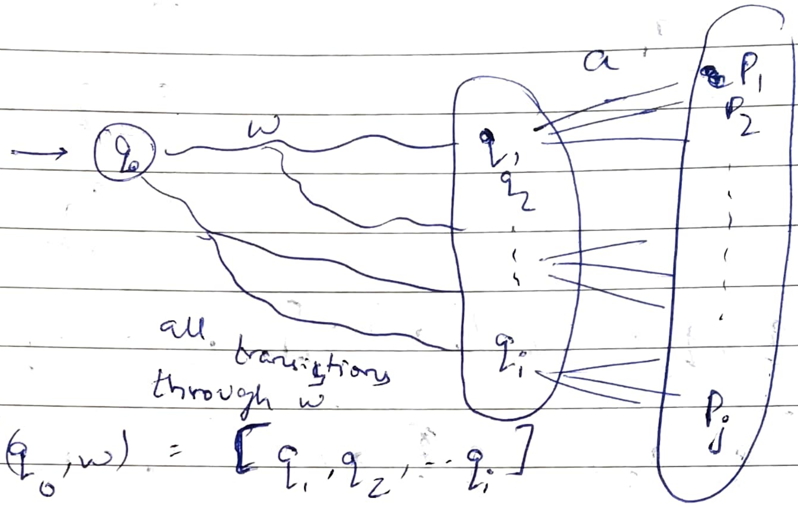
- given NFA $M(Q, \Sigma, \delta, q_0, F)$ (ignoring ε transistions)
  construct DFA $M'(Q', \Sigma, \delta', q_0', F')$ (remains in same state if ε is used for transistion)
  such that $L(M) = L(M')$

Basic idea



$\delta'(q_0, w) = [q_1, q_2, \dots q_i]$

In NFA

$\delta(\{q_1, q_2, \dots q_i\}, a) = \{p_1, p_2, \dots, p_j\}$

In DFA

$\delta'([q_1, q_2, \dots q_i], a) = [P_1, P_2 \dots P_j]$

$\Downarrow$ looks like

$[q_1, q_2 \dots, q_i] \xrightarrow{a} [P_1, P_2 \dots\dots, P_j]$

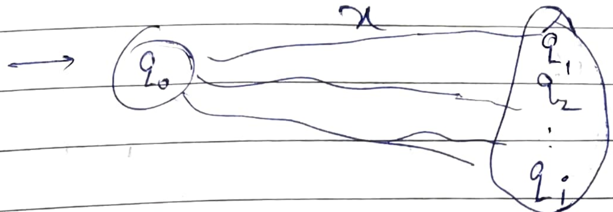
$\cancel{q_0' \in [q_0]}$ $\quad q' = [q_0]$

Brackets is used for combination of states. One state can also be expressed in it. eg $[q_1]$, $[q_2, q_3]$.

$q' \in F'$ iff $q \in q'$ and $q \in F$

q' $\downarrow$ some set from power set eg $[q_1, q_2, q_3]$, $[q_4, q_5]$

$\delta'([q_0], x) = [q_1, q_2, \ldots, q_i]$ iff
$\delta(q_0, x) = \{q_1, q_2, \ldots, q_i\}$



Proof by induction on x (length of input)

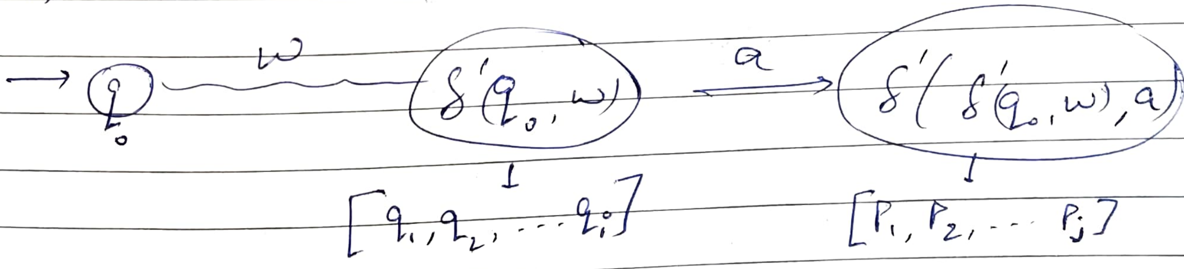
basis : $|x| = 0$
$$\delta'([q_0], \varepsilon) = [q_0]$$
$$\delta(q_0, \varepsilon) = \{q_0\}$$

I.H : for $|x| \leq m$, above condition is true

I.S : $|x| \leq m+1$
we split x into $x = wa$

$$\delta'(q_0, x) = \delta'(q_0, wa) = \delta'(\delta'(q_0, w), a)$$

DFA



As $w \leq m$
By IH
$$\delta'(q_0, w) = [q_1, q_2, \ldots, q_i]$$

$$\delta'(\delta'(q_0, w), a) = \delta'([q_1, q_2, \ldots q_i], a)$$
$$= [p_1, p_2, \ldots p_j]$$

iff

$$\delta(\{q_1, q_2, \ldots q_i\}, a) = \{p_1, p_2, \ldots p_j\}$$

So

$$\delta([q_0], x) = [p_1, p_2, \ldots p_j]$$

iff

$$\delta(q_0, x) = \{p_1, p_2, \ldots, p_j\}$$

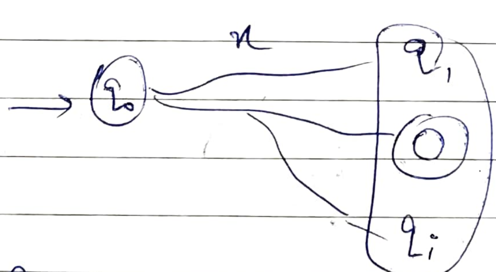
For final states

~~DFA~~ DFA

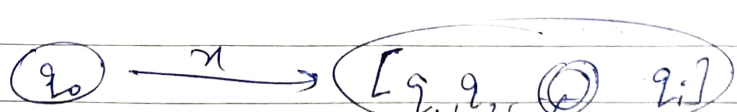
$\delta'([q_0], x) \in F'$ iff → $x \in L(M')$

$\delta(q_0, x) \in F$ → $x \in L(M)$

In NFA



In DFA



this state was in F in M.

So this state belongs to F'

So we have proved NFA with no ε transitions are $\leq$ DFA.

For NFA we have corresponding DFA $M'(Q', \Sigma, \delta', q_0', F')$

- We will now see NFAs with ε transitions

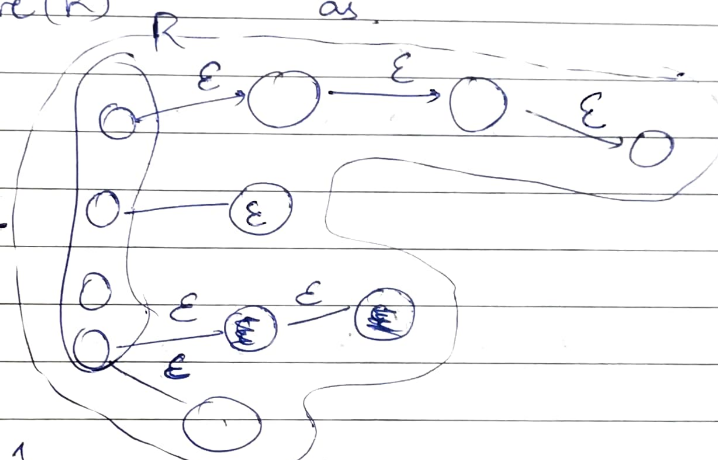
$$\text{NFA } M(Q, \Sigma, \hat{\delta}, q_0, F)$$

with no ε transitions. $\leftarrow$ NFA $M'(Q, \Sigma, \delta', q_0, F')$

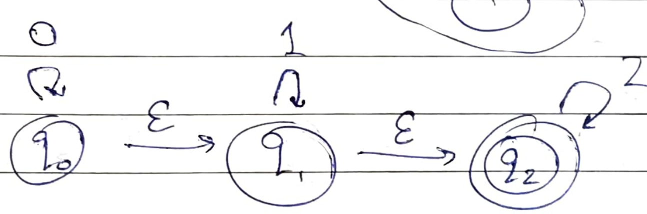
we define ε closure(R) $\rightarrow$ set of states as

eg

Set of all this states is ε closure(R)



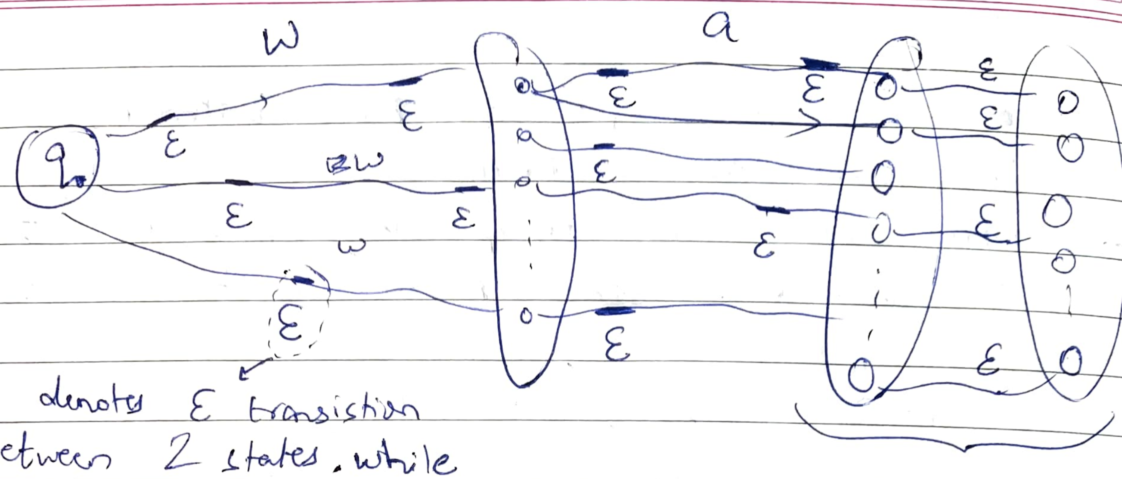
eg.



$$\varepsilon\text{-closure}(q_0) = \{q_0, q_1, q_2\}$$
$$\varepsilon\text{-closure}(q_1) = \{q_1, q_2\}$$
$$\varepsilon\text{-closure}(q_2) = \{q_2\}$$

$$\varepsilon \text{ closure of } (\{q_0, q_1\}) = \varepsilon\text{-closure}(q_0) \cup \varepsilon\text{-closure}(q_1)$$
$$= \{q_0, q_1, q_2\}$$

$$\varepsilon\text{-closure}(\phi) = \phi$$



this denotes ε transition in between 2 states, while consuming w input.

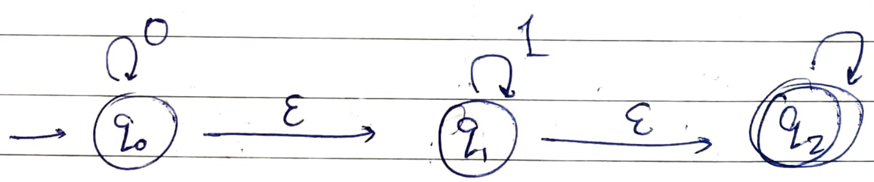
$$\hat{\delta}(q, \varepsilon) = \varepsilon\text{-closure}(q_0)$$

$$\hat{\delta}(q, wa) = $$

we dont define, ~~ex~~formulate them, but logically they represent them.

$$L(M) = \{\, w \mid \hat{\delta}(q_0, w) \text{ contains a final state in } F \,\}$$

eg.



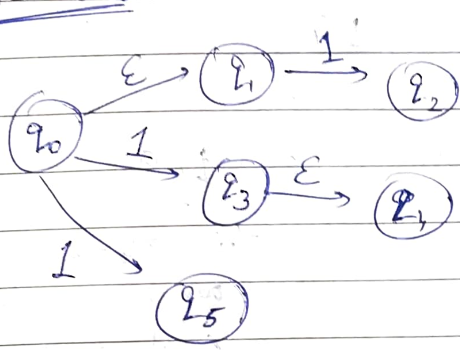
$\hat{\delta}(q_0, 0) = \{q_0, q_1, q_2\}$ $\qquad$ $\hat{\delta}(q_0, 2) = \{q_2\}$

$\hat{\delta}(q_0, 1) = \{q_1, q_2\}$ $\qquad$ $\hat{\delta}(q_1, 1) = \{q_1, q_2\}$

~~for $|x| \geq 0$, $\delta'(q_0, x) = \hat{\delta}(q_0, x)$~~ (single symbol ... string)

We will now see how to deal with NFAs having ε transitions

Basic idea



$M(Q, \Sigma, \delta, q_0, F)$

~~This NFA could be converted to~~

$$\delta(q_0, 1) = \{q_2, q_3, q_4, q_5\}$$

For NFA $\rightarrow$ $M'(Q, \Sigma, \delta', q_0', F')$ not having ε transitions

$$q_0' = \varepsilon(q_0) = \{q_0, q_1\}$$

$$\delta'(q_0', 1) = \cancel{\ldots}$$

$$= \varepsilon\text{-closure of } (\delta(q_0, 1) \cup \delta(q_1, 1))$$

$$= \varepsilon\text{-closure of } (\{q_3, q_5\} \cup \{q_2\})$$

$$= \varepsilon\text{-closure of } (\{q_2, q_3, q_5\})$$

$$= \{q_2, q_3, q_4, q_5\}$$

So in M' we include the ε-closures.

We will prove for NFA M with ε transitions can be transformed to NFA M' without ε transitions by creating such ~~transs~~ equivalent transitions.

Proof

NFA $\xrightarrow{\text{with}}$ ε transistion

$M(Q, \Sigma, \hat{\delta}, q_0, F)$

NFA $\xrightarrow{\text{with}}$ no ε transistion

$M'(Q, \Sigma, \delta', q_0, F')$

s.t. $L(M') = L(M)$

$$\forall_{q \in Q} \forall_{a \in \Sigma} \underbrace{\delta'(q, a)}_{\text{this expression returns a set of final state}} = \underbrace{\hat{\delta}(q, a)}_{\text{this expression returns a set of final state.}}$$

Here we are equating, the set of states obtained after transistion, not the transistion itself

And remember, we didn't give any formal defn for ~~δ~~ ~~δ~~ $\hat{\delta}(q, a)$, it was defined pictorially, including the ε-closures.

for $|x| > 0$ $\delta'(q_0, x) = \hat{\delta}(q_0, x)$

We will prove that for EVERY TRANSISTION, ITS RETURN VALUE, SET OF STATES, are equal.

- Proof by Induction

basis: $|x| = 1$ $\delta'(q_0, a) = \hat{\delta}(q_0, a)$

IH: for $|x| \leq m$, ~~th~~ $\delta'(q_0, x) = \hat{\delta}(q_0, x)$

$x = m+1$ we split x into wa, where $|w| \leq m$, a is one symbol.

I.S: $\delta'(q_0, wa) = \delta'(\underbrace{\delta'(q_0, w)}, a)$

applying IH

$= \delta'(\underbrace{\hat{\delta}(q_0, w)}_{\text{Let this be some set } S}, a)$

$= \delta'(S, a)$

using its defn

$= \bigcup_{q \in S} \underbrace{\delta'(q, a)}_{\text{using IH}}$

$= \bigcup_{q \in S} \hat{\delta}(q, a)$

$= \hat{\delta}(q, wa)$

relation with ε closure.

q_0 is start state, q is ε-closure of q_0

- We will relate the final states F, F'.

$\delta'(q_0, x)$ contains a state of F' iff $\hat{\delta}(q_0, x)$ contains a state of F

$\Rightarrow$ for $x = \varepsilon$ $\quad \overset{\to \text{machine } M'}{\delta'(q_0, \varepsilon) = \{q_0\}}$

$\overset{\to \text{machine } M}{\delta'(q_0, \varepsilon) = \varepsilon\text{-closure}(q_0) = \{\ldots, q_0, \ldots\}}$

Here if $q_0 \in F'$ then q_0 here belongs to F.

But lets say in M, some other state apart from q_0 is in F, so we must then include some state of M' in F', so here we include q_0.

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$$F' = \begin{cases} F \cup \{q_0\} & \text{if } \varepsilon \text{ closure } (q_0) \text{ contains a state of } F. \\ \end{cases}$$

$\Rightarrow x \neq \varepsilon$

first part

$\hat{\delta}(q_0, x)$ contains a state of F

$\delta'(q_0, x)$ also contains a state of F'.

True pictorially.

Second part,

$\delta(q_0, x)$ contains a state of F'

then

$\hat{\delta}(q_0, x)$ also contains a state of F.

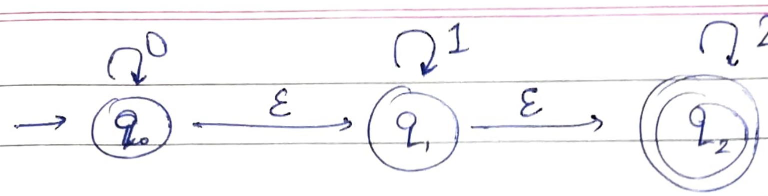
True pictorially.

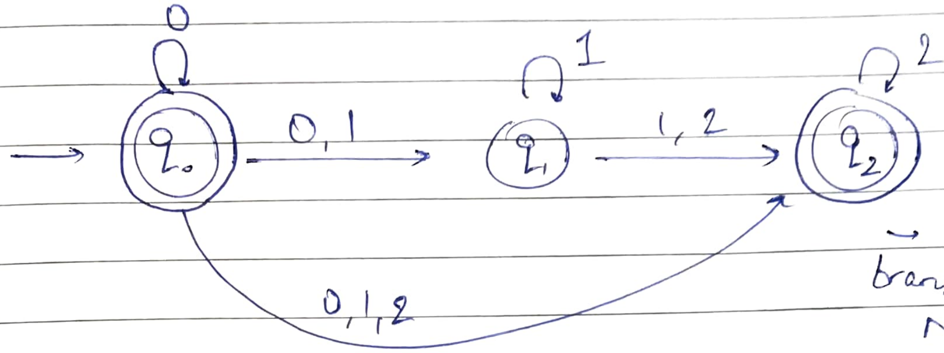
Final F, F' relation

$$F' = \begin{cases} F \cup \{q_0\} & \text{if } \varepsilon\text{-closure}(q_0) \text{ contains a state of } F \\ F \end{cases}$$

Hence we were able to construct an NFA with no ε transitions.

eg.





→ no ε transition NFA

$$\hat{\delta}(q_0, \varepsilon) = \varepsilon\text{-closure}(\{q_0\}) = \{q_0, q_1, q_2\}$$

$$\hat{\delta}(q_0, 0) = \{q_0, q_1, q_2\}$$

$$\hat{\delta}(q_0, 1) = \{q_1, q_2\}$$

$$\hat{\delta}(q_0, 2) = \{q_2\}$$

$$\hat{\delta}(q_1, 0) = \phi$$

$$\hat{\delta}(q_1, 1) = \{q_1, q_2\}$$

$$\hat{\delta}(q_1, 2) = \{q_2\}$$

$$\hat{\delta}(q_2, 0) = \phi$$

$$\hat{\delta}(q_2, 1) = \phi$$

$$\hat{\delta}(q_2, 2) = \{q_2\}$$

For all of them we make equivalent ~~no ε~~ of δ' transition

ε-closure contains a state in F, so F' is $F \cup \{q_0\}$