Name: __________________
Grade: _____
Competitions interested in (select as many as you like):
□ HMMT (Harvard–MIT)
□ CHMMC (Caltech Harvey Mudd)
□ (A)SMT (Stanford)
□ BMT (Berkeley)

2015 Gunn HMMT/CHMMC TST (Team Selection Test)

1. _________ Susan has one fair 6-sided die and one fair 2-sided coin. The die starts out with the 1 facing up. On each turn, Susan flips the coin. If the coin lands heads, she then rolls the die and proceed to the next turn. If it lands tails, Susan proceeds to the next turn. What is the probability that die will have the 1 facing up after four turns?

2. _________ There are three distinguishable representatives from each of four countries: Amurica, Brazil, China, and the Democratic People's Republic of Korea. The 12 representatives stand in a line with a representative from Amurica at the front of the line. If a representative from Brazil cannot directly stand behind a representative from Amurica, a representative from China cannot directly stand behind a representative from Brazil, a representative from the Democratic People's Republic of Korea cannot directly stand behind a representative from China, and a representative from Amurica cannot directly stand behind a representative from the Democratic People's Republic of Korea, how many ways can the representatives get in line? You may leave your answer in terms of factorials.

3. _________ Compute the smallest positive integer that is 4 mod 7, 6 mod 11, and 2 mod 13.

4. _________ What is the largest $n$ such that $2^n$ divides $128!$?

5. _________ Two regular tetrahedra with distinct vertices are inscribed inside a unit cube such that they have parallel sides. What is the volume of their intersection?

6. _________ Triangle $ABC$ has lengths $AB = 8$, $BC = 15$. The area of $ABC$ is 37. What is $\cos^2(\angle ABC)$?

7. _________ Let $A$ and $B$ be two externally tangent circles with radii 2 and 5, respectively. Let $C$ be the point of tangency. A line is externally tangent to $A$ and $B$ at points $X$ and $Y$. What is the area of triangle $CXY$?

8. _________ (AIME 2004 I #13) The polynomial $P(x) = (1 + x + x^2 + \cdots + x^{17})^2 - x^{17}$ has 34 complex roots of the form $z_k = r_k[\cos(2\pi a_k) + i\sin(2\pi a_k)], k = 1, 2, 3, \ldots, 34$, with $0 < a_1 \leq a_2 \leq a_3 \leq \cdots \leq a_{34} < 1$ and $r_k > 0$. Given that $a_1 + a_2 + a_3 + a_4 + a_5 = m/n$, where $m$ and $n$ are relatively prime positive integers, find $m + n$.

9. _________ (AIME 2004 II #15) A long thin strip of paper is 1024 units in length, 1 unit in width, and is divided into 1024 unit squares. The paper is folded in half repeatedly. For the first fold, the right end of the paper is folded over to coincide with and lie on top of the left end. The result is a 512 by 1 strip of double thickness. Next, the right end of this strip is folded over to coincide with and lie on top of the left end, resulting in a 256 by 1 strip of quadruple thickness. This process is repeated 8 more times. After the last fold, the strip has become a stack of 1024 unit squares. How many of these squares lie below the square that was originally the 942nd square counting from the left?