

# MA-222

Note Title

06-08-2021

Thm: Archimedean Property.

Lec-4

$$a, b \in \mathbb{N}$$

$$\exists n \in \mathbb{N}$$

$$na \geq b$$

Pf:

$$\text{sup. val. i.e.}$$

$$\forall n \in \mathbb{N}$$

$$na < b$$

$$S = \{b - na \mid n \in \mathbb{N}\} \subseteq \mathbb{N}$$

$$\Rightarrow \underline{b - na} \geq 0 \quad \forall n \in \mathbb{N}$$

$$S \neq \emptyset$$

By MOP,  $S$  has a least elt. say  $n_0$ .

$n_0 = b - m_0 a$  for some  $m_0 \in \mathbb{N}$ .

$$b - \underbrace{(m+1)}_{\in S} a \in S.$$

$$b - (m+1)a = (b - m_0 a) - a = n_0 - a \leq n_0 -$$

~~$n_0$~~   $\in S$

contrad to the leastness of  $n_0$ .

Thus  $\exists n \in \mathbb{N}$  s.t.  $na \geq b$ .  $\square$

# Principle of mathematical induction

Ver 1: Let  $P(n)$  be a statement about a natural no.  $n$ .

if suppose  $P(1)$  is true.

if whenever  $P(k)$  holds then  $P(k+1)$  also holds.

Then  $P(n)$  is true for all natural no.  $n$ .

Adv. ver.  $P(n)$  be true  
 $S = \{n \in \mathbb{N} \mid P(n) \text{ is true}\}$

Supp. if  $1 \in S$ .

if  $k \in S$  then  $k+1 \in S$ .

then  $S = \mathbb{N}$ .

Ver 2: (Extended P.M.P.)

$P(n)$ : stmt.

$S = \{n \in \mathbb{N} \mid P(n) \text{ holds}\}$

Sup  $S$  has the following properties:

if  $\exists n_0 \in S$

base case

induction step

whenever  $k \in S$  ( $k \geq n_0$ )  $\Rightarrow k+1 \in S$ .

Then

$S = \{n_0, n_0+1, \dots\} = \mathbb{N} - \{1, \dots, n_0-1\}$

Example:

$\parallel 2^n < n!$

$\forall n \geq 4$

$n$	1	2	3	4	5	6
$2^n$	2	4	8	16	32	64
$n!$	1	2	6	24	120	720

WOP  $\implies$  PMT

Second PMT

$P(n)$  be a stmt.

$S = \{n \in \mathbb{N} \mid P(n) \text{ holds}\}$   
 $S$  has the following properties:

as  $\textcircled{N_0} \in S$ .

if  $\textcircled{N_0}, \textcircled{N_1}, \dots$

$\textcircled{N_k} \in S$

then  $k+1 \in S$ .

then  $S = \mathbb{N}$ .

$P(k) \implies P(k+1)$

$P(k-1)$   
 $P(k-2)$

$M \times N$

double induction

$P(m, n)$  : be a stmt about  $m$  &  $n$ .

or  $P(m_0, n_0)$  be true.

or  $\forall a \leq m_0, \quad P(a, n) \text{ is true}$   
then  $P(a, n+1)$  is true.  $\} \quad Q(n)$

or  $\forall b \geq n_0,$

then  $P \text{ holds } \forall m, n \in M -$   
 $\xrightarrow{\text{step 2}} n_0$

$P(m, n, d)$

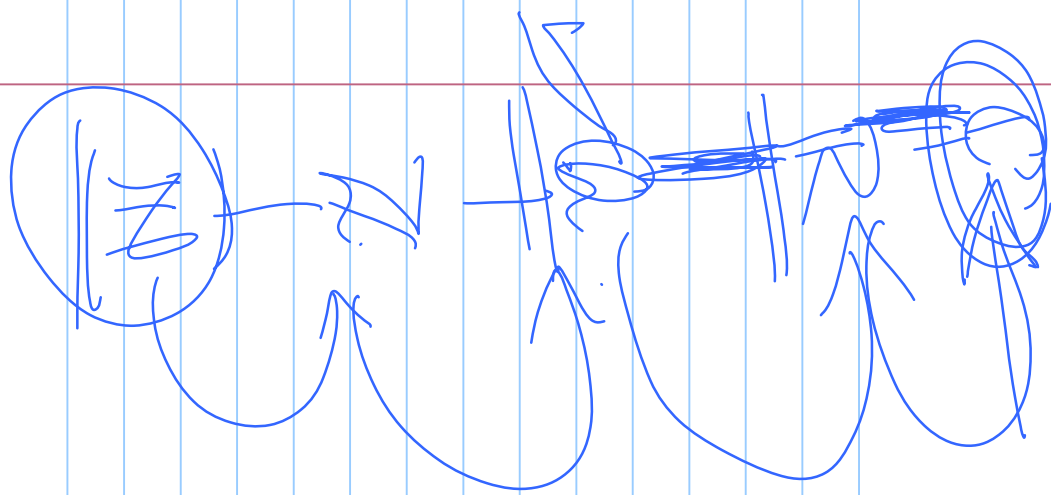
Triple induction

$$\cancel{m^2 + n^2} \geq 0$$

$P(n, n_2, \dots, n_r)$

$\mathbb{N} \times \mathbb{N} \times \dots \times \mathbb{N}$   
r-tuples.





0

$\mathbb{Z}$

$\mathbb{Z} \times \mathbb{Z}$

$\mathbb{Z}$

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middle number.

$i = \sqrt{-1}$

$(a, b) \sim (c, d)$   
 $ad = bc$

Geometer's  
Sketchpad  
Cinderella

$GX-5$

Galois theory

Field  $H_n$

$\mathbb{C} \times \mathbb{C} \times \mathbb{C}$   
 $\mathbb{C}^n$

$$\frac{\cancel{S}}{S/2}$$

$$\mathbb{Z}_6$$

$a \in S$

$$\cancel{[a]} = \{ \cancel{b \in S} \mid a \sim b \}$$

$$\cancel{(1,1)} \sim \cancel{(2,2)}$$

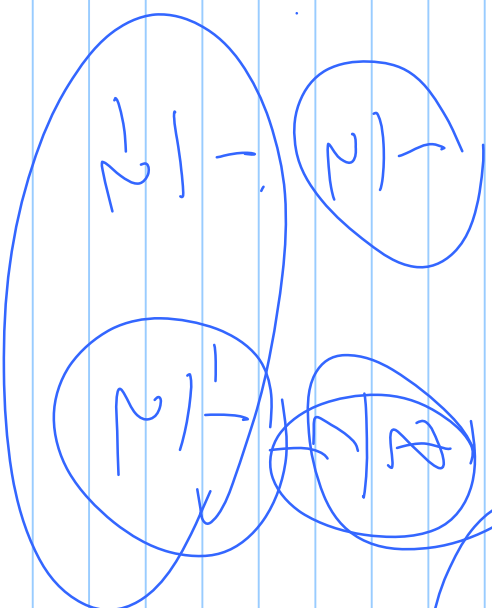
$$(2,1) \sim (-1,-2)$$

$$\frac{1}{2} - \frac{1}{2}$$

$$\frac{\cancel{1}}{\cancel{2}}$$

$$\frac{1}{2} = \frac{2}{4}$$

$$Q = \left\{ \frac{p}{q} \mid p, q \in \mathbb{Z}, q \neq 0 \right\}$$



$$Q = \left\{ \left[ \frac{a}{b} \right] \right\}$$