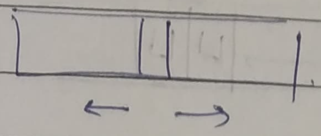
Continuing CHURCH TURING THESIS.

- DTM M_1, that has STAY PUT feature

$\{L, R, S\}$
↓
stay there



or remain there

$$\delta(q, x) = (q', y, S)$$

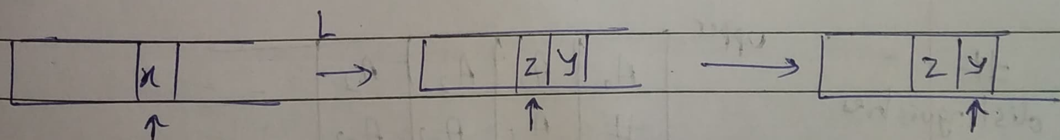
DTM M_2, Standard, no STAY PUT feature
movement ⇒ $\{L, R\}$

To prove: Both have equal computational power.

If M_2 does something (goes L, R), M_1 can do it.

If M_1 does something, can M_2 do the same
(L, R) can be done directly
let us look at STAY PUT case.

In M_1 ⇒ $\delta(q, x) = (q', y, S)$
In M_2



$$\delta(q, x) = (q'', y, L)$$
$$\delta(q'', z) = (q', z, R)$$

So here by 2 transitions, we implemented STAY PUT feature
Hence, no extra power with STAY PUT feature

• Two way infinite tape DTM vs Standard DTM (one way infinite)

... | ⊔ | ⊔ | ← w → | ⊔ | ⊔ | ...

| ← w → | ⊔ | ⊔ | ...

So question is, does this add extra computational power.

functions on one way infinite tape can be simulated ~~same~~ on two way (we will use its one way infinite part only)

Now can reverse can happen?
2 way ∞ tape functions be implemented on 1 way ∞ tape.

| ⊔ | ⊔ | ← w → | ⊔ | ⊔

| ← w → | ⊔ | ⊔ | ...

... | A_{-3} | A_{-2} | A_{-1} | A_0 | A_1 | A_2 | A_3 | ...

Suppose these are tape contents. we can represent it in single way ∞ tape by

upper	A_0	A_1	A_2	A_3	—
down	#	A_{-1}	A_{-2}	A_{-3}	—

distinguishes right way ∞ and left way ∞ tracks.

$\forall q \in M_1$

this means we are working on upper track

$(q, U) \in M_2 \rightarrow$ (while working in right way ∞ track in M_1)

$(q, D) \in M_2 \rightarrow$ (while working in left way track in M_1)

this means we are working on bottom/down track

$(q, U), (q, D)$ are different states in M_2

Note: Dont compare dimensions of states in M_1, M_2. In M_1, q is sufficient, but in M_2 (q, U) / (q, D) is needed to represent a unique state.

We will make $(A_0, \#)$ (A_1, A_{-1}), ... pairs ~~pairs~~ ~~is~~ as individual symbols ~~representing them~~.

$\forall\ \gamma \in T_1 \quad [\gamma, \#] \in T_2$

$\forall\ \gamma_1, \gamma_2 \in T_1 \quad [\gamma_1, \gamma_2] \in T_2$

$\forall\ \gamma \in T_1 \quad [\gamma, \sqcup] \in T_2$

$\forall\ \gamma \in T_1 \quad [\sqcup, \gamma] \in T_2$

$[\sqcup, \sqcup] \in T_2$

$\forall\ q \in F_1 \quad (q, U) \in F_2$
$(q, D) \in F_2$

eg M_1

...	$\sqcup$	$\sqcup$	$\sqcup$	α	v_2	v_3	$\sqcup$	$\sqcup$	...

M_2

α	w_2	...	w_k	$\sqcup$	...
$\#$	$\sqcup$	...	$\sqcup$	$\sqcup$	...

See next page examples for better under-standing

eg 1

$\delta(q_0, \alpha) = (q_1, \beta, R)$
[in M_1]

$\delta([q_0, U], [\alpha, \#]) = ([q_1, U], [\beta, \#], R)$
[in M_2]

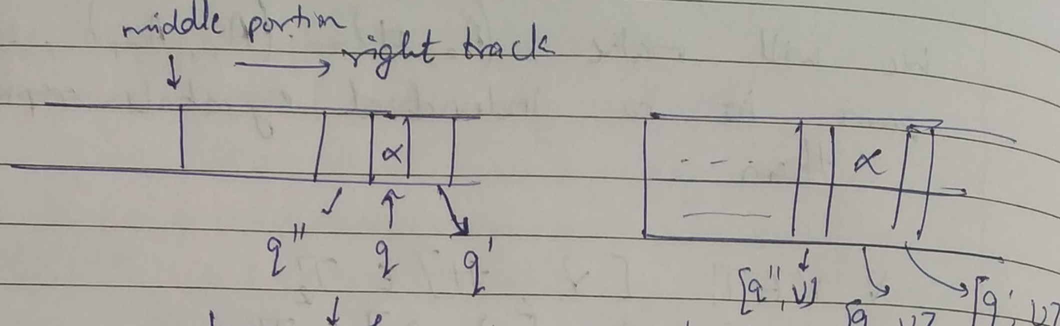
eg 2

$\delta(q_0, \alpha) = (q_1, \beta, L)$ (in M_1)
~~(in M_1)~~

$\delta([q_0, U], [\alpha, \#]) = ([q_1, D], [\beta, \#], R)$

simulation of upper track

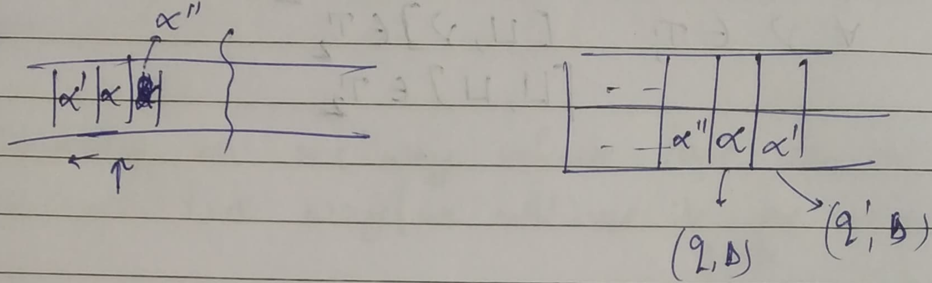
eg3



here focus is upper track

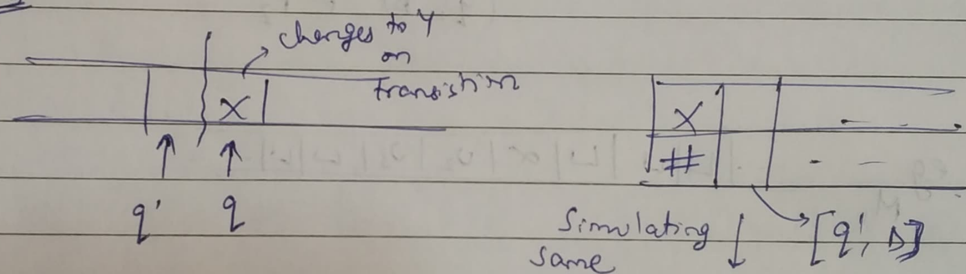
simulation of lower track

eg4



moving across boundary

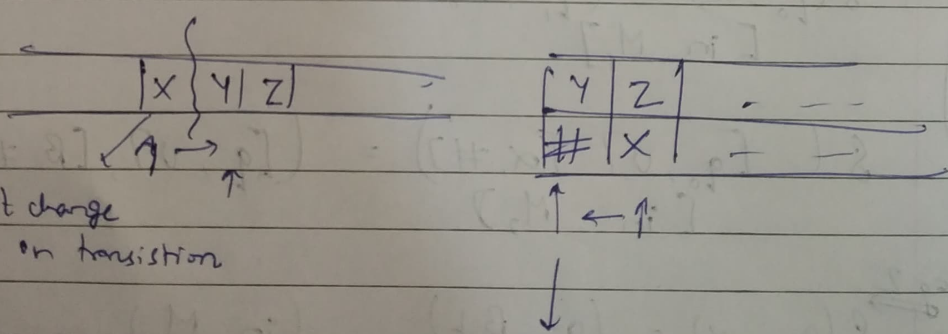
eg5



$\delta(q, x) = (q', Y, L)$

$\delta([q, U], [X, \#])$
$=$
$([q', D], [Y, \#], R)$

eg6.



let it change to K on transition

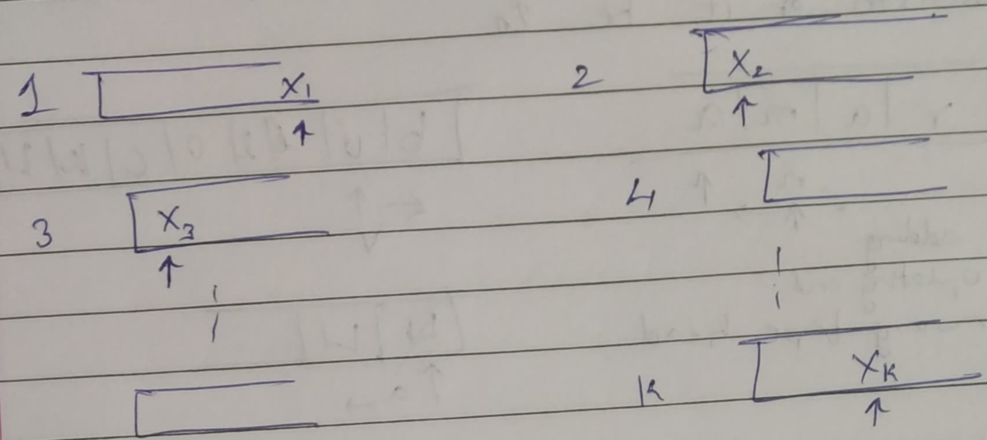
So

$\delta(q, x) = (q', K, R)$

$\cancel{\delta [q, D]}$ $\delta((q, D), [Z, X])$
$=$
$([q', U], [Z, K], L)$

Conclusion ~~tape~~
2 way ∞ DTM has same computational power as standard DTM (one way ∞)

- DTM with K tapes.



Transision function.

$$\delta(q, (x_1, x_2, \ldots, x_k)) = \left(q', (x_1', x_2', \ldots, x_k'), (R, L, R, \ldots, L)\right)$$

~~We~~ ~~Try to~~ We are trying to simulate this using standard DTM M_2

just copy non-blank contents of k diff tapes to M_2 tape separated by #

M_2 tape. | # | tape 1 contents | # | tape 2 contents | # | ... | ...

so contents are put on tape.

now we must have something to represent tape head.

So let normal symbol is γ, then $\dot{\gamma}$ signifies tape head is on it.

$\boxed{\forall \gamma \in T_1 \quad \gamma \in T_2, \ \dot{\gamma} \in T_2}$

e.g. | r | a | m | a | ⊔ | ⊔ | ... | b | u | l | l | o | c | k | ⊔ | ...

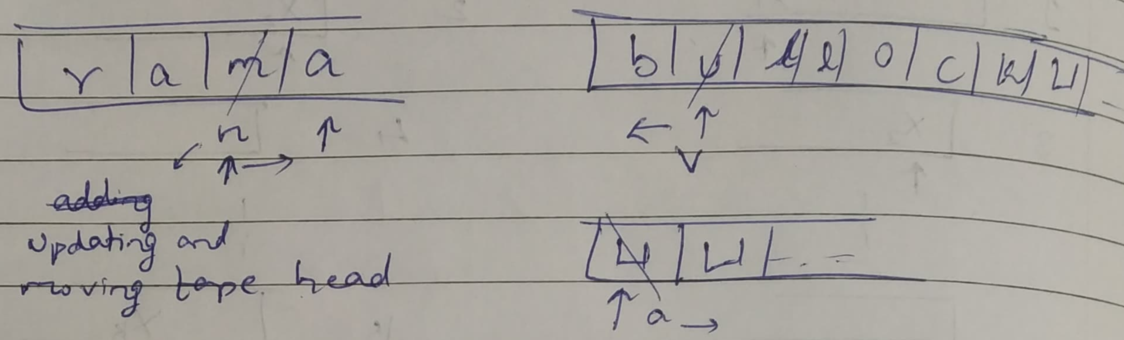
| ⊔ | ⊔ | ⊔ |

So M_2 tape has # r a $\dot{m}$ a # b $\dot{u}$ l l o c k #

↳ $\dot{⊔}$ # ....

(continuation)

referring to ramu, bullock example. let the transistion of it be to



so M_2 will be

$\# \, r a n \dot{a} \, \# \, \dot{b} v l l o c k \, \# \, a \, \dot{⊔} \, \# \ldots$

$\delta(q, [m, v, ⊔]) = (q', [n, v, a], [R, L, R])$

So DTM with k tapes also doesn't have extra computational power.
It is same as that of a standard DTM