Individual Tiebreakers (Division B)

Gunn Math Competition 2025

Instructions and Basic Format

- This round contains 3 short-answer questions to be solved in 15 minutes, by you. Each problem is worth more points than all previous points, but by no significant amount, so be mindful of where you spend your time. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
- **NO CALCULATORS (or abaci).** Protractors, rulers, and compasses are permitted. Do not cheat in any way. When caught, you will be blacklisted from the competition and will not be able to participate for the rest of the day.
- Carry out any reasonable calculations. For instance, you should evaluate $\frac{1}{2} + \frac{1}{3}$, but you do not need to evaluate large powers such as 7^8.
- Write rational numbers in lowest terms. Decimals are also acceptable, provided they are exact. You may use constants such as π, e, $\sin 10°$, and $\ln 2$ in your answers.
- Move all square factors outside radicals. For example, write $3\sqrt{7}$ instead of $\sqrt{63}$.
- Denominators do *not* need to be rationalized. Both $\frac{\sqrt{2}}{2}$ and $\frac{1}{\sqrt{2}}$ are acceptable.
- Do not express an answer using a repeated sum or product.

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Problems

1. Take a sector of a circle of radius 20, shaped like a pizza slice, and fold it so it makes a cone with an open base. Attach a hemisphere that to the open base of the cone (to make it like an ice-cream cone). Given that the entire shape's surface area is 192π, consider its volume. It can be expressed in the form $\pi(a\sqrt{b}+c)$, where a, b, and c are positive integers and b is square-free. Calculate $a+b+c$.

2. Let $f(a,b)$ denote the period of the sequence of remainders of powers of a when divided by b. For example, for $a = 3$, and $b = 5$, we have $3^0 \equiv 1 \pmod 5, 3^1 \equiv 3 \pmod 5, 3^2 \equiv 4 \pmod 5, 3^3 \equiv 2 \pmod 5$ and $3^4 \equiv 1 \pmod 5$, giving that sequence $1, 3, 4, 2, 1$ which then repeats every 4 terms, yielding $f(3,5) = 4$. Compute the following:
$$\sum_{n=1}^{82} f(n, 83).$$

3. Compute the following sum:
$$\sum_{k=1}^{\infty} \arctan\left(\frac{k-1}{k^3-1}\right).$$