1 Trigonometry Review

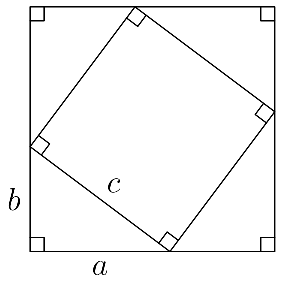

Figure 1: Scenario in Problem 1.

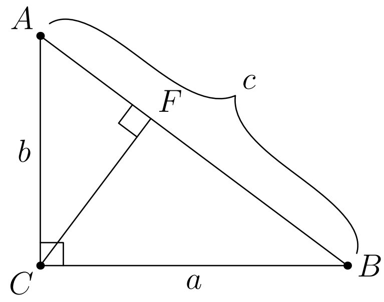

Figure 2: Scenario in Problem 2.

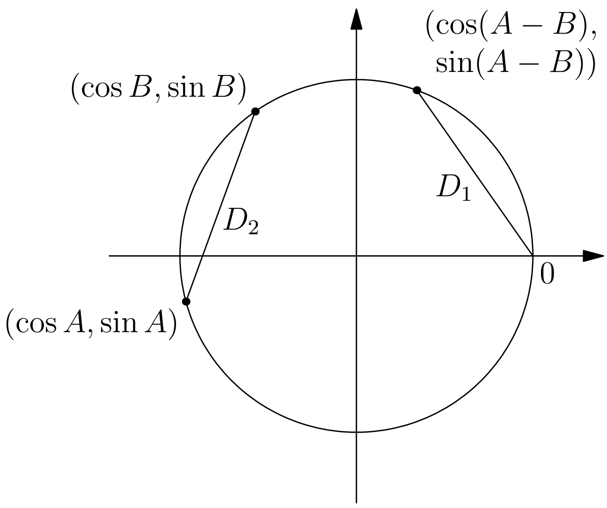

Figure 3: Scenario in Problem 3.

This chapter reviews material you learned last year that you will need as background knowledge for our study of linear algebra. If you don't know this material already, make sure to learn it.

1. Prove the Pythagorean theorem using "conservation of area." Start with Figure 1.

2. Prove the Pythagorean theorem using a right triangle with an altitude drawn to its hypotenuse, as shown in Figure 2, making use of similar right triangles.

3. We now prove the trigonometric identities.

   (a) Draw and label a right triangle and a unit circle, then write trig definitions for $\cos$, $\sin$, $\tan$, and $\sec$ in terms of your drawing.

   (b) Use a right triangle and the definitions of $\sin$ and $\cos$ to find and prove a value for $\sin^2\theta + \cos^2\theta$.

   (c) Use the picture of the unit circle in Figure 3 to find and prove a value for $\cos(A-B)$. Note that D_1 and D_2 are the same length because they subtend the same size arc of the circle. Set them equal and work through the algebra, using the distance formula and part (b) of this problem.

4. Write down as many trig identities as you can—no need to prove these.

$$\begin{array}{rrr} \sin(A+B) = & \sin(A-B) = & \cos(A+B) = \\ \tan(A+B) = & \tan(A-B) = & \sin(2A) = \\ \cos(2A) = & \tan(2A) = & \sin\left(\frac{A}{2}\right) = \\ \cos\left(\frac{A}{2}\right) = & \tan\left(\frac{A}{2}\right) = & \end{array}$$

5. Let's review complex numbers and DeMoivre's theorem.

(a) Recall that you can write a complex number both in Cartesian and polar forms. Let

$$a + bi = (a, b) = (r\cos\theta, r\sin\theta) = r\cos\theta + ir\sin\theta.$$

What is r in terms of a and b?

(b) Expand $(a + bi)(c + di)$ the usual way.

(c) Let $a + bi = r_1(\cos\theta + i\sin\theta)$ and $c + di = r_2(\cos\phi + i\sin\phi)$. Multiply them, and use the angle addition formulas to show that multiplying two complex numbers involves multiplying their lengths and adding their angles. This is DeMoivre's theorem!

(d) Use part (c) to simplify $(\sqrt{3} + i)^{18}$.

6. Here is a review of 2D rotation.

   (a) Recall that we can graph complex numbers as ordered pairs in the complex plane. Now, consider the complex number $z = \cos\theta + i\sin\theta$, where θ is fixed. What is the magnitude of z?

   (b) Multiplying $z \cdot (x + yi)$ yields a rotation of the point (x, y) counterclockwise around the origin by the angle θ. Notice that rotating the graph counterclockwise around the origin has the same effect as rotating the coordinate axes clockwise around the origin by the same angle θ. What if we wanted to rotate clockwise by θ instead?

7. Rotate the following conics by (i) $30°$, (ii) $45°$, and (iii) θ:

   (a) $x^2 - y^2 = 1$ (b) $\frac{x^2}{16} - \frac{y^2}{9} = 1$ (c) $y^2 = 4Cx$

You should have mastery of this material so that we can immediately investigate novel and interesting ideas. These often have surprising connections to the trigonometry and transformational geometry you learned last year. For example, we will soon find another convenient way to perform a rotation of coordinates.