MA-222

Note Title 27-08-2021

Lec-12

$\mathbb{N}$

- $m+n \in \mathbb{N}$
- $(m+n)+l = m+(n+l)$
- $m+n = n+m$

Whole nos.

Extra elt

0

- $m+0 = m$
  $= 0+m$

$\mathbb{Z}$

for every $m \in \mathbb{N}$

$\exists\ (-m)$ such that

- $m+(-m) = 0$
  $= (-m)+m.$

$\underline{\mathbb{N}}$

- $m \cdot n \in \mathbb{N}$.
- $(m \cdot n) \cdot l = m \cdot (n \cdot l)$
- $m \cdot n = n \cdot m$

$m \cdot 1 = m = 1 \cdot m$
$\forall\, m \in \mathbb{N}$.

Whole no
$m \cdot 0 = 0$

$\underline{\mathbb{Z}}.$

$m \cdot (-m) = -m^2$

$\underline{\mathbb{Q}}$

$\forall\, m \in \mathbb{N}\ \exists\ \frac{1}{m}$ s.t
$m \cdot \frac{1}{m} = 1$

$\mathbb{N} \rightsquigarrow \frac{1}{\mathbb{N}} \rightsquigarrow \mathbb{N} \cup \frac{1}{\mathbb{N}} \quad (-\mathbb{N}) \quad (-\frac{1}{\mathbb{N}}) \rightsquigarrow \underline{\mathbb{Q}}\ ?$

$a, b \in S$. If $a \cdot b \in S \longrightarrow$ Nothing

otherwise $S = S \cup \{a \cdot b\}$

$\frac{1}{a} \in S$

if not $S = S \cup \{\frac{1}{a}\}$

$\mathbb{Q} \setminus \{0\}$

$\mathbb{Q}^*$

$\frac{p}{q} \cdot \frac{r}{s} \in \mathbb{Q}$

Associativity

$1 \cdot \frac{p}{q} = \frac{p}{q} \cdot 1 \in \frac{p}{q}$

$\frac{p}{q} \cdot \frac{q}{p} = 1 = \frac{q}{p} \cdot \frac{p}{q}$

$$M = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \;\middle|\; a, b, c, d \in \mathbb{R} \right\}$$

$A + B \in M$

$(A + B) + C = A + (B + C)$

$A + \underline{O} = A = A + O$

$A + (-A) = O = (-A) + A$

$\{A \in M \mid \det A \neq 0\}$

$A \cdot B \in M$

$(A \cdot B) \cdot C = A \cdot (B \cdot C)$

$A \cdot I = A$

$A \cdot B = I$
$= B \cdot A$

Defn: Binary operation $*$ on a set G is a function $* : G \times G \to G$.

Notation: $*(a, b) = a * b$.

- Associative
- Commutative

$+$ on $\mathbb{Z}, \mathbb{N}, \mathbb{W}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$, $M_n(\mathbb{R})$, $M_{m \times n}(\mathbb{R})$.

- $\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}, M$

But not on $\mathbb{N}$ & $\mathbb{W}$

$-(3,2) \in \mathbb{N}$

But $-(2,3) \notin \mathbb{N}$.

$3-2$

$2-3$

$-$ is nonassociative on $\mathbb{Z}$

$-$ is noncommutative on $\mathbb{Z}$.

$(a-b)-c \neq a-(b-c)$

$a-b \neq b-a$

Ex ~~39~~ Find nonassociative commutative bin op on $\mathbb{Z}$?

Associative but noncomm on $\mathbb{Z}$.

$*(m,n) = mn - m - n$

$m^2 + n^2 - 2mn$

$\mathbb{R}^3$ cross product on vector in $\mathbb{R}^3$.

$\mathbb{R}^n$ inner product. → When will this be a bin. op?

Defn: **Group:**

A nonempty set G together with a binary op $*$ on G is said to be a group if

0) $a*b \in G$ ← closure / G is closed wrt $*$.

i) $*$ is associative

ii) $\exists\, e \in G$ s.t. $a*e = e*a = a \;\forall a \in G$.

e is called identity of G wrt $*$

iii) for every $a \in G$, $\exists\, b \in G$ s.t. $a*b = e = b*a$

this b is called inverse of a wrt $*$

Notn a^{-1}.

$(G, *)$

$\downarrow$

G.

Defn: A group $(G, *)$ is said to be commutative OR Abelian if $*$ is commutative on G.

Otherwise G is called noncommutative or non Abelian gp.

Examples: $\mathbb{Z}$, $\mathbb{R}$, $\mathbb{Q}$, $\mathbb{C}$, $M_{m\times n}(\mathbb{R})$,

nonexamples: $\mathbb{N}$, $\mathbb{W}$, $(\mathbb{Z}, \cdot)$ $(M_{m\times n}(\mathbb{R}), \cdot)$

$(\mathbb{Z}, -)$

- $a - b \in \mathbb{Z}$
- $(a-b)-c \neq a-(b-c)$
- $a - 0 =$