

Assignment 2 ICSE class 12 2017

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Q1 (v) Evaluate:

$$\int \frac{1}{x^2} \sin^2 \left(\frac{1}{x} \right) dx$$

Solution Let

$$\frac{1}{x} = t \quad (0.1)$$

$$\implies -\frac{1}{x^2} dx = dt$$

$$\implies \frac{1}{x^2} dx = -dt \quad (0.2)$$

From (0.1) and (0.2)

$$\int \frac{1}{x^2} \sin^2 \left(\frac{1}{x} \right) dx = - \int \sin^2 (t) dt \quad (0.3)$$

$$\cos (2t) = 1 - 2 \sin^2 (t)$$

$$\implies \sin^2 (t) = \frac{1 - \cos (2t)}{2} \quad (0.4)$$

From (0.4)

$$- \int \sin^2 (t) dt = -\frac{1}{2} \int 1 - \cos (2t) dt \quad (0.5)$$

$$\int 1 - \cos (2t) dt = t - \frac{\sin (2t)}{2} + C$$

$$\implies -\frac{1}{2} \int 1 - \cos (2t) dt = \frac{\sin (2t)}{4} - \frac{t}{2} + C \quad (0.6)$$

from (0.3),(0.5) and (0.6)

$$\int \frac{1}{x^2} \sin^2 \left(\frac{1}{x} \right) dx = \frac{\sin (2t)}{4} - \frac{t}{2} + C$$