CS207 Design and Analysis of Algorithms

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Backtracking

- Useful for constraint satisfaction problems (CSPs)
- (A CSP has a set of variables, each with a domain of values, and a set of constaints on these variables specified through relations on them. We need to find values for all variables, so that all constraints are satisfied. E.g., Sudoku)
- Incrementally builds candidates to the solutions
- Abandons a candidate ("backtracks") as soon as it becomes clear that the candidate cannot be completed into a valid solution

The n-Queens Problem

- Given to you are an $n \times n$ chess board, and n queens
- You are required to place the quuens on the board so that no two attack each other
- The board will have no other piece save these n-queens
- That means no two queens are on the same row, the same column or the same diagonal
- In particular, every row has exactly one queen
- Let Q_i be the queen of the i-th row

The 4-Queens Problem

	Q_1		
			Q_2
Q_3			
		Q_4	

- We denote this solution by the tuple $(2, 4, 1, 3)$
- Means: Q_1 is in column 2, Q_2 is in column 4 etc.
- (2) is a partial candidate solution; only Q_1 has been placed
- $(2, 4)$ is a partial candidate solution; only Q_1 and Q_2 have been placed
- $(2, 4, 1)$ is a partial candidate solution; only Q_1, Q_2 and Q_3 have been placed
- $(2, 4, 1, 3)$ is a full solution
- Say, we need to find all the solutions

The 4-Queens Problem

- Let us begin with (1)

Q_1			

- $(1, 1)$ and $(1, 2)$ are invalid
- Let us consider $(1, 3)$

Q_1			
		Q_2	

- $(1, 3, i)$ is invalid for every i; so, $(1, 3)$ cannot be completed into a full solution

The 4-Queens Problem

- We backtrack from $(1, 3)$ to (1)
- Let us explore $(1, 4)$ next

Q_1			
			Q_2

- $(1, 4, 1)$ is invalid; let us consider $(1, 4, 2)$

Q_1			
			Q_2
	Q_3		

- $(1, 4, 2, i)$ is invalid for every i; so, $(1, 4, 2)$ cannot be completed into a full solution; backtrack to $(1, 4)$

The 4-Queens Problem

- $(1, 4, 3)$ and $(1, 4, 4)$ are invalid; $(1, 4)$ has failed
- We backtrack from $(1, 4)$ to (1)
- All extensions of (1) have been explored; we backtrack to $()$
- Let us explore (2) next
- $(2, 1)$, $(2, 2)$, $(2, 3)$ are invalid
- Let us consider $(2, 4)$
- First, let us look at $(2, 4, 1)$
- $(2, 4, 1, 1)$ and $(2, 4, 1, 2)$ are invalid
- $(2, 4, 1, 3)$ **is a solution**

	Q_1		
			Q_2
Q_3			
		Q_4	

The 4-Queens Problem

- $(2, 4, 1, 4)$ is invalid
- We backtrack from $(2, 4, 1)$ to $(2, 4)$
- $(2, 4, 2)$, $(2, 4, 3)$, $(2, 4, 4)$ are invalid
- We backtrack from $(2, 4)$ to (2), and then to $()$
- And so on continues the backtracking algorithm
- By left-right symmetry we know that (3) and (4) are the mirror images of (2) and (1) respectively
- So $(3, 1, 4, 2)$ is a solution as well
- There are only two solutions

	Q_1		
			Q_2
Q_3			
		Q_4	

		Q_1	
Q_2			
			Q_3
	Q_4		