# Team Round, Division A

## Division A, 60 Minutes in Teams of 4

1. $ABCD$ is an isosceles trapezoid with $AD \parallel BC$. $AD = 4, BC = 10$, and a circle $\omega$ with center $O$ is inscribed inside $ABCD$. $\omega$ is tangent to $AB$ at point $P$, where $AP = 2$ and $BP = 5$. Compute $\cos^2(\angle AOP)$.

2. In the middle of the Earth Tectonics labs, the power goes off in Chanew's chemistry classroom. Dr. Mellows has three red lava lamps and three blue lava lamps. She arranges them in a row on her desk randomly, and then randomly turns three of them on. What is the probability that the leftmost lamp is blue and off, and the rightmost lamp is red and on?

3. Compute, to the nearest integer, the ratio of the number of ways to arrange the letters in "GUNN-MATHCOMP" so that exactly one of the substrings 'GUNN', 'MATH', or 'COMP' appears to the number of ways so that at least two of the substrings appear.

4. Suppose you have a polynomial, $p(x)$, of degree 4 with integer coefficients such that $p(0) = 7, p(1) = 11$, and $p(2) = 13$. Compute the minimum possible positive value of $p(25)$.

5. Suppose that points $A, B$, and $C$ lie on the curve $y = x^2$ and $B$ has coordinates $(4, 16)$. The $x$ coordinates of $A$ and $C$ sum to 8, and $\angle ABC = 90°$. Let $G = (G_x, G_y)$ be the centroid of $\triangle ABC$ and $O$ be the origin. Compute $G_x + G_y$.

6. Two points are randomly chosen a certain distance $x$ from the center of a circle of radius 1. Given the probability that the midpoint of the 2 points lies inside or on the circle is $\frac{1}{6}$, compute $x$.

7. Let $\triangle ABC$ be an equilateral triangle of side length 2. Point $D$ satisfies $DA = DB = DC = 3$. Find the inradius of tetrahedron $ABCD$.

8. Alice and Bob are worst enemies in a class of six students. Each student has at least one friend, no two students share more than one friend, and no three students are all friends with each other. In how many ways can the six students be friends with each other?

9. Let $\omega_1$ be a circle of radius 6 centered at $O_1$, and let $\omega_2$ be a circle of radius 8 centered at $O_2$. They intersect at points $A$ and $B$. Let $C$ be the second intersection of $\omega_1$ and the line tangent to $\omega_2$ at $A$. Let $D$ be the second intersection of $\omega_2$ and the line tangent to $\omega_1$ at $A$. Let line $CD$ intersect the circumcircle of $AO_1O_2$ at $E$ and $F$, where $E$ is closer to $C$. Given that $O_1A = O_2F$ and $O_1O_2$ is parallel to $BF$, compute the area of quadrilateral $ABFO_2$.

10. Let $f(n)$ denote the number of solutions, $\{a_1, a_2, a_3, \ldots, a_{4n}\}$, to the congruence

$$\sum_{i=1}^{4n} a_i^2 \equiv 1 \pmod{2025}$$

where all $a_i$ are between 1 and 2025. Find the unique positive integer $n$ such that

$$v_2(f(n)) + v_3(f(n)) + v_5(f(n)) = 2021.$$