



MA 106 Endsem Exam : Part A : Question Paper
Indian Institute of Technology Bombay

A

Roll No.:
Name:

Division.: D

Tutorial: T

Apr. 19, 2023

8.30 - 10.30 AM

9.30 AM (Part A)

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

- ⊕ There are 14 questions in Part A. Each question contains a **single correct answer** of one mark.
- ⊕ You need to indicate your answer by bubbling the box in the OMR (Optical Mark Recognition) sheet. The right way to fill a box like is as . Use only a ball-point pen to fill the correct choice in the OMR sheet.
- ⊕ **Answer provided in the OMR sheet will only be evaluated.** You can mark answer in the question paper for your reference but it will not be evaluated.
- ⊕ The OMR sheet will be collected back at 9.30 AM.
- ⊕ **Notation:** $\mathbb{R}^{m \times n}$: the set of all $m \times n$ real matrices, A^t : the transpose of A , E_λ : the eigenspace for an eigenvalue λ , $\chi_A(x)$: the characteristic polynomial of A , $\text{Null}(A)$: the nullspace of A , $\mathcal{C}(A)$: the column space of A .

Q-1) Consider $V = \mathbb{R}^{2 \times 2}$ and the inner product $\langle A, B \rangle = \text{tr}(A^t B)$. Let $J = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $p_I(J)$ denote the orthogonal projection of J along the identity matrix I . Then $p_I(J)$ is

- A I B 0 C J D $2I$

Soln.: We have $P_I(J) = \frac{\langle I, J \rangle}{\langle I, I \rangle} I = \frac{\text{tr}(J)}{2} I = I$. ⊗

Q-2) Define $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ by $T(u) = Au$ where $A = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 4 & 7 \\ -2 & 2 & 0 \end{bmatrix}$. Then

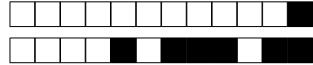
- A Null(T) is a line.
 B Null(T) = $\{0\}$.
 C Im(T) is a line.
 D Null(T) is a plane.

Soln.: Note that $\det A = 0$ and $\begin{vmatrix} 1 & 3 \\ 3 & 4 \end{vmatrix} \neq 0$. Therefore, $\text{rank}(A) = 2$ and so $\text{nullity}(A) = 1$. ⊗

Q-3) Let $P_2(\mathbb{R})$ be the space of polynomials over \mathbb{R} of degree ≤ 2 . Define $T : P_2(\mathbb{R}) \rightarrow \mathbb{R}^3$ by $T(f(x)) = (f(0), f(-1), f(1))^t$. Then $\text{rank}(T)$ is

- A 4 B 1 C 2 D 3

Soln.: Let $f(x) = a + bx + cx^2$, $a, b, c \in \mathbb{R}$. Then we have $f(-1) = a - b + c$, $f(0) = a$, $f(1) = a + b + c$. If $T(f) = 0$, then $a = b = c = 0$. This shows that f is one-one. Hence, T is an isomorphism. So, $\text{rank}(T) = 3$. ⊗



Q-4) Let A be a 3×3 real matrix and $A^2 = A$. Then $\dim(\text{Null}(A) \cap \mathcal{C}(A))$ is

- [A] 3 [B] 1 [C] 0 [D] 2

Soln.: If $v = Ax \in \text{null}(A) \cap \mathcal{C}(A)$, for some $x \in \mathbb{R}^3$, then $0 = Av = A^2x = Ax = v$. ✖

Q-5) The number of 2×2 nilpotent real matrices is

- [A] infinite [B] 2 [C] 1 [D] 3

Soln.: For every $n \in \mathbb{Z}$ we have that $\begin{bmatrix} 0 & n \\ 0 & 0 \end{bmatrix}$ is nilpotent. ✖

Q-6) Let A be a 3×3 real matrix with $\text{tr}(A) = 0$. Let $\chi_A(2) = \chi_A(3) = 0$. Then

- [A] $\chi_A(x) = (x - 2)(x - 3)$ [C] $\chi_A(x) = (x - 2)^2(x - 3)$
[B] $\chi_A(x) = (x + 5)(x - 2)(x - 3)$ [D] $\chi_A(x) = (x - 2)(x - 3)^2$

Soln.: Since 2, 3 are roots of the $\chi_A(x)$, and since the sum of all the roots of $\chi_A(x) = \text{tr}(A) = 0$, we have that -5 is a root of $\chi_A(x)$. ✖

Q-7) Consider positive real numbers a, b, c and matrix $A = \begin{bmatrix} a & b & c \\ a & b & c \\ a & b & c \end{bmatrix}$. Let $\lambda = a + b + c$. Then

- [A] $\dim E_0 = 1$, $\dim E_\lambda = 1$. [C] $\dim E_0 = 2$, $\dim E_\lambda = 2$.
[B] $\dim E_0 = 2$, $\dim E_\lambda = 1$. [D] $\dim E_0 = 1$, $\dim E_\lambda = 2$.

Soln.: We have $\text{rank}(A) = 2$ and $\text{nullity}(A) = 2$. So, 0 is an eigenvalue of A with $\dim(E_0) = 2$. Also, $A[1 \ 1 \ 1]^t = (a + b + c)[1 \ 1 \ 1]^t$. So, $\lambda = a + b + c$ is an eigenvalue of A . Since $\dim(E_0) + \dim(E_\lambda) = 3$, we get $\dim(E_\lambda) = 1$. ✖



Q-8) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbb{R}^{2 \times 2}$ and $(a-d)^2 + 4bc > 0$. Then

A None of these answers are correct.

C Eigenvalues of A are not real.

B A is not diagonalizable over \mathbb{R} .

D There is a basis of \mathbb{R}^2 consisting of eigenvectors of A .

Soln.: Consider $\chi_A(x) = \begin{vmatrix} x-a & -b \\ -c & x-d \end{vmatrix} = x^2 - (a+d)x + (ad - bc)$.

Roots of $\chi_A(x)$ are

$$\lambda_1 = \frac{1}{2} \left[(a+d) + \sqrt{(a+d)^2 - 4(ad-bc)} \right] = \frac{1}{2} \left[(a+d) + \sqrt{(a-d)^2 + 4bc} \right],$$

$$\lambda_2 = \frac{1}{2} \left[(a+d) - \sqrt{(a+d)^2 - 4(ad-bc)} \right] = \frac{1}{2} \left[(a+d) - \sqrt{(a-d)^2 + 4bc} \right].$$

From the given condition we get $\lambda_1 \neq \lambda_2$. Thus, A is diagonalizable as the eigenvectors corresponding to λ_1 and λ_2 are linearly independent. \ddagger

Q-9) Let $V = \mathbb{R}^{2 \times 2}$ and $P \in V$ be a nonzero matrix. Consider the linear transformation $T : V \rightarrow V$ given by $T(A) = AP - PA$. Then

A T is 1-1.

C T is an isomorphism.

B None of these answers are correct.

D T is onto.

Soln.: Clearly $T(P) = 0$. Hence, T is not 1-1. Thus, T is not onto either. Hence, none of these answers are correct. \ddagger

Q-10) If A is a 3×3 real matrix and $A^2 + I = 0$ then

A $-1, i, -i$ are roots of $\chi_A(x)$.

C i and $-i$ are the only roots of $\chi_A(x)$.

B $1, i, -i$ are roots of $\chi_A(x)$.

D There is no such matrix.

Soln.: Since $\chi_A(x)$ has degree 3, it has a real root r . As $A_I^2 = 0$, we get $r^2 = -1$. This is a contradiction. Therefore no such matrix exists. \ddagger

Q-11) Let V be the vector space over \mathbb{R} of 2×2 Hermitian matrices. Then dimension of V is

A 4

B 2

C 1

D 3

Soln.: Any 2×2 Hermitian matrix A has the form $A = \begin{bmatrix} x & y+iz \\ y-iz & w \end{bmatrix}$ for some $x, y, z, w \in \mathbb{R}$. So,

$$A = x \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + y \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + z \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + w \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

This shows that $\dim(V) = 4$. \ddagger



Q-12) The conic described by the $5x^2 - 4xy + 8y^2 = 36$ is

- [A] a pair of lines. [B] a hyperbola. [C] a parabola. [D] an ellipse.

Soln.: The equation is

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 5 & -2 \\ -2 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 36.$$

Let $A = \begin{bmatrix} 5 & -2 \\ -2 & 8 \end{bmatrix}$. Then $\chi_A(x) = \det(xI - A) = (x - 4)(x - 9)$. As the eigenvalues are 4 and 9, the equation in u, r plane is given by

$$4u^2 + 9r^2 = 36 \implies \frac{u^2}{9} + \frac{v^2}{1} = 1$$

This is as an ellipse. ✖

Q-13) The rank of the linear transformation $T : \mathbb{R}^{2 \times 2} \rightarrow \mathbb{R}^{2 \times 2}$ given by $T(A) = PA$ where $A \in \mathbb{R}^{2 \times 2}$ and $P = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$ is

- [A] 2 [B] 3 [C] 1 [D] 4

Soln.: Clearly, $\text{Null}(T) = \{A \mid PA = 0\}$. It follows that

$$\begin{bmatrix} x & y \\ a & b \end{bmatrix} \in \text{Null}(T) \Leftrightarrow \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} x & y \\ a & b \end{bmatrix} = 0 \Leftrightarrow \begin{bmatrix} x + 3a = 0 \\ y + 3b = 0 \end{bmatrix} \Leftrightarrow x = -3a, \quad y = -3b.$$

Therefore,

$$\begin{bmatrix} x & y \\ a & b \end{bmatrix} = \begin{bmatrix} -3a & -3b \\ a & b \end{bmatrix} = a \begin{bmatrix} -3 & 0 \\ 1 & 0 \end{bmatrix} + b \begin{bmatrix} 0 & -3 \\ 0 & 1 \end{bmatrix}.$$

Hence, $\dim \text{Null}(T) = 2$ and so $\text{rank}(T) = 2$. ✖

Q-14) Let $V = \mathbb{R}^{3 \times 3}$ and $\langle A, B \rangle = \text{tr}(AB^t)$. Let D denote the subspace of diagonal matrices in V . Then $\dim D^\perp$ is

- [A] 6 [B] 9 [C] 5 [D] 3

Soln.: Since $\dim(D) = 3$ and $\dim(D) + \dim(V^\perp) = \dim(V) = 9$, we get that $\dim(V^\perp) = 6$. ✖



MA 106 Part A : Optical Mark Recognition (OMR) Sheet

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Bubble Your Rollnumber

0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9

B B

D

A

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Q-1) A B C DQ-8) A B C DQ-2) A B C DQ-9) A B C DQ-3) A B C DQ-10) A B C DQ-4) A B C DQ-11) A B C DQ-5) A B C DQ-12) A B C DQ-6) A B C DQ-13) A B C DQ-7) A B C DQ-14) A B C D



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Q-1> Let A be a 3×3 real matrix and $A^2 = A$. Then $\dim(\text{Null}(A) \cap \mathcal{C}(A))$ is

- A 1 B 0 C 3 D 2

Soln.: If $v = Ax \in \text{null}(A) \cap \mathcal{C}(A)$, for some $x \in \mathbb{R}^3$, then $0 = Av = A^2x = Ax = v$. ✖

Q-2> If A is a 3×3 real matrix and $A^2 + I = 0$ then

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 B i and $-i$ are the only roots of $\chi_A(x)$. D There is no such matrix.

Soln.: Since $\chi_A(x)$ has degree 3, it has a real root r . As $A_I^2 = 0$, we get $r^2 = -1$. This is a contradiction. Therefore no such matrix exists. ✖



Q-3) The conic described by the $5x^2 - 4xy + 8y^2 = 36$ is

- [A] an ellipse. [B] a parabola. [C] a pair of lines. [D] a hyperbola.

Soln.: The equation is

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[C] $\dim E_0 = 2$, $\dim E_\lambda = 1$.
[D] $\dim E_0 = 1$, $\dim E_\lambda = 1$.

Soln.: We have $\text{rank}(A) = 2$ and $\text{nullity}(A) = 2$. So, 0 is an eigenvalue of A with $\dim(E_0) = 2$. Also, $A[1 1 1]^t = (a + b + c)[1 1 1]^t$. So, $\lambda = a + b + c$ is an eigenvalue of A . Since $\dim(E_0) + \dim(E_\lambda) = 3$, we get $\dim(E_\lambda) = 1$. ✖

Q-5) Define $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ by $T(u) = Au$ where $A = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 4 & 7 \\ -2 & 2 & 0 \end{bmatrix}$. Then

- [A] $\text{Null}(T) = \{0\}$.
[B] $\text{Im}(T)$ is a line.
[C] $\text{Null}(T)$ is a line.
[D] $\text{Null}(T)$ is a plane.

Soln.: Note that $\det A = 0$ and $\begin{vmatrix} 1 & 3 \\ 3 & 4 \end{vmatrix} \neq 0$. Therefore, $\text{rank}(A) = 2$ and so $\text{nullity}(A) = 1$. ✖

Q-6) Let A be a 3×3 real matrix with $\text{tr}(A) = 0$. Let $\chi_A(2) = \chi_A(3) = 0$. Then

- [A] $\chi_A(x) = (x - 2)(x - 3)$
[B] $\chi_A(x) = (x - 2)(x - 3)^2$
[C] $\chi_A(x) = (x + 5)(x - 2)(x - 3)$
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Soln.: Since 2, 3 are roots of the $\chi_A(x)$, and since the sum of all the roots of $\chi_A(x) = \text{tr}(A) = 0$, we have that -5 is a root of $\chi_A(x)$. ✖



Q-7) The number of 2×2 nilpotent real matrices is

- A 1 B 3 C 2 D infinite

Soln.: For every $n \in \mathbb{Z}$ we have that $\begin{bmatrix} 0 & n \\ 0 & 0 \end{bmatrix}$ is nilpotent. ✖

Q-8) Let V be the vector space over \mathbb{R} of 2×2 Hermitian matrices. Then dimension of V is

- A 2 B 1 C 4 D 3

Soln.: Any 2×2 Hermitian matrix A has the form $A = \begin{bmatrix} x & y + iz \\ y - iz & w \end{bmatrix}$ for some $x, y, z, w \in \mathbb{R}$.
So,

$$A = x \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + y \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + z \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + w \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

This shows that $\dim(V) = 4$. ✖

Q-9) Consider $V = \mathbb{R}^{2 \times 2}$ and the inner product $\langle A, B \rangle = \text{tr}(A^t B)$. Let $J = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $p_I(J)$ denote the orthogonal projection of J along the identity matrix I . Then $p_I(J)$ is

- A $2I$ B 0 C J D I

Soln.: We have $P_I(J) = \frac{\langle I, J \rangle}{\langle I, I \rangle} I = \frac{\text{tr}(J)}{2} I = I$. ✖

Q-10) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbb{R}^{2 \times 2}$ and $(a-d)^2 + 4bc > 0$. Then

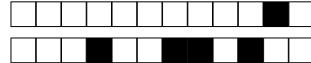
- A There is a basis of \mathbb{R}^2 consisting of eigenvectors of A . C A is not diagonalizable over \mathbb{R} .
 B Eigenvalues of A are not real. D None of these answers are correct.

Soln.: Consider $\chi_A(x) = \begin{vmatrix} x-a & -b \\ -c & x-d \end{vmatrix} = x^2 - (a+d)x + (ad-bc)$.

Roots of $\chi_A(x)$ are

$$\lambda_1 = \frac{1}{2} \left[(a+d) + \sqrt{(a+d)^2 - 4(ad-bc)} \right] = \frac{1}{2} \left[(a+d) + \sqrt{(a-d)^2 + 4bc} \right],$$
$$\lambda_2 = \frac{1}{2} \left[(a+d) - \sqrt{(a+d)^2 - 4(ad-bc)} \right] = \frac{1}{2} \left[(a+d) - \sqrt{(a-d)^2 + 4bc} \right].$$

From the given condition we get $\lambda_1 \neq \lambda_2$. Thus, A is diagonalizable as the eigenvectors corresponding to λ_1 and λ_2 are linearly independent. ✖



Q-11) Let $V = \mathbb{R}^{2 \times 2}$ and $P \in V$ be a nonzero matrix. Consider the linear transformation $T : V \rightarrow V$ given by $T(A) = AP - PA$. Then

- [A] None of these answers are correct.
[B] T is an isomorphism.

- [C] T is onto.
[D] T is 1-1.

Soln.: Clearly $T(P) = 0$. Hence, T is not 1-1. Thus, T is not onto either. Hence, none of these answers are correct. \ddagger

Q-12) Let $P_2(\mathbb{R})$ be the space of polynomials over \mathbb{R} of degree ≤ 2 . Define $T : P_2(\mathbb{R}) \rightarrow \mathbb{R}^3$ by $T(f(x)) = (f(0), f(-1), f(1))^t$. Then $\text{rank}(T)$ is

- [A] 1 [B] 2 [C] 4 [D] 3

Soln.: Let $f(x) = a + bx + cx^2$, $a, b, c \in \mathbb{R}$. Then we have $f(-1) = a - b + c$, $f(0) = a$, $f(1) = a + b + c$. If $T(f) = 0$, then $a = b = c = 0$. This shows that f is one-one. Hence, T is an isomorphism. So, $\text{rank}(T) = 3$. \ddagger

Q-13) The rank of the linear transformation $T : \mathbb{R}^{2 \times 2} \rightarrow \mathbb{R}^{2 \times 2}$ given by $T(A) = PA$ where $A \in \mathbb{R}^{2 \times 2}$ and $P = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$ is

- [A] 4 [B] 1 [C] 3 [D] 2

Soln.: Clearly, $\text{Null}(T) = \{A \mid PA = 0\}$. It follows that

$$\begin{bmatrix} x & y \\ a & b \end{bmatrix} \in \text{Null}(T) \Leftrightarrow \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} x & y \\ a & b \end{bmatrix} = 0 \Leftrightarrow \begin{bmatrix} x+3a & 0 \\ y+3b & 0 \end{bmatrix} \Leftrightarrow x = -3a, \quad y = -3b.$$

Therefore,

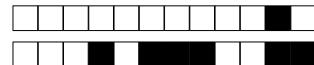
$$\begin{bmatrix} x & y \\ a & b \end{bmatrix} = \begin{bmatrix} -3a & -3b \\ a & b \end{bmatrix} = a \begin{bmatrix} -3 & 0 \\ 1 & 0 \end{bmatrix} + b \begin{bmatrix} 0 & -3 \\ 0 & 1 \end{bmatrix}.$$

Hence, $\dim \text{Null}(T) = 2$ and so $\text{rank}(T) = 2$. \ddagger

Q-14) Let $V = \mathbb{R}^{3 \times 3}$ and $\langle A, B \rangle = \text{tr}(AB^t)$. Let D denote the subspace of diagonal matrices in V . Then $\dim D^\perp$ is

- [A] 5 [B] 9 [C] 6 [D] 3

Soln.: Since $\dim(D) = 3$ and $\dim(D) + \dim(V^\perp) = \dim(V) = 9$, we get that $\dim(V^\perp) = 6$. \ddagger



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3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5
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8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9

B B

D

B

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Q-1) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbb{R}^{2 \times 2}$ and $(a-d)^2 + 4bc > 0$. Then

 A None of these answers are correct. C Eigenvalues of A are not real. B A is not diagonalizable over \mathbb{R} . D There is a basis of \mathbb{R}^2 consisting of eigenvectors of A .

Soln.: Consider $\chi_A(x) = \begin{vmatrix} x-a & -b \\ -c & x-d \end{vmatrix} = x^2 - (a+d)x + (ad-bc)$.

Roots of $\chi_A(x)$ are

$$\lambda_1 = \frac{1}{2} \left[(a+d) + \sqrt{(a+d)^2 - 4(ad-bc)} \right] = \frac{1}{2} \left[(a+d) + \sqrt{(a-d)^2 + 4bc} \right],$$

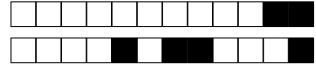
$$\lambda_2 = \frac{1}{2} \left[(a+d) - \sqrt{(a+d)^2 - 4(ad-bc)} \right] = \frac{1}{2} \left[(a+d) - \sqrt{(a-d)^2 + 4bc} \right].$$

From the given condition we get $\lambda_1 \neq \lambda_2$. Thus, A is diagonalizable as the eigenvectors corresponding to λ_1 and λ_2 are linearly independent. ✖

Q-2) Consider $V = \mathbb{R}^{2 \times 2}$ and the inner product $\langle A, B \rangle = \text{tr}(A^t B)$. Let $J = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $p_I(J)$ denote the orthogonal projection of J along the identity matrix I . Then $p_I(J)$ is

 A 0 B I C J D $2I$

Soln.: We have $P_I(J) = \frac{\langle I, J \rangle}{\langle I, I \rangle} I = \frac{\text{tr}(J)}{2} I = I$. ✖



Q-3) Let A be a 3×3 real matrix and $A^2 = A$. Then $\dim(\text{Null}(A) \cap \mathcal{C}(A))$ is

- [A] 0 [B] 3 [C] 2 [D] 1

Soln.: If $v = Ax \in \text{null}(A) \cap \mathcal{C}(A)$, for some $x \in \mathbb{R}^3$, then $0 = Av = A^2x = Ax = v$. ✖

Q-4) The rank of the linear transformation $T : \mathbb{R}^{2 \times 2} \rightarrow \mathbb{R}^{2 \times 2}$ given by $T(A) = PA$ where $A \in \mathbb{R}^{2 \times 2}$ and $P = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$ is

- [A] 4 [B] 1 [C] 2 [D] 3

Soln.: Clearly, $\text{Null}(T) = \{A \mid PA = 0\}$. It follows that

$$\begin{bmatrix} x & y \\ a & b \end{bmatrix} \in \text{Null}(T) \Leftrightarrow \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} x & y \\ a & b \end{bmatrix} = 0 \Leftrightarrow \begin{bmatrix} x + 3a & 0 \\ y + 3b & 0 \end{bmatrix} \Leftrightarrow x = -3a, \quad y = -3b.$$

Therefore,

$$\begin{bmatrix} x & y \\ a & b \end{bmatrix} = \begin{bmatrix} -3a & -3b \\ a & b \end{bmatrix} = a \begin{bmatrix} -3 & 0 \\ 1 & 0 \end{bmatrix} + b \begin{bmatrix} 0 & -3 \\ 0 & 1 \end{bmatrix}.$$

Hence, $\dim \text{Null}(T) = 2$ and so $\text{rank}(T) = 2$. ✖

Q-5) If A is a 3×3 real matrix and $A^2 + I = 0$ then

- [A] 1, i , $-i$ are roots of $\chi_A(x)$.
[B] There is no such matrix.
[C] $-1, i, -i$ are roots of $\chi_A(x)$.
[D] i and $-i$ are the only roots of $\chi_A(x)$.

Soln.: Since $\chi_A(x)$ has degree 3, it has a real root r . As $A_I^2 = 0$, we get $r^2 = -1$. This is a contradiction. Therefore no such matrix exists. ✖

Q-6) Define $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ by $T(u) = Au$ where $A = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 4 & 7 \\ -2 & 2 & 0 \end{bmatrix}$. Then

- [A] $\text{Null}(T)$ is a plane.
[B] $\text{Null}(T)$ is a line.
[C] $\text{Im}(T)$ is a line.
[D] $\text{Null}(T) = \{0\}$.

Soln.: Note that $\det A = 0$ and $\begin{vmatrix} 1 & 3 \\ 3 & 4 \end{vmatrix} \neq 0$. Therefore, $\text{rank}(A) = 2$ and so $\text{nullity}(A) = 1$. ✖



Q-7) The conic described by the $5x^2 - 4xy + 8y^2 = 36$ is

- A a pair of lines. B an ellipse. C a hyperbola. D a parabola.

Soln.: The equation is

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 5 & -2 \\ -2 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 36.$$

Let $A = \begin{bmatrix} 5 & -2 \\ -2 & 8 \end{bmatrix}$. Then $\chi_A(x) = \det(xI - A) = (x - 4)(x - 9)$. As the eigenvalues are 4 and 9, the equation in u, r plane is given by

$$4u^2 + 9r^2 = 36 \implies \frac{u^2}{9} + \frac{r^2}{4} = 1$$

This is as an ellipse. ✖

Q-8) Consider positive real numbers a, b, c and matrix $A = \begin{bmatrix} a & b & c \\ a & b & c \\ a & b & c \end{bmatrix}$. Let $\lambda = a + b + c$. Then

- A $\dim E_0 = 1$, $\dim E_\lambda = 2$. C $\dim E_0 = 2$, $\dim E_\lambda = 2$.
 B $\dim E_0 = 2$, $\dim E_\lambda = 1$. D $\dim E_0 = 1$, $\dim E_\lambda = 1$.

Soln.: We have $\text{rank}(A) = 2$ and $\text{nullity}(A) = 2$. So, 0 is an eigenvalue of A with $\dim(E_0) = 2$. Also, $A[1 1 1]^t = (a + b + c)[1 1 1]^t$. So, $\lambda = a + b + c$ is an eigenvalue of A . Since $\dim(E_0) + \dim(E_\lambda) = 3$, we get $\dim(E_\lambda) = 1$. ✖

Q-9) The number of 2×2 nilpotent real matrices is

- A infinite B 1 C 3 D 2

Soln.: For every $n \in \mathbb{Z}$ we have that $\begin{bmatrix} 0 & n \\ 0 & 0 \end{bmatrix}$ is nilpotent. ✖

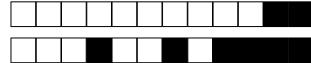
Q-10) Let V be the vector space over \mathbb{R} of 2×2 Hermitian matrices. Then dimension of V is

- A 1 B 4 C 3 D 2

Soln.: Any 2×2 Hermitian matrix A has the form $A = \begin{bmatrix} x & y + iz \\ y - iz & w \end{bmatrix}$ for some $x, y, z, w \in \mathbb{R}$. So,

$$A = x \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + y \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + z \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + w \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

This shows that $\dim(V) = 4$. ✖



Q-11) Let $V = \mathbb{R}^{2 \times 2}$ and $P \in V$ be a nonzero matrix. Consider the linear transformation $T : V \rightarrow V$ given by $T(A) = AP - PA$. Then

[A] T is 1-1.

[C] T is onto.

[B] None of these answers are correct.

[D] T is an isomorphism.

Soln.: Clearly $T(P) = 0$. Hence, T is not 1-1. Thus, T is not onto either. Hence, none of these answers are correct. \ddagger

Q-12) Let A be a 3×3 real matrix with $\text{tr}(A) = 0$. Let $\chi_A(2) = \chi_A(3) = 0$. Then

[A] $\chi_A(x) = (x - 2)(x - 3)^2$

[C] $\chi_A(x) = (x - 2)(x - 3)$

[B] $\chi_A(x) = (x - 2)^2(x - 3)$

[D] $\chi_A(x) = (x + 5)(x - 2)(x - 3)$

Soln.: Since 2, 3 are roots of the $\chi_A(x)$, and since the sum of all the roots of $\chi_A(x) = \text{tr}(A) = 0$, we have that -5 is a root of $\chi_A(x)$. \ddagger

Q-13) Let $P_2(\mathbb{R})$ be the space of polynomials over \mathbb{R} of degree ≤ 2 . Define $T : P_2(\mathbb{R}) \rightarrow \mathbb{R}^3$ by $T(f(x)) = (f(0), f(-1), f(1))^t$. Then $\text{rank}(T)$ is

[A] 1

[B] 2

[C] 4

[D] 3

Soln.: Let $f(x) = a + bx + cx^2$, $a, b, c \in \mathbb{R}$. Then we have $f(-1) = a - b + c$, $f(0) = a$, $f(1) = a + b + c$. If $T(f) = 0$, then $a = b = c = 0$. This shows that f is one-one. Hence, T is an isomorphism. So, $\text{rank}(T) = 3$. \ddagger

Q-14) Let $V = \mathbb{R}^{3 \times 3}$ and $\langle A, B \rangle = \text{tr}(AB^t)$. Let D denote the subspace of diagonal matrices in V . Then $\dim D^\perp$ is

[A] 6

[B] 9

[C] 5

[D] 3

Soln.: Since $\dim(D) = 3$ and $\dim(D) + \dim(V^\perp) = \dim(V) = 9$, we get that $\dim(V^\perp) = 6$. \ddagger



MA 106 Part A : Optical Mark Recognition (OMR) Sheet

Name:

Division.: D []

Stud. Sign.:

Roll No.:

Tutorial: T []

Invig. Sign.:

Bubble Your Rollnumber

0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9

 B B D

C

INSTRUCTIONS: **⊕** Use only blue/black colored ball-point pen to make a bubble.

⊕ The right way to fill a box like is as .

⊕ The OMR sheet will be evaluated by using computer, so **no writing is allowed anywhere in the OMR sheet except dedicated places.**

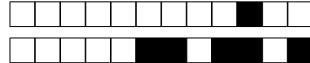
⊕ **No rewriting, no overwriting, no erasing, and no cancellation** are allowed.

⊕ Before bubbling your rollnumber, first convert the rollnmbre into nine characters by appending appropriate leading zero(s) at the beginning and bubble that nine characters rollnumber so that in the *i*th column you bubble the *i*th character only. For example, if your rollnumber is 22B1234 then first convert it into 0022B2134 and then bubble it accordingly.

⊕ Each question has single correct answer, so bubbling more than one option in a single question will fetch zero mark.

Bubble the box corresponds to your correct choice.

Q-1) A B C DQ-8) A B C DQ-2) A B C DQ-9) A B C DQ-3) A B C DQ-10) A B C DQ-4) A B C DQ-11) A B C DQ-5) A B C DQ-12) A B C DQ-6) A B C DQ-13) A B C DQ-7) A B C DQ-14) A B C D



MA 106 Endsem Exam : Part A : Question Paper
Indian Institute of Technology Bombay

D

Roll No.:
Name:

Division.: D

Tutorial: T

Apr. 19, 2023

8.30 - 10.30 AM

9.30 AM (Part A)

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

- ⊕ There are 14 questions in Part A. Each question contains a **single correct answer** of one mark.
- ⊕ You need to indicate your answer by bubbling the box in the OMR (Optical Mark Recognition) sheet. The right way to fill a box like is as . Use only a ball-point pen to fill the correct choice in the OMR sheet.
- ⊕ **Answer provided in the OMR sheet will only be evaluated.** You can mark answer in the question paper for your reference but it will not be evaluated.
- ⊕ The OMR sheet will be collected back at 9.30 AM.
- ⊕ **Notation:** $\mathbb{R}^{m \times n}$: the set of all $m \times n$ real matrices, A^t : the transpose of A , E_λ : the eigenspace for an eigenvalue λ , $\chi_A(x)$: the characteristic polynomial of A , $\text{Null}(A)$: the nullspace of A , $\mathcal{C}(A)$: the column space of A .

Q-1) The number of 2×2 nilpotent real matrices is

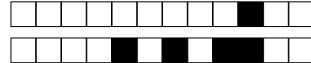
- A infinite B 3 C 2 D 1

Soln.: For every $n \in \mathbb{Z}$ we have that $\begin{bmatrix} 0 & n \\ 0 & 0 \end{bmatrix}$ is nilpotent. ✖

Q-2) Let $P_2(\mathbb{R})$ be the space of polynomials over \mathbb{R} of degree ≤ 2 . Define $T : P_2(\mathbb{R}) \rightarrow \mathbb{R}^3$ by $T(f(x)) = (f(0), f(-1), f(1))^t$. Then $\text{rank}(T)$ is

- A 4 B 3 C 2 D 1

Soln.: Let $f(x) = a + bx + cx^2$, $a, b, c \in \mathbb{R}$. Then we have $f(-1) = a - b + c$, $f(0) = a$, $f(1) = a + b + c$. If $T(f) = 0$, then $a = b = c = 0$. This shows that f is one-one. Hence, T is an isomorphism. So, $\text{rank}(T) = 3$. ✖



Q-3) The rank of the linear transformation $T : \mathbb{R}^{2 \times 2} \rightarrow \mathbb{R}^{2 \times 2}$ given by $T(A) = PA$ where $A \in \mathbb{R}^{2 \times 2}$ and $P = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$ is

- A 1 B 4 C 3 D 2

Soln.: Clearly, $\text{Null}(T) = \{A \mid PA = 0\}$. It follows that

$$\begin{bmatrix} x & y \\ a & b \end{bmatrix} \in \text{Null}(T) \Leftrightarrow \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} x & y \\ a & b \end{bmatrix} = 0 \Leftrightarrow \begin{bmatrix} x + 3a & 0 \\ y + 3b & 0 \end{bmatrix} \Leftrightarrow x = -3a, \quad y = -3b.$$

Therefore,

$$\begin{bmatrix} x & y \\ a & b \end{bmatrix} = \begin{bmatrix} -3a & -3b \\ a & b \end{bmatrix} = a \begin{bmatrix} -3 & 0 \\ 1 & 0 \end{bmatrix} + b \begin{bmatrix} 0 & -3 \\ 0 & 1 \end{bmatrix}.$$

Hence, $\dim \text{Null}(T) = 2$ and so $\text{rank}(T) = 2$. ✖

Q-4) Define $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ by $T(u) = Au$ where $A = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 4 & 7 \\ -2 & 2 & 0 \end{bmatrix}$. Then

- | | |
|---|---|
| <input type="checkbox"/> A $\text{Null}(T) = \{0\}$. | <input type="checkbox"/> C $\text{Null}(T)$ is a plane. |
| <input type="checkbox"/> B $\text{Im}(T)$ is a line. | <input checked="" type="checkbox"/> D $\text{Null}(T)$ is a line. |

Soln.: Note that $\det A = 0$ and $\begin{vmatrix} 1 & 3 \\ 3 & 4 \end{vmatrix} \neq 0$. Therefore, $\text{rank}(A) = 2$ and so $\text{nullity}(A) = 1$. ✖

Q-5) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbb{R}^{2 \times 2}$ and $(a-d)^2 + 4bc > 0$. Then

- | | |
|---|--|
| <input type="checkbox"/> A None of these answers are correct. | <input type="checkbox"/> C A is not diagonalizable over \mathbb{R} . |
| <input type="checkbox"/> B Eigenvalues of A are not real. | <input checked="" type="checkbox"/> D There is a basis of \mathbb{R}^2 consisting of eigenvectors of A . |

Soln.: Consider $\chi_A(x) = \begin{vmatrix} x-a & -b \\ -c & x-d \end{vmatrix} = x^2 - (a+d)x + (ad - bc)$.

Roots of $\chi_A(x)$ are

$$\lambda_1 = \frac{1}{2} \left[(a+d) + \sqrt{(a+d)^2 - 4(ad - bc)} \right] = \frac{1}{2} \left[(a+d) + \sqrt{(a-d)^2 + 4bc} \right],$$

$$\lambda_2 = \frac{1}{2} \left[(a+d) - \sqrt{(a+d)^2 - 4(ad - bc)} \right] = \frac{1}{2} \left[(a+d) - \sqrt{(a-d)^2 + 4bc} \right].$$

From the given condition we get $\lambda_1 \neq \lambda_2$. Thus, A is diagonalizable as the eigenvectors corresponding to λ_1 and λ_2 are linearly independent. ✖



Q-6) Let A be a 3×3 real matrix with $\text{tr}(A) = 0$. Let $\chi_A(2) = \chi_A(3) = 0$. Then

- [A] $\chi_A(x) = (x - 2)(x - 3)^2$
[B] $\chi_A(x) = (x + 5)(x - 2)(x - 3)$

- [C] $\chi_A(x) = (x - 2)^2(x - 3)$
[D] $\chi_A(x) = (x - 2)(x - 3)$

Soln.: Since 2, 3 are roots of the $\chi_A(x)$, and since the sum of all the roots of $\chi_A(x) = \text{tr}(A) = 0$, we have that -5 is a root of $\chi_A(x)$. \clubsuit

Q-7) Consider $V = \mathbb{R}^{2 \times 2}$ and the inner product $\langle A, B \rangle = \text{tr}(A^t B)$. Let $J = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $p_I(J)$ denote the orthogonal projection of J along the identity matrix I . Then $p_I(J)$ is

- [A] I [B] 0 [C] $2I$ [D] J

Soln.: We have $p_I(J) = \frac{\langle I, J \rangle}{\langle I, I \rangle} I = \frac{\text{tr}(J)}{2} I = I$. \clubsuit

Q-8) Let $V = \mathbb{R}^{2 \times 2}$ and $P \in V$ be a nonzero matrix. Consider the linear transformation $T : V \rightarrow V$ given by $T(A) = AP - PA$. Then

- [A] T is onto.
[B] T is 1-1.

- [C] None of these answers are correct.
[D] T is an isomorphism.

Soln.: Clearly $T(P) = 0$. Hence, T is not 1-1. Thus, T is not onto either. Hence, none of these answers are correct. \clubsuit

Q-9) Let $V = \mathbb{R}^{3 \times 3}$ and $\langle A, B \rangle = \text{tr}(AB^t)$. Let D denote the subspace of diagonal matrices in V . Then $\dim D^\perp$ is

- [A] 5 [B] 9 [C] 3 [D] 6

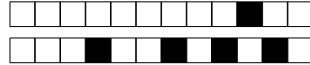
Soln.: Since $\dim(D) = 3$ and $\dim(D) + \dim(V^\perp) = \dim(V) = 9$, we get that $\dim(V^\perp) = 6$. \clubsuit

Q-10) Consider positive real numbers a, b, c and matrix $A = \begin{bmatrix} a & b & c \\ a & b & c \\ a & b & c \end{bmatrix}$. Let $\lambda = a + b + c$. Then

- [A] $\dim E_0 = 1$, $\dim E_\lambda = 2$.
[B] $\dim E_0 = 1$, $\dim E_\lambda = 1$.

- [C] $\dim E_0 = 2$, $\dim E_\lambda = 2$.
[D] $\dim E_0 = 2$, $\dim E_\lambda = 1$.

Soln.: We have $\text{rank}(A) = 2$ and $\text{nullity}(A) = 2$. So, 0 is an eigenvalue of A with $\dim(E_0) = 2$. Also, $A[1 \ 1 \ 1]^t = (a + b + c)[1 \ 1 \ 1]^t$. So, $\lambda = a + b + c$ is an eigenvalue of A . Since $\dim(E_0) + \dim(E_\lambda) = 3$, we get $\dim(E_\lambda) = 1$. \clubsuit



Q-11) If A is a 3×3 real matrix and $A^2 + I = 0$ then

- | | | | |
|----------------------------|--|----------------------------|--|
| <input type="checkbox"/> A | -1, i , $-i$ are roots of $\chi_A(x)$. | <input type="checkbox"/> C | 1, i , $-i$ are roots of $\chi_A(x)$. |
| <input type="checkbox"/> B | i and $-i$ are the only roots of $\chi_A(x)$. | <input type="checkbox"/> D | There is no such matrix. |

Soln.: Since $\chi_A(x)$ has degree 3, it has a real root r . As $A_I^2 = 0$, we get $r^2 = -1$. This is a contradiction. Therefore no such matrix exists. \ddagger

Q-12) Let V be the vector space over \mathbb{R} of 2×2 Hermitian matrices. Then dimension of V is

- | | | | | | | | |
|----------------------------|---|----------------------------|---|----------------------------|---|----------------------------|---|
| <input type="checkbox"/> A | 4 | <input type="checkbox"/> B | 3 | <input type="checkbox"/> C | 2 | <input type="checkbox"/> D | 1 |
|----------------------------|---|----------------------------|---|----------------------------|---|----------------------------|---|

Soln.: Any 2×2 Hermitian matrix A has the form $A = \begin{bmatrix} x & y+iz \\ y-iz & w \end{bmatrix}$ for some $x, y, z, w \in \mathbb{R}$.

So,

$$A = x \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + y \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + z \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + w \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

This shows that $\dim(V) = 4$. \ddagger

Q-13) Let A be a 3×3 real matrix and $A^2 = A$. Then $\dim(\text{Null}(A) \cap \mathcal{C}(A))$ is

- | | | | | | | | |
|----------------------------|---|----------------------------|---|----------------------------|---|----------------------------|---|
| <input type="checkbox"/> A | 1 | <input type="checkbox"/> B | 3 | <input type="checkbox"/> C | 2 | <input type="checkbox"/> D | 0 |
|----------------------------|---|----------------------------|---|----------------------------|---|----------------------------|---|

Soln.: If $v = Ax \in \text{null}(A) \cap \mathcal{C}(A)$, for some $x \in \mathbb{R}^3$, then $0 = Av = A^2x = Ax = v$. \ddagger

Q-14) The conic described by the $5x^2 - 4xy + 8y^2 = 36$ is

- | | | | | | | | |
|----------------------------|------------------|----------------------------|--------------|----------------------------|-------------|----------------------------|-------------|
| <input type="checkbox"/> A | a pair of lines. | <input type="checkbox"/> B | a hyperbola. | <input type="checkbox"/> C | an ellipse. | <input type="checkbox"/> D | a parabola. |
|----------------------------|------------------|----------------------------|--------------|----------------------------|-------------|----------------------------|-------------|

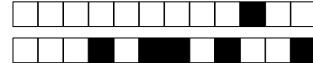
Soln.: The equation is

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 5 & -2 \\ -2 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 36.$$

Let $A = \begin{bmatrix} 5 & -2 \\ -2 & 8 \end{bmatrix}$. Then $\chi_A(x) = \det(xI - A) = (x - 4)(x - 9)$. As the eigenvalues are 4 and 9, the equation in u, r plane is given by

$$4u^2 + 9r^2 = 36 \implies \frac{u^2}{9} + \frac{v^2}{1} = 1$$

This is as an ellipse. \ddagger



MA 106 Part A : Optical Mark Recognition (OMR) Sheet

Name:

Division.: D []

Stud. Sign.:

Roll No.:

Tutorial: T []

Invig. Sign.:

Bubble Your Rollnumber

0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9

B B

D

D

- INSTRUCTIONS:
- ⊕ Use only blue/black colored ball-point pen to make a bubble.
 - ⊕ The right way to fill a box like is as .
 - ⊕ The OMR sheet will be evaluated by using computer, so **no writing is allowed anywhere in the OMR sheet except dedicated places.**
 - ⊕ **No rewriting, no overwriting, no erasing, and no cancellation** are allowed.
 - ⊕ Before bubbling your rollnumber, first convert the rollnmbre into nine characters by appending appropriate leading zero(s) at the beginning and bubble that nine characters rollnumber so that in the i th column you bubble the i th character only. For example, if your rollnumber is 22B1234 then first convert it into 0022B2134 and then bubble it accordingly.
 - ⊕ Each question has single correct answer, so bubbling more than one option in a single question will fetch zero mark.

Bubble the box corresponds to your correct choice.

- Q-1) A B C D
- Q-2) A B C D
- Q-3) A B C D
- Q-4) A B C D
- Q-5) A B C D
- Q-6) A B C D
- Q-7) A B C D

- Q-8) A B C D
- Q-9) A B C D
- Q-10) A B C D
- Q-11) A B C D
- Q-12) A B C D
- Q-13) A B C D
- Q-14) A B C D