(2.3) $V = |\operatorname{grad} H|^2 = \left(\frac{\partial H}{\partial n}\right)^2 + \left(\frac{\partial H}{\partial s}\right)^2$

- V is subharmonic in D

$(\because$ $\Delta V = \frac{\partial^2 V}{\partial n^2} + \frac{\partial^2 V}{\partial s^2} = \frac{\partial}{\partial n}\left(2 \frac{\partial H}{\partial n}\frac{\partial^2 H}{\partial n^2} + 2\frac{\partial H}{\partial s}\frac{\partial^2 H}{\partial n \partial s}\right)$

$$+ \frac{\partial}{\partial s}\left(2\frac{\partial H}{\partial n}\frac{\partial^2 H}{\partial s \partial n} + 2\frac{\partial H}{\partial s}\frac{\partial^2 H}{\partial s^2}\right)$$

$$= 2\Big[\underline{\left(\frac{\partial^2 H}{\partial n^2}\right)^2} + \frac{\partial H}{\partial n}\frac{\partial}{\partial n}\left(\frac{\partial^2 H}{\partial n^2}\right) + \underline{\left(\frac{\partial^2 H}{\partial n \partial s}\right)^2} + \frac{\partial H}{\partial s}\frac{\partial}{\partial n}\left(\frac{\partial^2 H}{\partial n \partial s}\right)$$

$$+ \underline{\left(\frac{\partial^2 H}{\partial s \partial n}\right)^2} + \frac{\partial H}{\partial n}\frac{\partial}{\partial s}\left(\frac{\partial^2 H}{\partial s \partial n}\right) + \underbrace{\left(\frac{\partial^2 H}{\partial s^2}\right)^2}_{\geq 0} + \frac{\partial H}{\partial s}\frac{\partial}{\partial s}\left(\frac{\partial^2 H}{\partial s^2}\right)\Big]$$

$$\geq 2\Big[\underbrace{\frac{\partial H}{\partial n}\frac{\partial}{\partial n}\left(-\frac{\partial^2 H}{\partial s^2}\right) + \frac{\partial H}{\partial n}\frac{\partial}{\partial s}\left(\frac{\partial^2 H}{\partial s \partial n}\right)}_{=0(?)} \quad (\because \Delta H = 0)$$

$$+ \underbrace{\frac{\partial H}{\partial s}\frac{\partial}{\partial s}\left(-\frac{\partial^2 H}{\partial n^2}\right) + \frac{\partial H}{\partial s}\frac{\partial}{\partial n}\left(\frac{\partial^2 H}{\partial n \partial s}\right)}_{=0(?)}\Big]$$

commuting 3rd order derivatives.

Why $\nexists$ curvature terms?

- (2.6 a) $$2\left[\frac{\partial H}{\partial n}\frac{\partial^2 H}{\partial n^2} + \frac{\partial H}{\partial s}\frac{\partial}{\partial s}\left(\frac{\partial H}{\partial n}\right) - K\left(\frac{\partial H}{\partial s}\right)^2\right] > 0$$

At P. $\frac{\partial v}{\partial n} > 0$

$$\Leftrightarrow 2\left[\frac{\partial H}{\partial n}\frac{\partial^2 H}{\partial n^2} + \frac{\partial H}{\partial s}\frac{\partial^2 H}{\partial n \partial s}\right] > 0$$

$$\frac{\partial^2 H}{\partial n \partial s} - \frac{\partial^2 H}{\partial s \partial n} = -K\frac{\partial H}{\partial s}$$

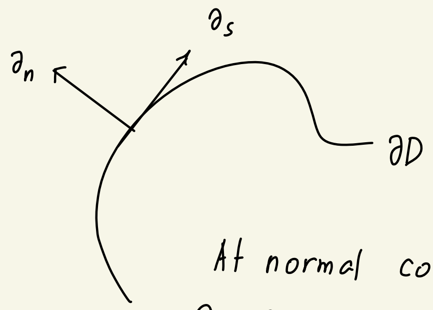
At normal coordinates $\Gamma^{\cdot}_{\cdot\cdot}$'s $= 0 \Leftrightarrow \nabla_{\partial_i} = \partial_i$

So indeed, K appears from commuting covariant derivatives

cf) In standard coordinates w/ automorphism $(x.y) \longleftrightarrow (n.s)$ already covered.

In normal coordinates, (w.r.t. $\partial_n . \partial_s$: $\Delta = \frac{\partial^2}{\partial n^2} + \frac{\partial^2}{\partial s^2}$)

• (2.7) $$\frac{\partial^2 H}{\partial n^2} + K \frac{\partial H}{\partial n} + \frac{\partial^2 H}{\partial s^2} = 0 \quad \text{at } P$$



At normal coord.

$\partial_n \cdot \partial_n = \partial_s \cdot \partial_s = 1$

$\partial_n \cdot \partial_s = \partial_s \cdot \partial_n = 0$

• (2.12) If $\dfrac{\partial H}{\partial s}(p) = 0$, $\dfrac{\partial^2 V}{\partial s^2} \leq 0$ at p.

($\because$

max at $p \Rightarrow \Delta V(p) \leq 0$

but
$$\underbrace{\frac{\partial^2 V}{\partial n^2}}_{\geq 0} = 2\left[\left(\frac{\partial^2 H}{\partial n^2}\right)^2 + \underbrace{\frac{\partial H}{\partial n}\frac{\partial^3 H}{\partial n^3}}_{= p_2^4 H^2 \geq 0} + \left(\frac{\partial^2 H}{\partial n \partial s}\right)^2 + \cancelto{0}{\frac{\partial H}{\partial s}\frac{\partial^3 H}{\partial n^2 \partial s}} \right]$$
)

• (2.13) $p_2^2 H \dfrac{\partial^2 H}{\partial s^2} + \left(\dfrac{\partial^2 H}{\partial s^2}\right)^2 \leq 0$ (still $\dfrac{\partial H}{\partial s} = 0$)

($\because$

$$\frac{\partial^2 V}{\partial s^2} = 2\left[\underbrace{\left(\frac{\partial}{\partial s}\left(\frac{\partial H}{\partial n}\right)\right)^2}_{\left(\frac{\partial}{\partial s}(p_2 H)\right)^2 = p_2^2 \left(\frac{\partial H}{\partial s}\right)^2 = 0} + \underbrace{\frac{\partial H}{\partial n}\frac{\partial^2}{\partial s^2}\left(\frac{\partial H}{\partial n}\right)}_{p_2^2 H \frac{\partial^2 H}{\partial s^2}} + \left(\frac{\partial^2 H}{\partial s^2}\right)^2 + \cancelto{0}{\frac{\partial H}{\partial s}\frac{\partial^3 H}{\partial s^3}} \right]$$
)

Q1 Why do we consider eigenvalue problems?

Q2 What's special about "Stekloff" eigenvalue problem? In particular, how's it related to minimal surfaces?

Q3 How could finding such eigenvalues help to solve a PDE of interest? In particular, which eigenvalue problem can be formulated on the setting of CMA?

왜 이 논문을 읽어보라고 했을까 생각해보았는가
max.principle의 활용과 curvature term control

$\exists$ probabilistic formulation

tensorial하게 바꾸면 Bochner formula와 Ricci curvature 이야기가 됨

이 논문에서의 세팅이 가장 기본적인 상황.

기하해석 공부를 어떤 맥락에서 할 것인지?