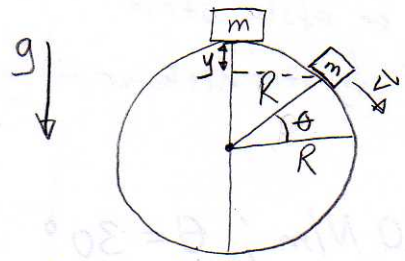
18. október 2011

Miðannarpróf

Lausn

Eðlisfr. 1V/1R

Dæmi 1: (30%)



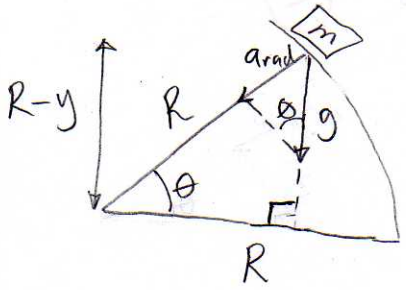
núningslaust

Varðveisla orku: (5)

$$\Delta U + \Delta K = 0$$

$$\Rightarrow mgy = \frac{1}{2}mv^2 \quad (5)$$

$$\Rightarrow v^2 = 2gy \quad (*)$$



Miðsóknar hröðun:

$$a_{rad} = \frac{v^2}{R} \quad (5)$$

Til að halda massanum í snertingu við kúluyfirborðið þarf

$$a_{rad} = \frac{v^2}{R} \leq \sin\theta \cdot g \,. \quad (5)$$

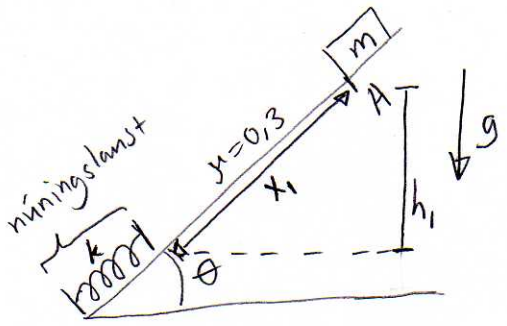
þ.e. hann missir snertingu þegar (5)

$$\frac{v^2}{R} = \sin\theta \cdot g = \frac{R-y}{R} \cdot g$$

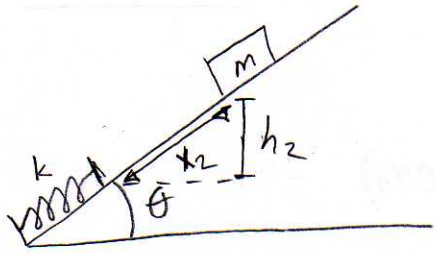
$$\overset{(*)}{\Rightarrow} \frac{2gy}{R} = \frac{R-y}{R} g \Rightarrow 2y = R - y$$

$$\Rightarrow 3y = R \Rightarrow \boxed{y = R/3} \quad (5)$$

Dæmi 2: (35%)



Fyrir



Eftir (x_2 er efsta staða eftir fyrsta árekstur við gorm)

a) (20%)

$x_1 = 5{,}0\ m$; $m = 5{,}0\ kg$; $k = 500\ N/m$; $\theta = 30°$

$\mu = 0{,}3$

$h_1 = x_1 \cdot \sin\theta = 2{,}5\ m$ $\quad$ $h_2 = x_2 \cdot \sin\theta$ (2)

Work Energy Theorem: (2)

$$mgh_1 + W_f = mgh_2 \quad (1)$$

(2) (2) (2)

W_f: vinna núningskrafts við það að renna niður og upp

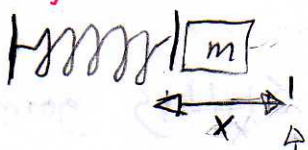
$$W_f = -F_N \cdot (x_1 + x_2) = -\mu mg\cos\theta\,(x_1 + x_2)$$

(2) (3)

(1) $\Rightarrow$ $x_1 \sin\theta - \mu\cos\theta\,(x_1 + x_2) = x_2 \sin\theta$

$$\Rightarrow x_2 = x_1 \frac{\sin\theta - \mu\cos\theta}{\sin\theta + \mu\cos\theta} = 1{,}58\ m \quad (5)$$

b) (15%)



↑ jafnvægisstaða gorms

(1) (2) (2) (2)

$$mgh_1 + W_f = \frac{1}{2}kx^2 - mg \cdot x \cdot \sin\theta \quad (2)$$

↑ vinna núningsx við það að renna niður

↑ $x > 0$

$$W_f = -\mu mg x_1 \cos\theta = -63{,}7\ \text{J} \quad (1)$$

$$mgh_1 = 122{,}5\ \text{J}$$

$$\Rightarrow mgh_1 + W_f = 58{,}8\ \text{J} \quad (1)$$

$$(2) \Rightarrow 250\ \text{N/m} \cdot x^2 - 24{,}5\ \text{N} \cdot x - 58{,}8\ \text{J} = 0 \quad (2)$$

Höfum jöfnu af gerð

$$ax^2 + bx + c = 0 \quad \text{hafa:} \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

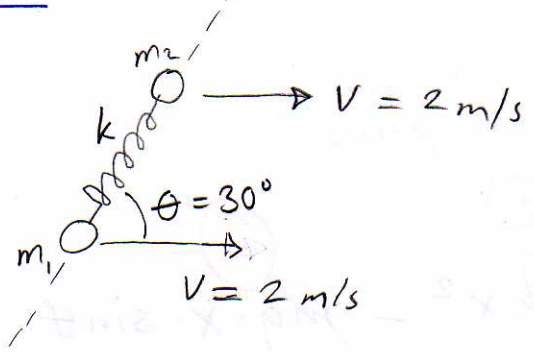
Svo (2)

$$x = \frac{24{,}5 \pm \sqrt{(24{,}5)^2 + 4 \cdot 250 \cdot 58{,}8}}{500} = \begin{cases} 0{,}53\ \text{m} & \leftarrow \text{svar} \\ (-0{,}44\ \text{m}) & (x > 0) \end{cases}$$

Mesta bjöppun gorms:

$$\boxed{x = 0{,}53\ \text{m}} \quad (2)$$

Dæmi 3: (35%)



Stytting gorms: $x = 25\ cm$

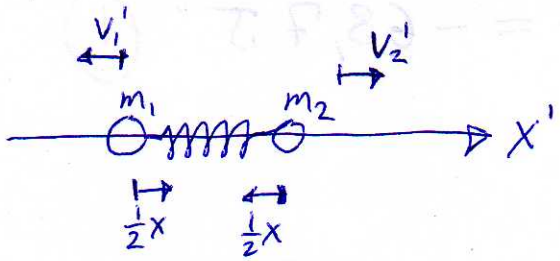
$k = 50\ N/m$

$m_1 = 3\ kg$

$m_2 = 2\ kg$

a) (15%)

Skoðum þetta í massamiðjukerfi (CM):



Eftir að massarnir missa snertingu við gorminn eru hraðar þeirra v_1' og v_2'.

Þar sem engin ytri kraftvogi verka á kerfið þá höfum við varðveislu skriðþunga:

$$p_1' = p_2' \Rightarrow m_1 v_1' = m_2 v_2' \quad (2)$$

svo $v_1' = \frac{m_2}{m_1} v_2'$ (*) (1)

Varðveisla orku: (Enginn núningur)

$$\frac{1}{2} k x^2 = \frac{1}{2} m_1 (v_1')^2 + \frac{1}{2} m_2 (v_2')^2 \quad (4)$$

$$\overset{(*)}{=} \frac{1}{2} m_2 \left(1 + \frac{m_2}{m_1}\right) \cdot (v_2')^2$$

$$\Rightarrow (v_2')^2 = \frac{k}{m_2} \frac{x^2}{1 + \frac{m_2}{m_1}} \quad (2)$$

svo

$$V_2' = \sqrt{\frac{k}{m_2} \cdot \frac{X^2}{1 + \frac{m_2}{m_1}}} = \sqrt{\frac{50\ N/m}{2\ kg} \cdot \frac{(0{,}25\ m)^2}{1 + \frac{2}{3}}}$$

$$= 0{,}968\ m/s \quad (1)$$

(*)

$$\Rightarrow V_1' = \frac{m_2}{m_1} = \frac{2}{3} V_2' = 0{,}645\ m/s \quad (1)$$

þ.a. hraði agnar 1 frá ögn 2 er:

$$\boxed{\Delta V = V_1' + V_2' = 1{,}61\ m/s} \quad (2)$$

b) (10%)

Þar sem við höfum engin ytri kraftvægi þá er hraði massamiðjunnar sá sami fyrir og eftir, þ.e.

$$\boxed{V_{cm} = V = 2\ m/s} \quad (10)$$

c) Ferð agnanna eftir atburðinn:

$$V_1 = \sqrt{(V_{cm} - \cos(\theta) \cdot V_1')^2 + (\sin\theta \cdot V_1')^2} = 1{,}48\ m/s \quad (5)$$

$$\simeq 1{,}5\ m/s$$

$$V_2 = \sqrt{(V_{cm} + \cos\theta \cdot V_2')^2 + (\sin\theta \cdot V_2')^2} = 2{,}88\ m/s \quad (5)$$

$$\simeq 2{,}9\ m/s$$