ECS 032 B: Homework 1

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1) def fun1(n):

```
x=0 → 1
for i in range (2, n/2): → n/2 - 2
    y= x·2 → 1
return x → 1
```

$T(n) = \frac{n}{2} - 2 + 2 = n/2$

$O(n)$

$nc \geq n/2$

$n \leq 2nc$

$n_0 = 1$

$c = 2$

2) def fun2(n):

```
x=0 → 1
for i in range (n): → n
    for j in range (n): → n
        for k in range (1, n): → (n-1)
            x = x+1 → 1
return x → 1
```

$T(n) = 1 + n \cdot n \cdot (n-1) \cdot 1 + 1$

$= 2 + n^3 - n^2$

$O(n^3)$

$c\,n^3 \geq 2 + n^3 - n^2$

$n^3 \leq cn^3 + n^2 - 2$

$n_0 = 2$

$c = 4$

3) def fun3(n):

```
x=0 → 1
i = 2 · n → 2n
while i >= 1:        } 3 iterations
    x = x - i
    i = i - 1
return x
```

$T(n) = 3(1+2n)$

$= 3 + 6n$

$O(n)$

$cn \geq 3 + 6n$

$n \leq \frac{c}{6}n - \frac{3}{6}$

$n_0 = 1$

$c = 10$

4) def fun4(n):

```
x = n → n
for i in range (n·n) → n^2
    x = x+2 → 1
return x → 1
```

$T(n) = n + n^2 + 1$

$O(n^2)$

$cn^2 \geq n + n^2 + 1$

$n \leq cn^2 - n^2 - 1$

$n_0 = 1$

$c = 4$

5) def fun5(lst)

```
x=0 → 1
for y is lst: n
    x = x+y → 1
return x
```

$T(n) = n+2$

$O(n)$

$cn \geq n+2$

$n \leq cn-2$

$n_0 = 1$

$c = 4$

2) 1 $T=10$ (constant)

$O(10)$ is equal to $\Omega(10)$ which is $\Theta(10)$ $T(n)$ is $O(n)$ and $\Omega(n)$ making it $\Theta(n)$

2. $T(n) = 2n^3 + 1$

$O(n^3)$

$cn^3 \geq 2n^3 + 1$

$\frac{cn^3 - 1}{2} \geq \frac{2n^3}{2}$

$n^3 \leq \frac{cn^3 - 1}{2} \rightarrow \frac{cn^3}{2} - \frac{1}{2}$

$n_0 = 1$

$c = 3$

$\Omega(n^3)$

$cn^3 \leq 2n^3 + 1$

$\frac{cn^3 - 1}{2} \geq \frac{2n^3}{2}$

$n^3 \geq \frac{cn^3}{2} - \frac{1}{2}$

$n_0 = 1$

$c = 2$

$T(n)$ is $O(n)$ and $\Omega(n)$ making it $\Theta(n)$

3. $T(n) = n^4 + 2n^2 + n \log n$

$O(n^4)$

$cn^4 \geq n^4 + 2n^2 + n \log n$

$n^4 \leq cn^4 - 2n^2 - n \log n$

$n^3 \leq cn^3 - 2n - \log n$

$n_0 = 1$

$c = 2$

$\Omega(n^4)$

$cn^4 \leq cn^4 + 2n^2 + n \log n$

$n^4 \geq cn^4 - 2n^2 \quad n \log n$

$n^3 \geq cn^3 - 2n - \log n$

$n_0 = 1$

$c = 1$

$T(n)$ is $O(n)$ and $\Omega(n)$ making it $\Theta(n)$

4. $T(n) = \log(5 \times 2^n) = \log_2(5) + n \log_2(2)$

$\Rightarrow$ highest order is n

$O(n)$

$cn \geq \log_2(5) + n \log_2(2)$

$n \log_2(2) \leq cn - \log_2(5)$

$n \leq cn - \log_2(5)$

$n_0 = 1$

$c = 4$

$\Omega(n)$

$cn \leq \log_2(5) + n \log_2(2)$

$n \log_2(2) \geq cn - \log_2(5)$

$1 \quad -2.3$

$n \geq cn - \log_2(5)$

$n_0 = 1$

$c = 1$

$T(n)$ is $O(n)$ and $\Omega(n)$ making it $\Theta(n)$

5. $T(n) = 3n^2 \log n$

$O(n^2 \log n)$

$c \cdot n^2 \log n \geq 3n^2 \log n$

$n_0 = 1$

$c = 4$

$\Omega(n^2 \log n)$

$c \cdot n^2 \log n \leq 3n^2 \log n$

$n_0 = 1$

$c = 2$

$T(n)$ is $O(n)$ and $\Omega(n)$ making it $\Theta(n)$