EÐLISFRÆÐI 1 V/R

Lausn á lokaprófi 2011

Dæmi 1 a) Reikna eins og „kastahreyfing"

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} v_x t \\ y_0 - \frac{1}{2} g t^2 \end{pmatrix} \qquad \begin{matrix} v_x = \text{fasti} = v_{x_0} \\ v_{y_0} = 0 \end{matrix}$$

$y_0 = 1\,m$ og þegar $y(t) = 0$ er: $0 = y_0 - \frac{1}{2} g t^2$

$$\Rightarrow t = (\pm)\sqrt{\frac{2y_0}{g}} = 0.45\,s$$

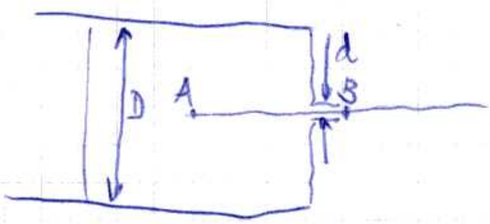
Á þessum tíma hefur boltinn komist til hægri

(x-stefna) um: $v_x \cdot t = 12\,m$

svo að

$$v_x = \frac{12\,m}{\sqrt{2y_0/g}} = \underline{\underline{26.6\,\frac{m}{s}}}$$

b) Bernoulli:



$$P_A + \rho g h_A + \frac{1}{2}\rho v_A^2 = P_B + \rho g h_B + \frac{1}{2}\rho v_B^2$$

$h_A = h_B$ svo að: $P_A = P_B + \frac{1}{2}\rho\left(v_B^2 - v_A^2\right)$ (*)

Nú er $p_B = p_0$ og $v_B = v_x$ frá a)-lið

Vitum (samfelldnijafna) að

$$A_A v_A = A_B v_B$$

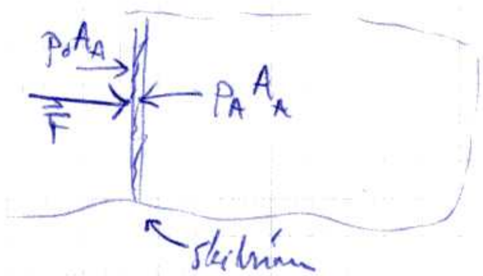
svo að $$v_A = \frac{A_B}{A_A} v_B = \frac{\pi \frac{d^2}{4}}{\pi \frac{D^2}{4}} \cdot v_B = \left(\frac{d}{D}\right)^2 \cdot v_B$$

$$= 10^{-2} \cdot v_B$$

Set í ⊛ og fæ:

$$p_A - p_B = p_A - p_0 = \frac{1}{2} \rho v_x^2 \left(1 - \left(\frac{d}{D}\right)^4\right)$$

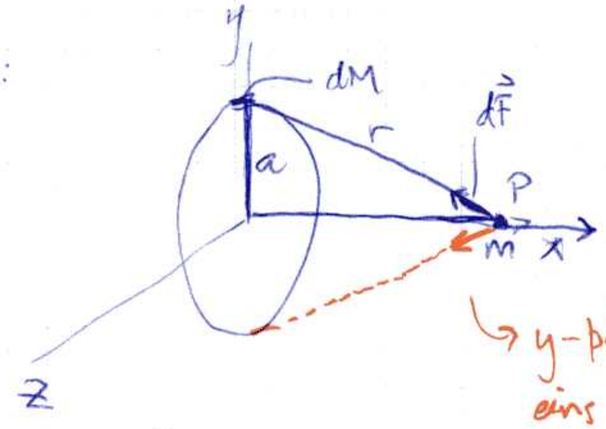
Krafturinn $\vec{F}$ er



$$F = (p_A - p_0) A_A = \frac{1}{2} \rho v_x^2 \left(1 - \left(\frac{d}{D}\right)^4\right) \cdot \frac{\pi D^2}{4} = \underline{\underline{694\ N}}$$

Dæmi 2:

a)



Öll örsmáþáttar element dM hefa sömu fjarlægð til punkts P

$$r = \sqrt{x^2 + a^2}$$

$$\Rightarrow \quad U = -\frac{GmM}{\sqrt{x^2+a^2}} \quad \left(\text{líka má} = -\frac{m}{\sqrt{x^2+a^2}} \int_0^{2\pi} \rho a \, d\theta \right.$$

$$\left. = -\frac{m}{\sqrt{x^2+a^2}} \cdot \rho a \cdot 2\pi \qquad \rho = \frac{M}{2\pi a} \right)$$

b) $\vec{F} = -\vec{\nabla} U$ og $F_x = -\frac{dU}{dx}$; $F_y = -\frac{dU}{dy} = 0$

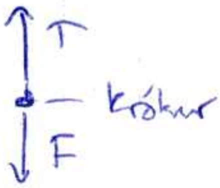
og $F_z = -\frac{dU}{dz} = 0$

$$F_x = \frac{d}{dx}\left(\frac{GmM}{\sqrt{x^2+a^2}}\right) = -GmM \frac{x}{(x^2+a^2)^{3/2}} \quad (*)$$

c) Sést vegna samhverfu að y- og z- og x- þáttur F verða að vera 0. x=0 í (*) gefur $F_x = 0$.

Dæmi 3:

a)



$$F = (M_d + M + m)g \qquad I = \frac{1}{2} M_d r^2$$

svo $$T = \left(\frac{2I}{r^2} + M + m\right) g = \underline{99.96\ N}$$

b) Orkuvarðveisla: ($E_{kin} = 0$ í byrjun)

$$Mgy_0 + mg\cancel{y_1} = Mg\cdot 0 + mg(\cancel{y_1} + y_0) + \frac{1}{2}Mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

hæð m $\quad$ 0 $\quad$ hækkar 5 m $= y_0$

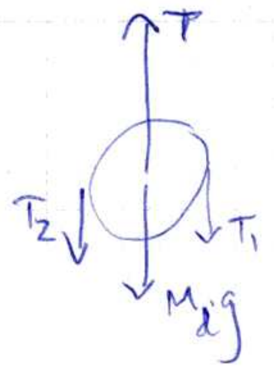
en $\omega = \frac{v}{r}$

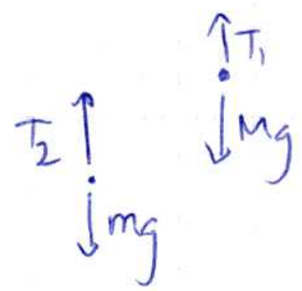
$$\Rightarrow \frac{1}{2}\left(M + m + \frac{I}{r^2}\right)v^2 = (M - m)gy_0$$

$$\Rightarrow v^2 = \frac{2(M-m)gy_0}{M + m + \frac{I}{r^2}}$$

$$\Rightarrow v = \underline{\underline{4.91\ \frac{m}{s}}}$$

c) Kraftjafnvægi:





① $Mg - T_1 = Ma_1$

② $mg - T_2 = ma_2 = -ma_1$ því $a_2 = -a_1$

③ $\tau_1 - \tau_2 = I\alpha$

þ.e. $T_1 r - T_2 r = I \frac{a_1}{r} \Rightarrow T_1 - T_2 = \frac{I}{r^2} a_1$

①-②: $(M-m)g = T_1 - T_2 + (M+m)a_1$

$$\Rightarrow a_1 = \frac{M-m}{M+m+\frac{I}{r^2}} \cdot g = 2.42 \ \frac{m}{s^2}$$

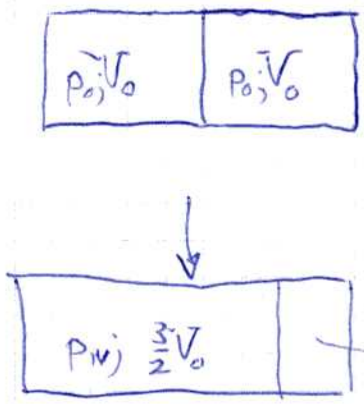
Svo fæst:

$$T = M_d \cdot g + T_1 + T_2 = 95.1 \ N$$

① $T_1 = M(g - a_1)$

② $T_2 = m(g + a_1)$

Dæmi 4:



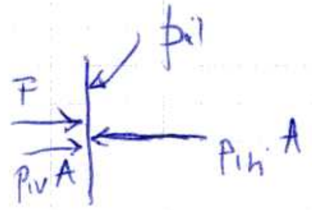
$p_0 V_0 = nRT_0 = \text{fasti}$
(bæði hólf)

a) Vinstri:

$$p_0 V_0 = \text{fasti} = p_{1v} \cdot \frac{3}{2} V_0$$

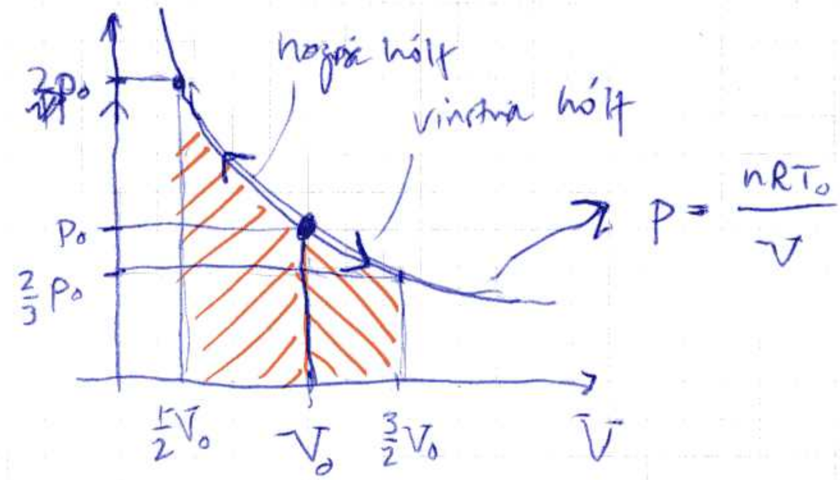
$$\Rightarrow \quad p_{1v} = \frac{2}{3} p_0$$

Hægri: $\quad p_0 V_0 = p_{1h} \cdot \frac{1}{2} V_0 \quad \Rightarrow \quad p_{1h} = 2 p_0$



$$F = (p_{1h} - p_{1v}) A = \left(2 - \frac{2}{3}\right) p_0 \cdot A = \underline{\underline{2000\ N}}$$

b)



c) Skyggðu svæðin sýna vinnu hvors hólfs um sig.

$W_v > 0$ og $W_h < 0$

Nú er $dU = \delta Q - p\,dV$ 1. LV.

$dU = 0$ því kjörgas og $U(T) = $ fasti því $T = $ fasti

Vinstri: $\delta Q_v = p\,dV > 0$ varmi til v-hólfs

Hægri $\delta Q_h = p\,dV < 0$ varmi frá h-hólfi

Nánar:

v: $$\int_{V_0}^{V_{iv}} p\,dV = nRT_0 \int_{V_0}^{\frac{3}{2}V_0} \frac{dV}{V} = nRT_0 \ln\frac{3}{2} = \ln 3 - \ln 2$$

h: $$\int_{V_0}^{V_{ih}} p\,dV = nRT_0 \int_{V_0}^{\frac{1}{2}V_0} \frac{1}{V}\,dV = nRT_0 \ln\frac{1}{2} = -\ln 2$$

Varmi í heild $Q = Q_v + Q_h = nRT_0 \underbrace{\left(\ln\frac{3}{2} + \ln\frac{1}{2}\right)}_{\ln\frac{3}{4} < 0}$

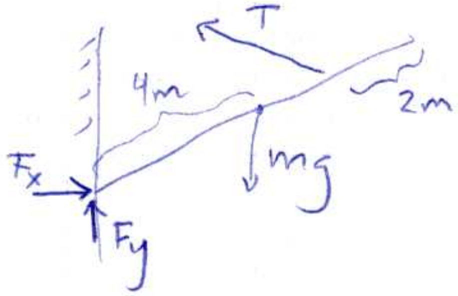
varmi fer út

ef T helst fasti

sem er gert ráð fyrir.

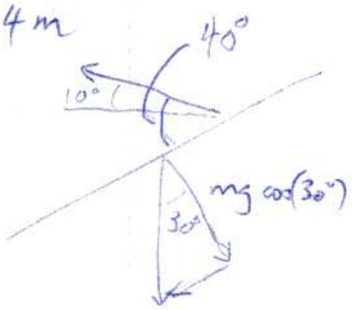
Dæmi 5:

a)



b) Vægi við vegg ↻+ (losna við F_x og F_y)

$$-T\sin(40^\circ)\cdot\underbrace{(L-2)}_{6m} + mg\cos(30^\circ)\cdot\underbrace{\frac{L}{2}}_{4m} = 0$$



$\Rightarrow$ $T = 13204\ N$

c) $\sum F_x = 0$

$$F_x - T\cos(10^\circ) = 0$$

$$F_x = 13003\ N$$

Dæmi 6:

$f_0 = 400$ Hz

$v = 343$ m/s

(i) „Veggur" hlustandi

$$f_L = \frac{v + \overbrace{v_L}^{v_L = 0}}{v + v_S} \underbrace{f_S}_{f_0} = \frac{343}{343 + 31.1} \cdot 400 = 366.7 \text{ Hz} =: f_v$$

(ii) Speglun af þeirri tíðni kemur að sjúkrabíl og numin tíðni er:

$$f_{bíll} = \frac{v + \overbrace{v_L}^{-31.1}}{v + \underbrace{v_S}_{0}} \underbrace{f_S}_{f_v} = \frac{343 - 31.1}{343} \cdot 366.7 = \underline{\underline{333.5 \text{ Hz}}}$$