

# Introduction to Programming (CS 101)

Spring 2024



## Lecture 11:

More about structs and recursion (recursion (recursion ...

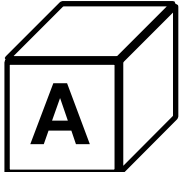
**Instructor:** Preethi Jyothi

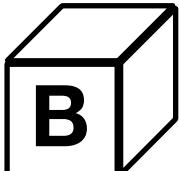
Based on material developed by Prof. Abhiram Ranade and Prof. Manoj Prabhakaran


# Recap (IA)


What is the output of the following program?

```
int alter(int &a) {  
    return a+=2;  
}  
  
main_program {  
    int a = 1;  
    cout << alter(a) << endl;  
}
```

 1

 2

 3

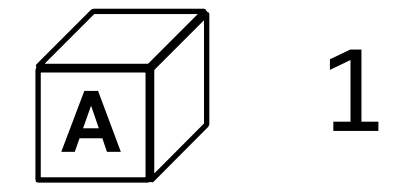
 `alter(a)` would work the same if `alter` returned `int` or `int&`. But `alter(alter(a))` would only work for the latter.

# Recap (IB)

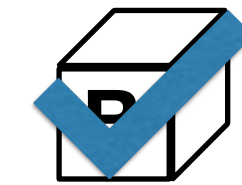
What is the output of the following program?

```
int& alter(int &a) {  
    return a+=2;  
}
```

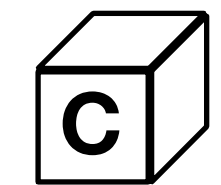
```
main_program {  
    int a = 1;  
    alter(a) = 2;  
    cout << a << endl;  
}
```



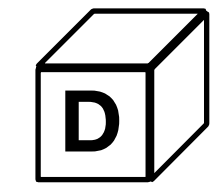
1



2



3



4



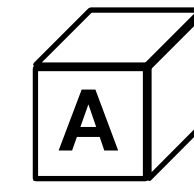
A function returning a reference can act like an lvalue.

# Recap (IC)

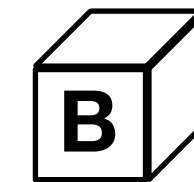
What is the output of the following program?

```
int& alter(const int &a) {  
    return a+=2;  
}
```

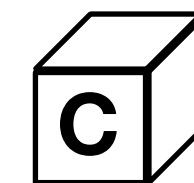
```
main_program {  
    int a = 1;  
    alter(a) = 2;  
    cout << a << endl;  
}
```



1



2



3



Compiler error

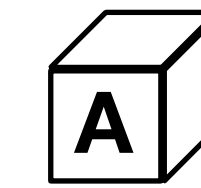


Compiler error! Cannot do `a+=2`  
since the reference `a` is defined to  
be a const-qualified type in `alter`

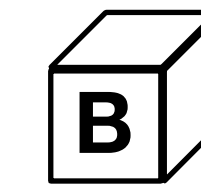
# Recap (ID)

What is the output of the following program?

```
int alter(int &b) {  
    return (b+1);  
}  
  
main_program {  
    int a = 1;  
    const int& b = alter(a);  
    cout << b << endl;  
}
```



Compiler error



1



2



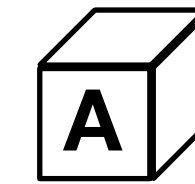
Can create a `const int&` to point to a temporary object in `b + 1`

# Recap (IE)

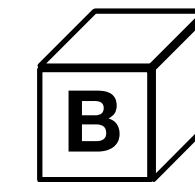
What is the output of the following program?

```
int alter(const int &b) {  
    return (b+1);  
}
```

```
main_program {  
    int a = 1;  
    a = alter(a) + 1;  
    cout << a << endl;  
}
```



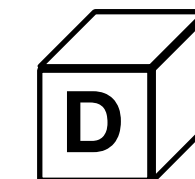
1



2



3



Compiler error

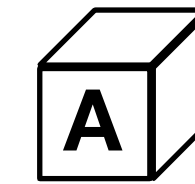


Cannot modify a `const int&` like `b`, but can directly modify the variable it points to i.e., `a`

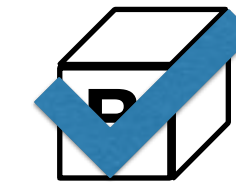
# Recap (IF)

What is the output of the following program?

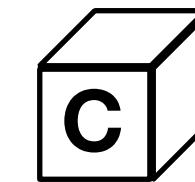
```
main_program {  
    float b = 4.1;  
    const int &c = b;  
    cout << c << endl;  
}
```



1



4



Compiler error



Can assign `const int&` to a temporary object (that results from casting `4.1` to `int`)



# struct variables

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# Creating your own data-type using `struct`

- A structure (defined using `struct`) denotes a collection of variables
  - The variables in the collection are referred to as ***members*** of the structure
- Syntax:

```
struct structure-name {  
    member-type1 member-name1 ;  
    member-typen member-namen ;  
};
```

- `struct`: Predefined keyword used to define a structure
- `structure-name`: User-defined structure name (naming rules same as for ordinary variables)
- `member-type1 member-name1`: Refers to a member variable's type and name

# struct

- Example of a structure:

```
struct Movie {  
    string title, genre;  
    char rating;  
    float IMDBscore, RTscore;  
    bool isEnglish;  
    :  
};
```

- Structure definition does not allocate space for the members
- `Movie m1, m2;` // this statement allocates memory for the variables `m1, m2` and their  
// respective members
- To access a structure's member, join the variable and the member name with a period.  
E.g., `m1.rating`, `m2.title`, etc.  
`m1.isEnglish = true; cout << m1.RTscore + m2.RTscore;`

# struct variables

- Rules for accessing (scope of) `struct` variables are the same as that for other primitive data types
- Lifetime of the `struct` variable is the block in which it's defined

```
int main() {  
    struct Student { int age, year; string branch; bool checkrcd;};  
    Student s1 = {.checkrcd = false};  
    cin >> s1.age >> s1.year >> s1.branch;  
    if(s1.year >= 2023 && s1.age >= 21) {  
        s1.checkrcd = true;  
        Student s1copy = {s1.age, s1.year, s1.branch, false};  
    }  
    cout << s1copy.age << endl; //Error; cannot find s1copy  
}
```



# Recursion

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# Designing a recursive function

- **Recursion:** When a function calls itself as part of its execution
  - Carefully think through the *base cases* i.e., the simplest non-recursive parts of the problem
  - What is the recursive step? Break the problem down into simpler instances of the same problem that eventually lead to a base case.
  - Check: Are all possible cases handled?
- Example: Compute the factorial of a non-negative number, recursively (without any loops)
  - Base case?
    - `if(n == 0) return 1;` or a more general `if(n <= 1) return 1;`
  - Recursive part?
    - We know for  $n \geq 1$ ,  $n! = n \times (n - 1)!$

# Designing a recursive function

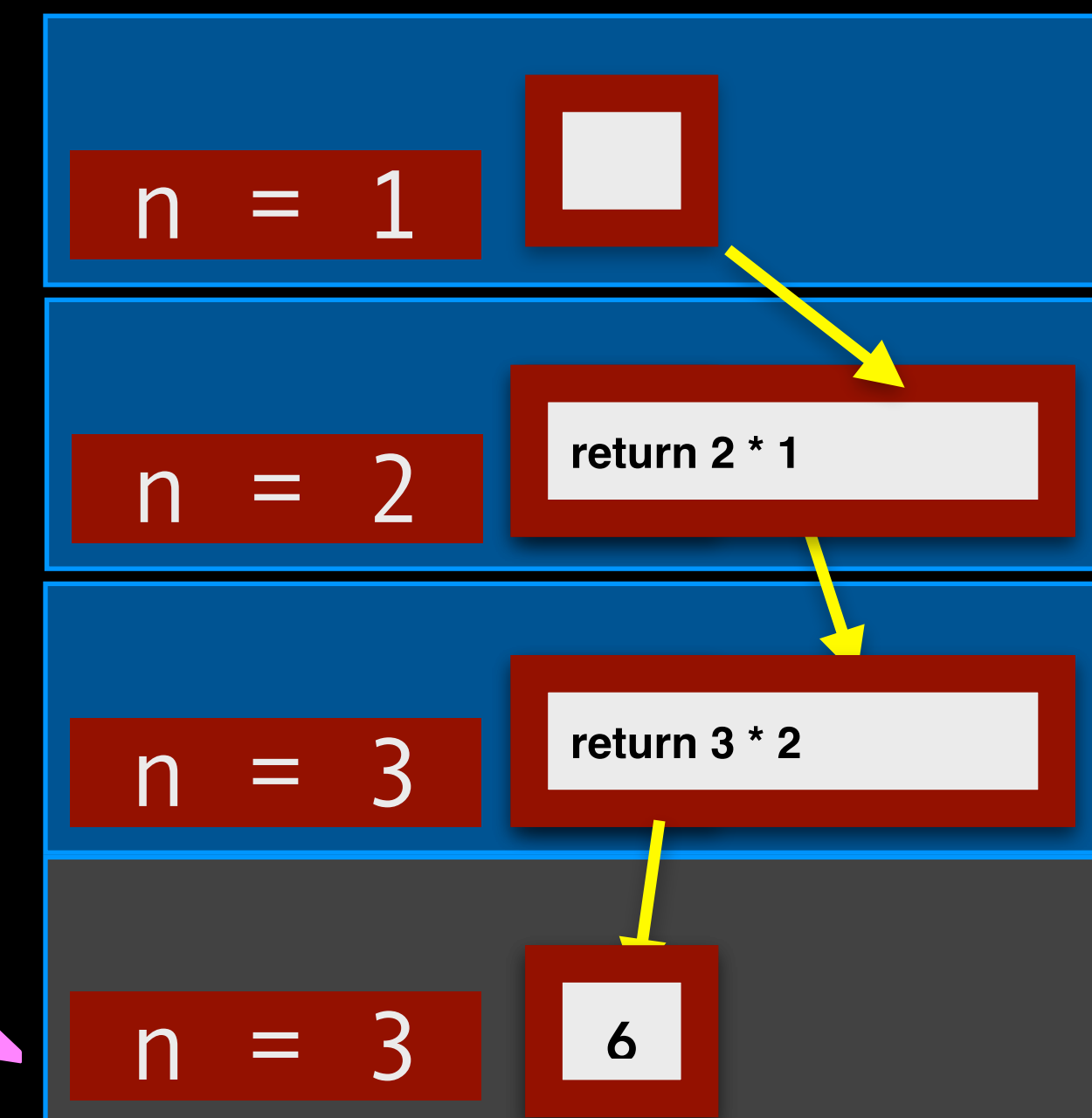
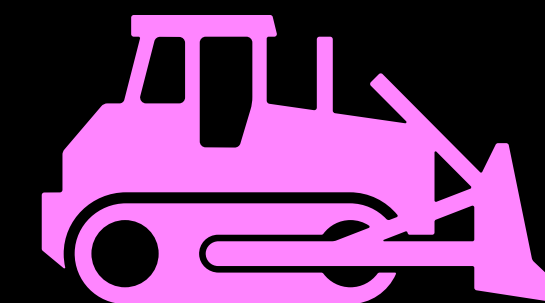
- **Recursion:** When a function calls itself as part of its execution
  - Carefully think through the *base cases* i.e., the simplest non-recursive parts of the problem
  - What is the recursive step? Break the problem down into simpler instances of the same problem that eventually lead to a base case.
  - Check: Are all possible cases handled?
- Example: Compute the factorial of a non-negative number, recursively (without any loops)

```
int factorial(int n) {  
    if(n <= 1) return 1;  
    else return n*factorial(n-1);  
}
```

# Visualizing a recursive function's calls on the stack

```
int factorial(int n) {  
    if(n <= 1) return 1;  
    else return n*factorial(n-1);  
}
```

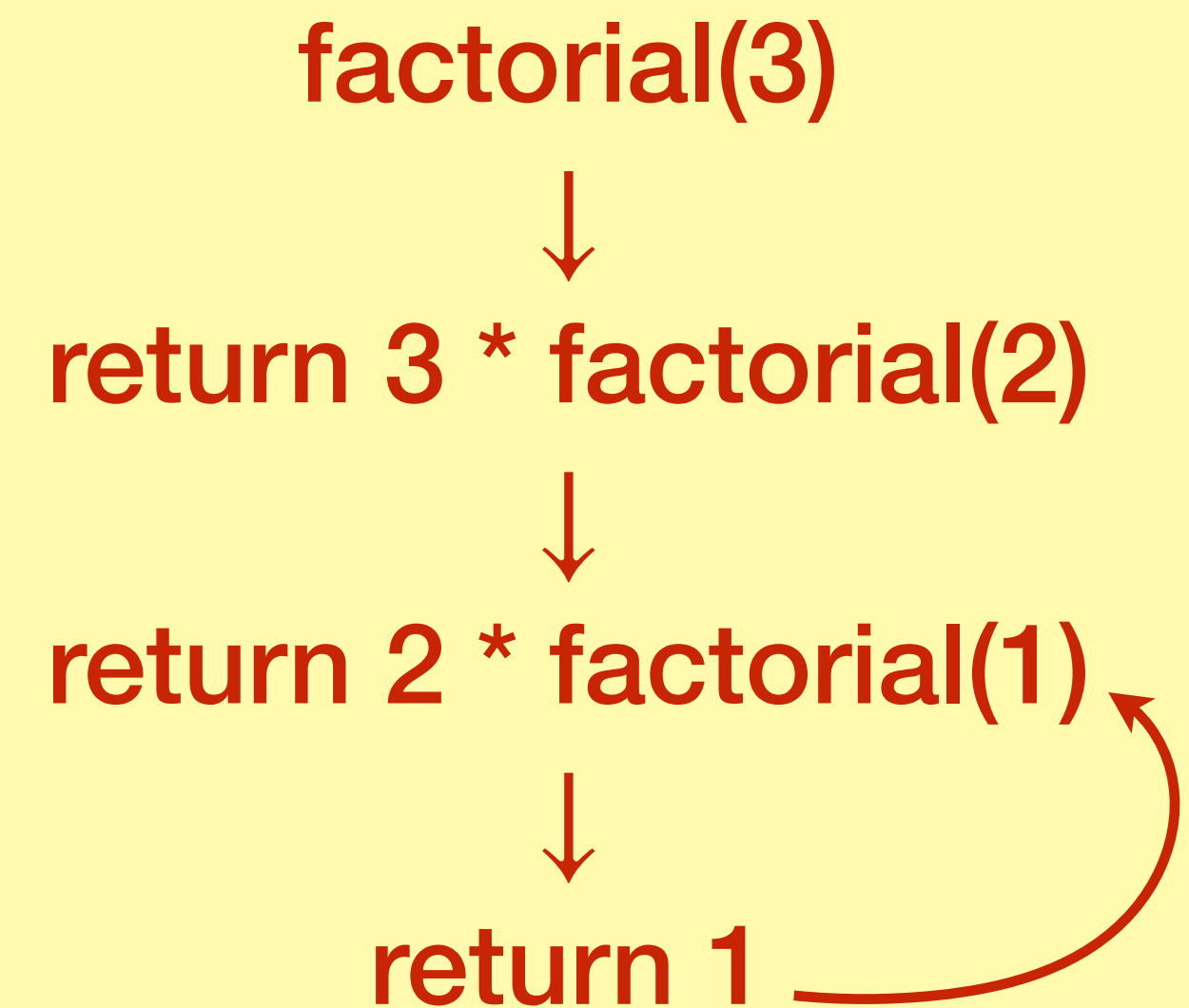
```
main_program {  
    int n;  
    cin >> n; //n = 3  
    cout << factorial(n);  
}
```



# Demonstration of recursion

```
int factorial(int n) {  
    if(n <= 1) return 1;  
    else return n*factorial(n-1);  
}
```

```
main_program {  
    int n;  
    cin >> n; //n = 3  
    cout << factorial(n);  
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```

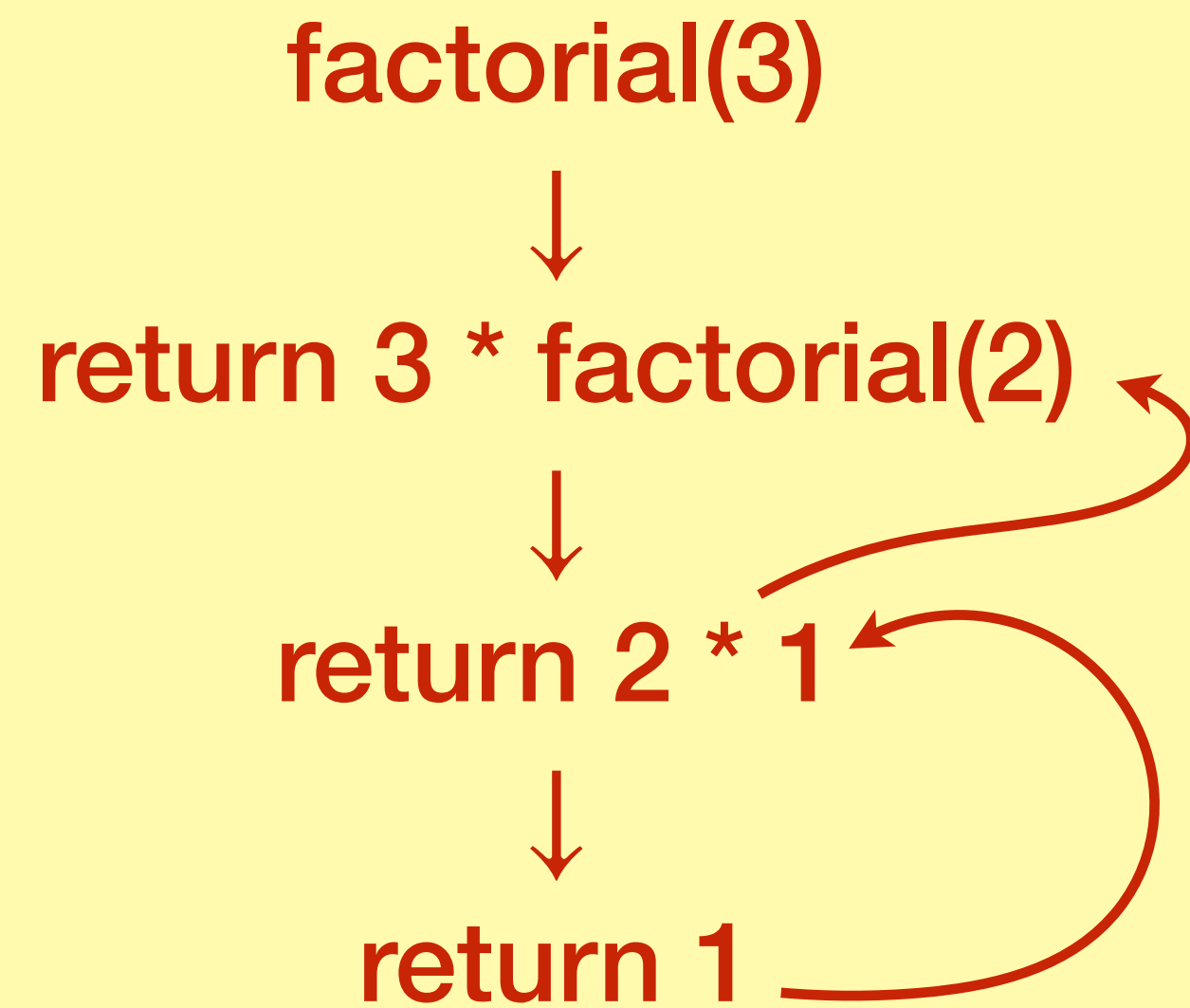




# Demonstration of recursion

```
int factorial(int n) {  
    if(n <= 1) return 1;  
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}
```

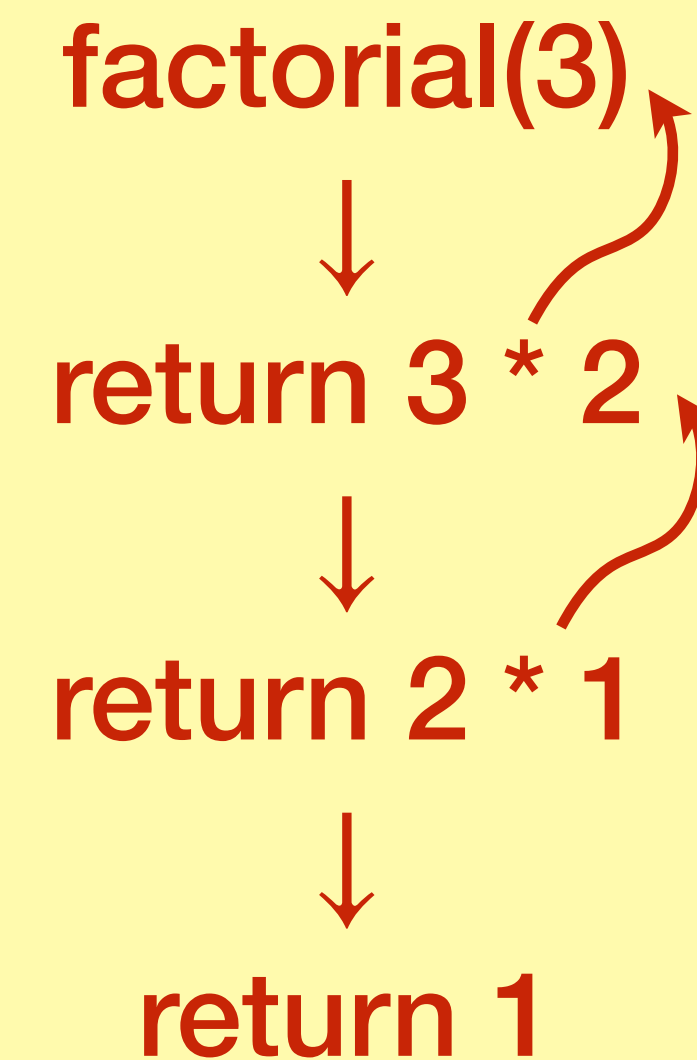
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    cout << factorial(n);  
}
```



# Demonstration of recursion

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int factorial(int n) {  
    if(n <= 1) return 1;  
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}
```

```
main_program {  
    int n;  
    cin >> n; //n = 3  
    cout << factorial(n);  
}
```



# Demonstration of recursion

```
int factorial(int n) {  
    if(n <= 1) return 1;  
    else return n*factorial(n-1);  
}
```

```
main_program {  
    int n;  
    cin >> n; //n = 3  
    cout << factorial(n);  
}
```

6  
↓  
return 3 \* 2  
↓  
return 2 \* 1  
↓  
return 1

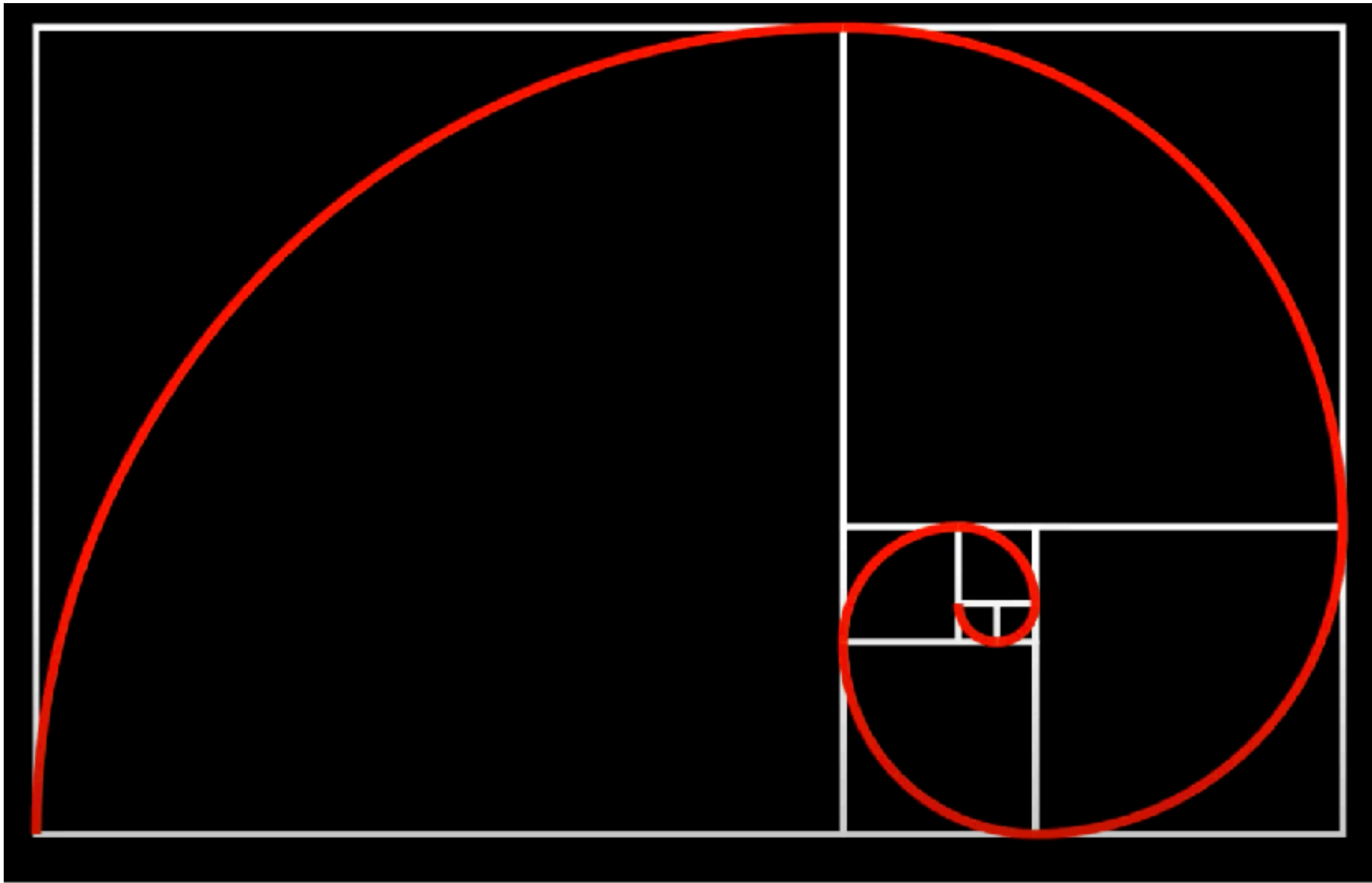


# Fibonacci sequence

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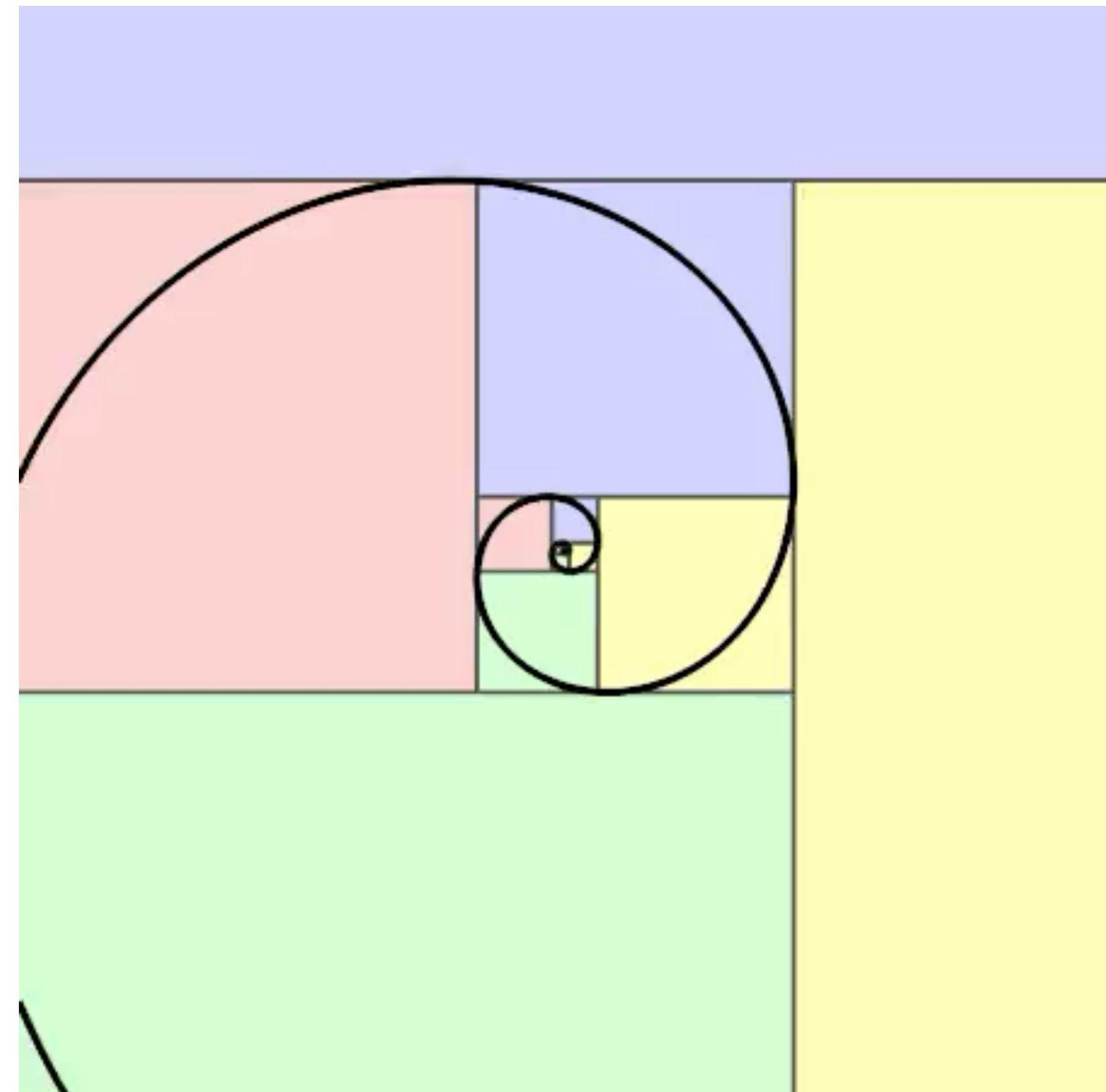
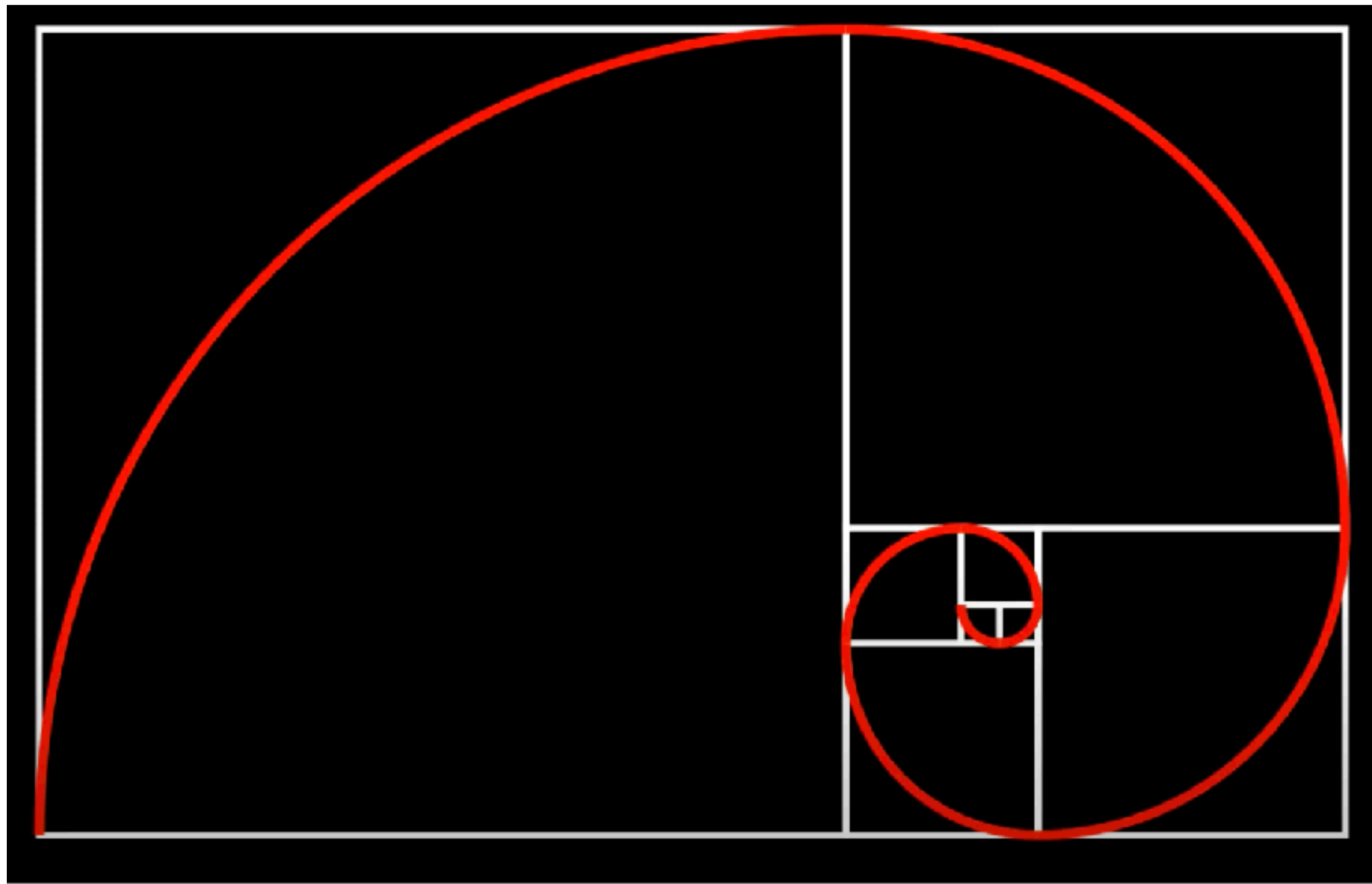
# Fibonacci sequence

- Each element is the sum of two elements before it
- Starting from 0 and 1, the sequence is: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...



# Fibonacci sequence

- Each element is the sum of two elements before it
- Starting from 0 and 1, the sequence is: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

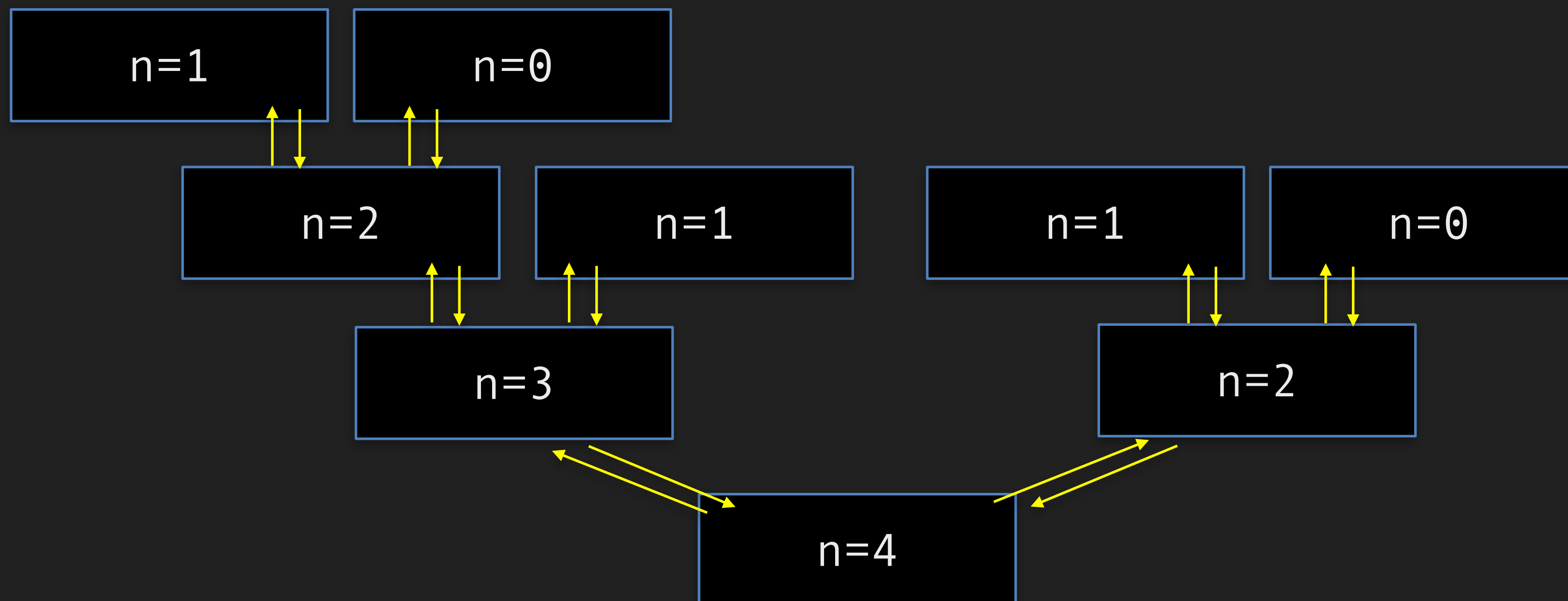


# Example: Fibonacci Sequence

```
int Fibonacci(unsigned int n) {  
    if(n==0) return 0;  
    if(n==1) return 1;  
    return Fibonacci(n-1) + Fibonacci(n-2);  
}
```

} Base cases

A very inefficient implementation!





# Combinations

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# n choose k

- Write a program to recursively compute the number of (unordered) ways to choose k objects from a set of n distinct objects. Relevant formula:  $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

- Single out an object (say **X**). Now think about the number of ways in which you want to:
  - Mandatorily include **X** in your selection
  - Mandatorily exclude **X** from your selection



Diagram illustrating the recursive step: Two arrows originate from the sub-cases 'Mandatorily include X' and 'Mandatorily exclude X'. The arrow from 'include X' points to the top part of the binomial coefficient  $\binom{n-1}{k-1}$ , and the arrow from 'exclude X' points to the bottom part  $\binom{n-1}{k}$ .


$$\binom{n-1}{k-1}$$

- Base cases:  $\binom{n}{0} = 1$  and  $\binom{n}{n} = 1$
- $$\binom{n-1}{k}$$

# n choose k

- Write a program to recursively compute the number of (unordered) ways to choose k objects from a set of n distinct objects. Relevant formula: 
$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

```
int combination(int n, int k) {  
    if(k == 0 || k == n) return 1;  
    return (combination(n-1,k-1) + combination(n-1,k));  
}
```



An inefficient  
implementation  
again!

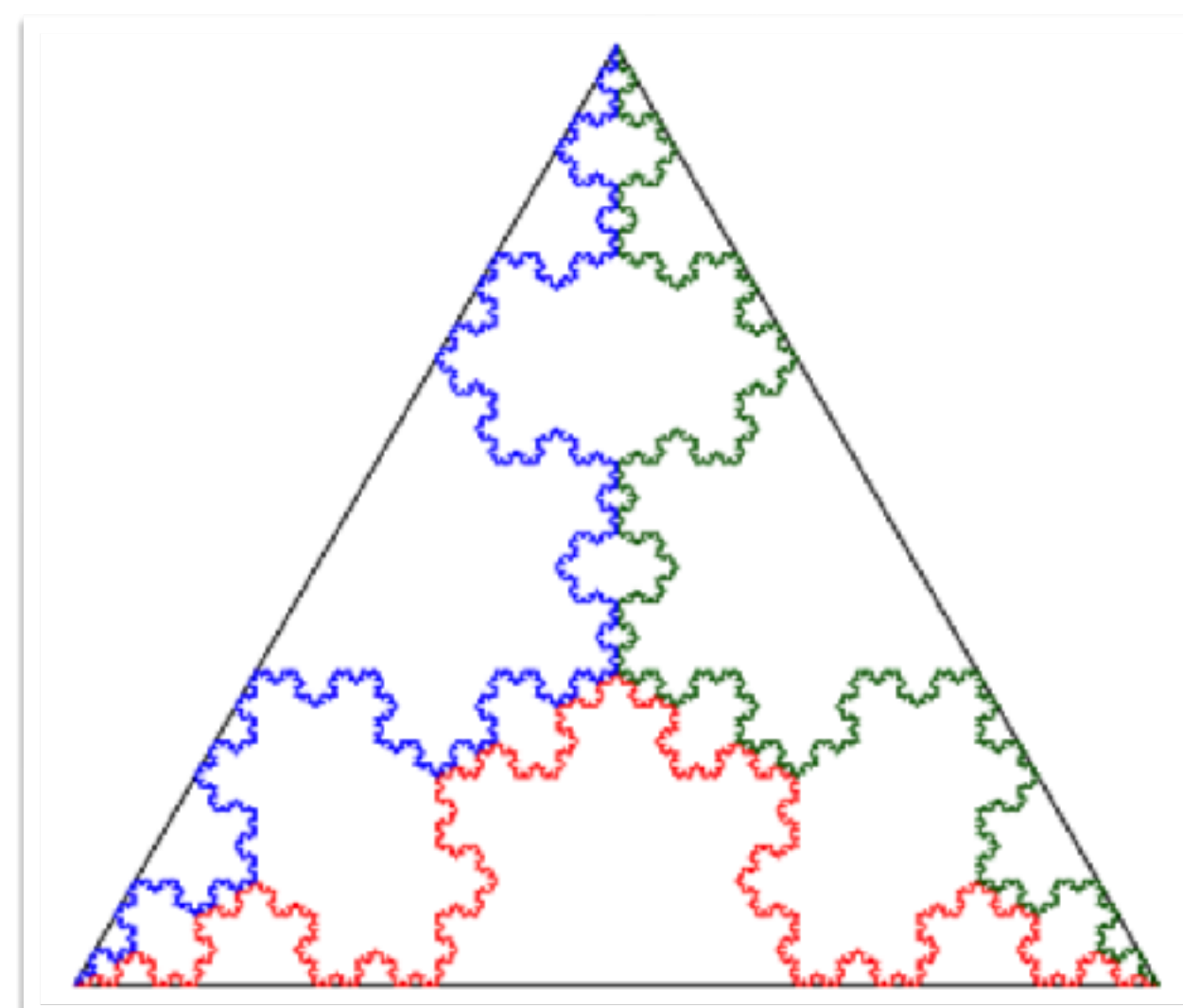
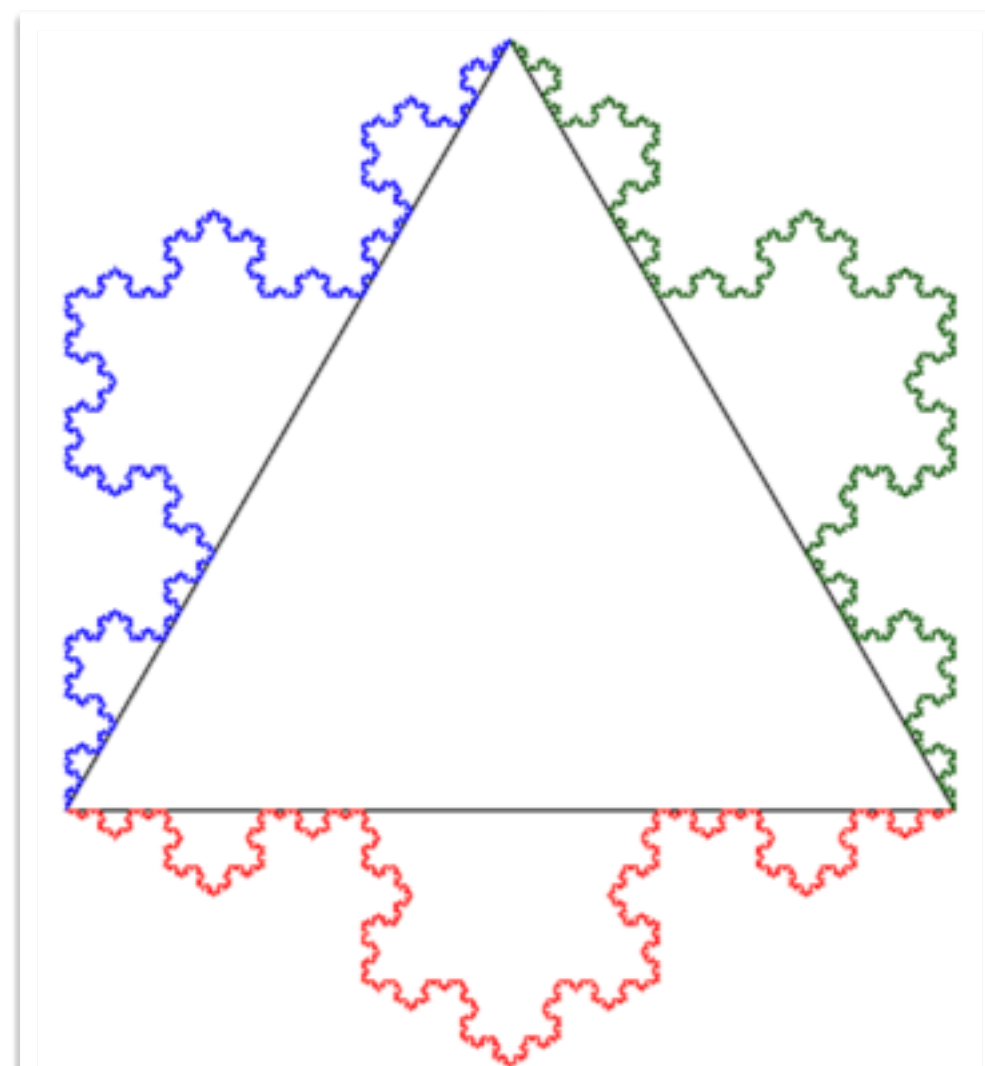
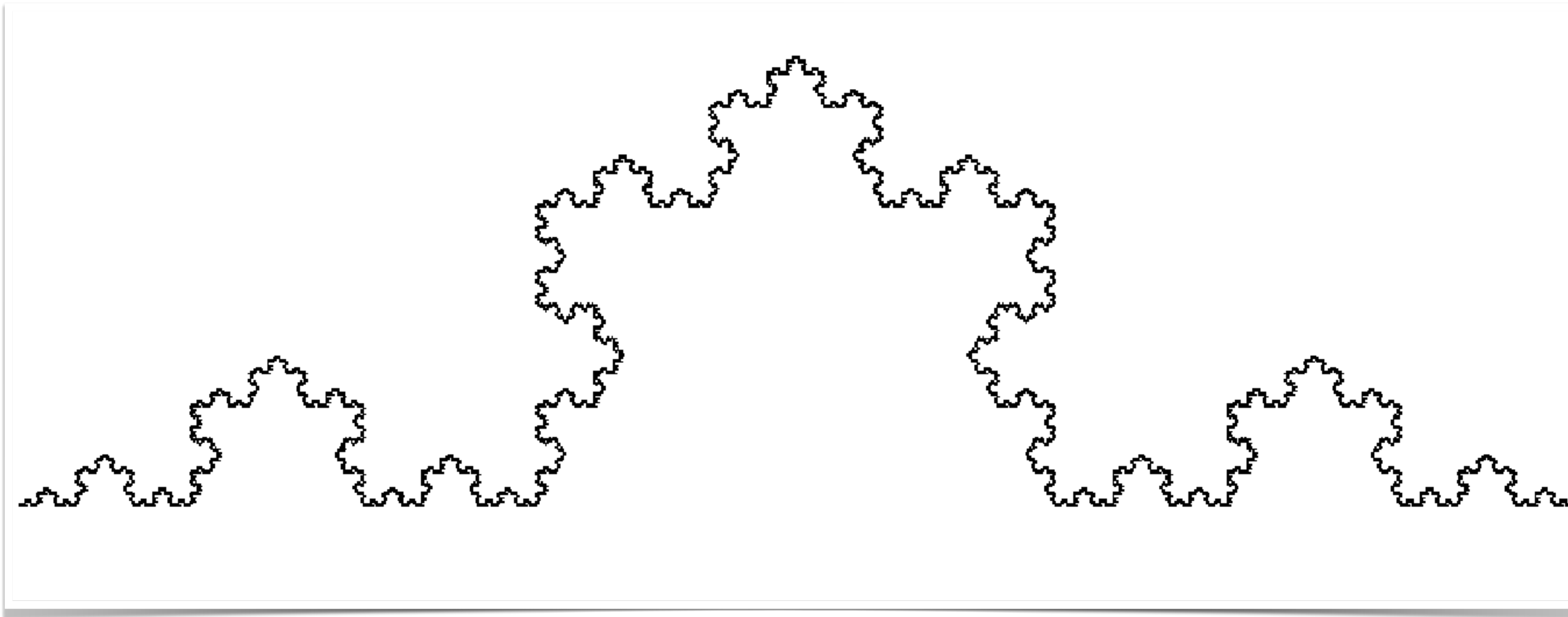


# Fractal

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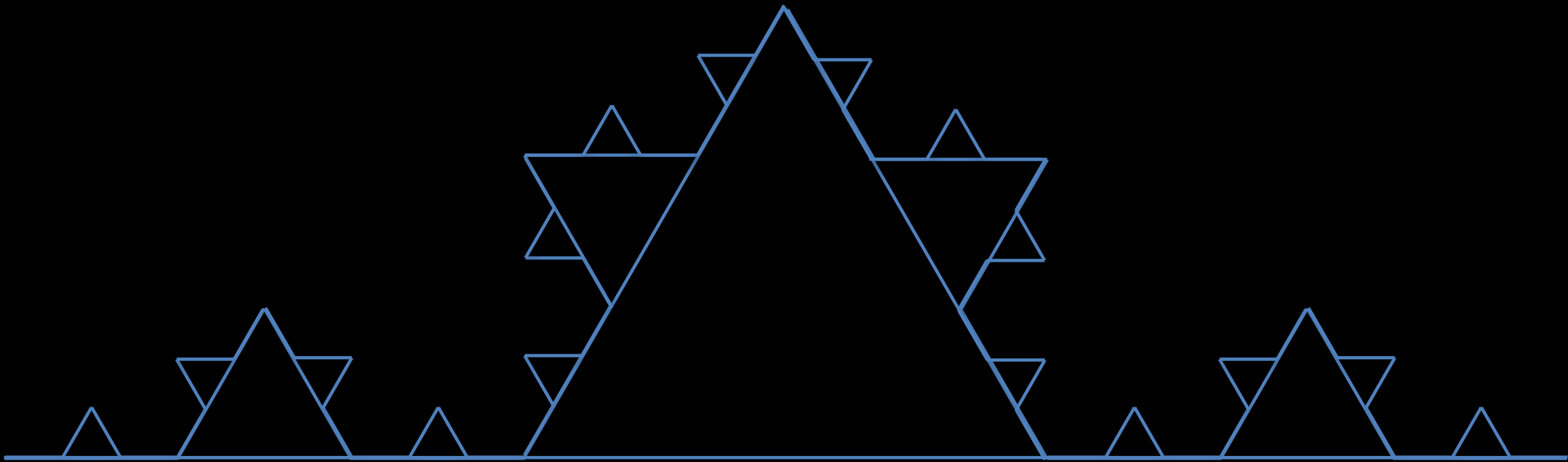
**"Beautiful, damn hard, increasingly useful. That's fractals."**  
**- Benoit Mandelbrot**

# Koch curve

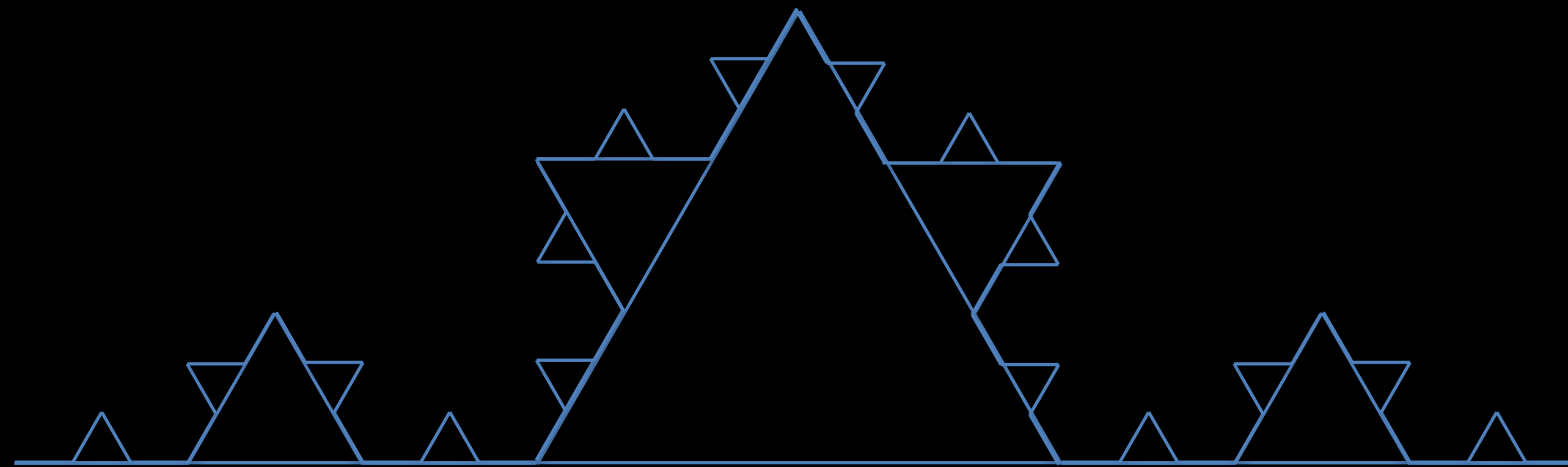


Koch snowflake and  
Koch anti-snowflake

# Unpacking the Koch curve



# Building a recursive function to draw a Koch curve



Recursive call

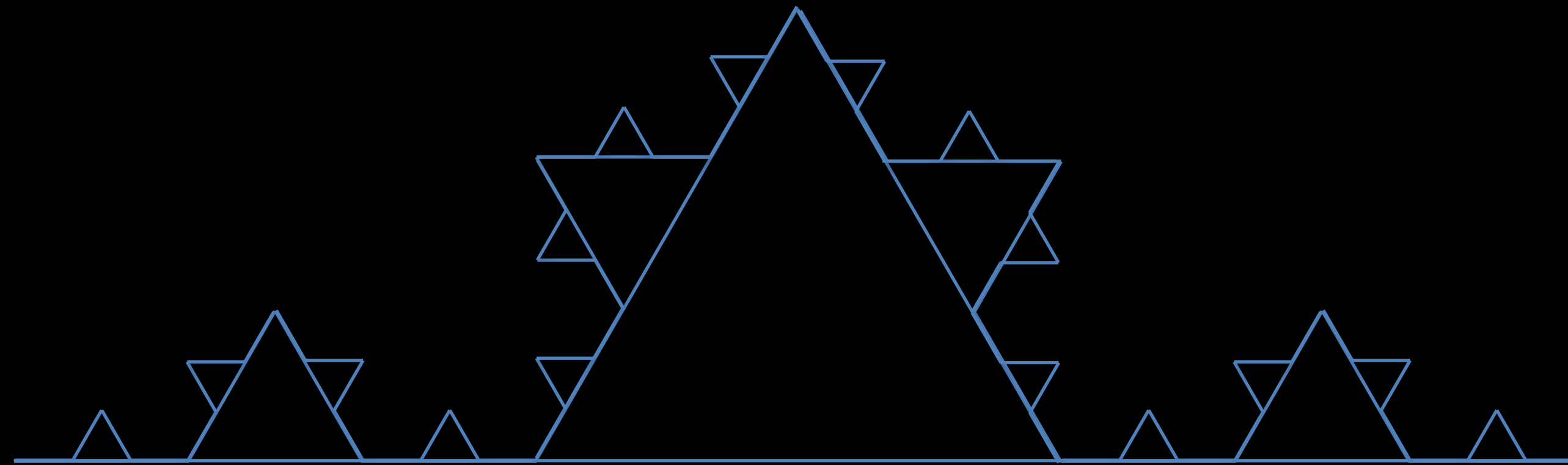
```
void draw(double L, int level) {  
    if (level == 0) { forward(L); return; }  
    if (level == 1) {  
        draw(L/3, 0); left(60);  
        draw(L/3, 0); right(120);  
        draw(L/3, 0); left(60);  
        draw(L/3, 0);  
    }  
}
```

```
    if (level == 2) {  
        draw(L/3, 1); left(60);  
        draw(L/3, 1); right(120);  
        draw(L/3, 1); left(60);  
        draw(L/3, 1);  
    }  
    if (level == 3) ...  
}
```

# Building a recursive function to draw a Koch curve

## Base case:

Ensures that  
the recursion is  
not infinite



```
void draw(double L, int level) {  
    if (level == 0) { forward(L); return; }  
    draw(L/3, level-1); left(60);  
    draw(L/3, level-1); right(120);  
    draw(L/3, level-1); left(60);  
    draw(L/3, level-1);  
}
```

Demo in class



# Tower of Hanoi

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# Tower of Hanoi

- Famous puzzle consisting of three rods and disks of varying diameters. Disks are stacked on one rod, ordered bottom-to-top from largest to smallest (in diameter). The goal is to move all the disks from one (**source**) rod to one of the other two rods (i.e., a **target** rod, and the remaining rod is referred to as a **spare** rod), while satisfying the following rules:
  1. Only one disk is moved at a time.
  2. A valid move is taking the topmost disk from a stack and placing it on another stack or rod.
  3. A disk cannot be placed on top of a smaller disk.



# Tower of Hanoi: Building the recursive solution

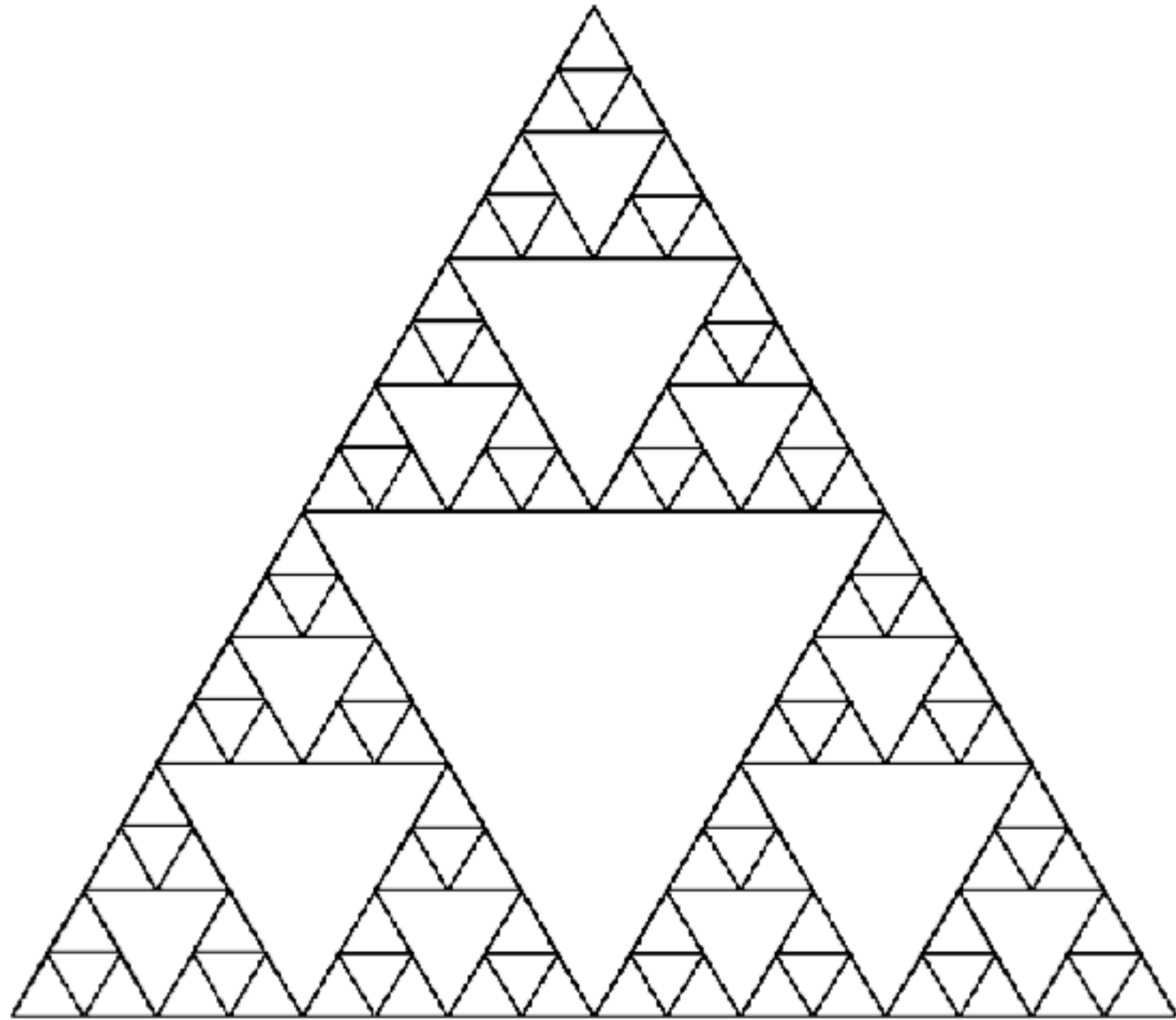
- Say we know how to solve Tower of Hanoi for  $n-1$  disks using `towerofHanoi( $n-1$ , source, target, spare)`. Then, to solve it for  $n$  disks:
  1. Move  $n-1$  disks from the source rod to the spare rod.  
That is, call `towerofHanoi( $n-1$ , source, spare, target)` //recursive call
  2. Disk  $n$  is the only one remaining on the source rod. Move it from source to target rod.  
This is a base case.
  3. Move  $n-1$  disks back from the spare rod to the target rod.  
That is, call `towerofHanoi( $n-1$ , spare, target, source)` //recursive call



Demo in class

# Homework Exercises

Write a recursive program to draw the following fractal with repeating equilateral triangles



Write a recursive function to print out all permutations of a string. Assume the string is a single word. If there are duplicates, you can print them all out.

If the input is "out", your code prints:

```
out
otu
uot
uto
tou
tuo
```



**Next class: Arrays**  
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