Assignment 2 ICSE class 12 2017

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Download all python codes from

https://github.com/GunjitMittal/Assignment2/tree/main/Assignment2/codes

Download all latex codes from

https://github.com/GunjitMittal/Assignment2/tree/main/Assignment2

1 Question

Evaluate:

$$\int \frac{1}{x^2}\sin^2\left(\frac{1}{x}\right)dx$$

2 Solution

Let

$$\frac{1}{x} = t \tag{2.1}$$

$$\implies -\frac{1}{x^2}dx = dt \tag{2.2}$$

$$\implies \frac{1}{x^2}dx = -dt \tag{2.3}$$

From (2.1) and (2.3)

$$\int \frac{1}{x^2}\sin^2\left(\frac{1}{x}\right)dx = -\int \sin^2(t)\,dt \tag{2.4}$$

$$\cos(2t) = 1 - 2\sin^2(t) \tag{2.5}$$

$$\implies \sin^2(t) = \frac{1-\cos(2t)}{2} \tag{2.6}$$

From (2.6)

$$-\int \sin^2(t)\,dt = -\frac{1}{2}\int 1-\cos(2t)\,dt \tag{2.7}$$

$$\int 1-\cos(2t)\,dt = t - \frac{\sin(2t)}{2} + C \tag{2.8}$$

$$\implies -\frac{1}{2}\int 1-\cos(2t)\,dt = \frac{\sin(2t)}{4} - \frac{t}{2} + C \tag{2.9}$$

from (2.4),(2.7) and (2.9)

$$\int \frac{1}{x^2}\sin^2\left(\frac{1}{x}\right)dx = \frac{\sin(2t)}{4} - \frac{t}{2} + C \tag{2.10}$$