Simple Linear Regression

Checking relationships between 2 variables

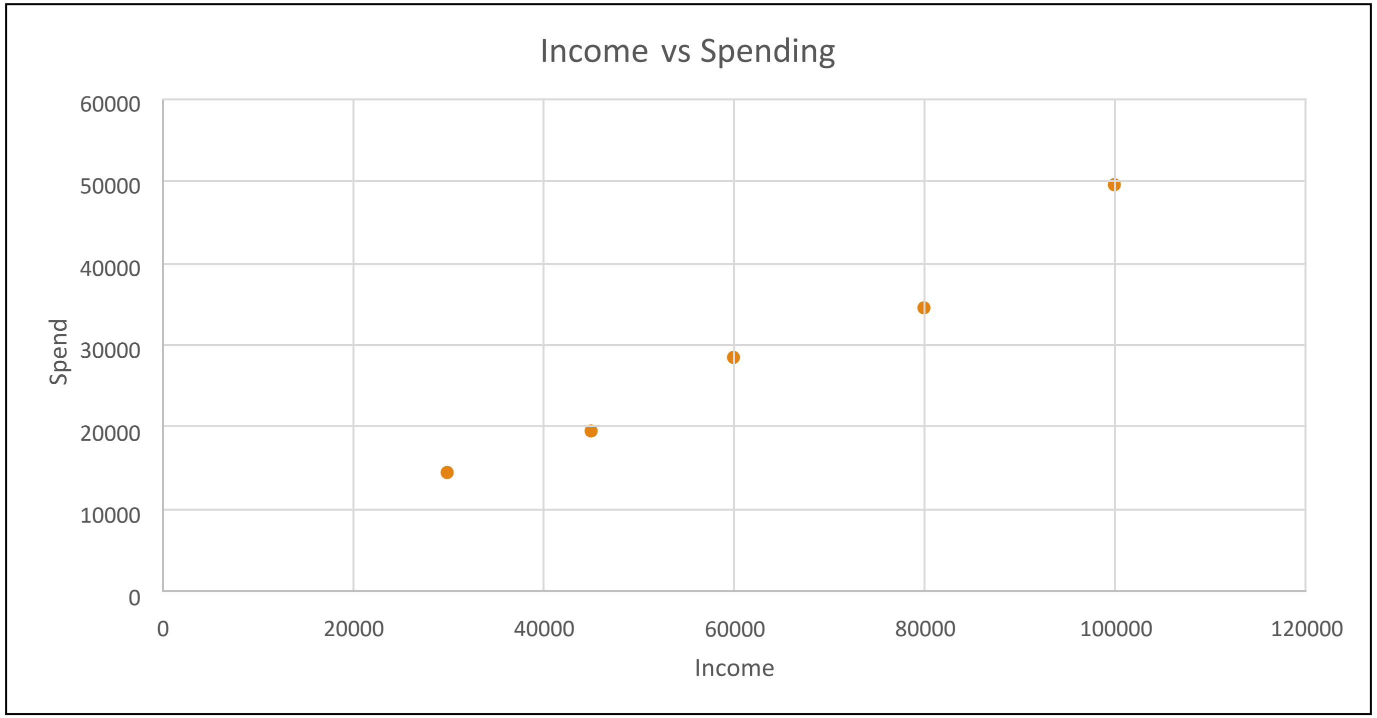
Income per month	Spending
100000	50000
80000	35000
60000	29000
45000	20000
30000	15000

Spending ~ Income

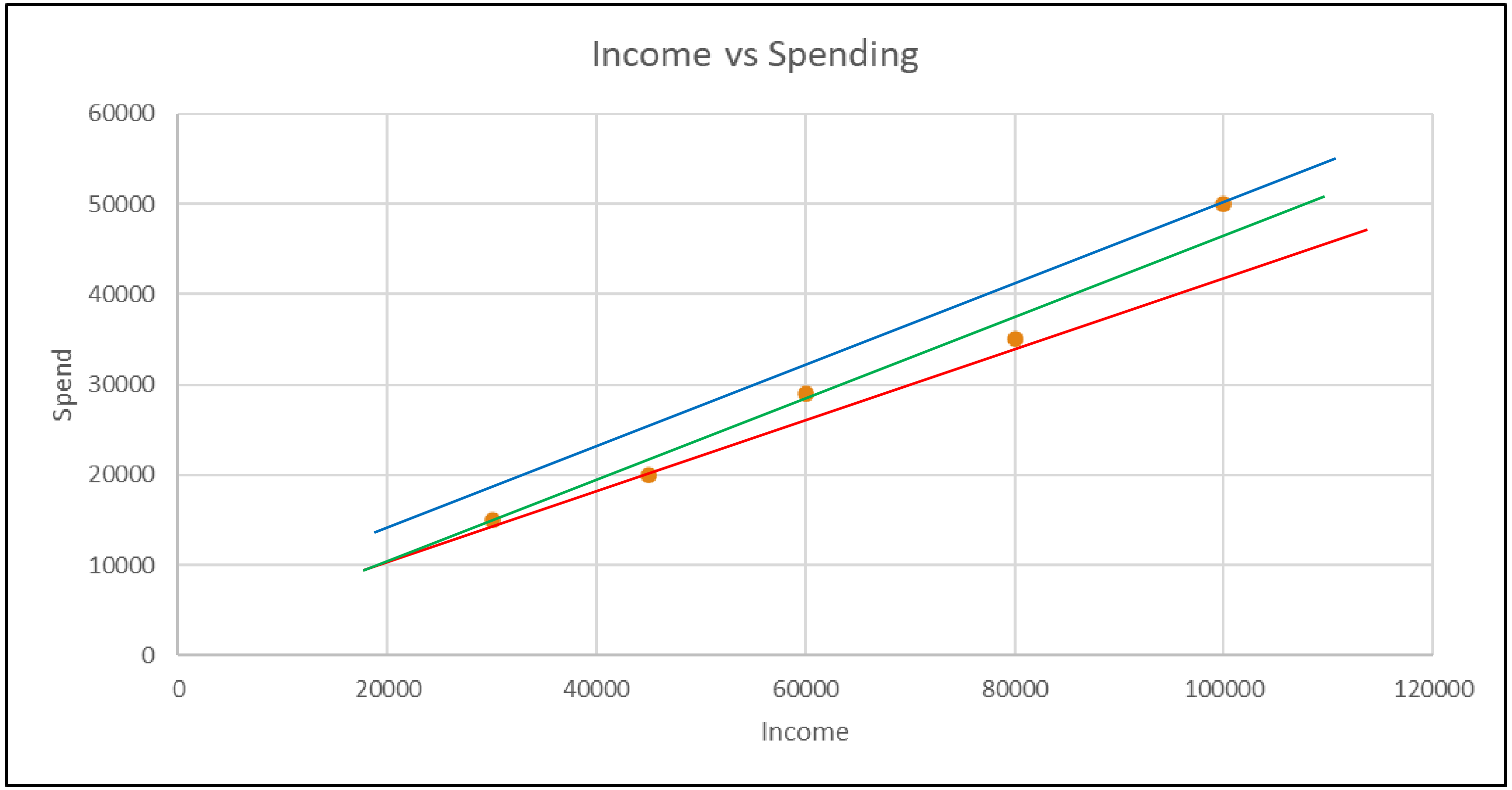
Independent Feature

Dependent Feature / Target Feature

Visualizing the Relationship between Income vs Spend

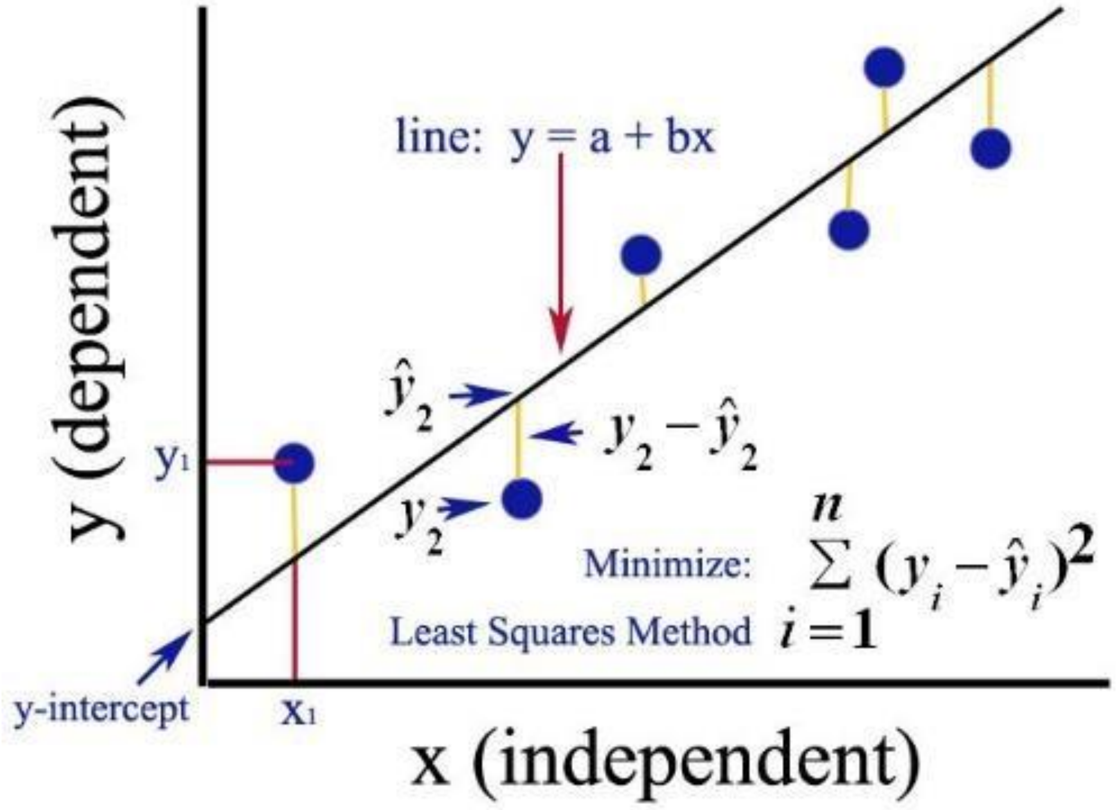


Predicting Spend based on income



Which line to fit for this problem?

Least Squared Error model



Cost function

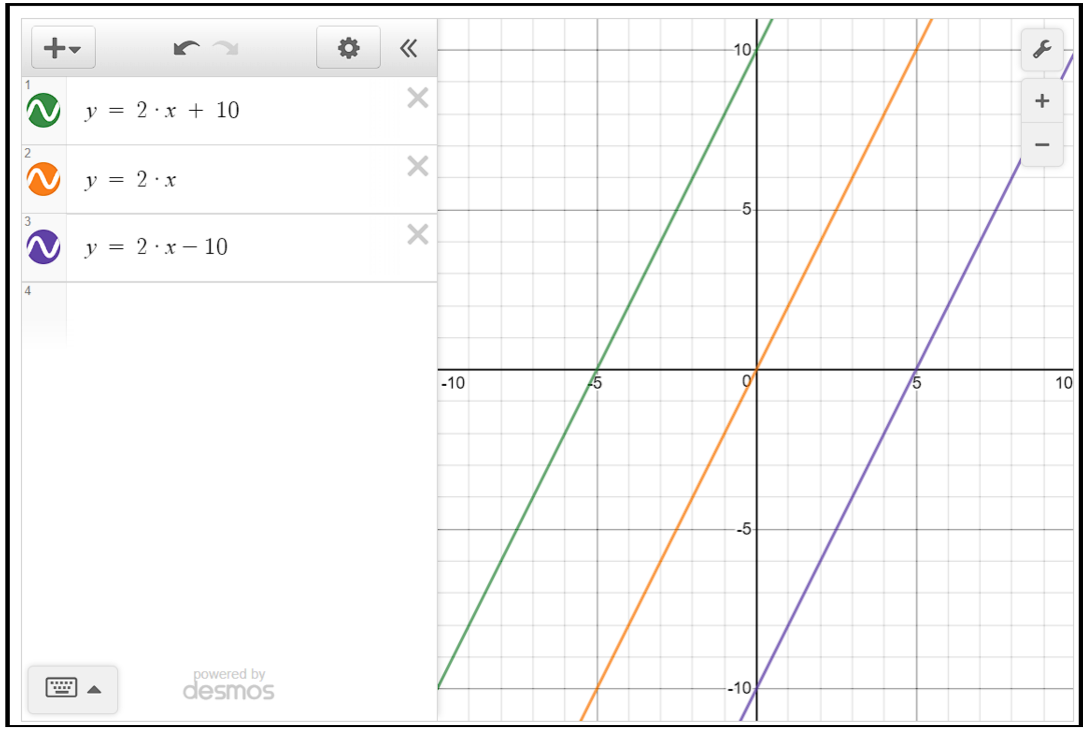
Equation Of Line

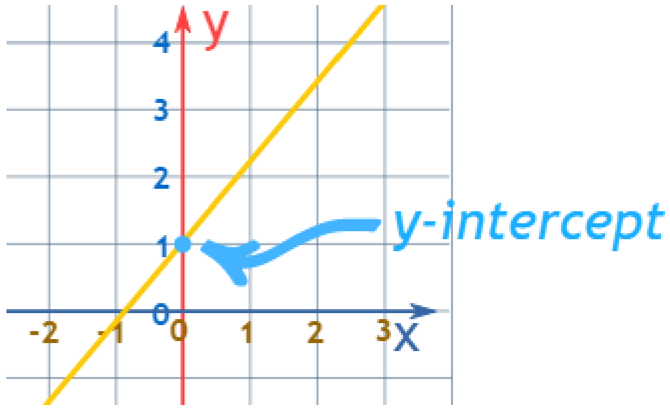
$$y = mx + c$$

y : Dependent Feature(Target)
x : Independent Feature
m : Slope of a line
c : y intercept of a line

What happens if we change Line intercept ?

At x = 0 , y = c (intercept)





What is slope?

If x increases by 1 or unit value , how much will y change?

Eg. y = 2*x + 3

x0 = 3 increases by one, x1 = 4

y0 = 2*3+3 = 9

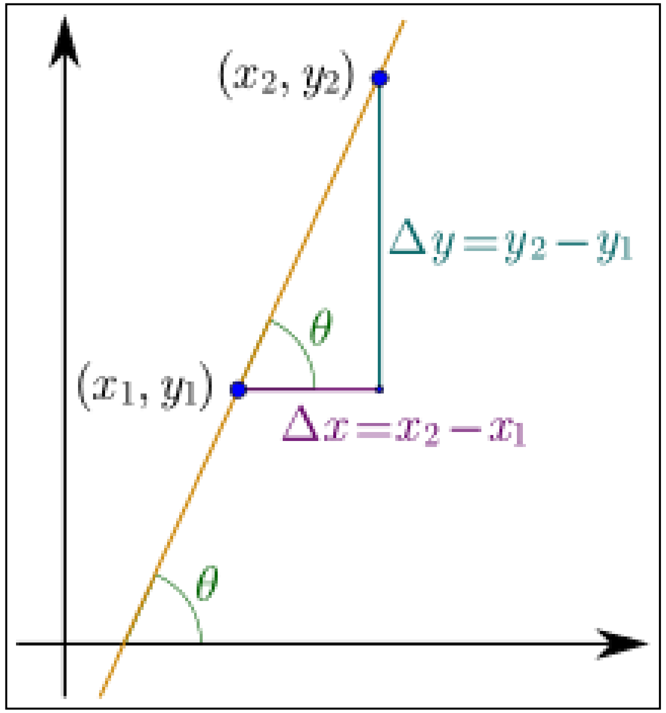
y1 = 2*4+3 = 11

y1-y0 = 11-9 = 2

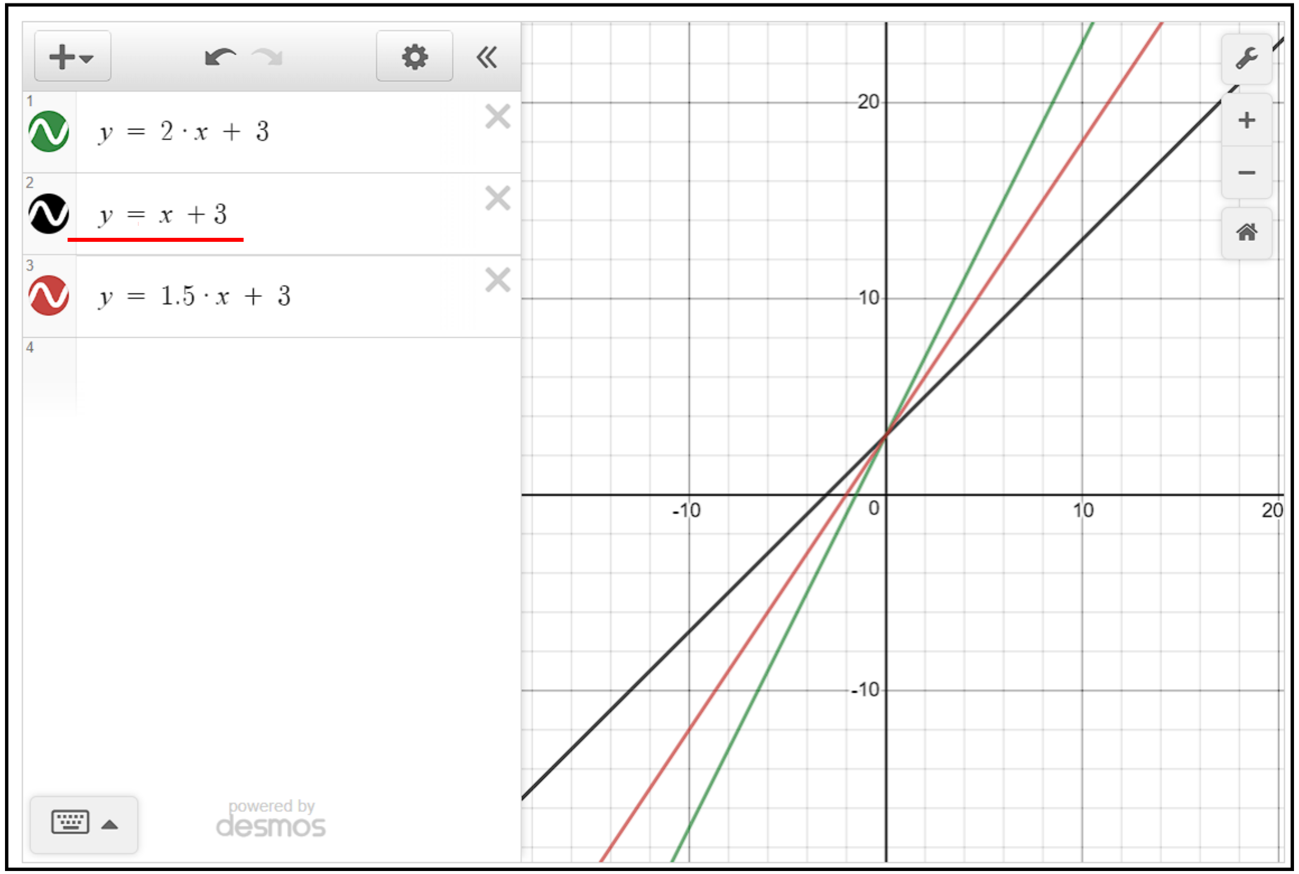
Slope = (y2-y1)/(x2-x1)

When x increased by 1 , y increased by 2

y = 2*x + 3 , Slope = 2



Lines with different slopes



Simple linear Regression Objective

- fit a line $yactual = \beta_0 + \beta_1 \cdot x + \varepsilon$
- $ypred = \beta_0 + \beta_1 \cdot x$
- Minimise the Squared error for given relationships
- Least Squares error method
- Formula for slope : $\beta_1 = \frac{cov(x,y)}{var(x)} = \frac{\Sigma(x-\bar{x})(y-\bar{y})/n}{\Sigma(x-\bar{x})^2/n}$
- Formula for Intercept : $\beta_0 = \bar{y} - \beta_1 \cdot \bar{x}$
- $\bar{x}$: ***Mean of all x values***
- $\bar{y}$: **Mean of all y values**

Solving income vs Spend problem

Income per month (x)	Spending (y)
100000	50000
80000	35000
60000	29000
45000	20000
30000	15000

$$Spending = \beta_0 + \beta_1 \cdot Income + \varepsilon$$

B0 and B1 Calculation

Income per month (x)	Spending (y)	X mean	Y mean	x-xmean	y-ymean	prod	(x-xmean)^2
100000	50000	63000	29800	37000	20200	747400000	1369000000
80000	35000	63000	29800	17000	5200	88400000	289000000
60000	29000	63000	29800	-3000	-800	2400000	9000000
45000	20000	63000	29800	-18000	-9800	176400000	324000000
30000	15000	63000	29800	-33000	-14800	488400000	1089000000

300600000

616000000

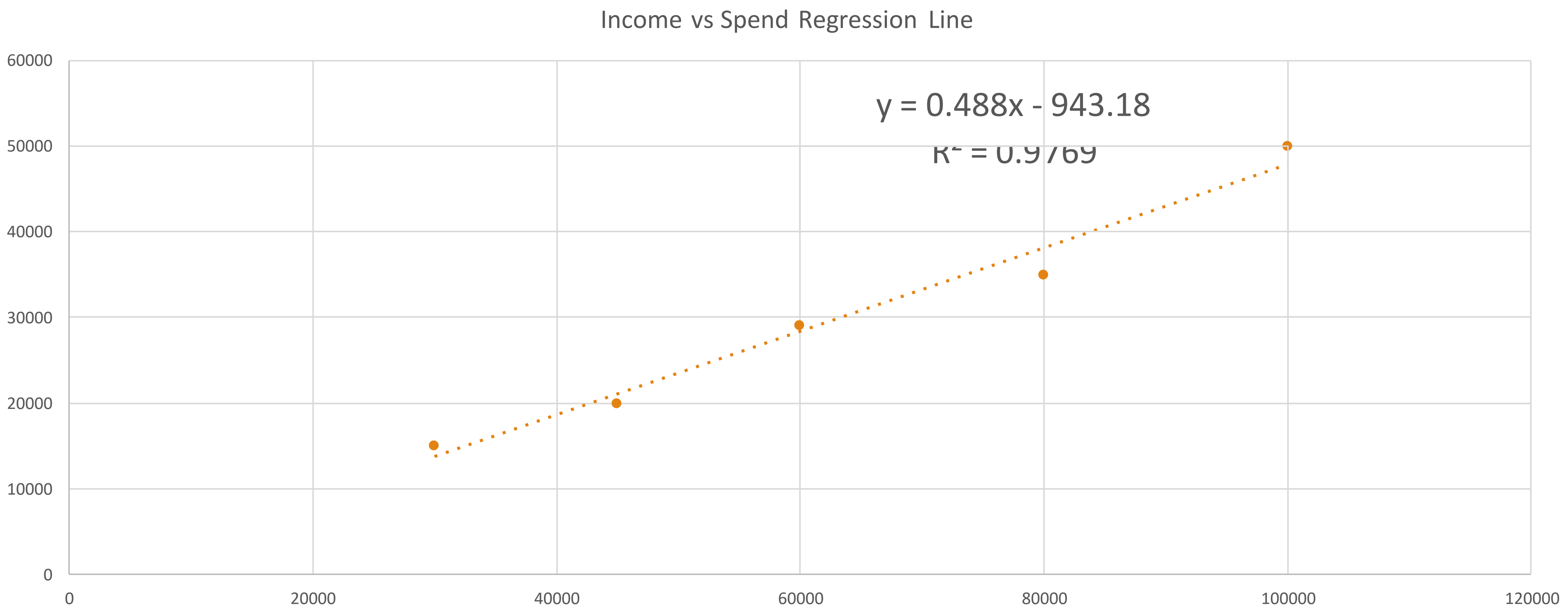
Formula for slope : $\boldsymbol{\beta_1} = \frac{cov(x,y)}{var(x)} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$

Formula for Intercept : $\boldsymbol{\beta_0} = \bar{y} - \boldsymbol{\beta_1} \cdot \bar{x}$

SUM	1503000000	3080000000
COV	300600000	616000000

B0	**0.4880**
B1	**-943.1818**

Regression Line



Metrics to evaluate Model (Regression only)

- Mean Squared Error (MSE)
- Root Mean Squared Error (RMSE)
- Mean Absolute Error (MAE)
- Mean Absolute Percentage Error (MAPE)
- R squared

Mean Squared Error

$$\text{MSE} = \underbrace{\frac{1}{n}\sum_{i=1}^{n}}_{\text{Mean}}(\underbrace{Y_i - \hat{Y}_i}_{\text{Error}})^{\overbrace{2}^{\text{Squared}}}$$

$$RMSE = \sqrt{\sum_{i=1}^{n}\frac{(\hat{y}_i - y_i)^2}{n}}$$

Income per month (x)	Spending (y)	Ycap	Error	Squared Error
100000	50000	47855.52	2144.48	4598796.70
80000	35000	38095.78	-3095.78	9583848.98
60000	29000	28336.04	663.96	440844.26
45000	20000	21016.23	-1016.23	1032731.07
30000	15000	13696.43	1303.57	1699298.47

B1	0.488
B0	-943.182

Sum	17355519.48
Count	5
Average	3471103.896
MSE	3471103.896
RMSE	1863.089879

Mean Absolute Error

$$\frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i|$$

Income per month (x)	Spending (y)	Ycap	Error	absolute error
100000	50000	47855.52	2144.48	2144.48
80000	35000	38095.78	-3095.78	3095.78
60000	29000	28336.04	663.96	663.96
45000	20000	21016.23	-1016.23	1016.23
30000	15000	13696.43	1303.57	1303.57

Sum	8224.03
Count	5
MAE	1644.81

Mean Absolute Percentage Error

$$MAPE = \frac{1}{N}\sum_{t=1}^{N} \left| \frac{E_t - A_t}{A_t} \right|$$

Income per month	Spending (y)	Ycap	Error	absolute error	Abs Perc error
100000	50000	47855.5	2144	2144.5	4.29%
80000	35000	38095.8	-3096	3095.8	8.85%
60000	29000	28336	664	663.96	2.29%
45000	20000	21016.2	-1016	1016.2	5.08%
30000	15000	13696.4	1304	1303.6	8.69%
				MAPE	5.84%

R squared metric

Formula

$$R^2 = 1 - \frac{RSS}{TSS}$$

R^2 = coefficient of determination

RSS = sum of squares of residuals

TSS = total sum of squares

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (\hat{y}_i - y_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y}_i)^2}$$

R squared metric Calculation

Income per month (x)	Spending (y)	Ycap	Error	Error^2	yi - ymean	(yi - ymean)^2
100000	50000	47855.52	2144.48	4598796.70	20200	408040000
80000	35000	38095.78	-3095.78	9583848.98	5200	27040000
60000	29000	28336.04	663.96	440844.26	-800	640000
45000	20000	21016.23	-1016.23	1032731.07	-9800	96040000
30000	15000	13696.43	1303.57	1699298.47	-14800	219040000

ymean	29800

RSS	17355519.48	TSS	750800000
R2	0.9769		

Thank You