Assignment 6 12th Class

Gunjit Mittal (AI21BTECH11011)

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Outline

Question

Let a pair of dice be thrown and the random variable X be the sum of the numbers that appear on the two dice. Find the mean or expectation of X.

Solution

The sample space of the experiment consists of 36 elementary events in the form of ordered pairs (x_i, y_i), where x_i= 1, 2, 3, 4, 5, 6 and y_i = 1, 2, 3, 4, 5, 6.
The random variable X i.e. the sum of the numbers on the two dice takes the values 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 or 12.
Now

$$\Pr(X = 2) = \Pr((1,1)) = \frac{1}{36} \tag{2.1}$$

$$\Pr(X = 3) = \Pr((1,2),(2,1)) = \frac{2}{36} \tag{2.2}$$

$$\Pr(X = 4) = \Pr((1,3),(2,2),(3,1)) = \frac{3}{36} \tag{2.3}$$

$$\Pr(X = 5) = \Pr((1,4),(2,3),(3,2),(4,1)) = \frac{4}{36} \tag{2.4}$$

(2.5)

$$\Pr(X=6) = \Pr((1,5),(2,4),(3,3),(4,2),(5,1)) = \frac{5}{36} \tag{2.6}$$

$$\Pr(X=7) = \Pr((1,6),(2,5),(3,4),(4,3),(5,2),(6,1)) = \frac{6}{36} \tag{2.7}$$

$$\Pr(X=8) = \Pr((2,6),(3,5),(4,4),(5,3),(6,2)) = \frac{5}{36} \tag{2.8}$$

$$\Pr(X=9) = \Pr((3,6),(4,5),(5,4),(6,3)) = \frac{4}{36} \tag{2.9}$$

$$\Pr(X=10) = \Pr((4,6),(5,5),(6,4)) = \frac{3}{36} \tag{2.10}$$

$$\Pr(X=11) = \Pr((5,6),(6,5)) = \frac{2}{36} \tag{2.11}$$

$$\Pr(X = 12) = \Pr((6,6)) = \frac{1}{36} \tag{2.13}$$

The probability distribution of X is

X or x_i	2	3	4	5	6	7	8	9	10	11	12
$\Pr(X)$ or p_i	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Therefore,

$$\mu = E(x) = \sum_{i=1}^{n} x_i p_i = 2 \times \frac{1}{36} + 3 \times \frac{2}{36} + 4 \times \frac{3}{36} +$$
$$5 \times \frac{4}{36} + 6 \times \frac{5}{36} + 7 \times \frac{6}{36} + 8 \times \frac{5}{36} + 9 \times \frac{4}{36} +$$
$$10 \times \frac{3}{36} + 11 \times \frac{2}{36} + 12 \times \frac{1}{36} \tag{2.14}$$

(2.15)

$$= \frac{2+6+12+20+30+42+40+36+30+22+12}{36} \tag{2.16}$$

$$= 7 \tag{2.17}$$

Thus, the mean of the sum of the numbers that appear on throwing two fair dice is 7.