

Random Numbers



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Abstract—This manual provides a simple introduction to the generation of random numbers

1 Uniform Random Numbers

Let U be a uniform random variable between 0 and 1.

1.1 Generate 10^6 samples of U using a C program and save into a file called uni.dat .

Solution: Execute the following C program.

wget https://github.com/gadepall/probability/ raw/master/manual/codes/exrand.c wget https://github.com/gadepall/probability/ raw/master/manual/codes/coeffs.h

1.2 Load the uni.dat file into python and plot the empirical CDF of *U* using the samples in uni.dat. The CDF is defined as

$$F_U(x) = \Pr\left(U \le x\right) \tag{1.1}$$

Solution: The following code plots Fig. 1.2

wget https://github.com/gadepall/probability/ raw/master/manual/codes/cdf plot.py

- 1.3 Find a theoretical expression for $F_U(x)$.
- 1.4 The mean of U is defined as

$$E[U] = \frac{1}{N} \sum_{i=1}^{N} U_i$$
 (1.2)

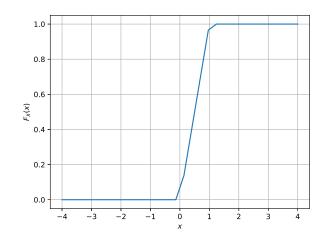


Fig. 1.2: The CDF of U

and its variance as

$$var[U] = E[U - E[U]]^2$$
 (1.3)

Write a C program to find the mean and variance of U.

1.5 Verify your result theoretically given that

$$E\left[U^{k}\right] = \int_{-\infty}^{\infty} x^{k} dF_{U}(x) \tag{1.4}$$

2 Central Limit Theorem

2.1 Generate 10⁶ samples of the random variable

$$X = \sum_{i=1}^{12} U_i - 6 \tag{2.1}$$

using a C program, where U_i , i = 1, 2, ..., 12 are a set of independent uniform random variables between 0 and 1 and save in a file called gau.dat

2.2 Load gau.dat in python and plot the empirical CDF of *X* using the samples in gau.dat. What properties does a CDF have?

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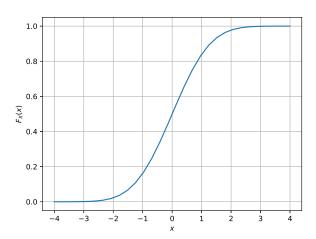


Fig. 2.2: The CDF of X

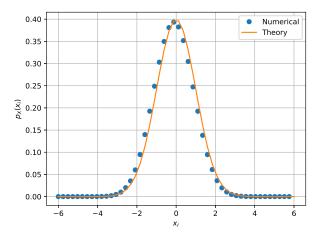


Fig. 2.3: The PDF of X

Solution: The CDF of *X* is plotted in Fig. 2.2

2.3 Load gau.dat in python and plot the empirical PDF of *X* using the samples in gau.dat. The PDF of *X* is defined as

$$p_X(x) = \frac{d}{dx} F_X(x) \tag{2.2}$$

What properties does the PDF have?

Solution: The PDF of *X* is plotted in Fig. 2.3

- 2.4 Find the mean and variance of *X* by writing a C program.
- 2.5 Given that

$$p_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right), -\infty < x < \infty, \quad (2.3)$$

repeat the above exercise theoretically.

3 From Uniform to Other

3.1 Generate samples of

$$V = -2\ln(1 - U) \tag{3.1}$$

and plot its CDF.

3.2 Find a theoretical expression for $F_V(x)$.