Random Numbers

G V V Sharma*

Contents

***Abstract*—This manual provides a simple introduction to various concepts in optimization.**

1 Uniform Random Numbers

Let U be a uniform random variable between 0 and 1.

1.1 Generate 10^6 samples of U using a C program and save into a file called uni.dat .

1.2 Load the uni.dat file into python and plot the empirical CDF of U using the samples in uni.dat.

1.3 Verify that your CDF in the above problem is correct by plotting the theoretical $F_U(x)$.

1.1 Central Limit Theorem

1.4 Generate $U_i, i = 1, 2, \dots, 12$, a set of independent uniform random variables between 0 and 1 using a C program.

1.5 Generate 10^6 samples of the random variable

$$S = \sum_{i=1}^{12} U_i - 6 \tag{1.1}$$

and save in a file called gau.dat

1.6 Load gau.dat in python and plot the empirical PDF of S using the samples in gau.dat. Does it look familiar? Comment.

* The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

1.2 From Uniform to Other

1.7 Generate samples of

$$V = -2\ln(1 - U) \tag{1.2}$$

and plot its CDF. Comment.

1.8 Generate the Rayleigh distribution from Uniform. Verify your result through graphical plots.

2 The Gaussian Distribution

2.1 Generate a Gaussian random number with 0 mean and unit variance.

Solution: Open a text editor and type the following program.

```
#!/usr/bin/env python

#This program generates a Gaussian random
    no with 0 mean and unit variance

#Importing numpy
import numpy as np

print (np.random.normal(0,1))
```

Save the file as gaussian_no.py and run the program.

2.2 The mean of a random variable X is defined as

$$E[X] = \frac{1}{N}\sum_{i=1}^{N} X_i \tag{2.1}$$

and its variance as

$$\mathrm{var}[X] = E[X - E[X]]^2 \tag{2.2}$$

Verify that the program in 2.1 actually generates a Gaussian random variable with 0 mean and unit variance.

Solution: Use the header in the previous program, type the following code and execute.

```
#This program generates a Gaussian random
    no with 0 mean and unit variance

#Importing numpy
import numpy as np


simlen = int(1e5) #No of samples

n = np.random.normal(0,1,simlen)#Random
    vector

mean = np.sum(n)/simlen #Mean value

print (mean)

var = np.sum(np.square(n − mean∗np.ones
    ((1,simlen))))/simlen

print (var)
```

2.3 Using the previous program, verify you results for different values of the mean and variance.

3 CDF and PDF

3.1 A Gaussian random variable X with mean 0 and unit variance can be expressed as $X \sim \mathcal{N}(0, 1)$. Its cumulative distribution function (CDF) is defined as

$$F_X(x) = \Pr(X < x), \tag{3.1}$$

Plot $F_X(x)$.
Solution: The following code yields Fig. 3.1.

```
#Importing numpy, scipy, mpmath and pyplot
import numpy as np
import matplotlib.pyplot as plt




x = np.linspace(−4,4,30)#points on the x axis
simlen = int(1e5) #number of samples
err = [] #declaring probability list
n = np.random.normal(0,1,simlen)

for i in range(0,30):
        err_ind = np.nonzero(n < x[i]) #
            checking probability condition
        err_n = np.size(err_ind) #
            computing the probability
        err.append(err_n/simlen) #storing
            the probability values in a list


plt.plot(x.T,err)#plotting the CDF
plt.grid() #creating the grid
plt.xlabel('$x$')
plt.ylabel('$F_X(x)$')
plt.show() #opening the plot window
```

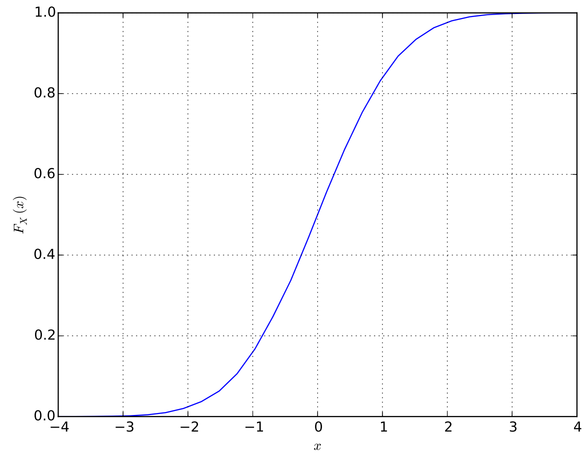

Fig. 3.1: CDF of X

3.2 List the properties of $F_X(x)$ based on Fig. 3.1.

3.3 Let

$$p_X(x_i) = \frac{F_X(x_i) - F_X(x_{i-1})}{h}, i = 1, 2, \dots h \tag{3.2}$$

for $x_i = x_{i-1} + h, x_1 = -4$. Plot $p_X(x_i)$. On the same graph, plot

$$p_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, -4 < x < 4 \tag{3.3}$$

Solution: The following code yields the graph in Fig. 3.3

```
https://github.com/gadepall/EE1390/raw/
    master/manuals/supervised/linear_class/
    codes/1.4.py
```

Thus, the PDF is the derivative of the CDF. For $X \sim \mathcal{N}(0, 1)$, the PDF is

$$p_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \quad -\infty < x < \infty \tag{3.4}$$

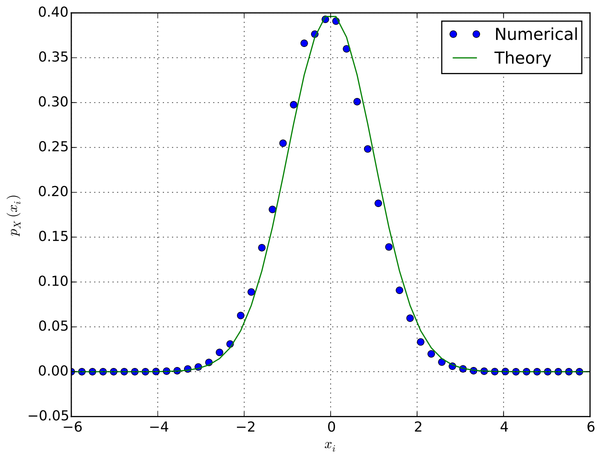

Fig. 3.3: The PDF of X

3.4 For $X \sim \mathcal{N}\left(\mu, \sigma^2\right)$,

$$p_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty \tag{3.5}$$

Plot $p_X(x)$ for different values of μ and σ in the same graph. Comment.