

Assignment 2 ICSE class 12 2017

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Q1 (v) Evaluate: $\int \frac{1}{x^2} \sin^2\left(\frac{1}{x}\right) dx$

Solution Let

$$\frac{1}{x} = t \quad (0.1)$$

$$-\frac{1}{x^2} dx = dt \quad (0.2)$$

$$\frac{1}{x^2} dx = -dt \quad (0.3)$$

From (0.1) and (0.3)

$$\int \frac{1}{x^2} \sin^2\left(\frac{1}{x}\right) dx = - \int \sin^2(t) dt \quad (0.4)$$

$$\cos(2t) = 1 - 2 \sin^2(t) \quad (0.5)$$

$$\sin^2(t) = \frac{1 - \cos(2t)}{2} \quad (0.6)$$

From (0.6)

$$- \int \sin^2(t) dt = -\frac{1}{2} \int 1 - \cos(2t) dt \quad (0.7)$$

$$\int 1 - \cos(2t) dt = t - \frac{\sin(2t)}{2} + C \quad (0.8)$$

from (0.8)

$$-\frac{1}{2} \int 1 - \cos(2t) dt = \frac{\sin(2t)}{4} - \frac{t}{2} + C \quad (0.9)$$

$$\therefore \int \frac{1}{x^2} \sin^2\left(\frac{1}{x}\right) dx = \frac{\sin(2t)}{4} - \frac{t}{2} + C \quad (0.10)$$