R : comm ring with 1. $P \subseteq R$. prime ideal. $R_P := (R-P)^{-1}R$

$R_P = \{ \frac{r}{a} : r \in R, a \in R-P \}$.

Let $\frac{r}{a} \in R_P$.

If $r \in R-P$, $\frac{1}{r} \in R_P$ s.t. $r \cdot \frac{1}{r} = 1 \in R_P$.

$\Rightarrow \frac{a}{r} \in R_P$ with $\frac{r}{a} \cdot \frac{a}{r} = 1$. $\therefore \frac{r}{a}$ is unit.

If $r \in P$, r is not unit in R since P is prime ideal ($\Rightarrow$ proper ideal)

Suppose $\exists \frac{r_0}{a_0} \in R_P$ the inverse of $\frac{r}{a}$, i.e. $\frac{r}{a} \cdot \frac{r_0}{a_0} = 1$.

$\Leftrightarrow r \cdot r_0 \cdot \frac{1}{a} \cdot \frac{1}{a_0} = 1$. $\Leftrightarrow r r_0 \frac{1}{a} \frac{1}{a_0} \cdot a_0 a = a_0 a$. $\Leftrightarrow r r_0 = a_0 a$.

case (i) Let $r_0 \in P$. then $a_0 a = r r_0 \in P$ but $a_0 \in R-P$, $a \in R-P$.

$\Rightarrow$ contradiction. ($\because$ P is prime ideal)

case (ii) Let $r_0 \in R-P$. then $\exists \frac{1}{r_0} \in R_P$, thus $a_0 a \frac{1}{r_0} = r \in P$

$\Rightarrow$ contradiction.

$\therefore \frac{r}{a}$ is not unit.

$I = \{ \frac{d}{a} : d \in P, a \in R-P \}$ is ideal of R_P. because

(i) $\forall \frac{d_1}{a_1}, \frac{d_2}{a_2} \in I$, $\frac{d_1}{a_1} - \frac{d_2}{a_2} = \frac{a_2 d_1 - a_1 d_2}{a_1 a_2} \in I$ ($\because$ $a_1 a_2 \in R-P$ and $a_2 d_1 - a_1 d_2 \in P$).

$\Rightarrow I$ is additive subgroup of R_P.

(ii) $\forall \frac{r_0}{a_0} \in R_P, \forall \frac{d}{a} \in I$, $\frac{r_0}{a_0} \cdot \frac{d}{a} = \frac{r_0 d}{a_0 a} \in I$ ($\because$ $a_0 a \in R-P$ and $r_0 d \in P$.)

$\therefore$ since every $x \in R_P - I$ is unit, I is the only maximal ideal of R_P.