Note Title 02-08-2021

MA-222

Lecture-2

$$\mathbb{N} = \{1, 2, 3, \ldots\} = \{0, 2, 3, \ldots\}$$

$$\mathbb{N} = \{x \mid x = \underbrace{1+1+\cdots+1}_{\text{finitely}}\}$$

$$\mathbb{N} = \{x \mid x = y+1 \text{ for some } y \in \mathbb{N}\}$$

$$\text{or } x = 0$$

$\underline{\underline{S(x) = x + 1}}$

$\underline{\underline{S(x) = x \cup \{x\}}}$

$1 = S(\emptyset)$

$2 = S(1) = 1 \cup \{1\} = \underline{\{1, \{1\}\}}$

$\underline{\underline{S(a) = a \cup \{a\}}}$

$S(1,2,3\} = \{1,2,3\} \cup \{\{1,2,3\}\} = \{1,2,3,\{1,2,3\}\}$

NOTHING.

$\emptyset = 0$

$S(\emptyset) = \emptyset \cup \{\emptyset\} = \{\emptyset\} = 1 = a$

$S(1) = 1 \cup \{1\} = 2 = a$

$= 3$

Sets

$\emptyset$

$S(\cancel{\emptyset}) = \{\emptyset\}$

$S(\cancel{1}\,2) = 1 \cup \{1\}$

$= S(\emptyset) \cup \{S(\emptyset)\}$

$= \{\emptyset\} \cup \{\{\emptyset\}\}$

$= \{\emptyset, \{\emptyset\}\}$

$S(2) = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\} = 2 \cup \{2\}$

$= \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

Symbol.

0

1 — 1 — 9

$\cancel{2}$ — 2 — 2

$3.$ — 3

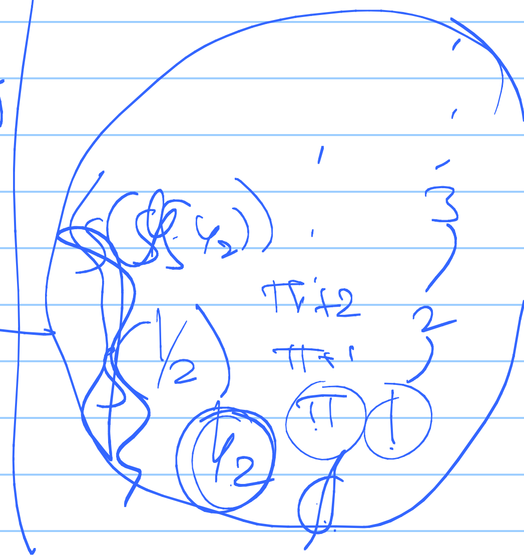
$\cancel{3}$ — 4 — 8

$\mathbb{N} = \{ x \mid x = \emptyset$ or $x = S(y)$ for some $y \in \mathbb{N} \}$

Inductive set: Let A be a set.
A is said to be inductive if

i) $\emptyset \in A$

ii) If $x \in A$ then $S(x) \in A$.



Ex $A = \{\emptyset, 0\} \cup \mathbb{R}_+$

Construction of $\mathbb{N}$.

- Start with an inductive set A.
- Define $\mathbb{N}_A = \bigcap_{\substack{B \subseteq A \\ \& \text{ B is inductive}}} B$

A & B : inductive sets.

Thm: $\mathbb{N}_A = \mathbb{N}_B = \mathbb{N}$

Def^n: The set $\mathbb{N}_A$ as obtained above is called the set of natural no.

Remark: $\mathbb{N}$ is an inductive set.

Peano's axioms:

- $\exists$ exactly one elt of $\mathbb{N}$ that is not a successor of any elt of $\mathbb{N}$. say 1
- $\forall x \in \mathbb{N}$, $S(x) \in \mathbb{N}$.
- $S(x) = S(y)$ then $x = y$. | S is one-one.
- For any inductive set S,

  $\mathbb{N} \subseteq S$.

- ~~If $X \subseteq$~~

– If $X \subseteq \mathbb{N}$ such that $\underline{1 \in X}$ & $\underline{\forall x \in X,}$

base step.

induction step. $\underline{S(x) \in X}$.

Then $X = \mathbb{N}$

This is called principle of <u>mathematical induction</u>

Properties of S-operator.

- $\underline{S \text{ is one-one}}$.
- $x, y \in \mathbb{N}$. then

$$x + S(y) = S(x+y).$$

In particular,

$$x + S(1) = S(x+1) = S(S(x)).$$

Define $+$ on $\mathbb{N}$.

$x \in \mathbb{N}$ & $\underline{\underline{n}}$

$$\underline{\underline{x + n}} = \underbrace{S(S(S(\ldots S(x))))}_{n \text{ times}}$$

$$\left.\begin{aligned} S(x) &= x+1 \\ S(S(x)) &= x+2 \\ &\vdots \end{aligned}\right\} \Big\|$$

$+$ is binary op. on $\mathbb{N}$.

Properties of $\mathbb{N}$:

- Associativity $\quad x+(y+z)=(x+y)+z$
- Commutativity $\quad \underline{x+y}=\underline{y+\underline{\underline{x}}}$
- Cancellation $\quad x+z=y+z$

then $x=y$

$x + x$

$x + x + x$

$x + x + x + \cdots + x$

finitely many.

$$\left.\begin{matrix} 2 \cdot x \\ 3 \cdot x \\ n \cdot x \end{matrix}\right\}$$