Deep Portfolio Theory

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Introduction

- ## Reference paper

  [J.B.Heaton et al.] Deep Portfolio Theory, *https://arxiv.org/abs/1605.07230*

- ## Deep Portfolio Theory

  [Goal]

  Provide a theory of deep portfolio

  - Develop a self-contained four step routine of *encode*, *calibrate*, *validate* and *verify* to formulate an automated and general portfolio selection process
  - Reduce model dependence to a minimum through a data driven approach which established the risk-return balance as part of the validation phase of a supervised learning routine
  - Construct an auto-encoder and multivariate portfolio payouts

  Deep factors

  - Lower (or hidden) layer abstractions which, through training, correspond to the independent variable
  - Dominant deep factors, which frequently have a non-linear relationship to the input data, ensure applicability of the subspace reduction to the independent variable

Preliminaries

Markowitz Model

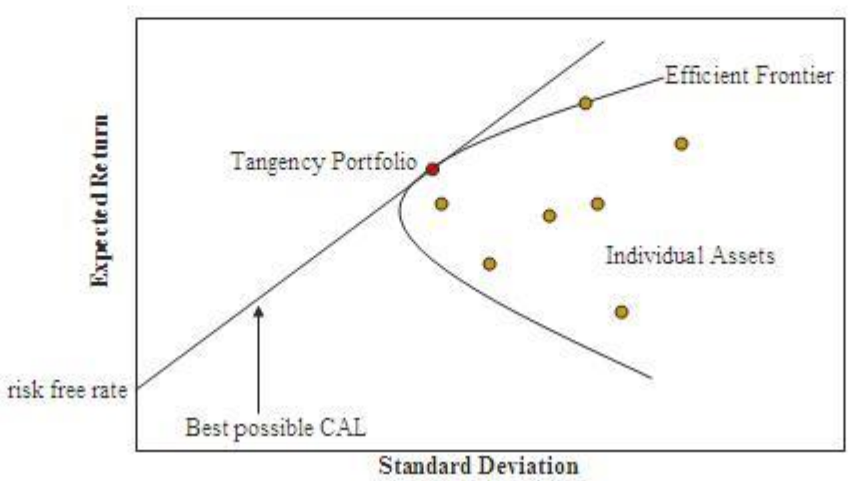
- Introduced by Harry Markowitz in a 1952 essay
- A mathematical framework for assembling a portfolio of assets such that the expected return is maximized for a given level of risk

[Expected return of portfolio]

$$E(R_p) = \sum_i w_i E(R_i)$$

[Variance of portfolio]

$$\sigma_p^2 = \sum_i \sum_j w_i w_j \sigma_i \sigma_j \rho_{ij}$$



Preliminaries

Black-Litterman model

- Introduced by Black & Litterman in 1991
- Create stable, mean-variance efficient portfolios, based on an investor's unique insights, which overcome the problem of input-sensitivity

[Expected return of assets]

$$E(R) = [(\tau\Sigma)^{-1} + P'\Omega^{-1}\mathrm{P}]^{-1}[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}\mathrm{Q}]$$

τ : a scalar

Σ : the covariance matrix of excess returns

P : a matrix that identifies the assets involved in the views

Ω : a diagonal covariance matrix of error terms from the expressed views representing the uncertainty in each view

Π : the implied equilibrium return vector

Q : the view vector

[Variance of assets]

$$cov(R) = [(\tau\Sigma)^{-1} + P'\Omega^{-1}\mathrm{P}]^{-1}$$

Design of Deep Portfolio Theory

Four step deep portfolio construction

[Goal]

To provide a self-contained procedure that illustrates the trade-offs involved in constructing portfolios to achieve a given goal, e.g., to beat a given index by a pre-specified level

- Assume that the available market data has been separated into two disjointed sets for training and validation

I. Auto-encoding

Find the market-map, $F_W^m(X)$,

$$\min_W \left|\left|X - F_W^m(X)\right|\right|_2^2 \text{ subject to } \left|\left|W\right|\right| \leq L^m$$

This auto-encodes X with itself and creates a more information-efficient representation of X

II. Calibrating

For a desired result (or target) Y, find the portfolio-map, $F_W^p(X)$

$$\min_W \left|\left|X - F_W^p(X)\right|\right|_2^2 \text{ subject to } \left|\left|W\right|\right| \leq L^p$$

This creates a (non-linear) portfolio from X for the approximation of objective Y

Design of Deep Portfolio Theory

- **Four step deep portfolio construction**

III. Validating

Find L^m and L^p to suitably balance the trade-off between the two errors

$$\epsilon_m = \left|\left|\hat{X} - F^m_{W_m^*}(\hat{X})\right|\right|_2^2 \quad and \quad \epsilon_p = \left|\left|\hat{Y} - F^p_{W_p^*}(\hat{Y})\right|\right|_2^2$$

IV. Verifying

Choose market-map F^m and portfolio-map F^p such that validation is satisfactory.

Deep Portfolio Theory

Deep Portfolio Theory

- A large amount of input data $X = (X_{it})_{i,t=1}^{N,T} \in \mathbb{R}^{TxN}$: a market of N stocks over T time periods, a skinny matrix

[Markowitz]

- A data reduction as taken a dataset of $N * T$ observations to a set of parameters of size N (means) and $N(N-1)/2$ for the variance-covariances
- Very poor solution: L^2-norm of the fit of the implied market prices using the historical mean will have a large error as it ignores all periods of large volatility and jumps

$$\bar{X} = \frac{1}{T}\sum_{t=1}^{T} X_{it}$$

$$XX' = \frac{1}{T}\sum_{t=1}^{T}(X_{it} - \bar{X})(X_{it} - \bar{X})'$$

[Black-Litterman]

- The auto-encoding step soles the optimization problem of finding $\hat{\mu}(X)$ and $\hat{\Sigma}(X)$ from a penalty formulation

$$||\mu - X||_{\Sigma}^{2} + \lambda||P\mu - q||_{\Omega}^{2}$$

Deep Portfolio Theory

Deep Auto-Encoder

$$
\begin{aligned}
Y_j(x) = F_W^m(X)_j &= \sum_{k=1}^{K} W_2^{jk} f\left(\sum_{i=1}^{N} B_1^{ki} x_i\right) \\
&= \sum_{k=1}^{K} W_2^{jk} Z_j \text{ for } Z_j = f\left(\sum_{i=1}^{N} W_1^{ki} x_i\right)
\end{aligned}
$$

Since we are trying to fit the model $X = F_W(X)$

$$
\begin{aligned}
&\mathcal{L}(W) = \arg\min_W \|X - F_W(X)\|^2 + \lambda\phi(W) \\
\text{with} \quad &\phi(W) = \sum_{i,j,k} |W_1^{jk}|^2 + |W_2^{ki}|^2,
\end{aligned}
$$

$$
\arg\min_{W,Z} \|X - W_2 Z\|^2 + \lambda\phi(Z) + \|Z - f(W_1, X)\|^2
$$

- **Deep Auto-Encoder**

$$Y_j(x) = F_W^m(X)_j = \sum_{k=1}^{K} W_2^{jk} f\left(\sum_{i=1}^{N} B_1^{ki} x_i\right)$$
$$= \sum_{k=1}^{K} W_2^{jk} Z_j \text{ for } Z_j = f\left(\sum_{i=1}^{N} W_1^{ki} x_i\right)$$

Since we are trying to fit the model $X = F_W(X)$

$$\mathcal{L}(W) = \arg\min_W \|X - F_W(X)\|^2 + \lambda\phi(W)$$
$$\text{with} \quad \phi(W) = \sum_{i,j,k} |W_1^{jk}|^2 + |W_2^{ki}|^2,$$

$$\arg\min_{W,Z} \|X - W_2 Z\|^2 + \lambda\phi(Z) + \|Z - f(W_1, X)\|^2$$

Experiments

Data

- Weekly returns data for the component stocks of the biotechnology IBB index for the period Jan. 2012 to Apr. 2016

Auto-encoding & Calibration : Jan. 2012 ~ Dec. 2013

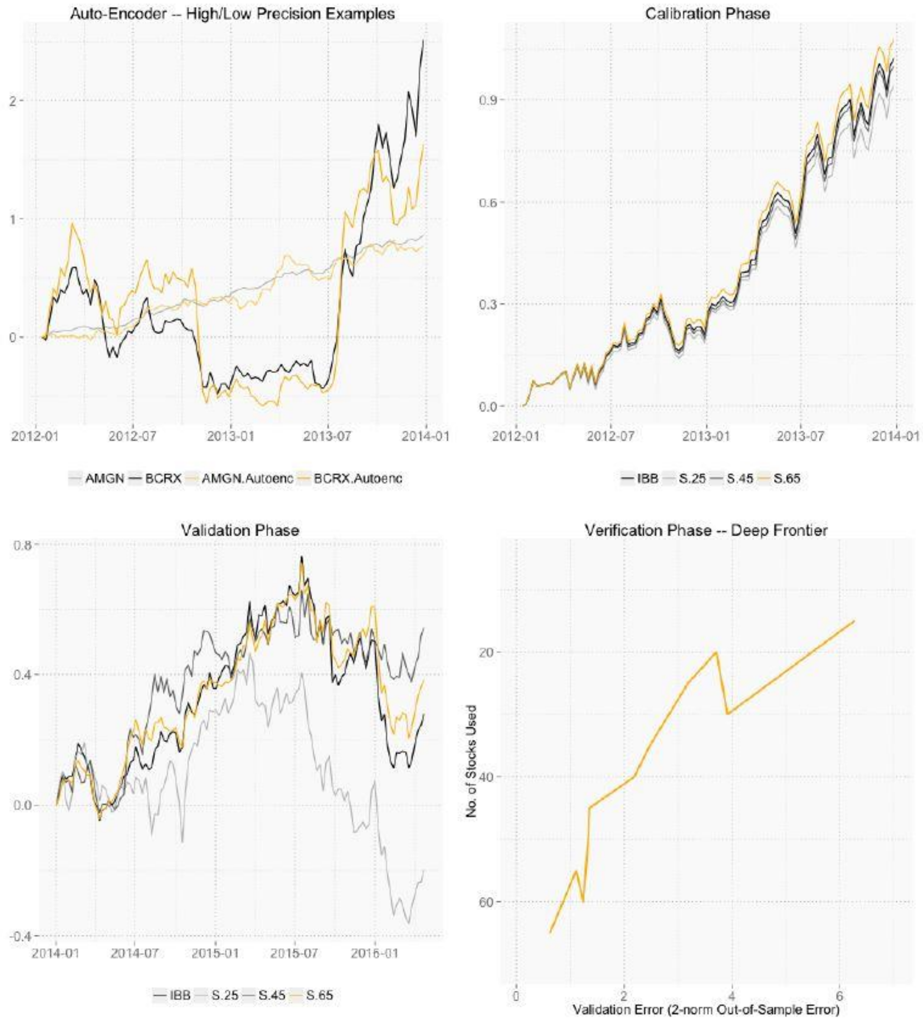
Validation & Verification : Jan. 2014 ~ Apr. 2016

Procedure

1. After auto-encoding the universe of stocks, consider the 2-norm difference between every stock and its auto-encoded version → *communal information*
2. Proximity of a stock to its auto-encoded version → A measure for the similarity of a stock with the stock universe
3. The 10 most communal stocks + x-number of most non-communal stocks
4. Infer weights to track the IBB index in calibration phase
5. Validate the results and draw Deep frontier

Experiments

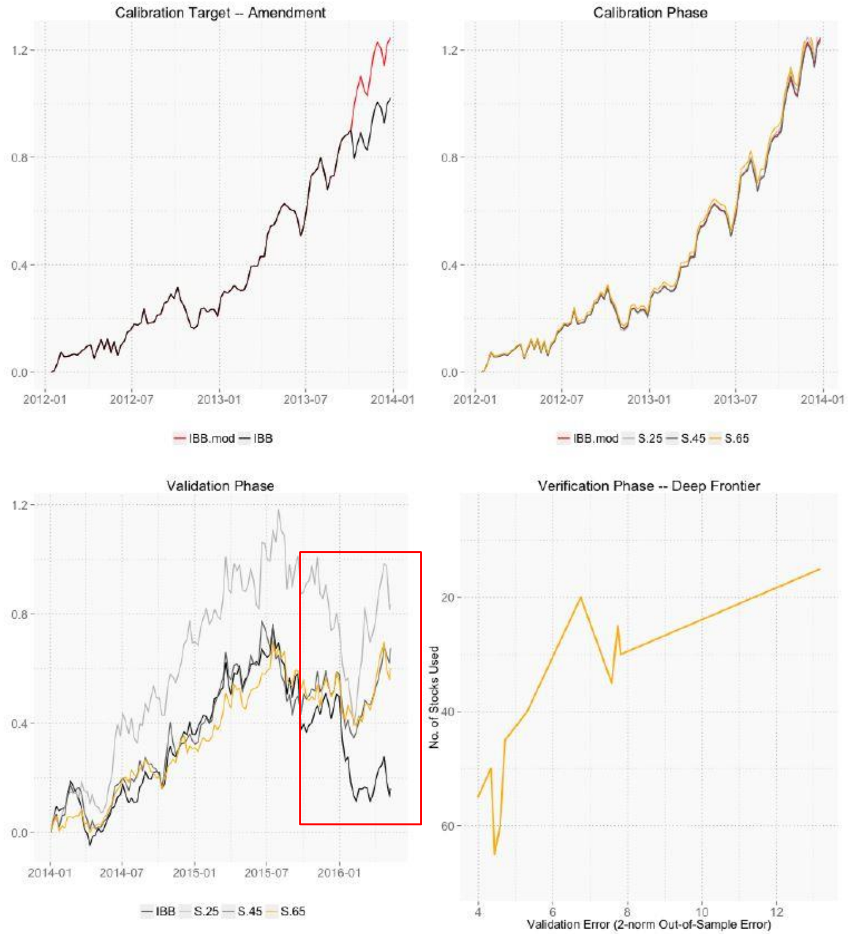
- **Result**



Experiments

Beating the Index!!

- Amend the target data during the calibration phase by replacing all returns smaller than -5% by exactly 5%
- aim to create an index tracker with anti-correlation in periods of large drawdowns



Domain adaptation in Finance

[Goal]

Develop pricing methodology of new markets based on existing markets

[Problem]

1. Existence of data

2. Distributions of training data ≠ Distributions of test data

$$\sum_{i=1}^{n_{tr}} \frac{p_{te}\left(x_i^{tr}\right)}{p_{tr}\left(x_i^{tr}\right)} \left(\hat{f}\left(x_i^{tr}\right) - y_i^{tr}\right)^2$$

[Experiment]

Training : S&P500 call option

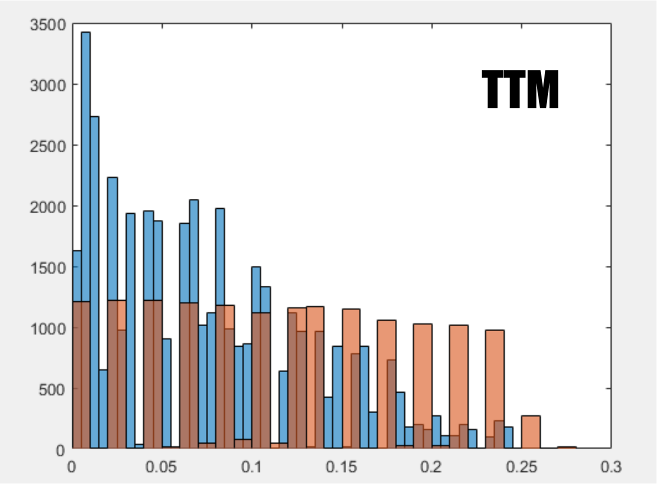
Test : KOSPI200 call option

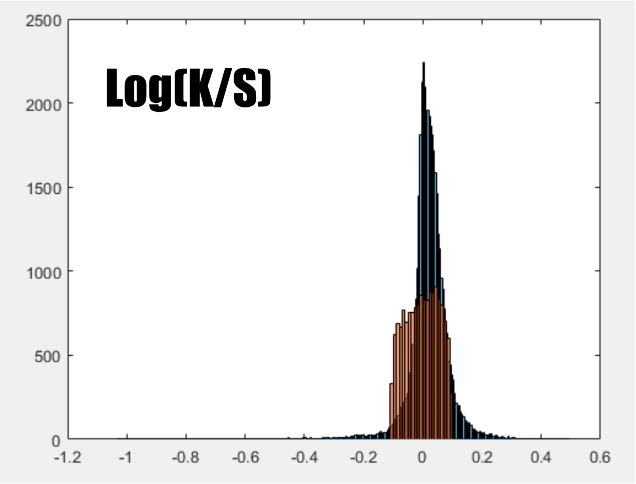
Feature : Time to Maturity(TTM), Strike Price/Spot Price(K/S) and interest rate(IR)

Importance weight : average of TTM and K/S

Current researches

- **Histogram**





- **Result**

	W/O weights	W/ weights
N_hidden	50	50
L2 regularization	0.001	0.001
Epoches	20000	20000
Learning rate	0.001	0.0005
Minibatches	20	20
Output multiplier	1000	1000
MAPE	36.73%	30.49%