

L-005

Input 1

5	20
80	51
79	
11	
40	

Search Value (Num, Value) {  
 linearly through all  
 entries

// compare value with each element  
 for (curr = 0, curr < Num.len, curr++)  
 {  
   if (Num[curr] == value) {

return curr

}

curr Num (curr)  
 0 20  
 1 51

}

return -1 // Error

}

C1.n + C2

How to convert relation to function

- > Consider only Worst case
- > Average case
- > Best case

Value  
absent

Value  
~~absent~~ problem  
at  
end

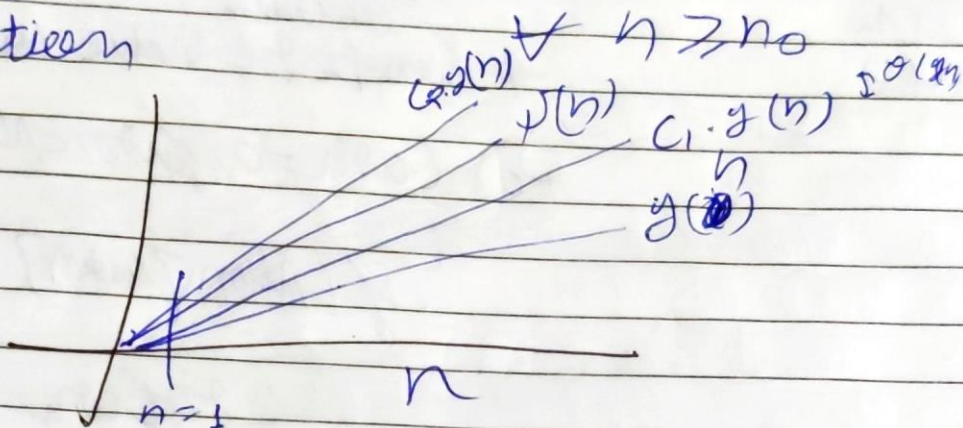
flight  
industrial plant



$$f(n) \in O(g(n))$$

$f$   
 makes  
 problem size  
 to  
 resource & time  
 consumption

$$0 \leq C_1 g(n) \leq f(n) \leq C_2 g(n) \quad n \geq n_0$$



$$f(n) = n^2 + 2n + 5$$

$$g(n) = n^2$$

$$h(n) = n$$

$$f(n) = n^2 + 2n + 5$$

$$g(n) = n^2$$

$$C_1 = 1, C_2 = 8, n_0 = 1$$

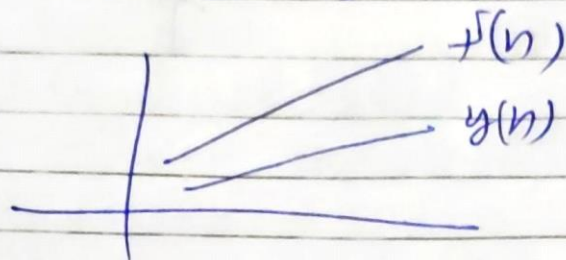
$n$	$f(n)$	$g(n)$	$C_1 g(n)$	$C_2 g(n)$
0	5	0	0	0
1	8	1	1	8
2	13	4	4	32
3	20	9	9	72
4	29	16	16	128
5	40	25	25	200

$$f(n) \in O(g(n)) \quad C_1 = 1 \quad C_2 = 8 \quad n_0 = 1$$



$$f(n) \in O(g(n)) \quad c, n_0$$

$$0 \leq f(n) \leq c \cdot g(n) \quad \forall n \geq n_0$$



$$f(n) = n^2 + 5n + 2 \quad g(n) = n^2 \quad f(n) \in O(g(n))?$$

$c \quad n_0$

$$f(n) = 5n + 2 \quad g(n) = n^2 \quad c = 2 \quad n_0 = 3$$

$n$	$5n+2$	$n^2$	$2 \cdot n^2$	$0.5n^2$
0	2	0	0	0
1	7	1	2	0.5
2	12	4	8	2
3	17	9	18	4.5
4	22	16	32	8
5	27	25	50	12.5
6	32	36		18

$$c = 0.5$$



$\Theta$   
 asymptotic

=

$O$   
 asymptotic  
 upper bound

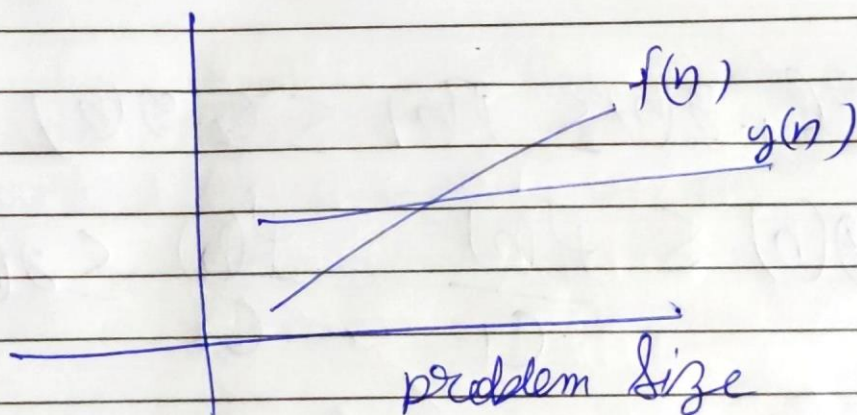
$\leq$

$\Omega$   
 asymptotic

$\geq$

$$f(n) \in \Omega(g(n)) \quad (n \geq n_0)$$

$$0 \leq c \cdot g(n) \leq f(n) \quad \forall n \geq n_0$$



$$f(n) = n^2 + 5n + 2, \quad g(n) = n^2 \quad f(n) \in \Omega(g(n))$$

$c = 1 \quad n_0 = 0$

$$f(n) \geq c \cdot g(n) \quad c = 500$$

Transitivity

$$A:B, B:C \rightarrow A:C$$

$$f(n) \in \underset{\Omega}{O}(g(n)), \quad g(n) \in \underset{\Omega}{O}(h(n)) \rightarrow f(n) \in \underset{\Omega}{O}(h(n))$$



# Reflexivity

A:

$$f(n) \in O(f(n))$$

$$f(n) \in O(f(n))$$

$$f(n) \in \Omega(f(n))$$

## Symmetry

$$A: B \Leftrightarrow B: A$$

$$f(n) \in O(g(n)) \Leftrightarrow g(n) \in O(f(n))$$

$$0 \leq C_1 \cdot g(n) \leq f(n) \leq C_2 \cdot g(n) \quad \forall n \geq n_0$$

$$g(n) \leq \frac{f(n)}{C_1}$$

$$\frac{f(n)}{C_2} \leq g(n) \quad \forall n \geq n_0$$

$$1/C_1 = C_1'$$

$$1/C_2 = C_2'$$

$$\text{Symmetry } f(n) \in O(g(n)) \Leftrightarrow g(n) \in O(f(n)) \quad \text{false}$$

Counter example



# Fibonacci Numbers

~~fib~~

$$\text{fib}(1) = 0 \quad \text{fib}(2) = 1 \quad \text{fib}(n) = \text{fib}(n-1) + \text{fib}(n-2)$$

$$\text{FibR}(n) \quad T(n) = T(n-1) + T(n-2)$$

$$T(n) \approx (1.6)^n$$

```

if (n == 1) {
    return 0;
}
if (n == 2) {
    return 1;
}

```

n	Fib(n)
1	0
2	1
3	1
4	2
5	3
6	5
7	8
8	13

```

answer = FibR(n-1) + FibR(n-2);
return answer;

```

3 recursion

~~FibS(n) {~~

~~if (n == 1) {~~  
~~return 0;~~

~~if (n == 2) {~~  
~~return 1;~~

first pass



To compute  $n^{\text{th}}$  number start from 3<sup>rd</sup> number  
 on 2 Then compute with previous 5<sup>th</sup>

all year reach  $n^{\text{th}}$

FibS(n) {

if (n == 1) {  
 return 0  
 }  
 if (n == 2) {  
 return 1  
 }  
 fib n 1 = 1  
 fib n 2 = 0

for (loop = 3, loop ≤ n, loop = loop + 1) {

G.h

answer = fib n 1 + fib n 2

Fib n 2 = fib n 1

Fib n 1 = answer

}  
 return answer  
 }

$T(n) \in O(n)$

$$T(n) = C_1 + \underline{C_2 \cdot n} + C_3$$

$\approx 2 \cdot (n-1)$



$10^{10}$  ins  
 $10^8$  sec  
 delay

n	Fib S	Fib R
1	1	$(1.6)^1 \rightarrow 1.6$
10	10	$(1.6)^{10} \rightarrow 10^2$
100	100	$(1.6)^{100} \rightarrow 10^{20}$
1000	1000	$(1.6)^{1000} \rightarrow 10^{200}$
10000	10000	$(1.6)^{10000} \rightarrow 10^{2000}$

$\rightarrow 10^{2000}$  sec  
 less than 1 sec.

$$T(n) \approx 2T(n-1)$$

$$\approx 2^n$$



L-7

ADT  
data type

# Abstractions

Bits  $\longrightarrow$  digital circuit

Files  $\longrightarrow$  secondary storage

Machine code  $\longrightarrow$  operations by processor

Basic data types  $\longrightarrow$  ~~data~~ data handled by processor

$\rightarrow$  Ease of Understanding

$\rightarrow$  Independence from implementation

$\rightarrow$  Separation of concerns

Specification and implementation

$\Rightarrow$  Ability to build more abstractions

## Abstract Data Type (ADT)

$\rightarrow$  Data

$\rightarrow$  Operations

int  $\rightarrow$  integer

Data

16/32/64

(bits)

$\rightarrow$  circuits  $\rightarrow$  transistors

Operations

$+, -, \times, /, ++, --$



- Access to data is only through pointers
- Implementation details need not be shared
- Naturally translates to Object oriented programming languages.

## Array ADT

Data:  $\langle \text{index}, \text{value} \rangle$

Collection of pairs  $\uparrow$

Operations:

CreateNew (Capacity, type)

Store Value (index, val)

Access Value (index)

Search Value (Value)

Insert Array (Value)

• Store some type of data

~~• Contiguous storage~~

~~• Fixed storage~~

• Access element with index

• Most of the programming languages provide it

• The com have already other elementary index

• Where's element out of index

~~• Signature fix~~

core refinement

$\downarrow$  might have