MATH3202 ASSIGNMENT 1

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PACIFIC PARADISE

TABLE OF CONTENTS

(GUNIT SINGH'S DATA HAS BEEN USED FOR THE PURPOSE OF THIS COLLABORATION)

1.0 SECTION A: REPORT TO BOSS

1.1 FORMULATION

Sets

V	Set of Vaccine Depots (IDs) {ID-A, ID-B, ID-C}
A	Set of arcs from IDs to LVCs
L	Set of Local Vaccination Centres (LVCs) {LVC0, LVC1, …, LVC7}
B	Set of arcs from LVCs to CCDs
C	Set of Census Collection Districts (CCDs) {CCD0, CCD1, …, CCD24}
T	Set of the time in weeks {Week 1, Week 2, …, Week 6}

Data

c_v	Importation cost of the vaccine at depot $v \in V$ ($)
p_c	The population/demand at CCD for $c \in C$
$l1_a$	Length of arc $a \in A$ (km)
$l2_b$	Length of arc $b \in B$ (km)
d_a	Cost/km of travel between arc $a \in A$ (=$0.2/km)
e_b	Cost/km of travel between arc $b \in B$ (=$1/km)
$t1_a \in L$	The LVC that arc $a \in A$ flows to
$f1_a \in V$	The ID that arc $a \in A$ flows from
$t2_b \in C$	The CCD that arc $b \in B$ flows to
$f2_b \in L$	The LVC that arc $b \in B$ flows from
Id_v	The maximum capacity at each ID $v \in V$ (=31000 doses)
lv_l	The maximum number of doses that can be administered at a LVC $l \in L$ (=13000 doses)
lw_l	The maximum number of weekly doses that can be delivered to an LVC $l \in L$ (=1800 doses/week)
r_t	The delay cost for each citizen unvaccinated by the end of the week $t \in T$ ($10/week)
cf_t	The max difference between the minimum and maximum cumulative fraction vaccinated at the CCD at week $t \in T$ (=0.1)

Decision Variable

$x_{a,t}$ Number of vaccines flowing through arc $a \in A$ in week $t \in T$

$y_{b,t}$ Number of vaccines flowing through arc $b \in B$ in week $t \in T$

z_t Number of people unvaccinated after week $t \in T$

Objective Function

Minimise total cost of the operation:

$$\min\left[\sum_{t\in T}\left(\sum_{v\in V}\sum_{\substack{a\in A\\ s.t.\ f1_a=v}} c_v x_{a,t} + \sum_{a\in A} l1_a d_a x_{a,t} + \sum_{b\in B} l2_b e_b y_{b,t}\right) + \sum_{t\in T} r_t z_t\right]$$

The total cost to be minimised has four components (all summed over the 6-week period):

- Importation of vaccines to the ID's
- Transportation of vaccines from ID's to LVC
- Cost of accessing the vaccines from CCD to LVC for each citizen
- Delayed cost of vaccines

Constraints

$$\sum_{t\in T}\sum_{\substack{b\in B\\ s.t.\ t2_b=c}} y_{b,t} = p_c \qquad \forall c \in C \qquad (1)$$

$$\sum_{\substack{a\in A\\ s.t.\ t1_a=l}} x_{a,t} = \sum_{\substack{b\in B\\ s.t.\ f2_b=l}} y_{b,t} \qquad \forall l \in L, \quad \forall t \in T \qquad (2)$$

$$y_{b,t} = 0\ s.t. l2_b = 0 \qquad \forall b \in B, \quad \forall t \in T \qquad (3)$$

$$\sum_{t\in T}\sum_{\substack{a\in A\\ s.t.\ f1_a=v}} x_{a,t} \le id_v \qquad \forall v \in v \qquad (4)$$

$$\sum_{t\in T}\sum_{\substack{a\in A\\ s.t.\ t1_a=l}} x_{a,t} \le lv_l \qquad \forall l \in L, \qquad (5)$$

$$\sum_{\substack{a\in A\\ s.t.\ t1_a=l}} x_{a,t} \le lw_l \qquad \forall l \in L, \quad \forall t \in T \qquad (6)$$

$$u_t = \sum_{c\in C} p_c - \sum_{\substack{t'\in T\\ s.t.\ t'\le t}}\sum_{b\in B} y_{b,t'} \qquad \forall t \in T \qquad (7)$$

$$\left(\sum_{\substack{t'\in T\\ s.t.\ t'\le t}}\sum_{\substack{b\in B\\ s.t.\ t2_b=c}} y_{b,t'}\right)/p_c - \left(\sum_{\substack{t'\in T\\ s.t.\ t'\le t}}\sum_{\substack{b\in B\\ s.t.\ t2_b=c'}} y_{b,t'}\right)/p_{c'} \le cf \qquad \forall c' \in C, \forall c \in C, \forall t \in T \qquad (8)$$

$$x_{a,t}, y_{b,t}, z_t \ge 0 \qquad \forall a \in A, \forall b \in B, \forall t \in T \qquad (9)$$

Note: s.t. is an abbreviation for "such that".

Constraints Descriptions

Constraints (1) says the total amount of vaccines going to a CCD from its neighbouring LVC's must be equivalent to the population of the CCD.

Constraint (2) governs the demand of vaccines at an LVC, as the vaccines flowing out of it in a week must be equivalent to those flowing into it.

Constraint (3) addresses the fact that the citizens of CCD's are only able to obtain the vaccine from a neighbouring LVC (as a distance of 0km in the data represents a non-neighbouring LVC).

Constraints (4) and (5) says that the total vaccines imported to an ID and the total vaccines administered at an LVC, does not exceed its capacity throughout the operation.

Constraints (6) ensures that the maximum number of weekly doses that can be delivered at an LVC at any given week is not exceeded.

Constraints (7) produces the number of unvaccinated people each week, which is used in the objective function to account for delayed vaccinations.

Constraint (8) ensures that the differences between the cumulative fraction of the population vaccinated in each CCD at a given week does not vary by more than 10%.

Constraint (9) is a general constraint, defining the variables to be greater than or equal to 0 (this is an automatic constraint in Gurobi).

Code

The python file which model's this formulation through Gurobi is attached separately and produces the optimal solution for the problem as $13287841.

2.0 SECTION B: REPORT TO CLIENT

2.1 INTRODUCTION

This report details GHB Solutions' response to the vaccination distribution problem posed by the client: Pacific Paradise (PP). Gurobi Python; a powerful mathematical optimisation solver, has been implemented to determine the most cost-efficient vaccine distribution strategy (VDS) as part of Pacific Paradise's response to the COVID-19 pandemic. Throughout the modelling process, communications were provided by PP and as such, the solution was updated four times over a period of three weeks. Ultimately, the team arrived at a final optimised total cost of $13 287 841. The following section discusses how all the client's requirements have been included and satisfied by the optimisation, and provide insight into the main factors that affected the solution at each stage.

2.2 COMMUNICATION 1

Description

Assumption: each person only requires one/will only be given one vaccine to be fully protected at this stage of the pandemic response strategy.

Communication 1 introduces the task as a minimisation optimisation problem. The objective function indicates that costs arise by three means: ID importation cost, vaccine transportation cost, and citizen travel cost. Through Gurobi programming, the overall cost can be minimised by iterating different travel combinations with varying stock at each ID. The following main themes were discerned from the communication, and each point below briefly summarises how each theme has been addressed in the formulation.

1. Distribution network geometry: vaccines are sourced from IDs (set V) and transported to LVCs (set L), where citizens flock towards from CCDs (set C) to be administered. Each ID has access to all of the LVCs, as represented in data $t1_a$ and $f1_a$, but not all CCDs are connected to every LVCs, as seen in data $t2_b$ and $f2_b$
2. Transportation/travel expenses: Distances between IDs and LVCs are important as there is an associated cost of $0.2/km/dose (d_a) to transport the vaccines; the data set $l1_a$ multiplied by this d_a and the optimised y_a takes care of this factor in the model. Analogously, distances from CCDs to LVCs are contained within the data $l2_b$, and these $l2_b$ multiplied by the citizen travel cost e_b = $1/km/citizen and the optimised z_b gives the total transportation price incurred to citizens to receive the vaccine. Notably, a considerable sum of these distances $l2_b$ are equal to zero; not because the distance is physically equal to this, but because not every LVC can be accessed by each CCD.
3. Population demands: all Pacific Paradise residents will be vaccinated during the VDS, and so each CCD will have different vaccine demands due to population differences. p_c represents this demand, and impacts the popularity of each LVC to receive the vaccine.
4. ID importation costs: the vaccine enters Pacific Paradise at three IDs which each have varying rates, denoted by c_v. This impacts the stock of each ID, which in turn will impact the transportation distances and costs to get this stock to the LVCs for administration.

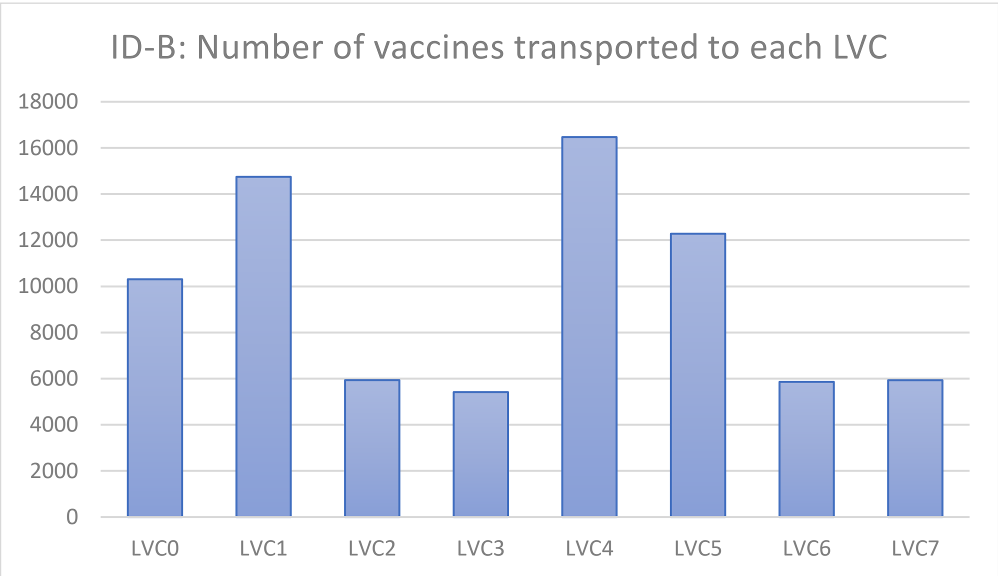
Insights

By far the most prominent aspect of the data is all vaccines being imported into vaccine depot B. This is purely because ID-B enjoys the cheapest vaccine importation cost at $105/dose; $39 cheaper than ID-C and $52 cheaper than ID-A. As a result, all LVC's received their vaccines from ID-B for this communication.

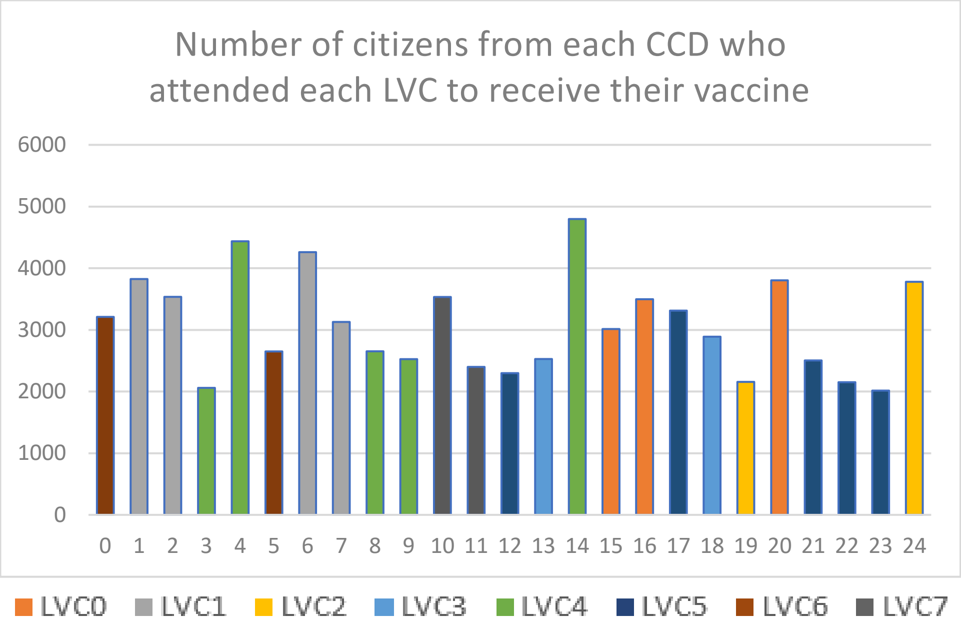
Subsequent to this, Graph 2.2.1 below illustrates that LVC's 0, 1, 4 and 5 receive majority of the vaccines (69.9% of the 76946 total - see Table 2 in appendices for exact figures). This is spearheaded by LVC4, which receives 16 469 vaccines. Gurobi has returned this value for two reasons: LVC4 is very near ID-B relative to other LVC's (6km), and also because LVC4 is very close to the two most populated CCD's; CCD4 (16.1km – 4436 people) and CCD14 (15.5km – 4797 people), meaning it has more people to vaccinate in its vicinity compared to other centres.

Graph 2.2.2 mirrors the findings summarised in graph 2.2.1, showing that LVC's 1 and 4 dominate the supply of vaccinations. It is important to note (and displayed in Table 3 in the appendices) that the entire population for each CCD travels to only one LVC. In other words, the citizens of each CCD do not split up when they go to get vaccinated; at least for this initial communication. LVC's 1, 4 and 5 supply fourteen out of the twenty-four CCD's, while the other five LVC's supply at most three CCD's each. It is anticipated that Gurobi has come to this solution because of the demand from nearby CCD's and their populations; a larger cost is intuitively associated with a larger population, especially if people are forced to travel further away than they need to and thus incurring higher travel fees.

This communication gave a result of $9,981,300.



Graph 2.2.1: Number of vaccines transported from ID-B to each LVC



Graph 2.2.2: Number of citizens from each CCD who attended each LVC to receive their vaccine

2.3 COMMUNICATION 2

Description

The second iteration of this formulation looks to cap the number of vaccines going through the ID's and LVC's. This formulation gives a more practical model for Pacific Paradise to monitor as realistically the transportation loads can't be infinite and are capped.

To meet these requirements, two additional constraints have been added to the code. These constraints ensure that the solution created will only output the cheapest option with no depot getting over 31000 doses and no LVC will receive over 13000 doses.

Insights

The new updated data alters the movement of vaccines through the depots, depot B has significantly dropped from receiving 76946 in communication 1 to 31000 in communication 2. Secondly, after communication 2 depot A has also reached its limit of 31000 doses; increasing from 0. Therefore, as both Depot A and B have both received their maximum number of doses the leftover 14946 are transported to Depot C. This was expected as the importation costs increase across the three Depots with Depot B being the cheapest and Depot C being the most expensive. The differences in these importation costs is greater than the transport costs, which resulted in Gurobi opting to minimise the costs by the cheaper Depots first.

Using LVC0 as an example to show these costs is Table 2.3.1 below:

Table 2.3.1: LVC0 example calculations

ID	Importation cost ($)	Transportation cost	Total cost ($)
A	144	$59.1 * 0.2 = 11.82$	155.82
B	105	$90.2 * 0.2 = 18.04$	123.04
C	157	$38.6 * 0.2 = 7.72$	164.72

Table 2.3.2: Table of doses at each depot

Depot	From Communication 1	From Communication 2	Change
A	0	31000	+31000
B	76946	31000	-45946
C	0	14946	+14946

The new requirement restricting the LVC's usage hasn't impacted the data as much as restricting the depots, as the vaccines were comparatively more spread out in communication 2. Not all doses are going to one LVC this time, but rather all eight LVC's are receiving vaccines initially.

From communication 1 there were only two LVC's exceeding 13000 doses, therefore resulting in Pacific Paradise only needing to find an alternative route for 5216 doses. This alteration has also resulted in a third LVC receiving their full amount of 13000 doses being LVC3.

This communication gave a result of $11,534,485.

2.4 COMMUNICATION 3

Description

This third iteration of the formulation examines setting out a plan in six-week blocks accounting for the fact that Pacific Paradise's LVCs will in practise be restricted to receiving only 1800 doses a week. An additional constraint has been included to account for this, which limits the number of vaccines getting transported to the LVC's $(y_{a,t})$; limiting it with a less than or equal to 1800.

Insights

This new constraint has drastically changed the formulation output; before the addition of this constraint all but two of the LVC's were receiving over the practical number of doses each week. As a result, the vaccines will be spread across a greater number of LVC's. It is also important to note that the CCD's utilised all reach zero vaccines at different times, with some running out in week four, others in week five, and the remainder in week six. This is due to the inconsistencies in the deportation of the vaccines, which currently are operating with no control systems.

Another noteworthy factor is the first 6 LVC's that receive their maximum doses of 1800; listed on the right in Table 2.4.1. These are the first LVC's to reach their limit, which occurs due to their ideal locations that reduce the transportation costs associated with moving the vaccines. The only other LVC to receive any doses is LVC 7, with all the other LVC's not receiving any due to their inefficient locations in relation to the three depots.

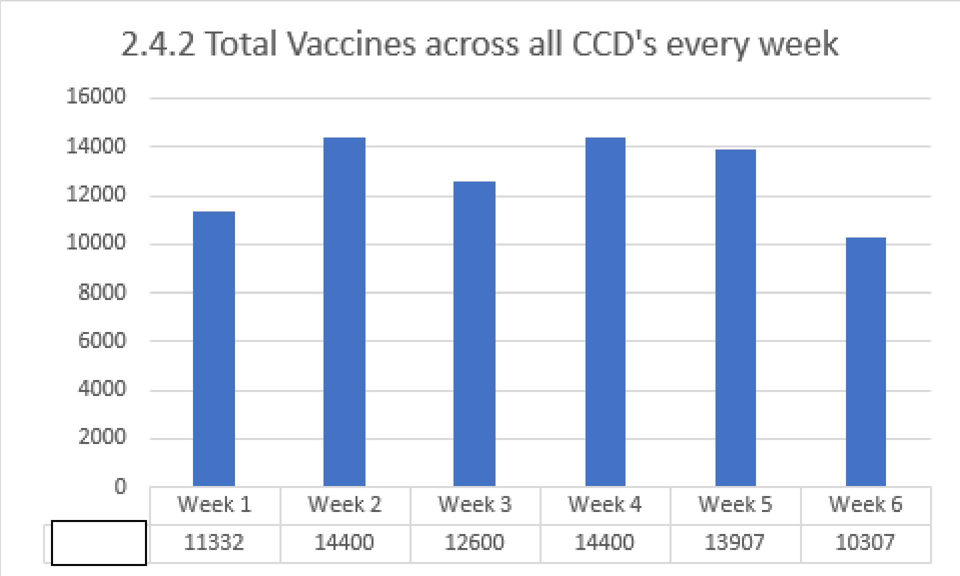
Table 2.4.1

LVC's to reach 1800 doses in week one
(ID-LVC) (0, 0)
(ID-LVC) (0, 1)
(ID-LVC) (0, 6)
(ID-LVC) (1, 3)
(ID-LVC) (1, 4)
(ID-LVC) (2, 5)

Graph 2.4.2 illustrates how the vaccines are sold off as well as representing the number of vaccines stored from week to week. The maximum possible amount of 14 400 is reached in weeks 2 and 4, and week 6 is the lowest amount, as to be expected.

This communication gave a result of $11,570,306.

Graph 2.4.2 Total vaccines across all CCD's for each week



2.5 COMMUNICATION 4

Description

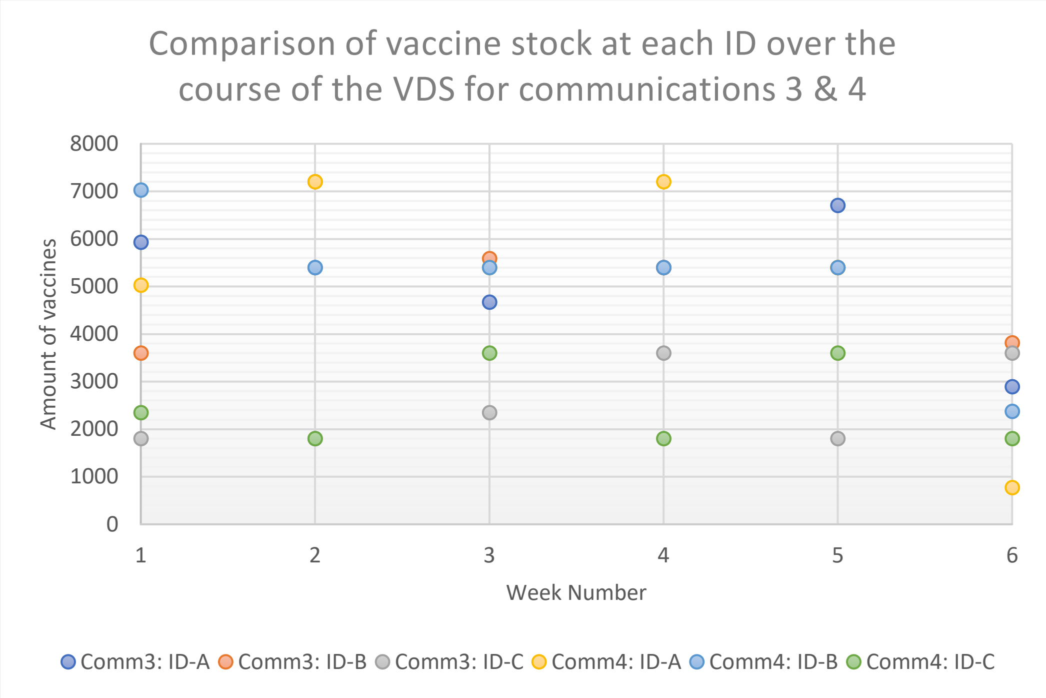
The fourth communication introduces a cost for delayed vaccinations, quantifying that for every week a person has to wait to get vaccinated, $10 will be added to the operation's total cost. New components were added to the formulation to incorporate this, including this forementioned cost r_t, a new decision variable for the number of people still unvaccinated u_t, and a new objective function with an additional term $\sum_{t \in T} u_t r_t$. There is also a new constraint which calculates u_t by subtracting the number of citizens already vaccinated from the total population, for every given week t.

Insights

As well as altering the pattern of vaccine distribution quite significantly, communication 4 has a large impact on the cost of the VDS directly through the additional term in the objective function; $\sum_{t \in T} u_t r_t$. Of the $13 274 353 total cost, $1 687 300; or 12.7%, came from the delayed vaccination expenses. This extra constraint has forced the model to encourage early vaccination as much as possible, and has lead to an overall decrease in the amount of people vaccinated in the last week. In communication 3, 10 307 people were vaccinated in week 6, however in communication 4 this number dropped to 4 946. This is also supported by Graph 2.5.1 below, where in week 6, all three ID's have less vaccine stock in communication 4 compared to communication 3.

However, despite this communication placing a penalty on delayed vaccinations, Table 2.5.1 below indicates that some of the other constraints from previous communications simply don't allow a larger portion of people to be vaccinated in the earlier weeks. Table 2.5.1 lists both the cumulative amount and percentage of people vaccinated in each week, as well as the percentage increase, and it is clear that this increase does not change at all; even to the fifth decimal place, for weeks 1-5. The most likely constraint which is limiting this increase to a seemingly set value of 18.714%, would be the 1800 maximum weekly dosage limit for each LVC.

This communication gave a result of $13,274,353.



Graph 2.5.1: Comparison of vaccine stock at each ID over the course of the VDS for communications 3 & 4

Table 2.5.1: The cumulative fraction of people vaccinated at the end of each week

Week	Number unvaccinated	Number vaccinated	Percentage	Percentage increase from previous week
1	62546	14400	18.714%	18.714%
2	48146	28800	37.429%	18.714%
3	33746	43200	56.143%	18.714%
4	19346	57600	74.858%	18.714%
5	4946	72000	93.572%	18.714%
6	0	76946	100.000%	6.428%

2.6 COMMUNICATION 5

Description

The fifth communication aims to ensure that the fraction of the population vaccinated in each CCD is relatively consistent. The goal is for the percentage vaccinated in each CCD over each week individually never exceeds a difference of 0.1 (i.e., 10%).

For this communication to be satisfied the distribution of the initial vaccines through the depots needed to be managed better to result in a more balanced distribution of doses amongst the depots.

Insights

Table 2.6.1 vaccinated max/min percentages of each CCD over each week

Weeks	Min	Max
1	0.1479	0.2479
2	0.3198	0.4198
3	0.5183	0.6183
4	0.7014	0.8014
5	0.8660	0.9660
6	1	1

As shown in Table 2.6.1 above the requirements laid out by Pacific Paradise have been met with all the maximum and minimum population percentages having a range of exactly 0.1, excluding week 6 as by this stage the whole population at all CCD's are vaccinated. This constraint is binding in all but week 6.

Another key factor to note is the presence of non-integer results within the data from communication 5; the first time this has been observed. This is because of the varying percentages of people vaccinated at each CCD changing every week, resulting in non-integer solutions, which was expected by adding this addition constraint. Ensuring each CCD gets vaccinated at roughly the same rate is beneficial because it means no district is being given priority over any other, and this may help with people being able to travel between districts. For example, no one would be in a situation where they are reluctant to travel to another CCD because that CCD is far less vaccinated.

This further benefits Pacific Paradise's management control, by ensuring all CCDs are vaccinated after six weeks and no CCD's will be done early as was occurring in previous communications. This makes it easier to control the vaccine distributions by ensuring the necessary amounts for each CCD are being received.

This communication gave a result of $13,287,841.

2.7 RESULTS SUMMARY

Table 2.7.1: Summary of results for each communication

Communication No	Optimised Total Cost ($) [to the nearest $]
1	9 981 300
2	11 534 485
3	11 570 306
4	13 274 353
5	13 287 841

The cost increases with each iteration, generally because of the fact that there are more constraints to account for. At present, GHB Solutions proposes that the vaccination distribution strategy from communication 5 be implemented in Pacific Paradise, however the company welcomes any further adjustments from the client if they are required. One further comment to be made is that this formulation does not take into account human error or human decision-making, which could include issues like the inability for someone to travel on-time to get vaccinated in the designated week, or people who prefer not to get vaccinated. It is not possible to model or predict these behaviours, and so GHB Solutions advises Pacific Paradise that these constraints be kept in mind when carrying out the VDS.

2.8 APPENDICES

2.8.1 COMMUNICATION 1

Table 1: Number of vaccines imported for each vaccine depot

ID-A	ID-B	ID-C
0	76946	0

Table 2: Number of vaccines transported from each ID to each LVC

	LVC0	LVC1	LVC2	LVC3	LVC4	LVC5	LVC6	LVC7
ID-A	0	0	0	0	0	0	0	0
ID-B	10307	14747	5935	5416	16469	12280	5860	5932
ID-C	0	0	0	0	0	0	0	0

Table 3: Number of citizens from each CCD who attended each LVC to receive their vaccine

CCD	Population	LVC0	LVC1	LVC2	LVC3	LVC4	LVC5	LVC6	LVC7
0	3210							3210	
1	3824		3824						
2	3535		3535						
3	2059					2059			
4	4436					4436			
5	2650							2650	
6	4260		4260						
7	3128		3128						
8	2652					2652			
9	2525					2525			
10	3533								3533
11	2399								2399
12	2298						2298		
13	2527				2527				
14	4797					4797			
15	3011	3011							
16	3495	3495							
17	3312						3312		
18	2889				2889				
19	2156			2156					
20	3801	3801							
21	2504						2504		
22	2151						2151		
23	2015						2015		
24	3779			3779					