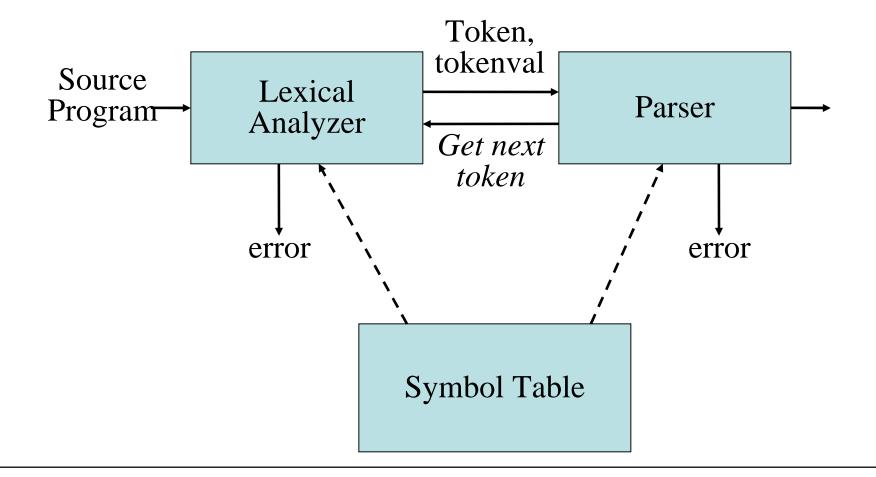


Lexical Analyzer

- Lexical Analyzer reads the source program character by character to produce tokens.
- Normally a lexical analyzer doesn't return a list of tokens at one shot, it returns a token when the parser asks a token from it.



Tokens, Lexemes, Patterns

- A token is a classification of lexical units
 - For example: id and num
- Lexemes are the specific character strings that make up a token
 - For example: abc and 123
- *Patterns* are rules describing the set of lexemes belonging to a token
 - For example: "letter followed by letters and digits" and "non-empty sequence of digits"

Token

- Token represents a set of strings described by a pattern.
 - Identifier represents a set of strings which start with a letter continues with letters and digits
 - The actual string (newval) is called as *lexeme*.
 - Tokens: identifier, number, addop, delimeter, ...
- Since a token can represent more than one lexeme, additional information should be held for that specific lexeme. This additional information is called as the *attribute* of the token.
- For simplicity, a token may have a single attribute which holds the required information for that token.
 - For identifiers, this attribute a pointer to the symbol table, and the symbol table holds the actual attributes for that token.
- Some attributes:
 - <id,attr> where attr is pointer to the symbol table
 - <assgop,_> no attribute is needed (if there is only one assignment operator)
 - <num,val> where val is the actual value of the number.
- Token type and its attribute uniquely identifies a lexeme.
- Regular expressions are widely used to specify patterns.

Terminology of Languages

- Alphabet : a finite set of symbols (ASCII characters)
- String:
 - Finite sequence of symbols on an alphabet
 - Sentence and word are also used in terms of string
 - ϵ is the empty string
 - |s| is the length of string s.
- Language: sets of strings over some fixed alphabet
 - \emptyset the empty set is a language.
 - { ε } the set containing empty string is a language
 - The set of well-formed C programs is a language
 - The set of all possible identifiers is a language.
- Operators on Strings:
 - Concatenation: xy represents the concatenation of strings x and y. $s \varepsilon = s$ $\varepsilon s = s$
 - $s^n = s s s ... s (n times) s^0 = \varepsilon$

Operations on Languages

- Concatenation:
 - $L_1L_2 = \{ s_1s_2 | s_1 \in L_1 \text{ and } s_2 \in L_2 \}$
- Union

-
$$L_1 \cup L_2 = \{ s \mid s \in L_1 \text{ or } s \in L_2 \}$$

• Exponentiation:

$$-\quad L^0=\{\epsilon\} \qquad \quad L^1=L \qquad \qquad L^2=LL$$

• Kleene Closure

$$- L^* = \bigcup_{i=0}^{\infty} L^i$$

• Positive Closure

$$- \quad \mathbf{L}^+ = \bigcup_{i=1}^{\infty} L^i$$

Example

•
$$L_1 = \{a,b,c,d\}$$
 $L_2 = \{1,2\}$

- $L_1L_2 = \{a1,a2,b1,b2,c1,c2,d1,d2\}$
- $L_1 \cup L_2 = \{a,b,c,d,1,2\}$
- L_1^3 = all strings with length three (using a,b,c,d)
- L_1^* = all strings using letters a,b,c,d and empty string
- L_1^+ = doesn't include the empty string

Regular Expressions

- We use regular expressions to describe tokens of a programming language.
- A regular expression is built up of simpler regular expressions (using defining rules)
- Each regular expression denotes a language.
- A language denoted by a regular expression is called as a **regular set**.

Regular Expressions (Rules)

Regular expressions over alphabet Σ

Reg. Expr	Language it denotes
3	{3}
$a \in \Sigma$	{a}
$(\mathbf{r}_1) \mid (\mathbf{r}_2)$	$L(r_1) \cup L(r_2)$
$(\mathbf{r}_1) (\mathbf{r}_2)$	$L(r_1) L(r_2)$
$(r)^*$	$(L(r))^*$
(r)	L(r)

$$\bullet \quad (\mathbf{r})^+ = \ (\mathbf{r})(\mathbf{r})^*$$

•
$$(r)$$
? = $(r) \mid \epsilon$

Regular Expressions (cont.)

• We may remove parentheses by using precedence rules.

```
- * highest
- concatenation next
- | lowest
```

• $ab^*|c$ means $(a(b)^*)|(c)$

• Ex:

```
\begin{array}{ll} - & \Sigma = \{0,1\} \\ - & 0|1 => \{0,1\} \\ - & (0|1)(0|1) => \{00,01,10,11\} \\ - & 0^* => \{\epsilon\,,0,00,000,0000,....\} \\ - & (0|1)^* => \mbox{ all strings with 0 and 1, including the empty string} \end{array}
```

Regular Definitions

- To write regular expression for some languages can be difficult, because their regular expressions can be quite complex. In those cases, we may use *regular definitions*.
- We can give names to regular expressions, and we can use these names as symbols to define other regular expressions.
- A *regular definition* is a sequence of the definitions of the form:

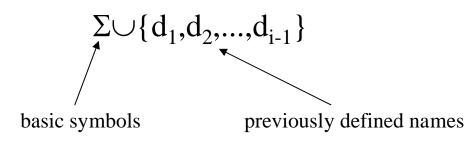
$$d_1 \rightarrow r_1$$

 $d_2 \rightarrow r_2$

$$d_n \rightarrow r_n$$

where d_i is a distinct name and

r_i is a regular expression over symbols in



Regular Definitions (cont.)

• Ex: Identifiers in Pascal

```
letter \rightarrow A | B | ... | Z | a | b | ... | z
digit \rightarrow 0 | 1 | ... | 9
id \rightarrow letter (letter | digit) *
```

- If we try to write the regular expression representing identifiers without using regular definitions, that regular expression will be complex.

$$(A|...|Z|a|...|z) ((A|...|Z|a|...|z) | (0|...|9))^*$$

• Ex: Unsigned numbers in Pascal

```
\begin{aligned} &\text{digit} \rightarrow 0 \mid 1 \mid ... \mid 9 \\ &\text{digits} \rightarrow \text{digit} ^+ \\ &\text{opt-fraction} \rightarrow (\text{ . digits }) ? \\ &\text{opt-exponent} \rightarrow (\text{ E (+|-)}? \text{ digits }) ? \\ &\text{unsigned-num} \rightarrow \text{digits opt-fraction opt-exponent} \end{aligned}
```

Disambiguation Rules

- 1) longest match rule: from all tokens that match the input prefix, choose the one that matches the most characters
- 2) rule priority: if more than one token has the longest match, choose the one listed first Examples:
- for8 is it the for-keyword, the identifier "f", the identifier

"fo", the identifier "for", or the identifier "for8"?

Use rule 1: "for8" matches the most characters.

• for is it the for-keyword, the identifier "f", the identifier

"fo", or the identifier "for"?

Use rule 1 & 2: the for-keyword and the "for"

identifier have the longest match but the

for-keyword is listed first.

How Scanner Generators Work

- Translate REs into a finite state machine
- Done in three steps:
 - 1) translate REs into a no-deterministic finite automaton (NFA)
 - 2) translate the NFA into a deterministic finite automaton (DFA)
 - 3) optimize the DFA (optional)

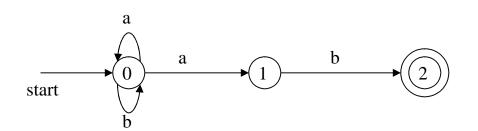
Finite Automata

- A *recognizer* for a language is a program that takes a string x, and answers "yes" if x is a sentence of that language, and "no" otherwise.
- We call the recognizer of the tokens as a *finite automaton*.
- A finite automaton can be: deterministic(DFA) or non-deterministic (NFA)
- This means that we may use a deterministic or non-deterministic automaton as a lexical analyzer.
- Both deterministic and non-deterministic finite automaton recognize regular sets.
- Which one?
 - deterministic faster recognizer, but it may take more space
 - non-deterministic slower, but it may take less space
 - Deterministic automatons are widely used lexical analyzers.
- First, we define regular expressions for tokens; Then we convert them into a DFA to get a lexical analyzer for our tokens.
 - Algorithm1: Regular Expression → NFA → DFA (two steps: first to NFA, then to DFA)
 - Algorithm2: Regular Expression → DFA (directly convert a regular expression into a DFA)

Non-Deterministic Finite Automaton (NFA)

- A non-deterministic finite automaton (NFA) is a mathematical model that consists of:
 - S a set of states
 - $-\Sigma$ a set of input symbols (alphabet)
 - move a transition function move to map state-symbol pairs to sets of states.
 - s_0 a start (initial) state
 - F a set of accepting states (final states)
- ϵ transitions are allowed in NFAs. In other words, we can move from one state to another one without consuming any symbol.
- A NFA accepts a string x, if and only if there is a path from the starting state to one of accepting states such that edge labels along this path spell out x.

NFA (Example)



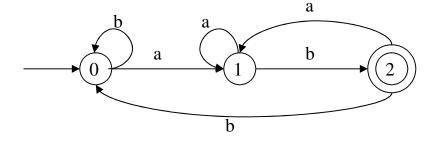
Transition graph of the NFA

0 is the start state
$$s_0$$
 {2} is the set of final states F $\Sigma = \{a,b\}$ $S = \{0,1,2\}$ Transition Function: $a \qquad b \qquad 0 \qquad \{0,1\} \qquad \{0\}$ 1

The language recognized by this NFA is (a|b)* a b

Deterministic Finite Automaton (DFA)

- A Deterministic Finite Automaton (DFA) is a special form of a NFA.
 - no state has ε- transition
 - for each symbol a and state s, there is at most one labeled edge a leaving s. i.e. transition function is from pair of state-symbol to state (not set of states)



The language recognized by this DFA is also (a|b)* a b

Implementing a DFA

• Le us assume that the end of a string is marked with a special symbol (say eos). The algorithm for recognition will be as follows: (an efficient implementation)

```
s \leftarrow s_0
                           { start from the initial state }
                           { get the next character from the input string }
c ← nextchar
while (c != eos) do
                           { do until the en dof the string }
   begin
       s \leftarrow move(s,c)
                           { transition function }
       c ← nextchar
   end
if (s in F) then
                           { if s is an accepting state }
   return "yes"
else
   return "no"
```

Implementing a NFA

```
S \leftarrow \epsilon\text{-closure}(\{s_0\}) \qquad \{ \text{ set all of states can be accessible from } s_0 \text{ by } \epsilon\text{-transitions } \}
c \leftarrow \text{nextchar}
\text{while } (c != \text{eos}) \{
\text{begin}
s \leftarrow \epsilon\text{-closure}(\text{move}(S,c)) \quad \{ \text{ set of all states can be accessible from a state in } S
c \leftarrow \text{nextchar} \qquad \text{by a transition on } c \}
\text{end}
\text{if } (S \cap F != \Phi) \text{ then} \qquad \{ \text{ if } S \text{ contains an accepting state } \}
\text{return "yes"}
\text{else}
\text{return "no"}
```

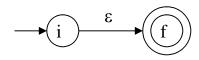
• This algorithm is not efficient.

Converting A Regular Expression into A NFA (Thomson's Construction)

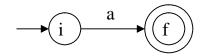
- This is one way to convert a regular expression into a NFA.
- There can be other ways (much efficient) for the conversion.
- Thomson's Construction is simple and systematic method. It guarantees that the resulting NFA will have exactly one final state, and one start state.
- Construction starts from simplest parts (alphabet symbols). To create a NFA for a complex regular expression, NFAs of its sub-expressions are combined to create its NFA,

Thomson's Construction (cont.)

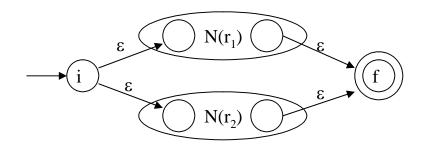
• To recognize an empty string ε



ullet To recognize a symbol a in the alphabet Σ



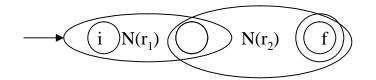
- If $N(r_1)$ and $N(r_2)$ are NFAs for regular expressions r_1 and r_2
 - For regular expression $r_1 | r_2$



NFA for $r_1 | r_2$

Thomson's Construction (cont.)

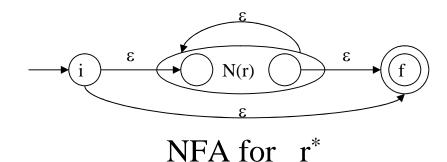
• For regular expression $r_1 r_2$



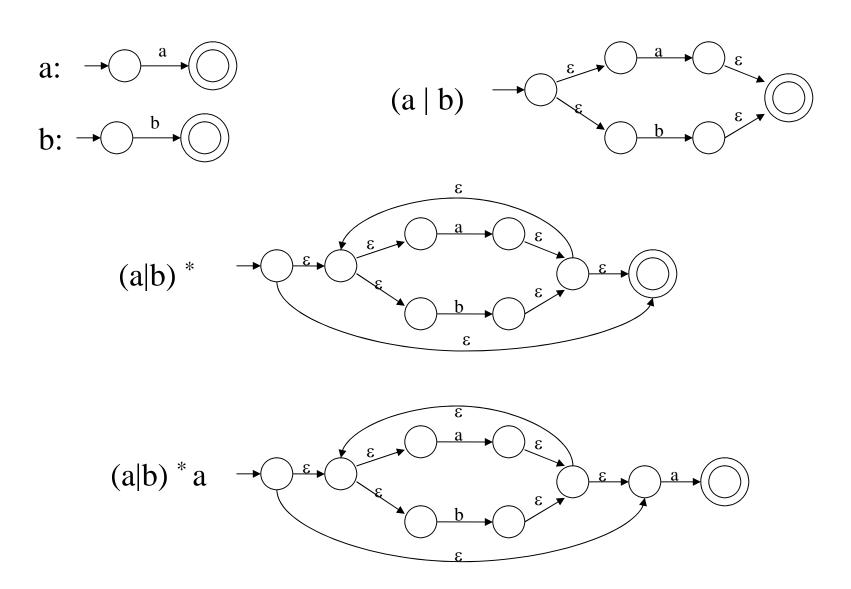
Final state of $N(r_2)$ become final state of $N(r_1r_2)$

NFA for $r_1 r_2$

• For regular expression r*



Thomson's Construction (Example - (a|b) *a)

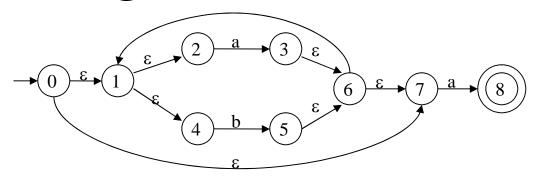


Converting a NFA into a DFA (subset construction)

```
put \varepsilon-closure(\{s_0\}) as an unmarked state into the set of DFA (DS)
while (there is one unmarked S_1 in DS) do
                                                                   \varepsilon-closure(\{s_0\}) is the set of all states can be accessible
    begin
                                                                   from s_0 by \epsilon-transition.
        mark S<sub>1</sub>
        for each input symbol a do
                                                               set of states to which there is a transition on
                                                               a from a state s in S<sub>1</sub>
            begin
               S_2 \leftarrow \varepsilon-closure(move(S_1,a))
               if (S_2 \text{ is not in DS}) then
                    add S<sub>2</sub> into DS as an unmarked state
               transfunc[S_1,a] \leftarrow S_2
            end
      end
```

- a state S in DS is an accepting state of DFA if a state in S is an accepting state of NFA
- the start state of DFA is ε -closure($\{s_0\}$)

Converting a NFA into a DFA (Example)



Converting a NFA into a DFA (Example – cont.)

 S_0 is the start state of DFA since 0 is a member of $S_0 = \{0,1,2,4,7\}$ S_1 is an accepting state of DFA since 8 is a member of $S_1 = \{1,2,3,4,6,7,8\}$

