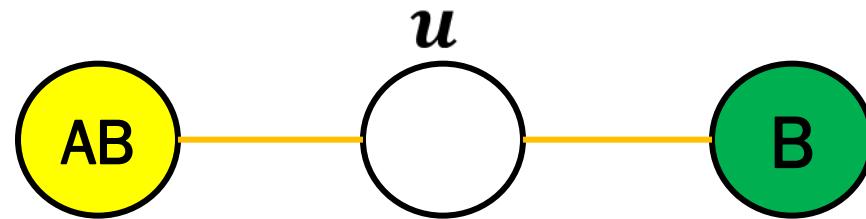
❑ Breakpoint Equations:

- a. B versus A: $a = 1, a < 1$: Prefer strategy B; $a > 1$: Prefer strategy A
- b. AB versus B: $a = c, a < c$: Prefer strategy B; $a > c$: Prefer strategy AB
- c. A versus AB: $c = 1, c < 1$: Prefer strategy AB; $c > 1$: Prefer strategy A

Generic Model: Case A



Generic Model: Case B



□ Three possible options for node u

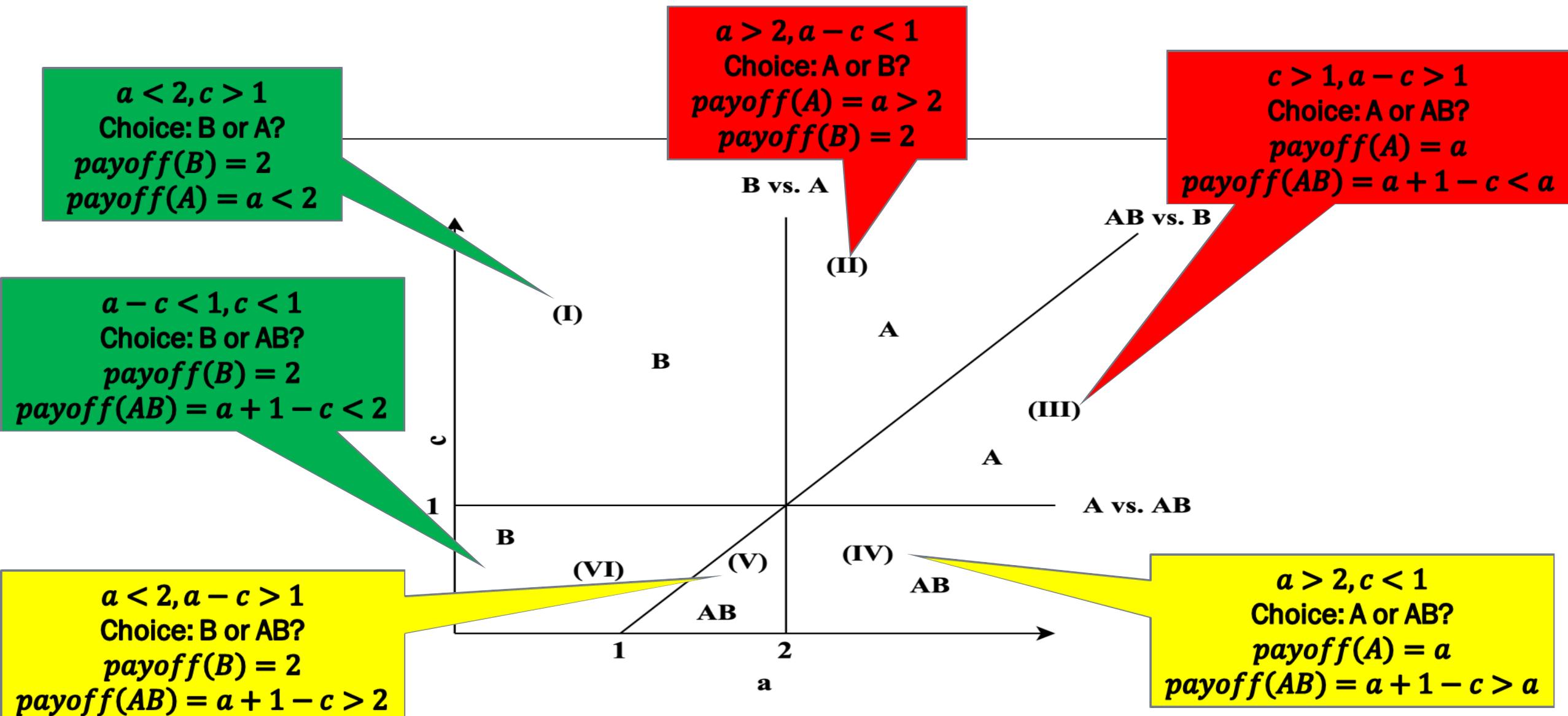
1. Adopt Behavior A; Payoff = $a + 0 = a$
2. Adopt Behavior B; Payoff = $1 + 1 = 2$
3. Adopt Behavior AB; Payoff = $\underline{a + 1 - c}$ if $\max(a, 1) = a$

(a + 1 - c) is circled in blue.

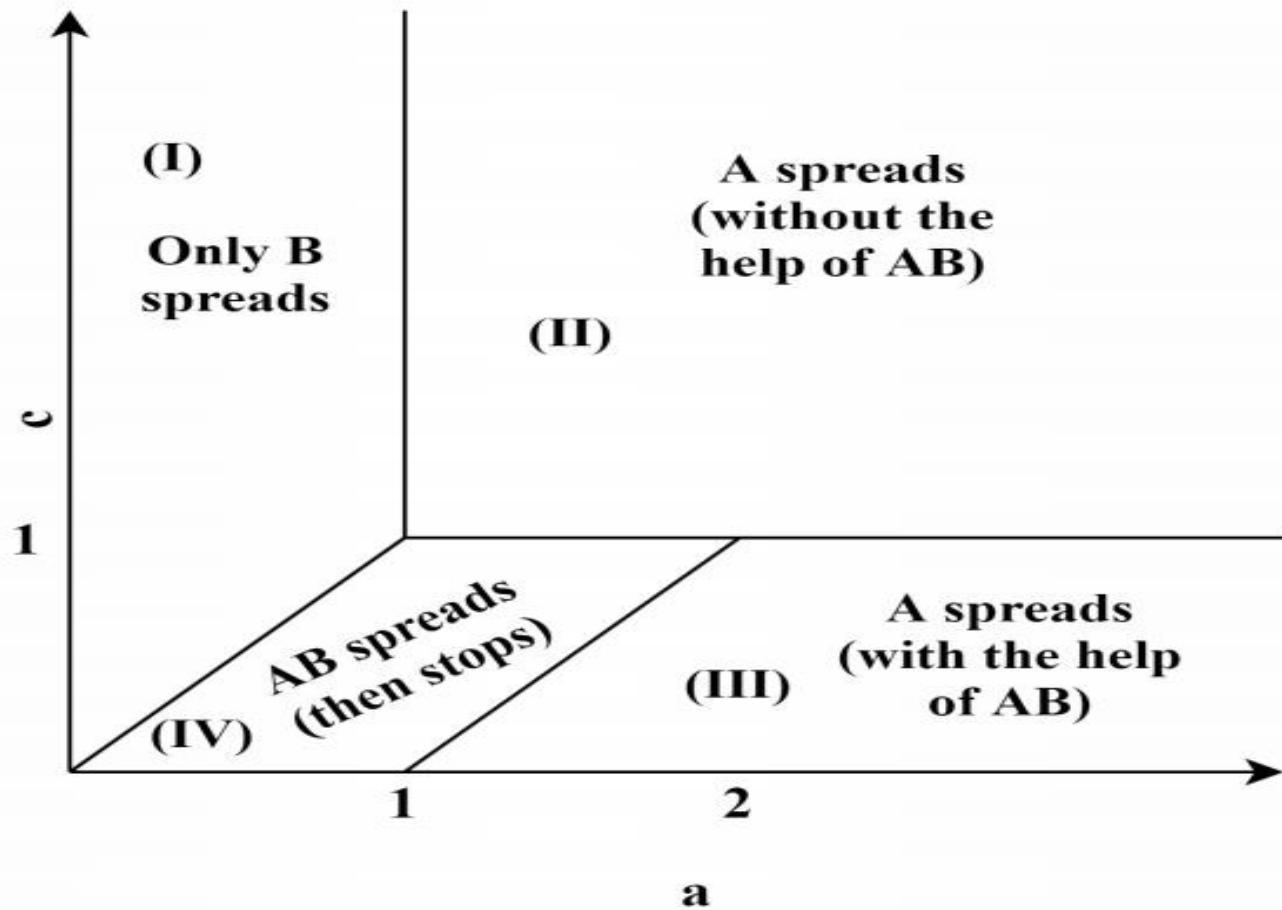
□ Breakpoint Equations:

- a. B versus A: $a = 2, a < 2$: Prefer strategy B; $a > 2$: Prefer strategy A
- b. AB versus B: $a - c = 1, a - c < 1$: Prefer strategy B; $a - c > 1$: Prefer strategy AB
- c. A versus AB: $c = 1, c < 1$: Prefer strategy AB; $c > 1$: Prefer strategy A

Generic Model: Case B



Generic Model: Combined



Case Study: The “Indignados” Movement/15-M Movement



<http://bit.ly/3sS4VVE>

- A series of protests, demonstrations, and occupations against austerity policies in Spain
- Began around the local and regional elections of 2011 and 2012 (starting [15 May 2011](#))
- According to [RTVE, the Spanish public broadcasting company](#), between 6.5 and 8 million Spaniards participated in these events
- Protesters/participants coordinated their movements using [Twitter](#)
- Dynamics of the participation of the citizens can be viewed as a cascade problem

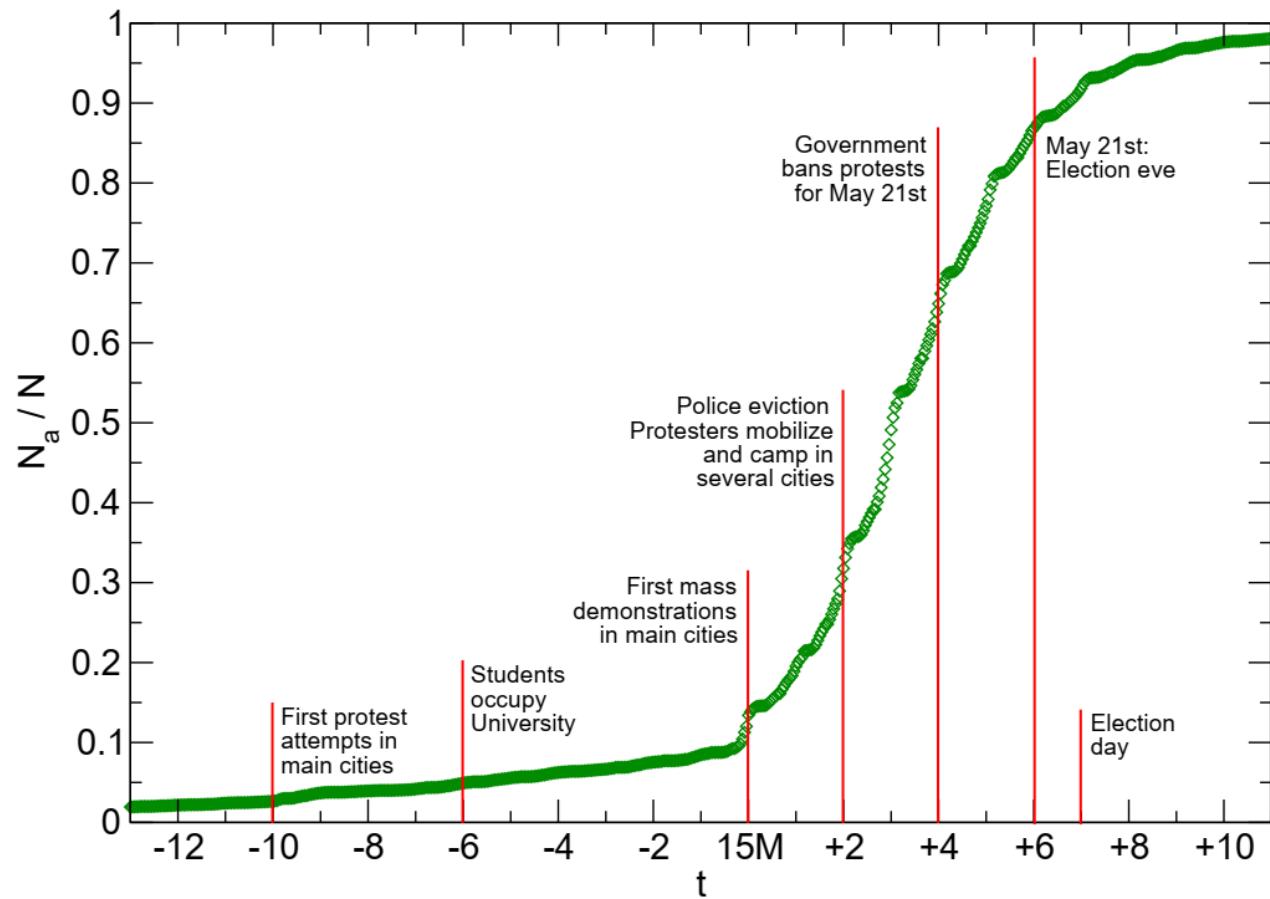
The “Indignados” Movement: Problem Formulation

- ❑ Answers to the following three questions are critical for the formulation
 - When does one generally start to tweet about the protests?
 - Does neighbor have any influence to the answer of the above question?
 - If yes, how much is the influence?
- ❑ González-Bailón et al. (2011) identified 70 hashtags used by the protesters
- ❑ Tweets containing these hashtags collected for 1 month
 - ❑ 581,750 tweets collected
 - ❑ 87,569 users were identified as relevant
- ❑ Condition for relevancy: any user who tweeted with the specified set of hashtags and has followers and followees who did the same

The “Indignados” Movement: Cascade Networks

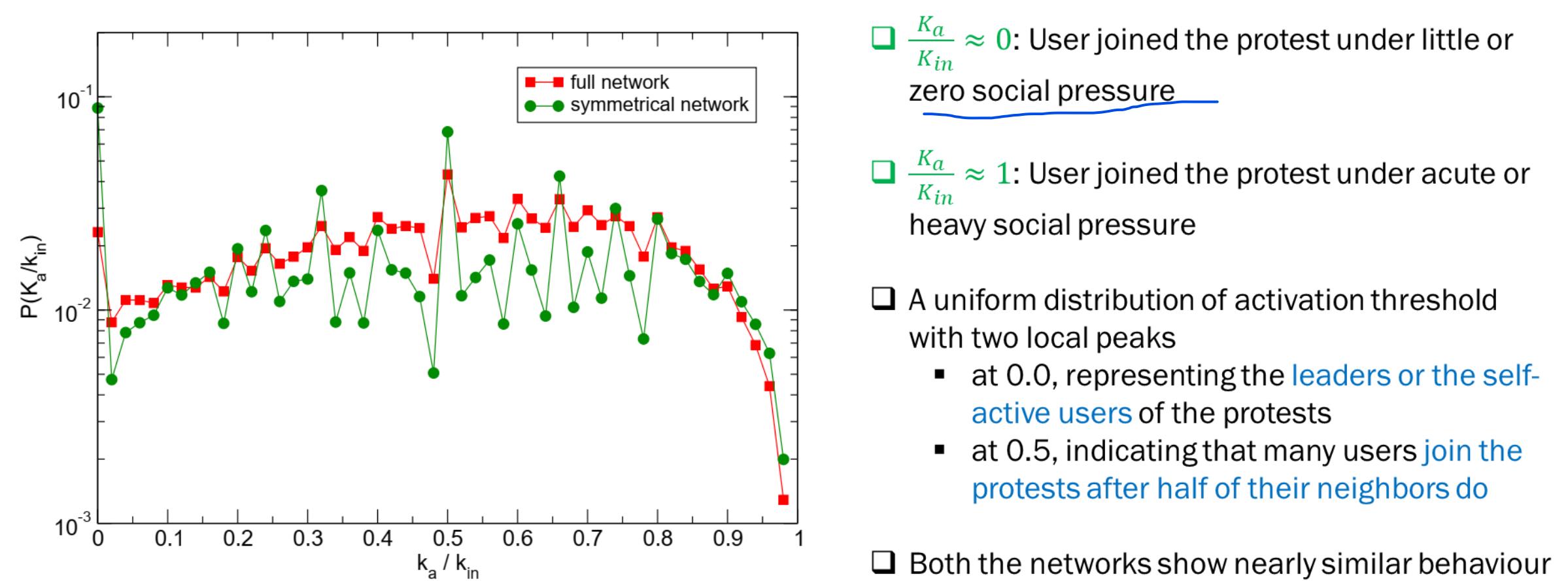
- ❑ Two kinds of networks in consideration:
 - **Full Network:** If a user follows another user, an edge will be formed between them connecting the two users in the network
 - **Symmetric Network:** If both the users follow each other, a single edge will be formed between the users in the network
- ❑ **User activation time** is refers to the time when the user starts tweeting about the protests
- ❑ K_{in} denotes the **number of neighbors** when a user becomes active
- ❑ K_a denotes the **number of active neighbors** when a user becomes active
- ❑ **Activation Threshold** $\left(\frac{K_a}{K_{in}}\right)$ refers to the fraction of active neighbors when a user becomes active
- ❑ **Recruitment** refers to the event wherein a user tweets about the ongoing protest

The “Indignados” Movement: Cascade Networks

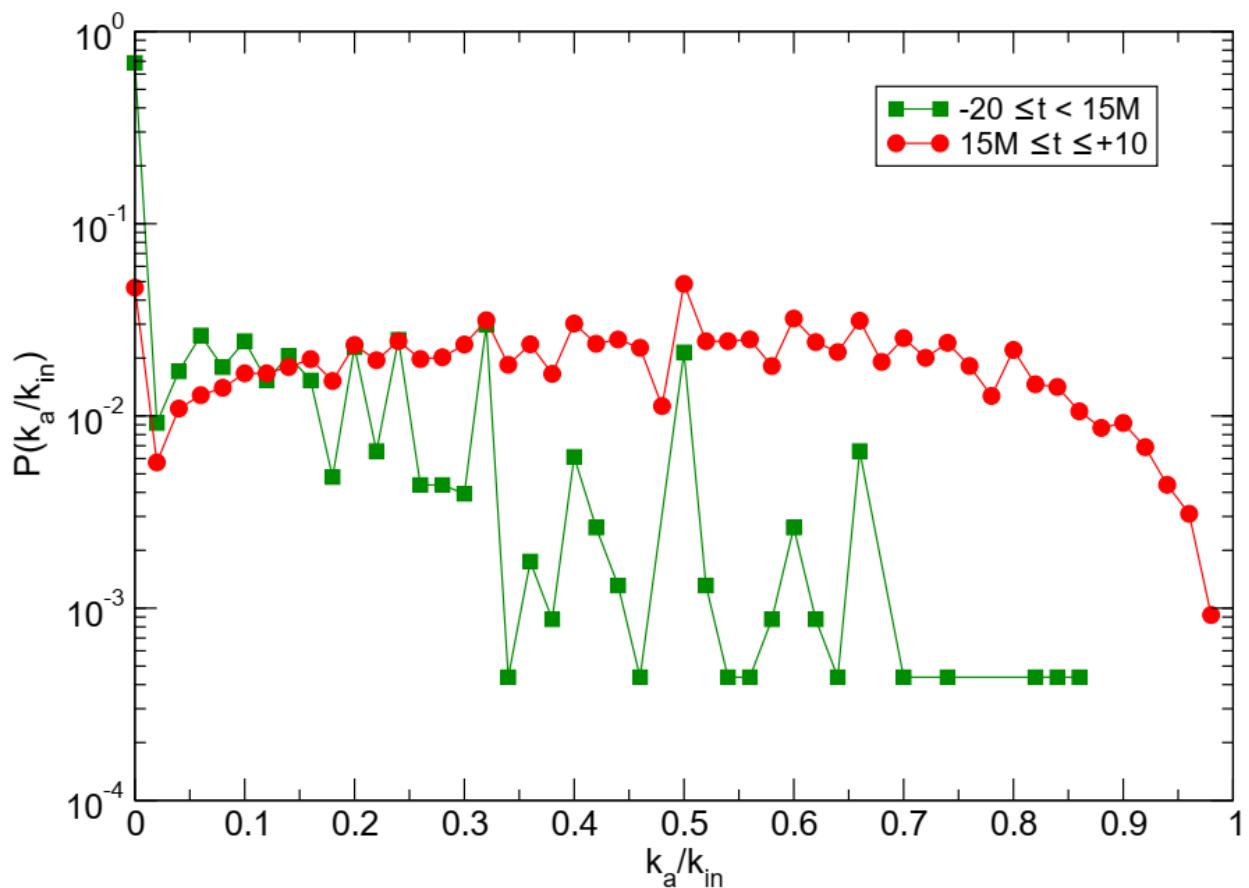


- Fraction of recruited users over time
- X-axis: the time spanning a month during the movement
- Y-axis: the number of active users normalized by the total users
- Curve reaches **0.98** by the end of the month

The “Indignados” Movement: Role of “Social Pressure” in Recruitment

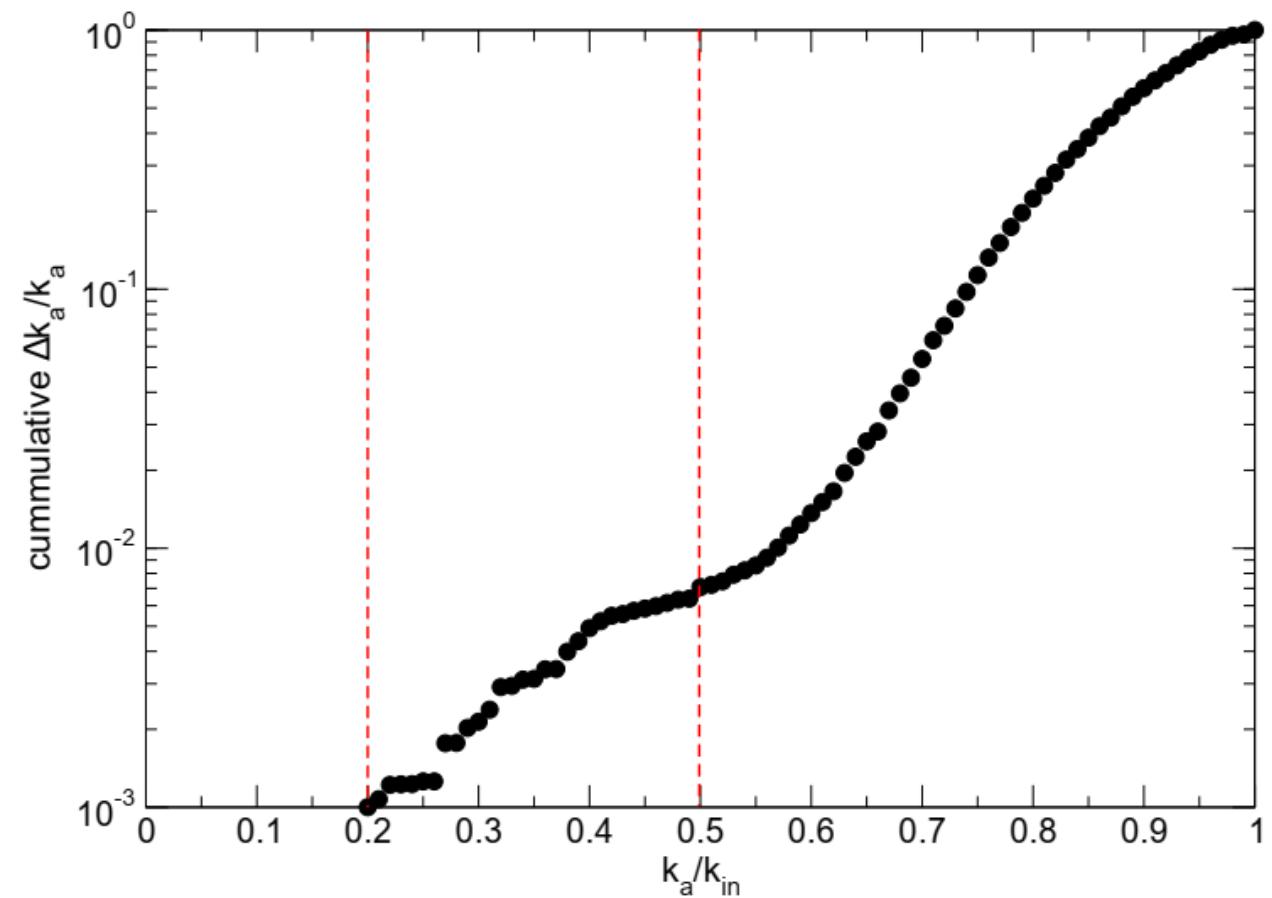


The “Indignados” Movement: Role of “Social Pressure” in Recruitment



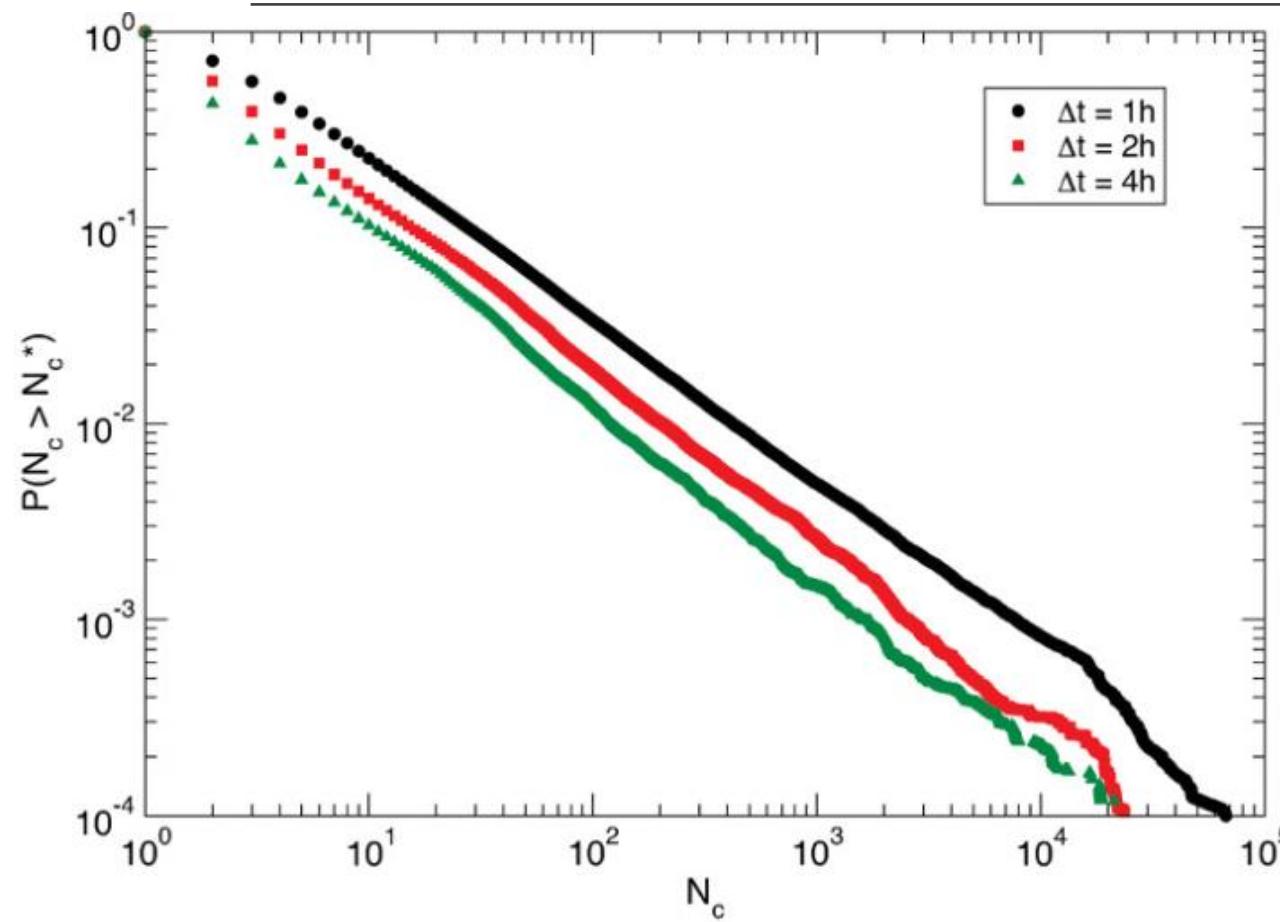
- ❑ The month-long timeline is divided into two time frames
 - ❑ 20-days timeframe before 15th May 2011
 - ❑ 10-days timeframe after 15th May 2011
- ❑ 15th May was the first day of the mass demonstrations
- ❑ Activation threshold distribution did not change much amongst the early activated, low-threshold users after 15th May
- ❑ Media coverage didn't significantly influence the recruitment of early activated low-threshold users

The “Indignados” Movement: Role of “Social Pressure” in Recruitment



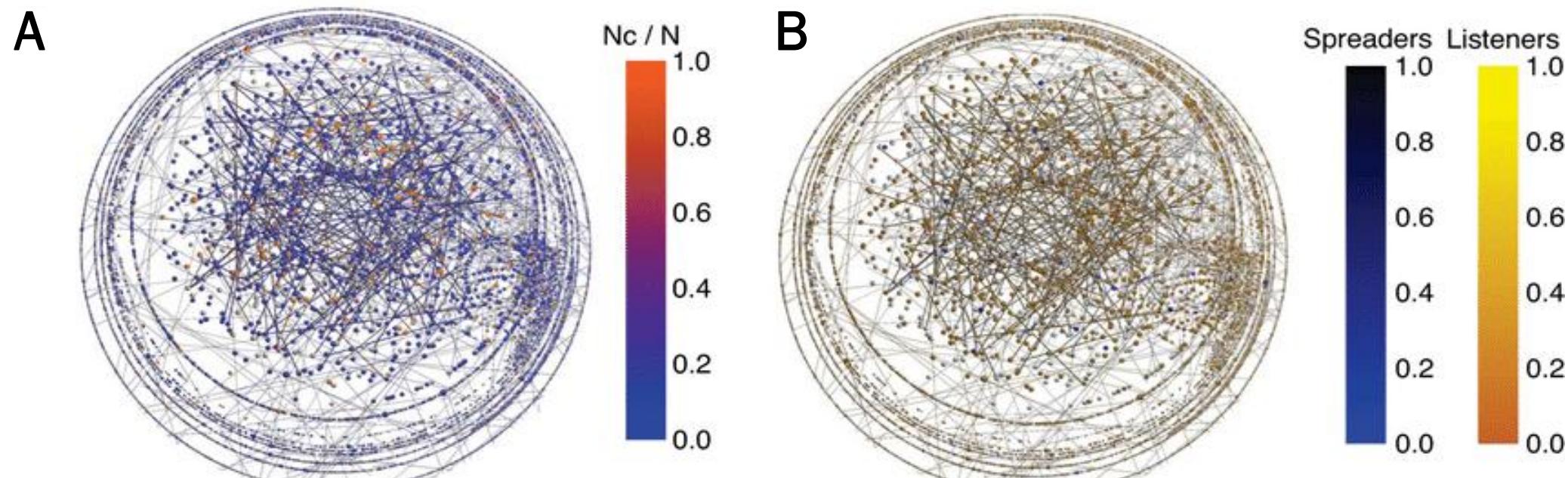
- If several neighbors of a user suddenly become active, then how likely is the user to become active?
- Burstiness is defined as the relative increase in the active neighbors
$$\frac{\Delta K_a}{K_a} = \frac{K_a^{t+1} - K_a^t}{K_a^t}$$
where t denotes a day
- Slope is steeper for high activation threshold users than the low activation threshold users
- High activation threshold users are more likely to join the protests if they see **a sudden increase in participation amongst their neighbors**

The “Indignados” Movement: Information Cascade



- If the messages of a user and her followers lie within the time difference Δt , then the user and the followers are said to form a cascade
- X-Axis denotes the size of cascades
- Y-Axis denotes the fraction of cascades having the corresponding size for different Δt
- Higher the size of the cascade, lower the number of such cascades

The “Indignados” Movement: Information Cascade



- Figure A: The nodes in the network arranged according to their k-core
- Cascades for higher K-core subgraphs (i.e., the central users) are responsible for starting successful cascades
- Figure B: Nodes in blue are users who participated in the diffusion of protest messages
- Figure B: Nodes in orange were exposed to the messages but did not send messages of their own

Decision-based Cascade Model: Limitations

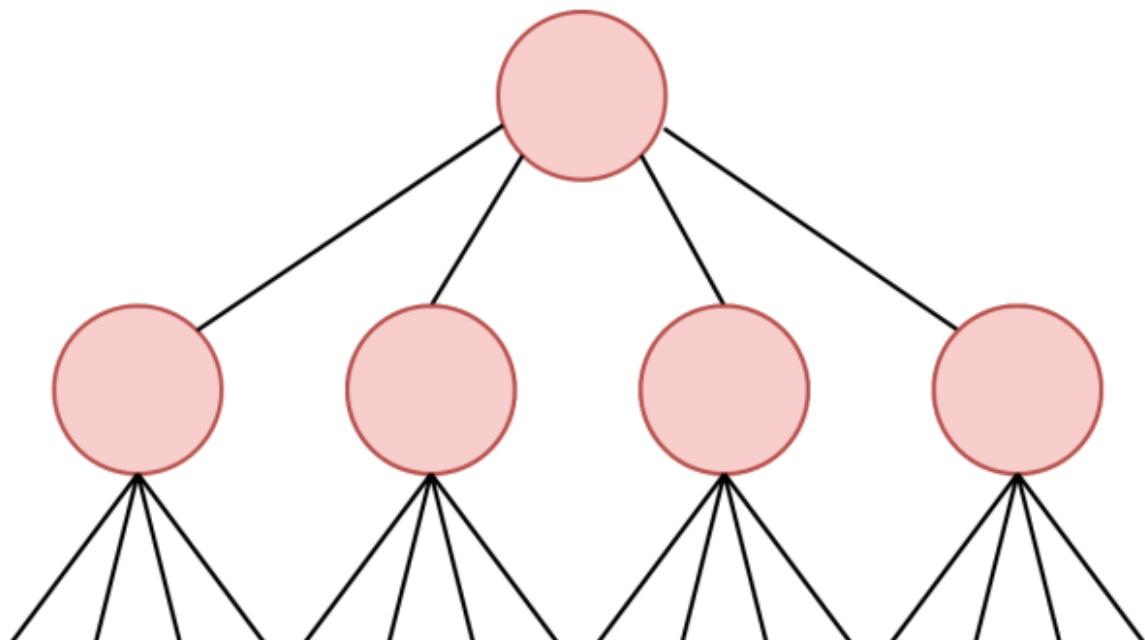
- ❑ Cascade will continue to grow only when its growth is associated with the highest reward amongst each of the nodes

- ❑ In many real-world scenarios, such hard decision making criteria or payoff functions are not available

- ❑ Infection spreading mechanism of a virus
 - We can model the spread of the virus as a cascade
 - Cascade growth is not in the hands of the node.

- ❑ Alternative Approach: [Probabilistic Cascade Model](#)

Probabilistic Cascade Model: Random Tree



A random tree with $d = 4$

- Basic assumptions
 - Person at the **root node** of the random tree is always infected
 - Each person in the random tree meets d new people. So, the random tree is a **d -nary tree**
 - Each person, on meeting an infected person, has the probability of getting infected as q ($q > 0$)
- For the virus to stay active and keep on spreading (cascade)
 - probability that a node at a depth h will be infected should be a positive real number
 - Same must hold for all h
$$\lim_{h \rightarrow \infty} P[\text{a node at depth } h \text{ is infected}] > 0$$
- The cascade would die out if
$$\lim_{h \rightarrow \infty} P[\text{a node at depth } h \text{ is infected}] = 0$$

Probabilistic Cascade Model: Random Tree

- ◻ If p_h be the probability of a node being infected at level h , then

$$p_h = 1 - (1 - q \cdot p_{h-1})^d$$

- ◻ The recurrence relation can have the following functional form:

$$f(x) = 1 - (1 - qx)^d$$

- ◻ The properties of f

- $f(x)$ is monotonic function
- $f'(x)$ is non-increasing
- $f'(x)$ is monotonic, non-increasing in $[0,1]$

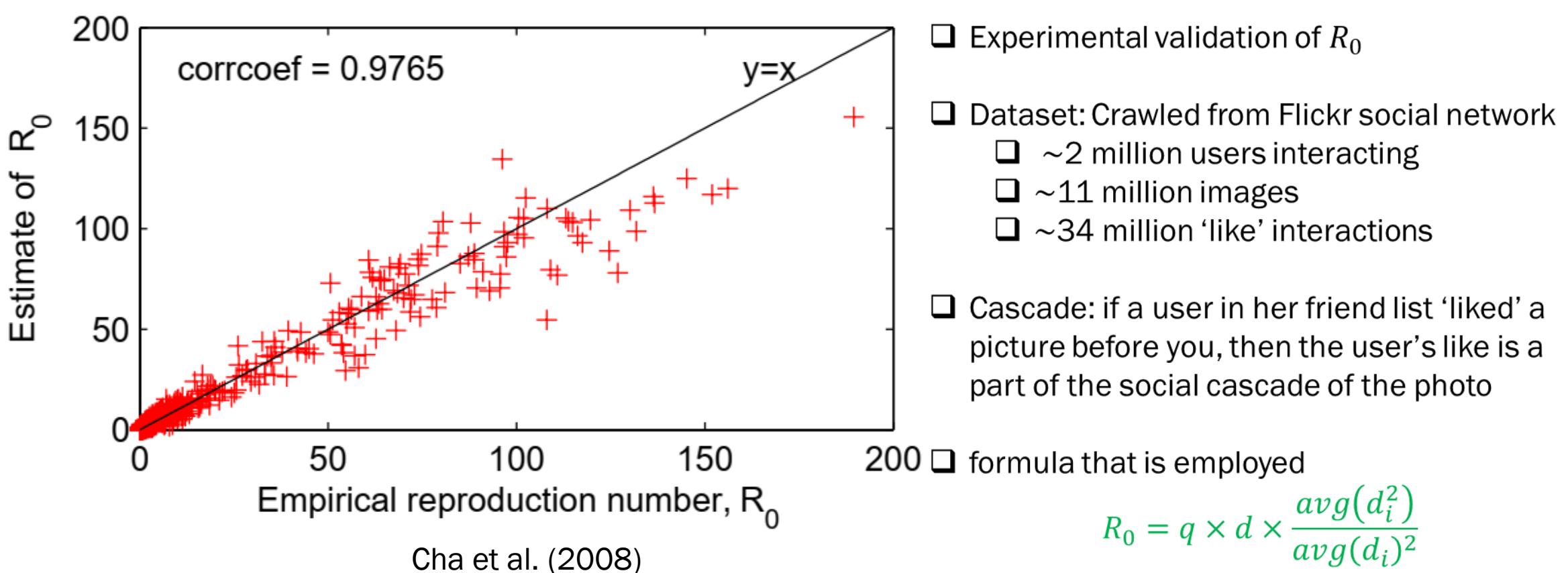
- ◻ $f(0) = 0$ and $f'(0) = q \cdot d$

Probabilistic Cascade Model: Random Tree

- Since $f'(x)$ is monotonic non-increasing, $f'(x) \leq q \cdot d$
- For epidemic to die out, $f(x) < x \Rightarrow q \cdot d < 1$
- The quantity $q \cdot d$ is called Reproductive number in the literature, denoted R_0
- If $R_0 \geq 1$, the epidemic grows in an exponential manner
- If $R_0 < 1$, the epidemic spread reduces constantly and eventually dies out

- two methods to contain the spread of the epidemic
 - reduce the value of $d \Rightarrow$ keep the already-infected nodes in isolation
 - reduce the value of $q \Rightarrow$ reduce transmission rate by promoting better hygiene practices

Probabilistic Cascade Model: The Reproductive Number

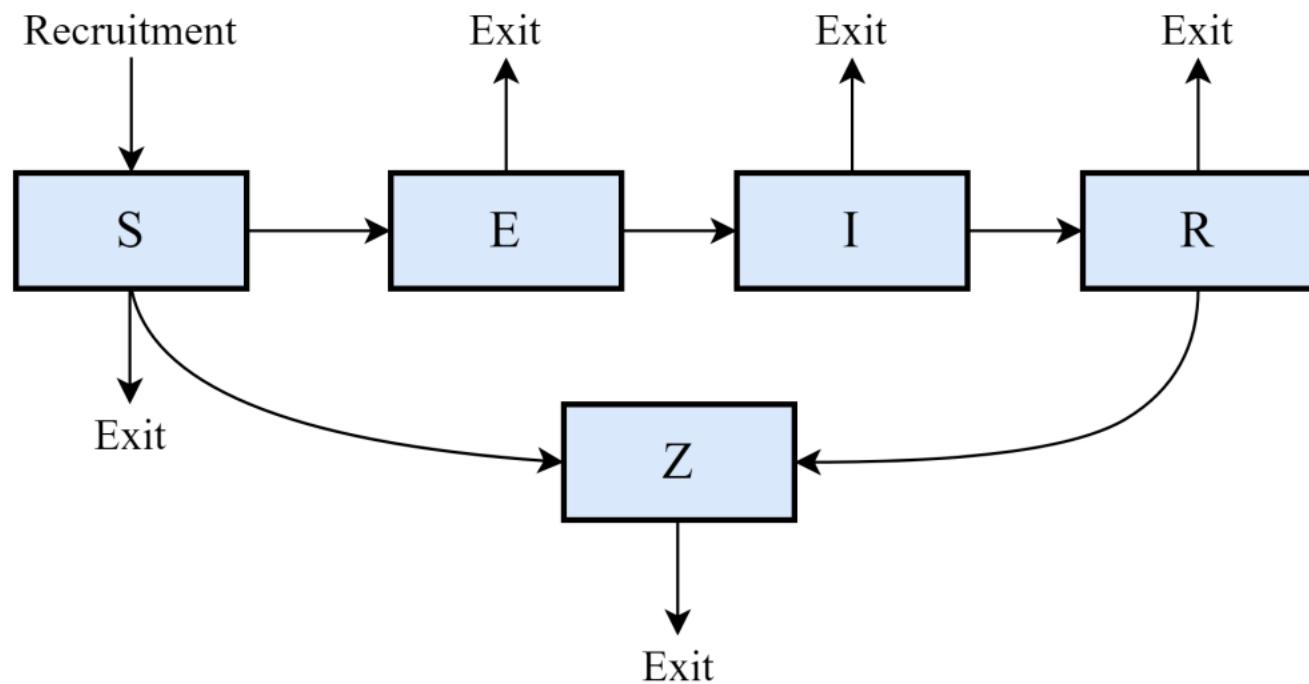


Compartmental Models of Epidemiology(Spreading models of viruses)

- ❑ Origination of such models in the early 20th century
- ❑ Seminal work by Kermack and McKendrick in 1927
- ❑ Models are most often run with ordinary differential equations
- ❑ Stochastic (random) framework, which are more realistic, are also possible
- ❑ Two important parameters:
 - ❑ Birth Rate (β): probability with which a neighbor node attacks another node with the virus
 - ❑ Death Rate (δ): probability with which an infected node heals

Compartmental Models of Epidemiology: SEIR Model

- ❑ A generalized framework to model the spread of epidemics
- ❑ SEIR (or S+E+I+R) is an acronym of
 - ❑ Susceptible (S): those who may become infected
 - ❑ Exposed (E): those who are infected, but not yet capable of spreading the infection/idea
 - ❑ Infected (I): those who are capable of further propagating the infection/idea
 - ❑ Recovered (R): those who have recovered from or become immune to the infection/idea
 - ❑ Immune(Z): Susceptible who no longer follow the infection/idea (Another possible state)
- ❑ Many possible variations of the model



Compartmental Models of Epidemiology: SIR Model (Chickenpox or plague virus model)



A node can go through only three stages: (i) Susceptible, (ii) Infected, and (iii) Recovered

Rate of change of ‘susceptible population’ is:

$$\frac{dS}{dt} = -\beta \times S \times I$$

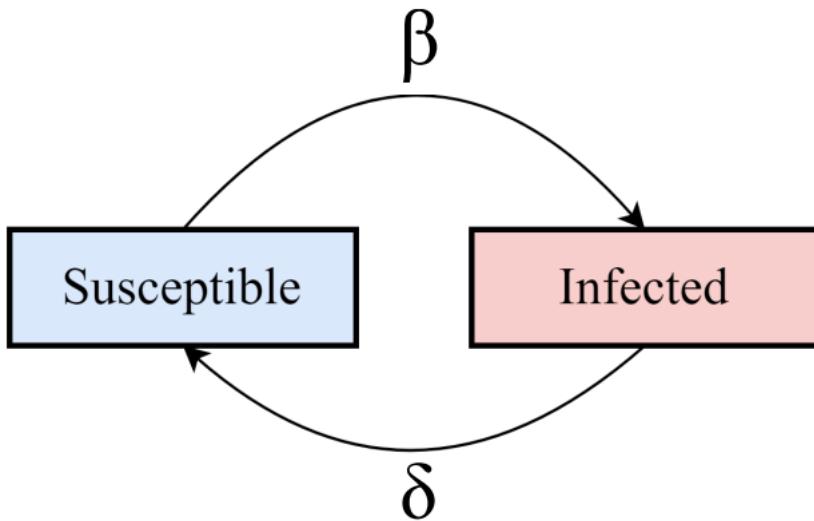
Rate of change of ‘recovered population’ is:

$$\frac{dR}{dt} = \delta \times I$$

Rate of change of ‘infected population’ is:

$$\frac{dI}{dt} = \beta SI - \delta I$$

Compartmental Models of Epidemiology: SIS Model(flu virus)



- a node can go through the phases of ‘susceptible’ to ‘infected’ to ‘susceptible’ again
- Common cold can recur with a high probability can be modelled by SIS
- Rate of change of ‘susceptible population’ is:
$$\frac{dS}{dt} = -\beta SI + \delta I$$
- Rate of change of ‘infected population’ is:
$$\frac{dI}{dt} = \beta SI - \delta I$$
- Strength of a virus = $\frac{\beta}{\delta}$
- Epidemic threshold, denoted by τ

Compartmental Models of Epidemiology: SIS Model

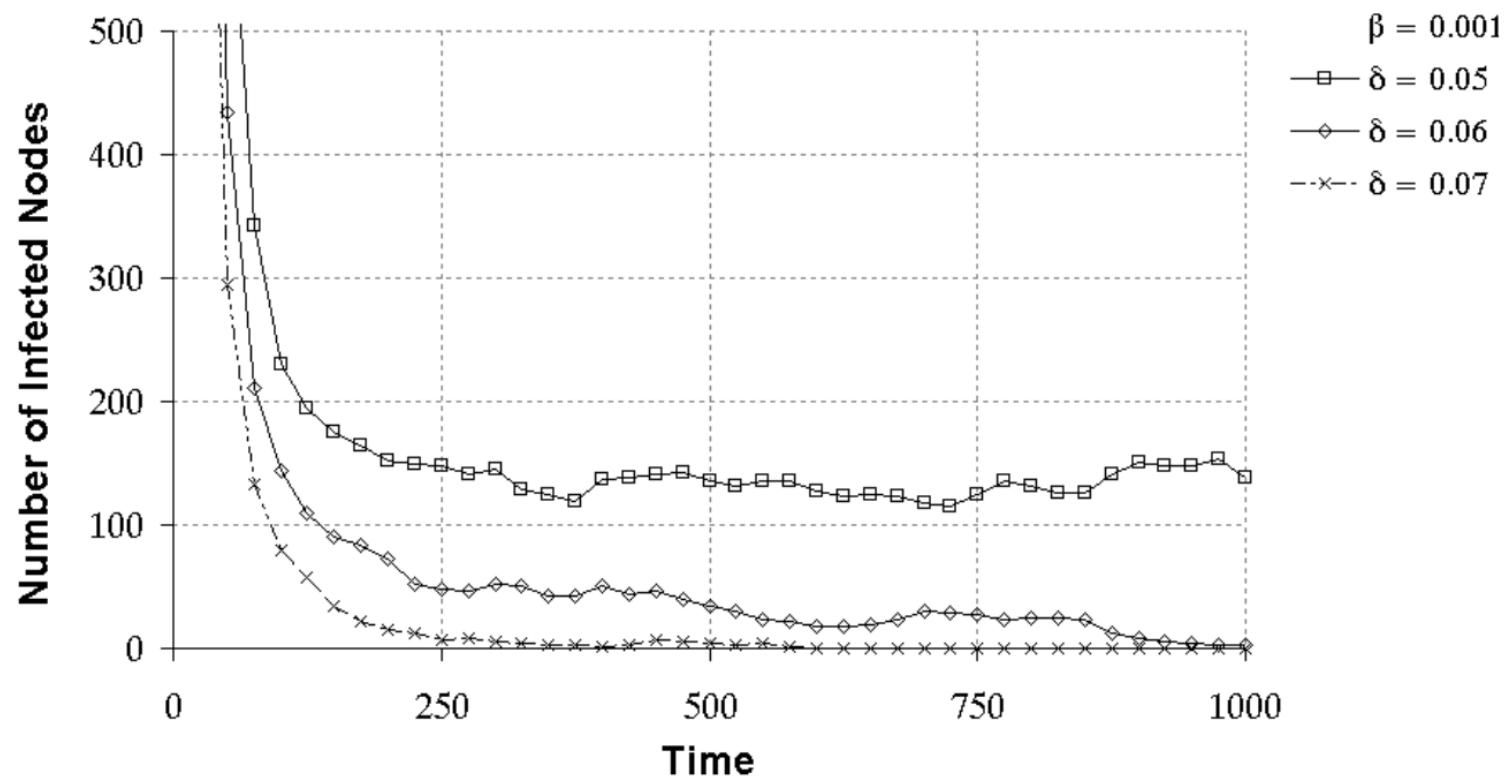
- ❑ The epidemic dies out if virus strength $< \tau$
- ❑ τ is nothing but the reciprocal of the largest eigenvalue of adjacency matrix representing the underlying network
- ❑ The epidemic dies out if

$$\frac{\beta}{\delta} < \tau = \frac{1}{\lambda_{1,A}}$$

where A : adjacency matrix of the underlying network

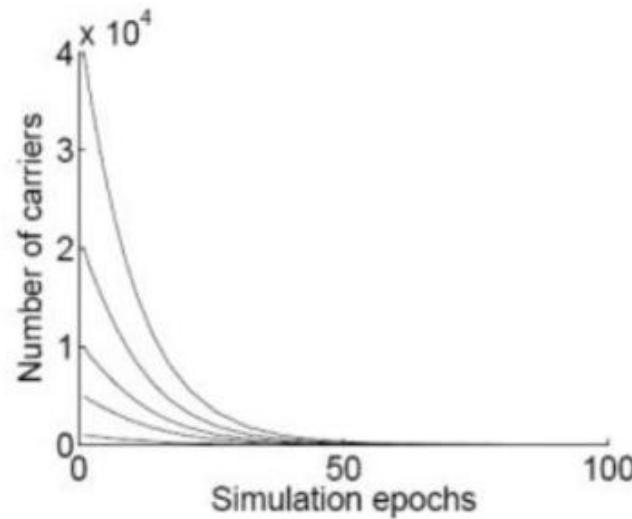
$\lambda_{1,A}$: largest eigen value of A

Compartmental Models of Epidemiology: SIS Model

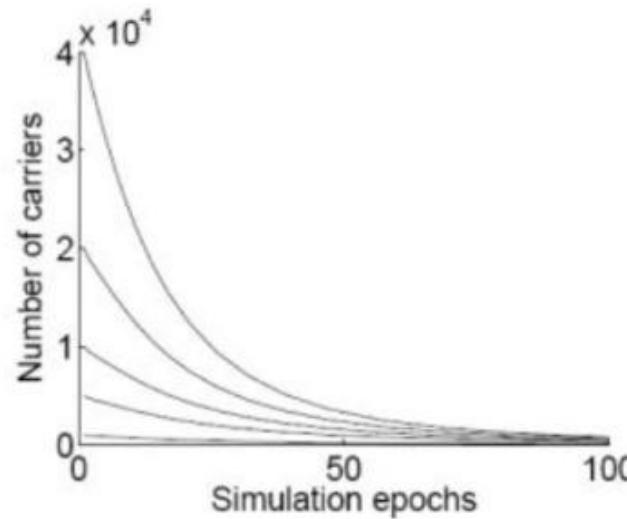


Reduction of Infected nodes for different β and δ

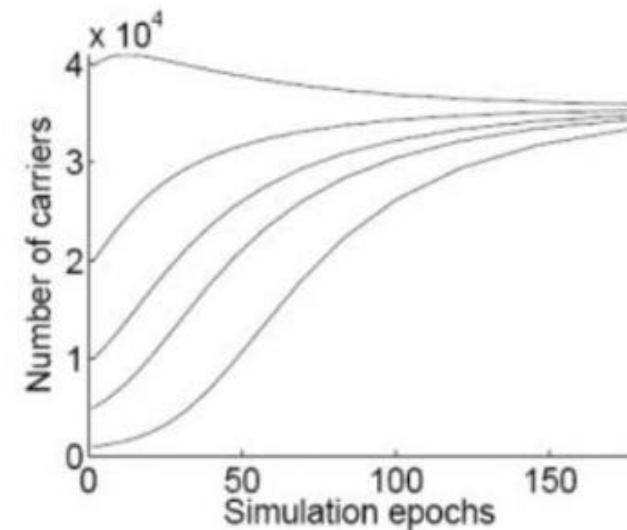
Compartmental Models of Epidemiology: SIS Model



(a) Below the threshold,
 $s=0.912$



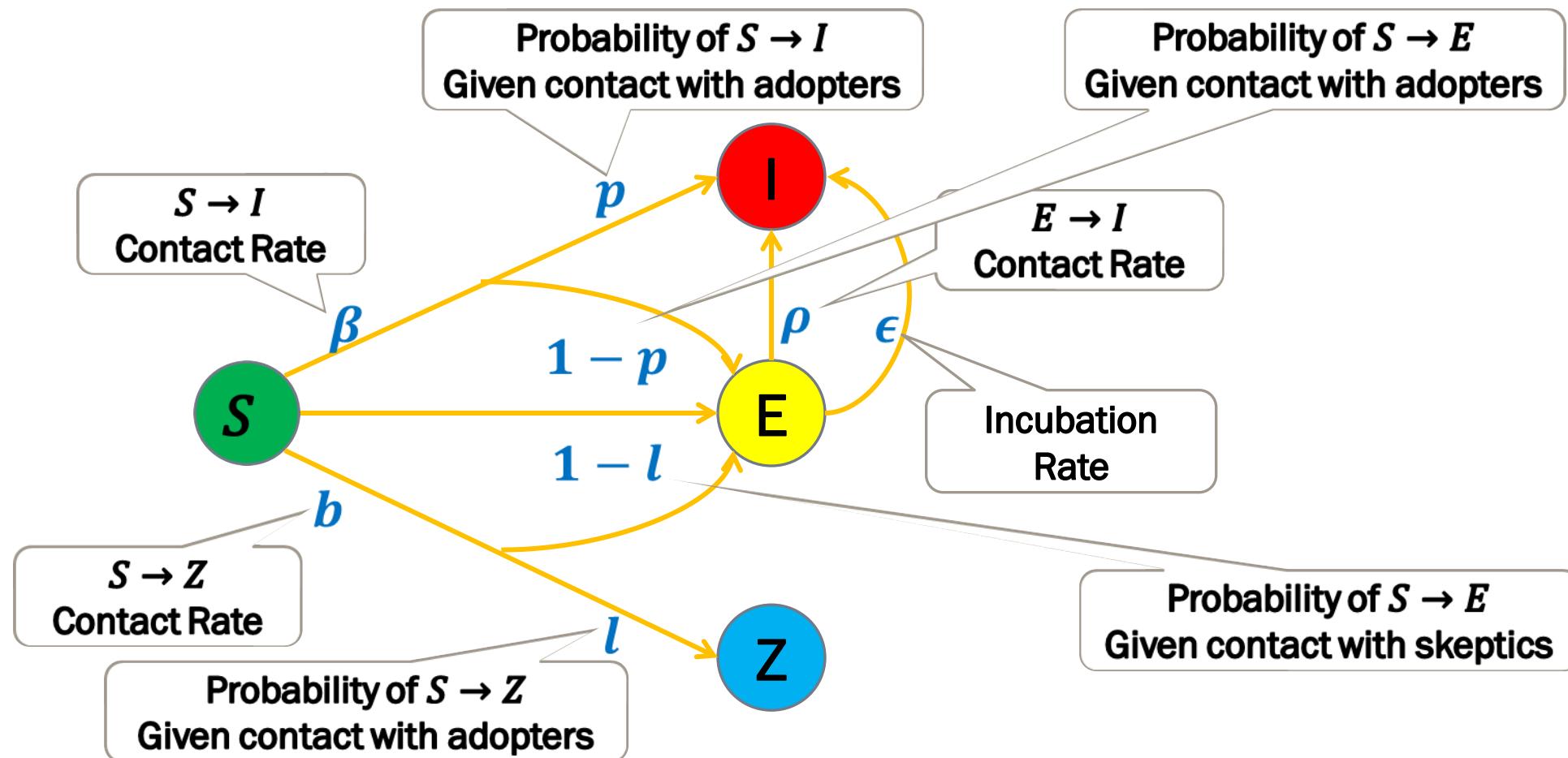
(b) At the threshold,
 $s=1.003$



(c) Above the threshold,
 $s=1.1$

Increasing or decreasing the number of initial carriers will make no difference in extending or reducing the duration of the epidemic

Analysing Rumour Spread: SEIZ Model



Analysing Rumour Spread: SEIZ Model

- ❑ A rumor in many ways is like a disease
 - Susceptible: those who are active on social media
 - Exposed: those who have seen/heard the rumor but did not believe it yet
 - Infected: those who believe the rumour
 - Skeptics: those who did not believe the rumor

- ❑ A new kind of metric defined for the event:

$$R_{SI} = \frac{(1-p)\beta + (1-l)b}{\rho + \epsilon}$$

- ❑ A kind of a flux ratio between the ratio of effects entering a node which is being examined to those leaving that node
 - ❑ high for real-life events
 - ❑ low for rumors

Analysing Rumour Spread: SEIZ Model

□ Mathematical representation of the model:

$$\triangleright \frac{dS}{dt} = -\beta S \frac{I}{N} - bS \frac{Z}{N}$$

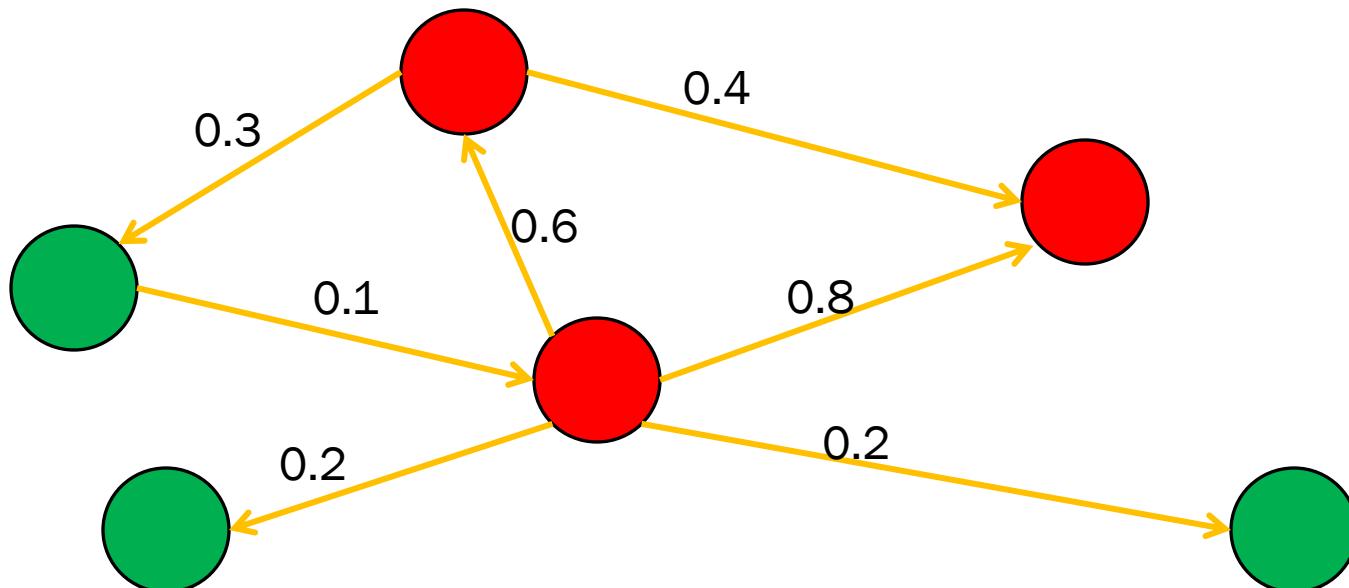
$$\triangleright \frac{dE}{dt} = (1-p)\beta S \frac{I}{N} + (1-l)bS \frac{Z}{N} - \rho E \frac{I}{N} - \epsilon E$$

$$\triangleright \frac{dI}{dt} = p\beta S \frac{I}{N} + \rho E \frac{I}{N} + \epsilon E$$

$$\triangleright \frac{dZ}{dt} = l\beta S \frac{Z}{N}$$

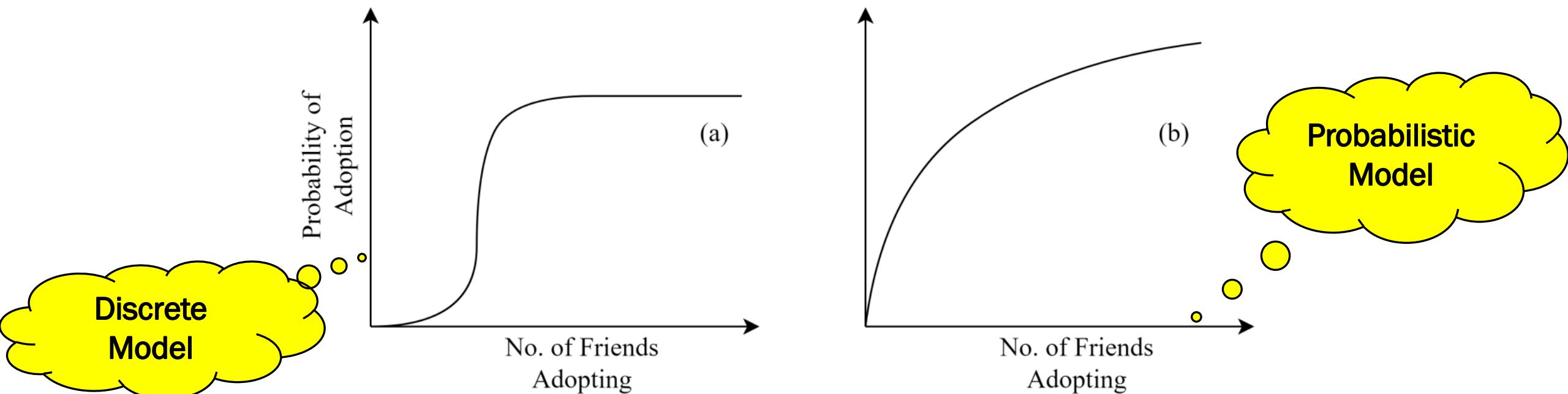
Independent Cascade Models

- ❑ Spread of infection with uniform probability between any node pair may not be realistic!
- ❑ Transmission of disease maybe more probable between certain pairs of nodes than other pairs
- ❑ An edge between u and v having a probability p_{uv} of transmission between them

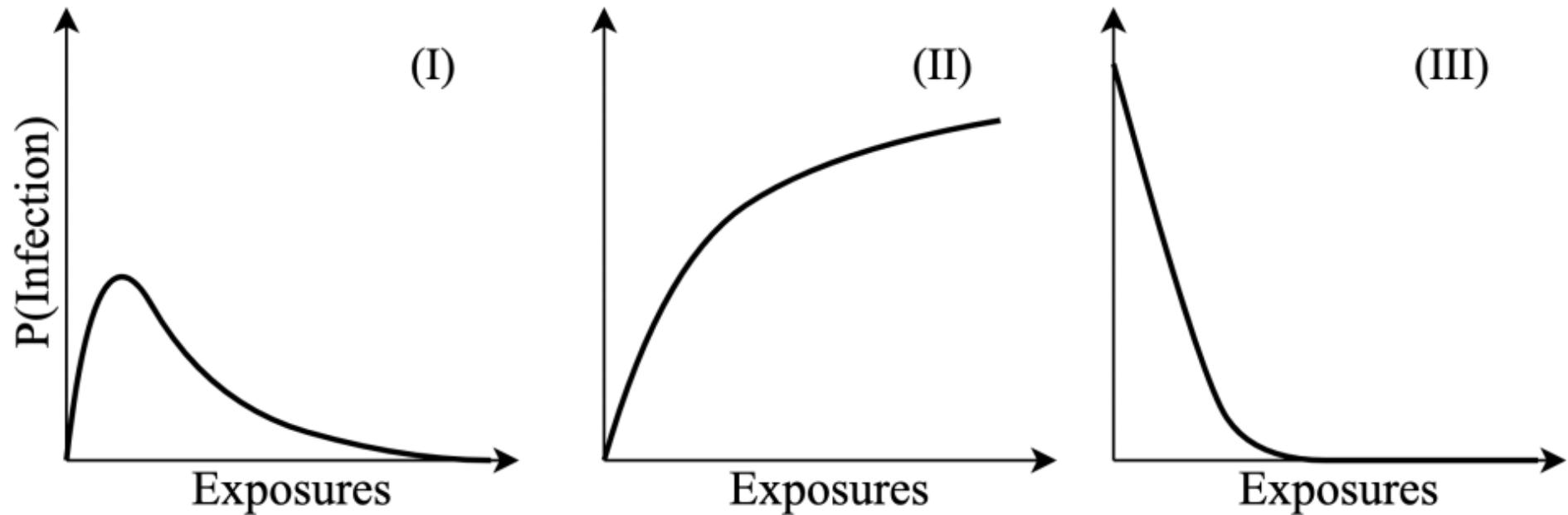


Independent Cascade Models

- ❑ **Exposure:** event of a node being exposed to a contagious incident
- ❑ **Adoption:** event of the node acting on the contagious incident
- ❑ **Hypothesis:** probability of adoption is influenced by the number of neighbors who have adopted

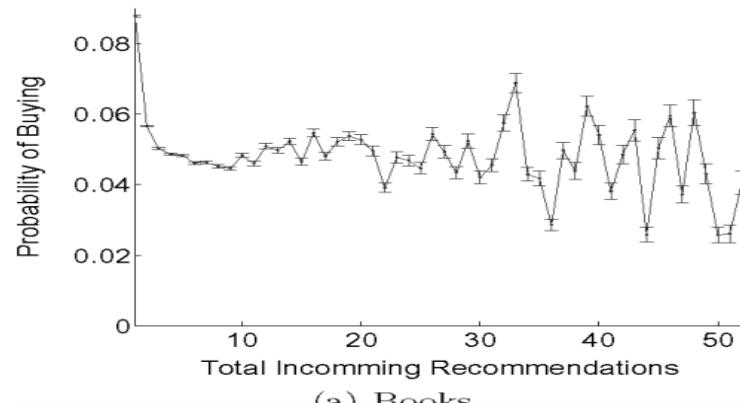


Independent Cascade Models: Exposure versus Adoption

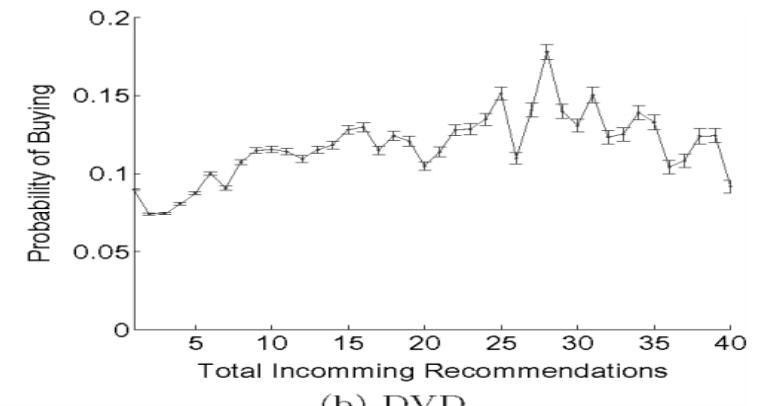


Independent Cascade Models: Exposure versus Adoption (Marketing)

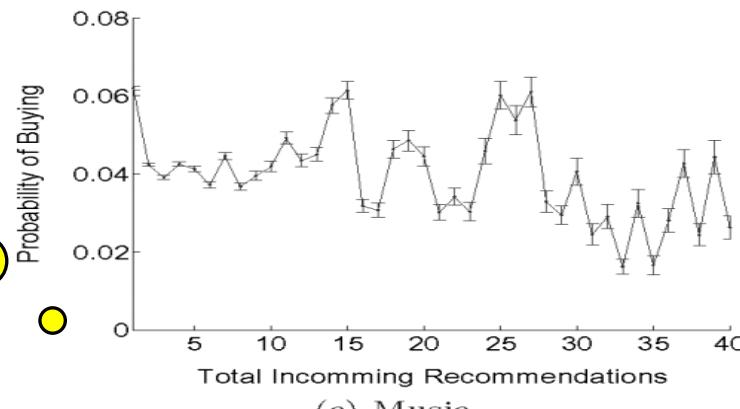
Obtained from study
of Referral programs
building marketing
habits



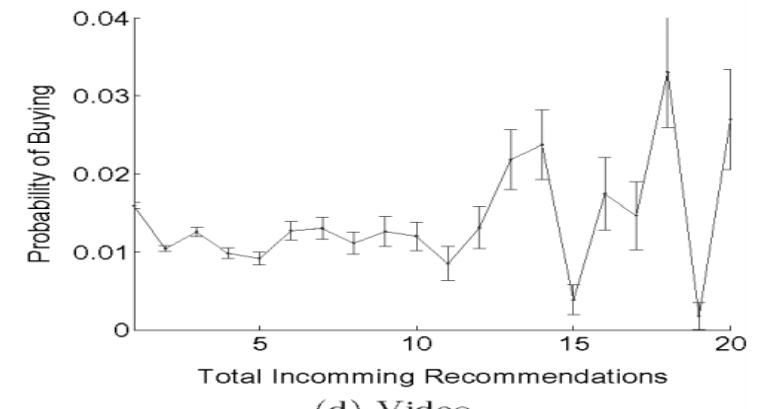
(a) Books



(b) DVD



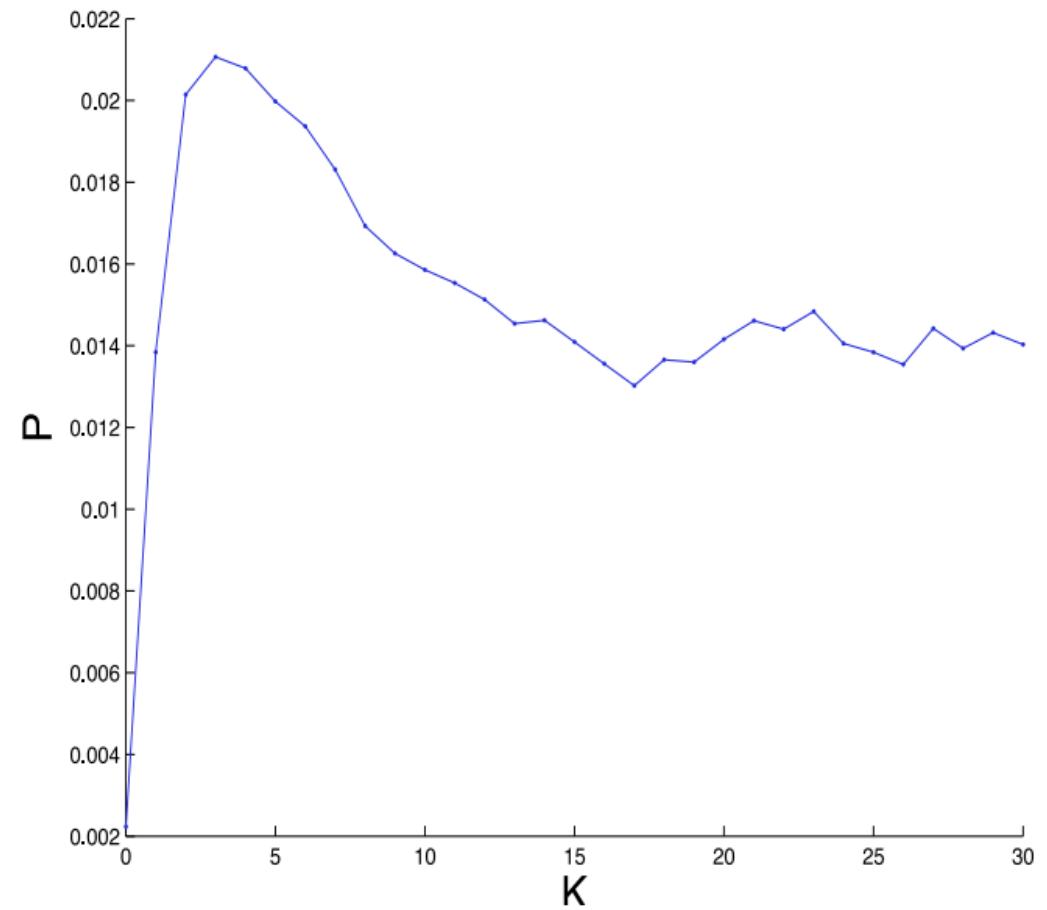
(c) Music



(d) Video

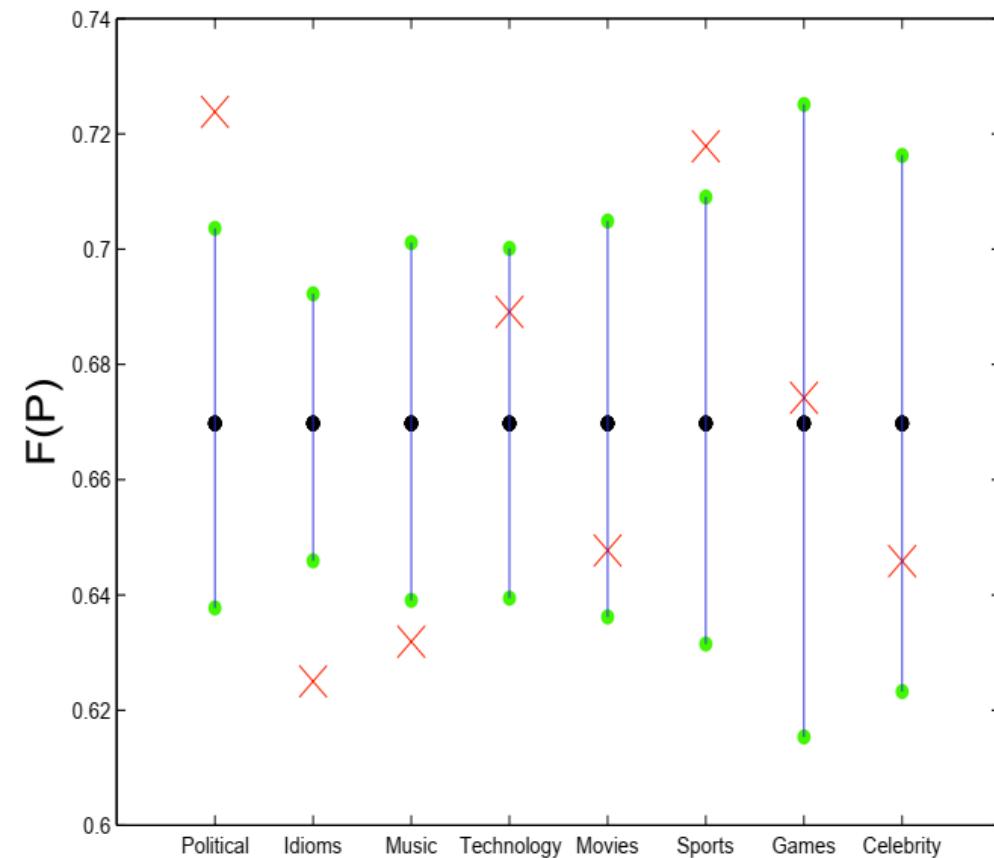
Independent Cascade Models: Exposure versus Adoption (Twitter)

- ❑ **Exposure:** seeing retweets of a particular tweet by different users
- ❑ **Adoption:** retweeting that particular tweet after seeing other retweets
- ❑ **Important Curve Parameters**
 - ❑ **Width of the curve:** The length of extension of the curve along the x-axis
 - ❑ **Stickiness of cascade:** It is the maximum height of the curve along y-axis
 - ❑ **Persistence of cascade:** This is defined as the ratio of the area under the curve and the rectangle defined by width and height of the curve



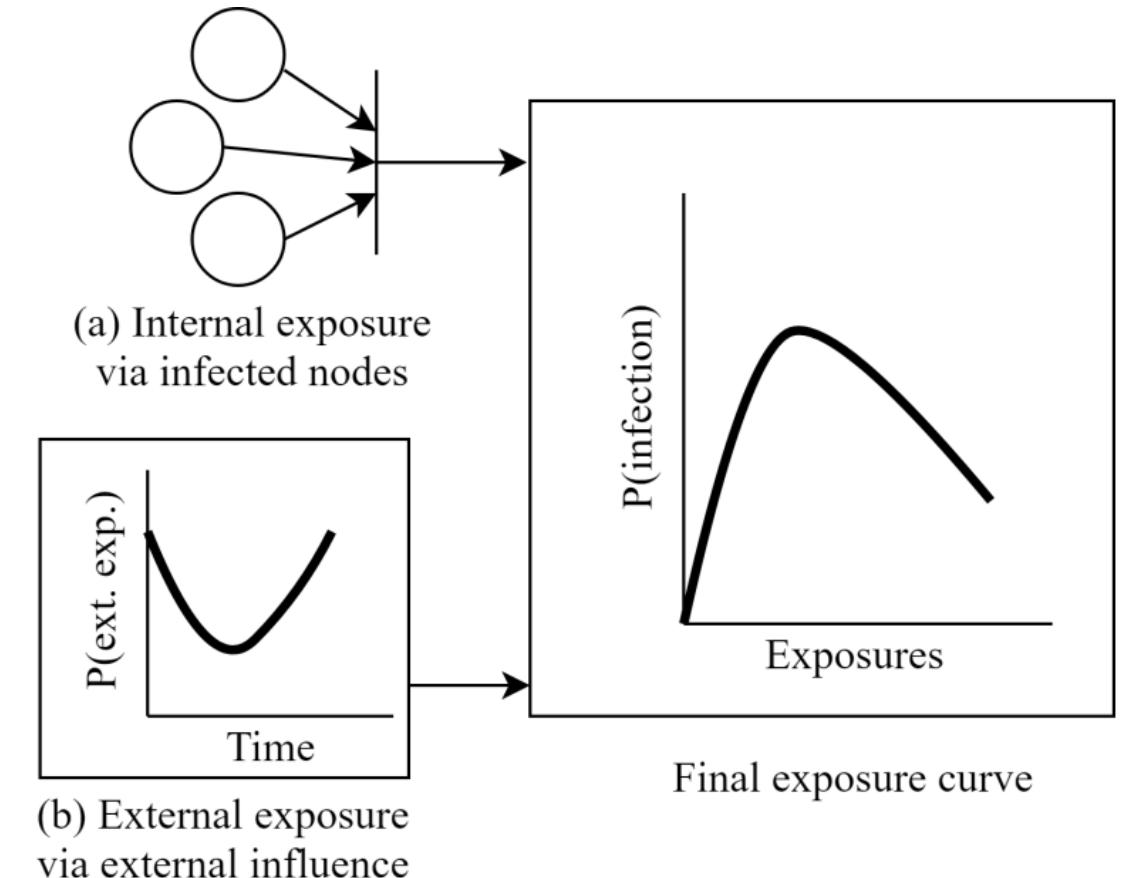
Independent Cascade Models: Exposure versus Adoption (Twitter)

- ❑ Comparing the exposure curves corresponding to different Twitter hashtags
 - ❑ Identified **8 topics**, each consisting of more than or equal to 20 hashtags
 - ❑ A **NUL set** consisting of random subset of the collected hashtags as a base set
- ❑ $F(P)$ denotes the **persistence levels**
- ❑ **Black dot** denotes the persistence of the NUL subset
- ❑ **Red cross** denotes the average persistence of the topic



Independent Cascade Models: Exposure versus Adoption (Twitter)

- an exposure can be injected externally also through media
- Available Input
 - a graph G
 - a set of node adoption times
- Study objective:
 - to model the external event
 - to find out the exposure curve



Cascade Prediction

- ❑ Can be divided into two main categories:
 - ❖ **Classification problem** – we predict if the diffused information/content would become popular in future
 - ❖ **regression problem** – we learn different numerical aspects of a cascade in future
 - ❑ final size,
 - ❑ growth
 - ❑ shape

Cascade Prediction: DeepCas

- ❑ proposed by Li et al. in 2017
- ❑ An end-to-end deep learning approach to predict the size of a cascade in future
- ❑ Uses a representation of the cascade network itself
- ❑ **Cascade Graph:** Let C be a set of cascades which originate in a network $G(V, E)$ at time t_0 . The graph $g_c^t = (V_c^t, E_c^t)$ denotes the cascade graph for a cascade $c \in C$ obtained after time t from t_0
- ❑ **Size Increment of Cascade:** Δs_c represents an increase in the size of cascade c in a Δt time interval, $\Delta s_c = |V_c^{t+\Delta t}| - |V_c^t|$
- ❑ DeepCas presents a neural network framework such that given a cascade graph g_c , it predicts the increment Δs_c

DeepCas: Flow Diagram

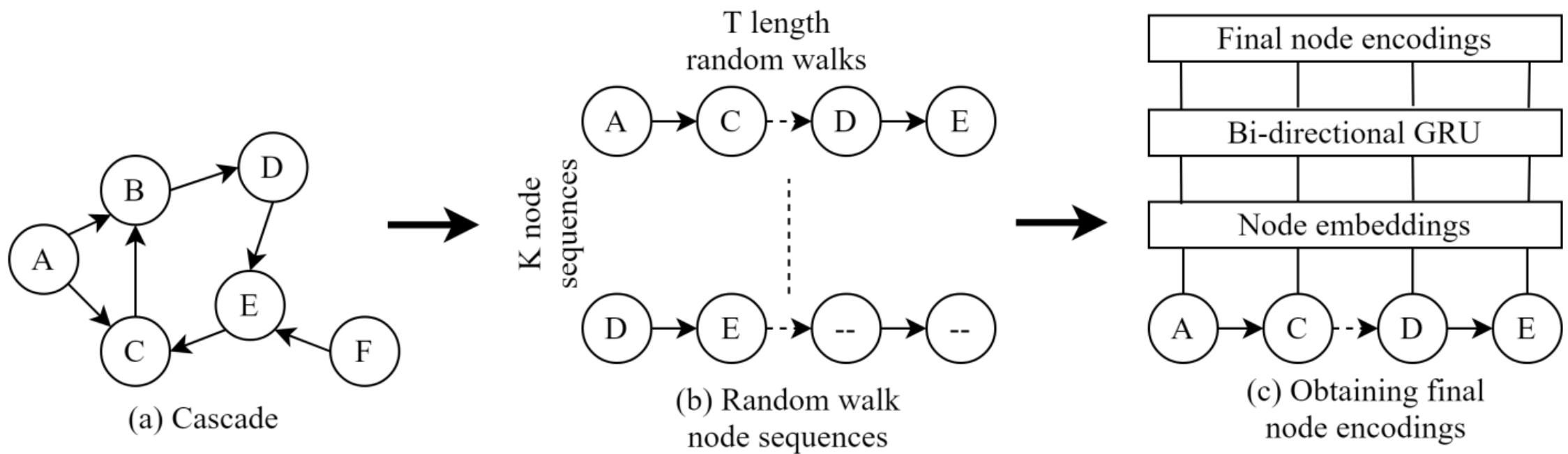
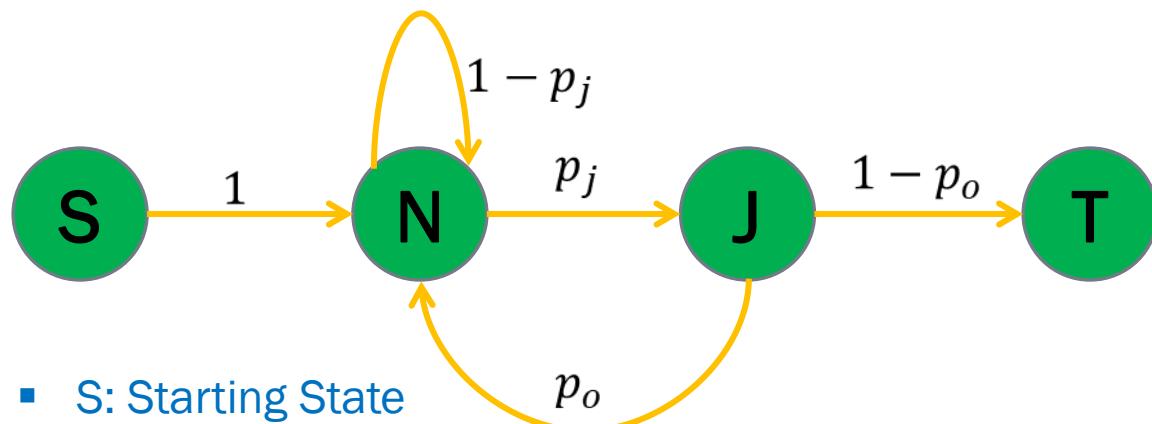


Figure 7.34: Flow diagram for DeepCas.

DeepCas: Markov Chain Model for Random Walk

- ❑ Node sequences are considered to incorporate the local and global network structure characteristics
- ❑ To sample node sequences, random walks on g_c using the Markov chain model is used



- S: Starting State
- N: Neighbor Node State
- J: Random Node Jump State;
- T: Terminal State

- ❑ Random walker is in state N at node v , then transition probability to select a neighbor node u to jump to:

$$p(u \in N_c(v)|v) = \frac{sc_t(u) + \alpha}{\sum_{s \in N_c(v)}(sc_t(s) + \alpha)}$$

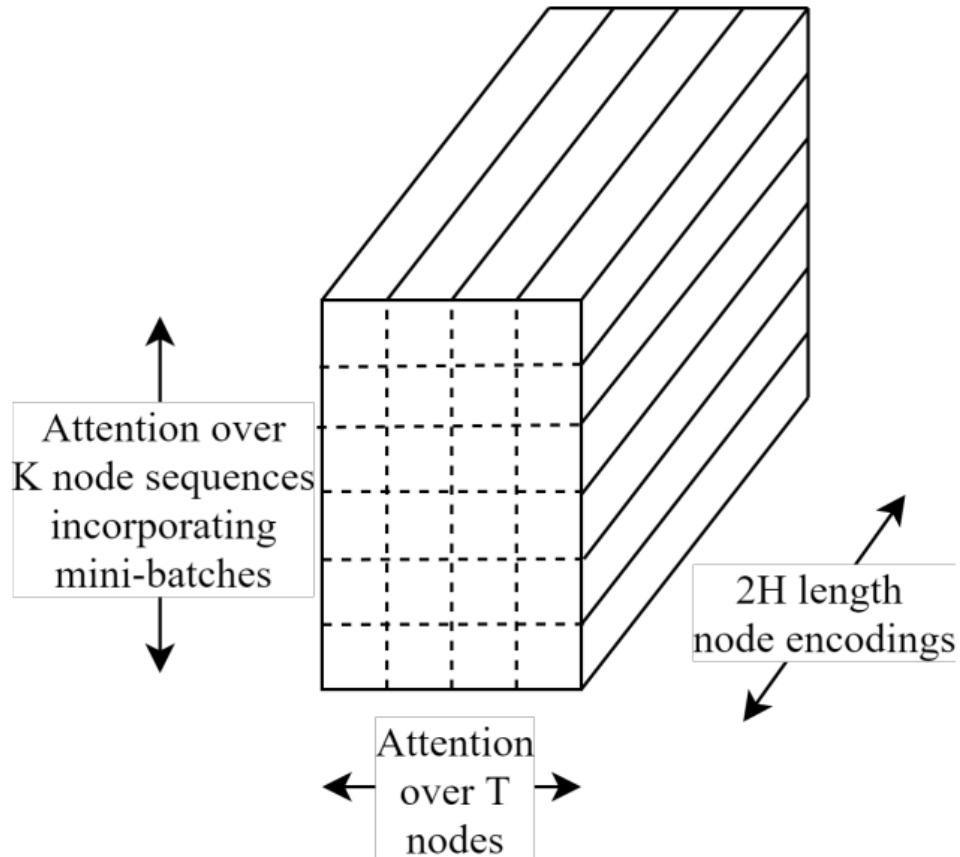
- ❑ random walker is at the random node jump state J , then transition probability of random jump:

$$p(u) = \frac{sc_t(u) + \alpha}{\sum_{s \in V_c}(sc_t(s) + \alpha)}$$

DeepCas: Node Embedding

- ❑ For determining **length of node sequences** (T) and **number of node sequences** (K), the probabilities p_j and p_o are learned using the neural network architecture
- ❑ The method use a node embedding to represent each node
 - N^{node} : number of nodes in a node sequence
 - $A \in \mathbb{R}^{H \times N^{node}}$: a given Embedding matrix
 - $q \in \mathbb{R}^{N^{node}}$: a vector
 - Then the vector embedding representation $x \in \mathbb{R}^H$ for each node is obtained as: $x = Aq$
- ❑ The authors used a **bi-directional Gated Recurrent Unit** (GRU) to obtain an encoding of the node sequences
 - ❑ $\overleftarrow{\overrightarrow{h}}_i^k \in \mathbb{R}^{2H}$: an encoding of the i^{th} node in the k^{th} node sequence resulted from concatenating the hidden vectors obtained from the forward and backward GRU

DeepCas: Final Cascade Graph Representation



❑ Use of attention mechanism to learn T and K

❑ Final representation of g_c :

$$h(g_c) = \sum_{k=1}^K \sum_{i=1}^T \left((1 - a_c)^{\lfloor k/B \rfloor} a_c \right) \lambda_i \overleftrightarrow{h}_i^k$$

❑ Objective function to minimize:

$$\mathcal{O} = \frac{1}{|C|} \sum_c (f(g_c) - \Delta s_c)^2$$

Cascade Prediction: DeepHawkes

- ❑ An end-to-end deep learning approach to predict the popularity of a piece of information diffused in an online social network via retweet prediction
- ❑ Model proposed by Cao et al. in 2017
- ❑ $\mathcal{C}^i = \{(u_j^i, v_j^i, t_j^i)\}$: cascade used to describe message $m_i \in M$
- ❑ (u_j^i, v_j^i, t_j^i) : j^{th} retweet such that v_j^i retweets message m_i of u_j^i wherein the time gap between the original post and the j^{th} retweet is t_j^i
- ❑ R_t^i : The **popularity** of m_i till time t
 - Total number of retweets within that time frame

DeepHawkes: Predict the Popularity

$$p_t^i = \sum_{j:t_j^i < t} \mu_j^i \varphi(t - t_j^i)$$

- p_t^i : the rate at which new retweets arrive for a message tweet m^i at time t
- t_j^i : time gap between the original post, and the j^{th} retweet
- μ_j^i : users who will be influenced by the j^{th} retweet
- $\varphi(\cdot)$: time decay function

□ Factors influencing the DeepHawkes formulation:

- **Influence of users**: messages retweeted by influential users tend to get further more retweets over time
- **Self-exciting mechanism**: every retweet of a message is responsible for attracting more retweets thereby increasing its popularity over time
- **Time decay effect**: influential exposure of retweets degrades as time passes

END