CS207 Design and Analysis of Algorithms

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Greedy Method

Greedy Method

- Works for some optimization problems, not all
- A greedy choice is one that looks best at the moment
- Steps:
  - Formulate a solution in which we make a *greedy* choice and are left with one subproblem.
  - Prove that there is always an optimal solution reachable thorugh the greedy choice; that is, the greedy choice is safe
  - Devise a way to combine an optimal solution to the subproblem with the greedy choice to obtain an optimal solution to the original problem

The Fractional Knapsack Problem

- Ali Baba goes into the cave of the 40 thieves.
- His donkey can carry a weight of at most W.
- He finds n heaps of metal dust arranged in order.
- Heap i is labelled by its weight w_i and its value v_i
- Ali Baba is greedy like his brother
- He will carry home the largest possible value.
- We need an algorithm for him to run
- Ali Baba can take a fraction of a heap, if he so wishes

Fractional Knapsack Solution

- Greedy choice works here
- If there is residual capacity, then it is better to fill it with gold rather than copper
- Assume that the heaps are sorted on "value per weight"
- Heap 1 of weight w_1 and value v_1 is the most precious metal
- If $W < w_1$, then take W/w_1 fraction of the first heap, and we are done
- Otherwise, take the whole of the first heap; the residual capacity is $W - w_1$; recurse with the remaining heaps
- It takes $O(n \log n)$ time to sort the heaps
- It takes only a scan of the resultant array to load the donkey now: $O(n)$ time
- Overall: $O(n \log n)$ time

Fractional Knapsack Solution

- Ali Baba is luckier than his brother; takes only $O(n \log n)$ time
- He gets away before the thieves come back
- In the fractional case, the most precious metal has to be picked to the maximum extent to get the optimal solution
- No other solution can be optimal, because in them some other metal can be replaced with the most precious one for a gain