

★ Properties of Measurement :-

- Identity & having unique meaning
- Magnitude :-
- Equal interval.
- A minimum value of zero :-

1. Nominal Scale of Measurement :- (Labels)

- does not have any numerical meaning
- can't be divided, multiplied, added or subtracted

— Used in research surveys and questionnaires

— Most fundamental research scale.

* Nominal with order : cold, warm, hot

* Nominal without order : male and female

* Dichotomous : having only two categories "Yes or No"

2. Ordinal scale of -

→ used to depict frequency, happiness, satisfaction

→ an ordered series of relationships or rank orders

→ can't be added to or subtract

3. Interval scale of measurement:-

→ can be added or subtracted but not ÷ or *

→ zero is an existing variable.

→ Used to quantify the difference between variables

→ Temperature and time

* Ratio scale of M -

- > Nominal, can be classified in orders, containing intervals and can be broken down into exact values
- > Height, weight, distance.

* Central tendency = statistical measure

- single value of entire data
- measures of central location
- summary statistics

* Mean

$$\text{sample mean} \approx \bar{x} = \frac{\sum x}{n}$$

$$\text{Population mean} : \mu = \frac{\sum x}{N}$$

x	0	1	2	3	4
f	3	6	6	3	1

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$\bar{x} = \frac{0+1f+12+9+4}{19}$$

$$= \frac{31}{19}$$

$$= 1.6316$$

★ Median

★ Problem 1:

$$\bar{x} = \frac{\sum x_i}{n} = \frac{21+23+23+54+67+21+25+21+54+72+75}{11} = \frac{456}{11} = 41.45$$

 \tilde{x}

21, 21, 21, 23, 23, 25, 54, 54, 67, 72, 75

Here $n=11$

$\therefore \left(\frac{n+1}{2}\right)^{\text{th}}$ observation will be median

 $\tilde{x} = 25$

Mode - 21.

★ Measures of variability of dispersion

similar values → little variability

- Range
- Variance
- Standard deviation

★ 40, 38, 42, 40, 39, 39, 43, 40, 39, 40 (Set I)

$$\bar{x} = \frac{\sum x_i}{n} = \frac{400}{10} = 40$$

For

$$\tilde{x} = 38, 39, 39, 39, 40, 40, 40, 40, 42, 43$$

$$\tilde{x} = 40$$

$$\text{Mode} = 40$$

$$46, 37, 40, 33, 42, 36, 40, 47, 34, 45$$

$$\bar{x} = 40, \tilde{x} = 40, \text{Mode} = 40$$

★ Range

$$R = x_{\max} - x_{\min}$$
$$= 43 - 38 = 5 \text{ for Set I}$$

★ The variance and Standard Deviation

Sample

$$s^2 = \frac{\sum (x - \bar{x})^2}{n-1}$$

$$s^2 = \frac{\sum x^2 - \frac{1}{n} (\sum x)^2}{n-1}$$

Sample S.D.

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}} = \sqrt{\frac{\sum x^2 - \frac{1}{n} (\sum x)^2}{n-1}}$$

Population

$$\sigma^2 = \frac{\sum (x - \mu)^2}{N} \quad \text{and} \quad \sigma = \sqrt{\frac{\sum (x - \mu)^2}{N}}$$

Set (7)

$$s^2 = \frac{(40-40)^2 + (38-40)^2 + (42-40)^2 + (40-40)^2 + \dots + (40-40)^2}{9}$$

$$s^2 = \frac{0 + 4 + 4 + 0 + 1 + 1 + 9 + 0 + 1 + 0}{9}$$

$$s^2 = \frac{20}{9} = 2.22$$

$$s = 1.49$$

Set 2. $s^2 = \frac{36 + 9 + 0 + 49 + 4 + 16 + 0 + 49 + 36 + 25}{9}$

$$s^2 = 24.889$$

$$s = 4.99$$

★ Stem and Leaf Diagrams

86 80 25 77 73 76 100 90 69 93
90 83 70 73 73 70 90 83 71 95
40 58 68 69 100 78 87 97 92 74

2	5
3	
4	0
5	8
6	9 8 9
7	7 3 6 0 3 3 0 1 8 4
8	6 0 3 3 7
9	0 3 0 0 5 7 2
10	0 0

ordered stem-leaf

2	5
3	
4	0
5	8
6	8 9 9
7	0 0 1 3 3 3 4 6 7 8
8	0 3 3 7 8
9	0 0 0 2 3 5 7
10	0 0

Frequency Histogram

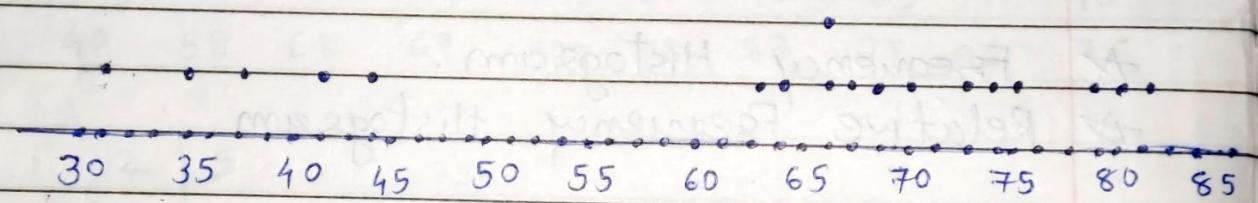
- ★ Frequency Histogram
- ★ Relative Frequency Histogram

20s	0.033
30s	0
40s	0.033
50s	0.033 0.033
60	0.1
70	0.333
80	0.1666
90	0.2333
100	0.0666

★ Dot plots for Numerical Data

64, 41, 44, 31, 37, 73, 72, 68, 835, 37, 81, 90, 82, 74,
79, 67, 66, 66, 70, 63

67, 21, 32, 88, 35, 71, 39, 35, 71, 63, 12, 46, 35, 39, 28,
65, 25, 24, 22



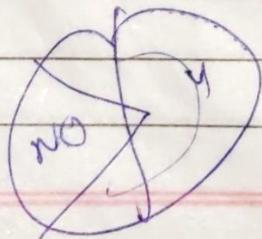
★ Bar chart:

when :- Categorical data

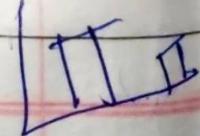
★ Pie charts:

Slices of pie separates categories.

small number of possible categories.



yes 60
no 60



50 workers over last 6 weeks

2, 2, 0; 0, 5, 8, 3, 4, 1, 0, 0, 7, 1, 7, 1, 5, 4, 0, 4, 0, 1, 8,
9, 7, 0, 1, 7, 2, 5, 5, 4, 3, 3, 0, 0, 2, 5, 1, 3, 0, 1, 0, 2, 4, 5,
0, 5, 7, 5, 1

	Frequency	R.F.
0	12	0.24
1	8	0.16
2	5	0.1
3	4	0.08
4	5	0.1
5	8	0.16
6	0	
7	5	0.1
8	2	0.04
9	1	0.02

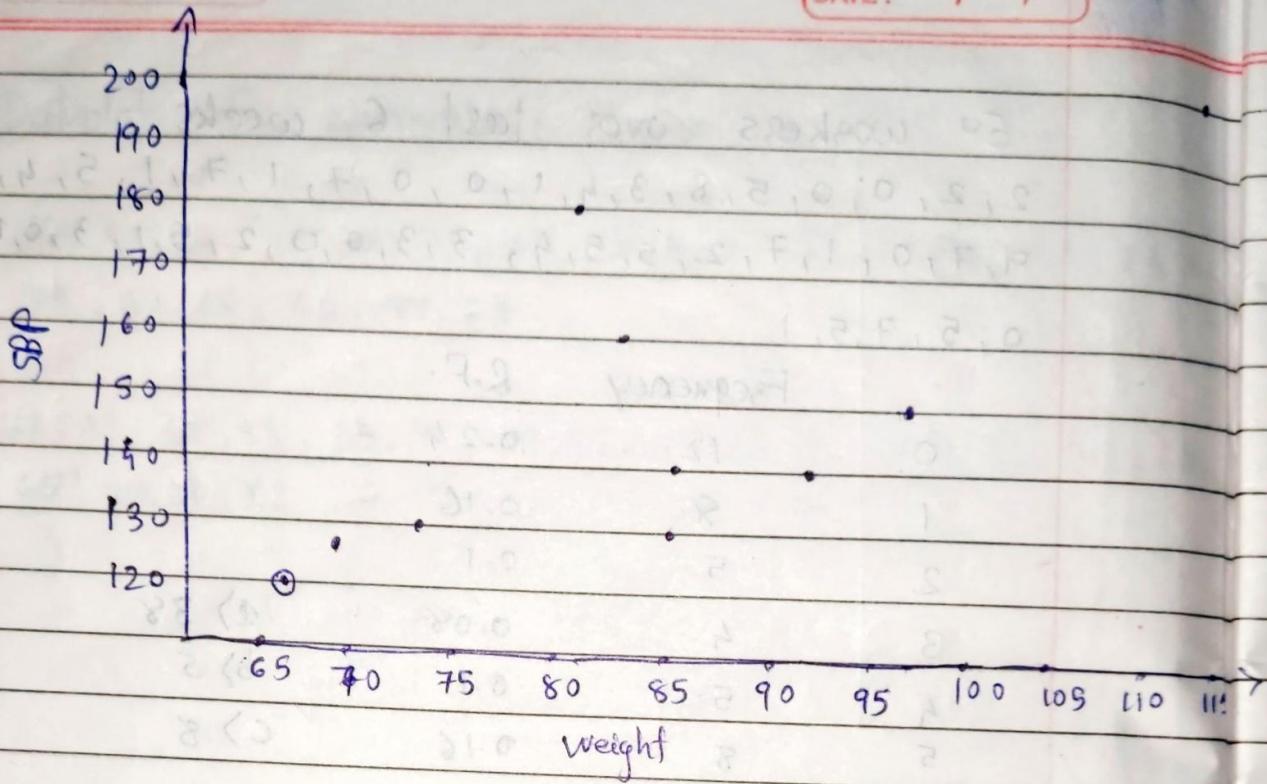
* Scatter Diagram

Two quantitative variable

One is independent (x) and second is dependent (y)

No frequency table. Points are joined.

67	69	85	83	87	81	97	92	114	85
120	125	140	160	130	180	150	140	200	130



* Simple correlation coefficient (r)

- Pearson's correlation or product moment correlation
- > measures the nature and strength
 - +ve increase
 - ve decrease

* $-1 \leq r \leq 1$

strong



perfect

intermediate

-0.75

-0.25

0

0.25

inte.



No

relation

strong
0.75



perfect.

$$r = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sqrt{\left(\sum x^2 - \frac{(\sum x)^2}{n} \right) \left(\sum y^2 - \frac{(\sum y)^2}{n} \right)}}$$

x	y	x^2	y^2	xy
7	12	49	144	84
6	8	36	64	48
8	12	64	144	96
5	10	25	100	50
6	11	36	121	66
9	13	81	169	117
$\sum x$	$\sum y$	$\sum x^2$	$\sum y^2$	$\sum xy$
= 41	= 66	= 291	= 742	= 461

$$r = \frac{461 - 451}{\sqrt{(291 - 280.167)(742 - 726)}}$$

$$= \frac{10}{\sqrt{10.833 \times 16}} = \frac{10}{\sqrt{173.328}} = \frac{10}{13.1654} = 0.7595$$

to measure -
positive relationship.

strong

x	y	x^2	y^2	xy
10	2	100	4	20
8	3	64	9	24
2	9	4	81	18
1	7	1	49	7
5	6	25	36	30
6	5	36	25	30
<u>32</u>	<u>32</u>	<u>230</u>	<u>204</u>	<u>129</u>

$$r = \frac{129 - 170.667}{\sqrt{(230 - 170.667)(204 - 170.667)}}$$

$$= \frac{-41.667}{\sqrt{59.333 \times 33.333}}$$

$$= -41.667$$

$$44.472$$

$$= -0.9369$$

Negative strongly correlated.

Q

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}}$$

\bar{x} = mean of x

\bar{y} = mean of y

★ Partial Correlation

- measure of the strength and direction of a linear relationship between two continuous variables
- known as covariates or control variables

Assumption 1: one dependent and one independent
both continuous

Assumption 2: one or more control variables.
known as covariate.

Assumption 3: continuous

4: There needs to be a linear relationship between all 3 vars.

5: no significant outliers

6: approximately normally distributed

$$\rho_{xy.z} = \frac{(r_{xy} - r_{yz}r_{zx})}{\sqrt{(1-r_{yz}^2)(1-r_{zx}^2)}}$$

$$R_{xyz} = \sqrt{\frac{(0.3988)^2 + (0.265)^2 - 2(0.3988)(0.265)(0.9179)}{1 - (0.3988)^2}}$$

$$= \sqrt{\frac{0.03525}{0.84095}} = \sqrt{0.04191} = 0.2047$$

* Regression Analyses

technique concerned with predicting some variables by knowing others.

- > Process of predicting one using another

y = criterion x = predictors

Slope = regression coefficient

Intercept = regression constant

$$\hat{y} = a + bx$$

$$\hat{y} = \bar{y} + b(x - \bar{x})$$

$$a = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}}$$

$$b = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}}$$

$$b = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}}$$

x	y	x^2	xy
7	12	49	84
6	8	36	48
8	12	64	96
5	10	25	50
6	11	36	66
9	13	81	117
41	66	291	461

$$\bar{x} = \frac{1}{6} (7 + 6 + 8 + 5 + 6 + 9) = 7.17$$

$$\bar{y} = \frac{1}{6} (12 + 8 + 12 + 10 + 11 + 13) = 10.83$$

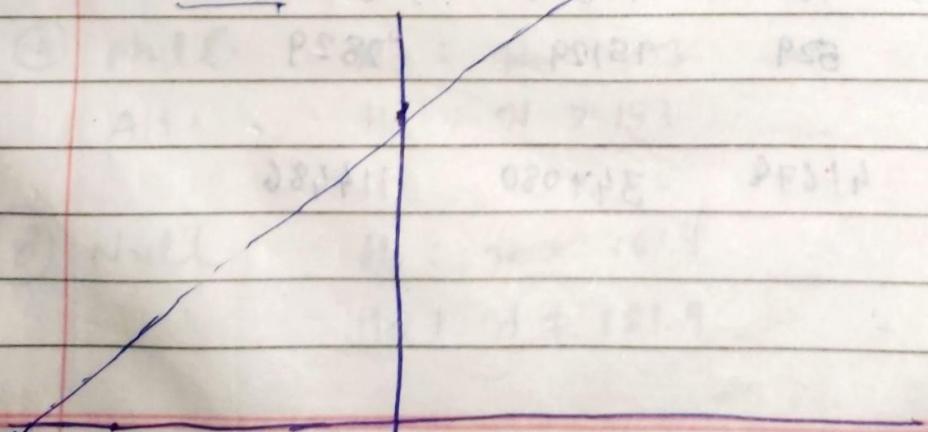
$$b = \frac{66 - \frac{66 \times 41}{6}}{291 - \frac{(41)^2}{6}} = \frac{461 - 451}{291 - 280.167} = 0.923$$

$$\hat{y} = 11 + 0.923(x - 6.83)$$

$$\hat{y} = 0.923x + 4.696$$

$$\hat{y} = 11 + 0.923(8.5 - 6.83) = 11 + 0.001 = 11.541$$

$$(x=8.5) \quad \hat{y} = 12.541$$



x	y	x^2	y^2	xy
20	120	400	14400	2400
43	128	1849	16384	5504
63	141	3969	19881	8883
26	126	676	15876	3276
53	134	2809	17956	7102
31	128	961	16384	3968
58	136	3364	18496	7888
46	132	2116	17424	6072
58	140	3364	19600	8120
70	144	4900	20736	10080
46	128	2116	16384	5888
53	136	2809	18496	7208
60	136	3600	21316	8760
20	124	400	15376	2480
63	143	3969	20449	9009
43	130	1849	16900	5590
26	124	676	15376	3224
19	121	361	14641	2299
31	126	961	15876	3906
23	123	829	15129	2829
Σ		852	2030	114486
\bar{x}	\bar{y}			
42.6	131.5			

$$c = \gamma + 6(22) \text{ No need of unit.}$$

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- ② The average July temperature in a region historically has been 74.5°F .

Null hypothesis $H = 74.5^{\circ}\text{F} = H_0$
Alt. " $H > 74.5^{\circ}\text{F}$

- ⑤ The average weight of a female airline passenger with luggage was 145 pounds ten years ago. The FAA believes it to be higher now.

$H_0 : \mu = 145 \text{ lb}$

$H_a : \mu > 145 \text{ lb}$

- ③ The average sti.

Q. 2.

- ④ Null hypothesis $H_0 : \mu = 38.2$

Alt. " $H_a : \mu < 38.2$

- ⑤ Null $H_0 : \mu = 5829$

Alt. " $H_a : \mu \neq 5829$

- ⑥ Null $H_0 : \mu = 133$

Alt. " $H_a : \mu > 133$

- ⑦ Null $H_0 : \mu = 161.9$

$H_a : \mu \neq 161.9$

Sampling Distribution.

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Sample is random \Rightarrow every statistic
is also random &
varies from sample to sample

$$A = \{152, 156, 160, 164\} \quad \bar{M} = 158$$

$$\sigma = \sqrt{20}$$

All possible samples

$$\begin{aligned} & \{(152, 152), (152, 156), (152, 160), (152, 164), \\ & (156, 152), (156, 156), (156, 160), (156, 164), \\ & (160, 152), (160, 156), (160, 160), (160, 164), \\ & (164, 152), (164, 156), (164, 160), (164, 164) \end{aligned}$$

$$\bar{M} = 152, 154, 156, 158, 154, 156, 158, 160, 156, \\ 158, 160, 162, 158, 160, 152, 164$$

\bar{x}	152	153	156	158	160	162	163
$P(\bar{x})$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$
	144	2964.5	4563	6241	4800	3280.5	1681

$$\begin{aligned} \text{E}(\bar{x}) &= \sum \bar{x} P(\bar{x}) \\ &= 144 + 29.25 + 45.5 + 62.5 + 39.5 + 30 \\ &\quad + 20.25 + 10.25 \end{aligned}$$

$$= 158.25$$

$$\mu = \mu_x$$

$$\sigma_x = \frac{\sigma_x}{\sqrt{n}}$$

n = sample size

$$P(a < z < b)$$

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$$\Phi(b) - \Phi(a)$$

$$\text{Var}(\bar{x}) = E(x^2) - (E(x))^2$$

$$P(-z) = 1 - \Phi(z)$$

$$= 24974 - 24964$$

$$\sigma_x = \sqrt{10} = 3.16\dots$$

Q Stemdeed Normal Dist.

$$\mu = 0, \sigma = 1$$

$$Z$$

$$\Phi_z = P(Z \leq z)$$

$$\textcircled{a} P(Z \leq 1.48) = \Phi(1.48) \quad b: P(Z \leq -0.25) = 1 - \Phi(-0.25)$$
$$= 0.9306 \quad = 1 - 0.5987$$
$$= 0.4013$$

$$\textcircled{b} P(Z \geq 1.60) = 1 - P(Z \leq 1.6) = 1 - 0.9452$$
$$= 0.0548$$

$$\textcircled{c} P(Z \geq -1.02) = 1 - P(Z \leq -1.02) = 0.8461$$
$$= 1 - (1 - P(Z \leq 1.02))$$

$$\textcircled{d} P(0.5 < Z < 1.57) = \Phi(1.57) - \Phi(0.5)$$
$$= 0.9418 - 0.6915$$
$$= 0.2503$$

$$\textcircled{e} P(-2.55 < Z < 0.09) = \Phi(0.09) - \Phi(-2.55)$$
$$= 0.5354 - (1 - 0.9946)$$
$$= 0.5305$$

Prob. for General Normal

$$P(a < X < b) = P\left(\frac{a-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{b-\mu}{\sigma}\right)$$

$a, b \in \mathbb{R}$

$$\mu = 10 \quad \sigma = 2.5$$

a. $P(X < 14)$

$$\begin{aligned} P\left(Z < \frac{14-10}{2.5}\right) &= P(Z < 1.6) \\ &= 0.9452 \end{aligned}$$

b. $P(8 < X < 14) = P(-0.8 < Z < 1.6)$

$$\begin{aligned} &= 0.9452 - 1 + 0.7881 \\ &= 0.7333 \end{aligned}$$

c. $\mu = 37500$

$\sigma = 4500$

$$\begin{aligned} P(30000 < X < 40000) &= P(-1.67 < Z < 0.556) \\ &= 0.7123 + 0.9525 - 1 \\ &= 0.6648 \end{aligned}$$

* CLT

sample ≥ 30

normally distribution

Sample mean $\bar{y}_x = \mu$

$$\text{S.D.} = S_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

* Large sample tests for a population mean

$$\bar{x} - \mu \quad \text{If } \sigma \text{ is known } z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

Standardized

Test

$$\text{If } \sigma \text{ is unknown } z = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

- 1 Identify the null and alternative hypothesis
 - 2 Identify the relevant test statistic and its distribution
 - 3 compute from the data the value of the test statistic
 - 4 construct the rejection region
 - 5 compare the value computed in 3 to the region constructed in 4 and make a decision.
- Formulate the decision in the context of the problem.

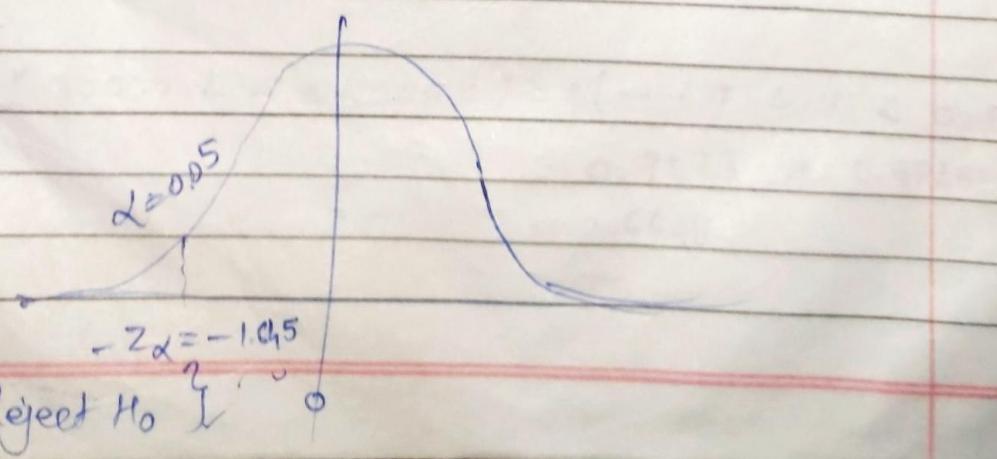
* It is hoped that a newly developed pain reliever will more quickly produce perceptible reduction in pain to patients after minor surgeries than a standard pain reliever. $\mu = 3.5$, $\sigma = 2.1$
 $\bar{x} = 3.1$, $s = 1.5$

$$H_0: \mu = 3.5$$

$$H_a: \mu < 3.5$$

$$@ \alpha = 0.05$$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{3.1 - 3.5}{1.5 / \sqrt{50}} = \frac{-0.4}{1.5} \times \sqrt{50} = 1.886$$



$$z = -1.886 \text{ Reject } H_0$$

A : number is 5

B : number is odd

$$\begin{aligned} P(A|B) &= \frac{P(A \cap B)}{P(B)} \\ &= \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3} \end{aligned}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$$

A₁ : Laboratory 1

A₂ : " 2

$$P(A_1) = 0.92$$

$$P(A_2) = 0.92$$

$$\therefore P(A_1 \cap A_2) = 0.8464$$

$$P(A_1 \cup A_2) = 0.92 + 0.92 - 0.8464 = 0.9936$$

$$1) R_x = \{0, 1, 2\}$$

$$S = \{HH, HT, TH, TT\}$$

$$P_x(x=0) = \frac{1}{4}$$

$$P_x(x=1) = \frac{2}{4} = \frac{1}{2}$$

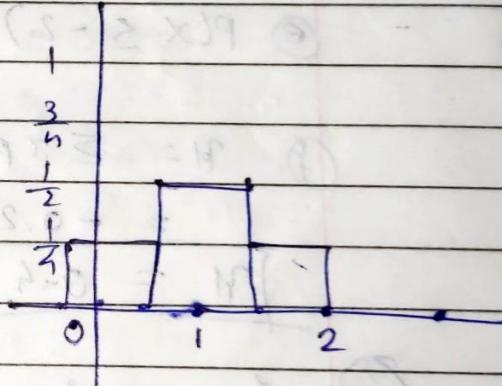
$$P_x(x=2) = \frac{1}{4}$$

x	0	1	2
$P(x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$$P(x \geq 1) = P(1) + P(2)$$

$$= \frac{1}{2} + \frac{1}{4}$$

$$= \frac{3}{4}$$



$$E(x) = -0.42 + 0.33 + 0.48 + 0.735$$

$$\mu = E(x) = 1.135$$

$$E(x^2) = 0.42 + 0.34 + 0.96 + 2.5725$$

$$= 4.2925$$

$$= 3.004275$$

x	-1	0	1	4
$p(x)$	0.2	0.5	a	0.1

$$\boxed{a = 0.2}$$

$$\textcircled{a} \quad \sum p(x) = 1 \quad (0 - x)^q$$

$$0.8 + a = 1$$

$$a = 0.2$$

$$\textcircled{b} \quad P(0) = 0.5$$

$$\textcircled{c} \quad P(X > 0) = P(1) + P(4) = 0.2 + 0.4 = 0.63$$

$$\textcircled{d} \quad P(X \geq 0) = 1 - P(X = -1) = 1 - 0.2 = 0.8$$

$$\textcircled{e} \quad P(X \leq -2) = 0$$

$$\textcircled{f} \quad M = \sum x p(x)$$

$$= -0.2 + 0 + 0.2 + 0.4$$

$$\underline{\underline{M = 0.4}}$$

$$\textcircled{g} \quad s^2 = \sum x^2 p(x) - (\sum x p(x))^2$$

$$\sum x^2 p(x) = 0.2 + 0.2 + 1.6 \\ = 2$$

$$s^2 = 2 - 0.16$$

$$s^2 = 1.84$$

$$\textcircled{h} \quad \underline{\underline{s = 1.3865}}$$

$$s = \underline{\underline{1.3865}}$$

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$$P(X=x) = {}^n C_x p^x q^{n-x}$$

★ Binomial dist.ⁿ

$$P(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} = {}^n C_x p^x q^{n-x}$$

$$\mu = np \quad \sigma^2 = npq$$

$$n=5, \quad p=0.17 \quad q=0.83$$

$$\sigma = \sqrt{npq}$$

$$P(0) = {}^5 C_0 (0.17)^0 (0.83)^5$$

$$P(0) = {}^5 C_0 (0.17)^0 (0.83)^5 = 0.3939$$

$$P(1) = {}^5 C_1 (0.17)^1 (0.83)^4 = 0.4034$$

$$P(2) = {}^5 C_2 (0.17)^2 (0.83)^3 = 0.1652$$

$$P(3) = {}^5 C_3 (0.17)^3 (0.83)^2 = 0.0338$$

$$P(4) = {}^5 C_4 (0.17)^4 (0.83)^1 = 0.0035$$

$$P(5) = {}^5 C_5 (0.17)^5 (0.83)^0 = 0.0001$$

0.40

0.35

0.30

0.25

0.20

0.15

0.10

0.05

0

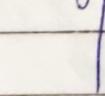
1

2

3

4

5



$$p=0.5 \quad q=0.5$$

$$(0.5)^x (0.5)^{10-x}$$

$$P(x) = {}^{10}C_x (0.5)^{10}$$

$$P(6) = {}^{10}C_6 (0.5)^{10}$$

$$\approx 0.2051$$

$$\textcircled{b} \quad P(X > 6) = 1 - P(X \leq 5)$$

$$= 1 - \left[{}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 + {}^{10}C_5 \right] (0.5)^{10}$$

$$\approx 0.3770$$

$$n=5 \quad p=1/3 \quad q=2/3$$

$$P(1) = {}^5C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^4$$

$$= 5 \times 0.33 \quad (0.67)^4 = 0.3325$$

$$P(X \leq 1) = {}^5C_0 \left(\frac{2}{3}\right)^5 + 0.3325 \\ = 0.4675$$

$${}^5C_x \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{5-x} \geq 0.95$$