MA-222

Note Title 30-08-2021

Lec-13.

$(\mathbb{Z}, -)$

$(a-b)-c \neq a-(b-c)$.

$(G, *)$

$\downarrow$

$\mathbb{Z}$ $\quad \mathbb{Q} \quad \mathbb{R} \quad \mathbb{C} \quad + \quad \underline{G}$

$\mathbb{Q}^+ \quad \mathbb{R}^+ \quad \mathbb{C}$

$\underline{\mathbb{Q}^* \quad \mathbb{R}^* \quad \mathbb{C}^*}$

$a * b = \frac{ab}{c}$

$c \in \mathbb{Q}^+$ (fixed).

$(\mathbb{Q}^+, *)$

find id

& a^{-1}?

$(\mathbb{Q}^+, *)$

$a * b = \frac{ab}{2}$

$(a * b) * c \stackrel{?}{=} a * (b * c)$

$e = 2$

$a^{-1} \in \mathbb{Q}^+$

$a * e = a \quad \forall a \in \mathbb{Q}^+$

$\frac{ae}{2} = a$

$\frac{e}{2} = 1$

$e = 2$

$(\mathbb{Q}^+, \circledast)$

Consider the gp $\mathbb{Q}^+$ wrt the op $*$ defined by — — —

$\mathbb{Z}_n$ = int mod n

$= \{\overline{0}, \ldots, \overline{n-1}\}$ forms a gp wrt $+$.

$\mathbb{Z}_n^* = \{\overline{a} \in \mathbb{Z}_n \mid (a, n) = 1\}$ forms a gp wrt $\cdot$

$\cancel{\mathbb{Z}_n =}$

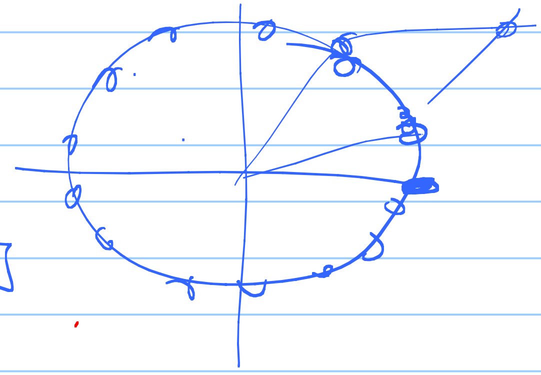
$\phi(n)$ = no of +ve integ coprime to n & less than n.

or equal to

$\phi(1) = 1$.

$S^1 = \{ z \in \mathbb{C} \mid |z| = 1 \}$ **unit circle**

Ex: is a gp w.r.t. ·



$\underline{U_n}$ = set of all n^{th} roots of unity

$= \{ e^{2\pi i \frac{k}{n}} \mid 0 \le k < n \}$ forms a gp w.r.t. ·

Quaternions

$$Q_8 = \{\pm 1, \pm i, \pm j, \pm k \mid i^2 = j^2 = k^2 = ijk = -1\}.$$

$-1 * -1 = 1$

$i * j = -k$

$i * -i = -1$

$j * -j = -1$

$j * i$

$i * -i$

	1	-1	i	-i	j	-j
1						
-1						
i						
-i						
j						
-j						

Ex Complete this table

Group table

Cayley table

mod 4

$\mathbb{Z}_4 \rightarrow$

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

$\{e, a, b, c\}$

$V =$

*	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

Chk: $G = \{e, a, b, c\}$. $(G, *)$ is a gp.

Klein-4 gp. K_4 or V (Viere)

$G = \{e, a\}$

$*$	e	a
e	e	a
a	a	e

$a * a = a$

$a * \square = e$

$G = \{e, a, b\}$

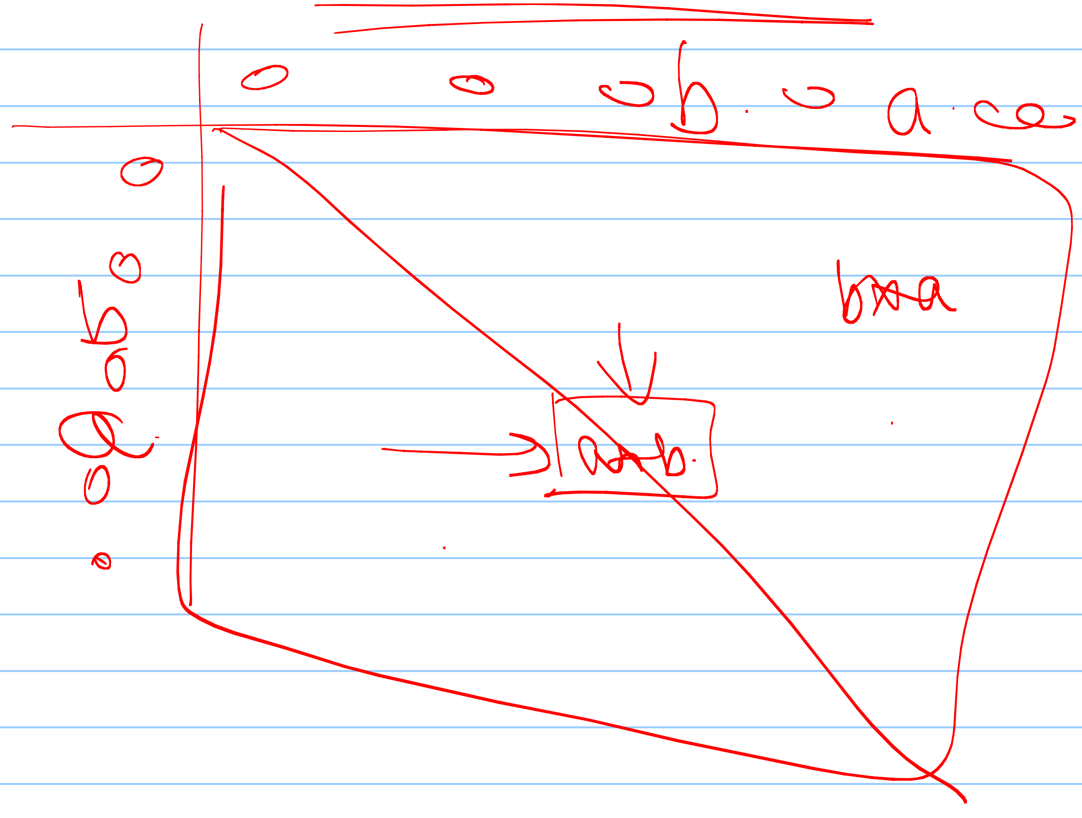
$*$	e	a	b
e	e	a	b
a	a		
b	b		

Dx

	b
a	$a * b = e$

$G = \{ \_\_\_\_ \}$

Abelian



$a * b$
$=$
$b * a$

Properties of Cayley table

$\#$) For every row (& column) every elt of G appears exactly once in this row (& column).

Permutation

$\#$) Cayley table is symmetric iff G is Abelian

$\#$) If a entry (r, c) is identity then (c, r) is also id.

__Define__ A nonempty subset H of a gp G is called a subgroup of G if H is a gp wrt the __same op.__ as G.

__Not__: $(H, *) \leq (G, *)$

or $H \leq G$.

Examples:

$$\mathbb{Z} \leq \mathbb{Q} \leq \mathbb{R} \leq \mathbb{C}.$$

$$U_n \leq S^1 \leq \mathbb{C}^* \quad \forall n \in \mathbb{N}$$

$$\tfrac{1}{2}\mathbb{Z} \nsubseteq \mathbb{Z} \qquad 5\mathbb{Z} = \{5k \mid k \in \mathbb{Z}\} \leq \mathbb{Z}$$

$$5\mathbb{Z} = (-5)\mathbb{Z}$$

Non examples:

$\mathbb{Z}_n \not\leq \mathbb{Z}_m$ $m \neq n$.

$M_2(\mathbb{R}) \not\leq M_3(\mathbb{R})$

$\boxed{\underset{\cdot}{\mathbb{R}^*} \subseteq \underset{+}{\mathbb{R}} \text{ but } \mathbb{R}^* \not\leq \mathbb{R}}$