REPORT TO THE BOSS

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FORMULATION FOR VACCINATION DISTRIBUTION

Sets

V	Set of Vaccine Depots (IDs) {ID-A, ID-B, ID-C}
A	Set of arcs from IDs to LVCs
L	Set of Local Vaccination Centres (LVCs) {LVC0, LVC1, …, LVC7}
B	Set of arcs from LVCs to CCDs
C	Set of Census Collection Districts (CCDs) {CCD0, CCD1, …, CCD24}

Data

$icost_v$	Importation cost of the vaccine at depot $v \in V$ (\$)
pop_c	The population/demand at a CCD for $c \in C$
$l1_a$	Length of arc $a \in A$ (km)
$l2_b$	Length of arc $b \in B$ (km)
$costIL$	Cost/km of travel between arc $a \in A$ (=\$0.2/km)
$costCL$	Cost/km of travel between arc $b \in B$ (=\$1/km)
$t1_a \in L$	The LVC that arc $a \in A$ flows to
$f1_a \in V$	The ID that arc $a \in A$ flows from
$t2_b \in C$	The CCD that arc $b \in B$ flows to
$f2_b \in L$	The LVC that arc $b \in B$ flows from
$maxID$	The maximum capacity at each ID $v \in V$ (=46,000 doses)
$upgrade_l$	The cost of upgrading LVC $l \in L$ $(\$)$
$closing_l$	The estimated savings from closing LVC $l \in L$ $(\$)$
$initialLVC$	The capacity of an LVC that is not upgraded (=13000 doses)
$increase$	Increase in capacity (decimal) if LVC upgraded (=0.5)
num	Number of LVC's a CCD $c \in C$ can be assigned to (=1)

Decision Variables

x_a	Number of vaccines flowing through arc $a \in A$
y_b	Number of vaccines flowing through arc $b \in B$
$s_l \in \{0,1\}$	Binary variable that represents 1 if LVC $l \in L$ is upgraded, 0 if not,
$cl_l \in \{0,1\}$	Binary variable that represents 1 if LVC $l \in L$ is closed, 0 if not.

$lc_b \in \{0,1\}$ Binary variable that represents 1 if CCD $t2_b = c \in C$ is assigned to get its vaccines from LVC $f2_b = l \in L$

Objective Function

Minimise total cost of the operation:

$$\min\left[\sum_{v\in V}\left(\sum_{\substack{a\in A\\ s.t.\ f1_a=v}} icost_v x_a\right)+\sum_{a\in A} l1_a costIL x_a+\sum_{b\in B} l2_b costCL y_b+\sum_{l\in L} upgrade_l s_l-\sum_{l\in L} closing_l cl_l\right]$$

The total cost to be minimised has five components:

- Importation of vaccines to the ID's
- Transportation of vaccines from ID's to LVC
- Cost of accessing the vaccines from CCD to LVC for each citizen
- Upgrading cost of an LVC
- Savings from closing of an LVC

Constraints

$$\sum_{\substack{b\in B\\ s.t.\ t2_b=c}} lc_b y_b = pop_c \qquad \forall c\in C \qquad (1)$$

$$(1-cl_l)\sum_{\substack{a\in A\\ s.t.\ t1_a=l}} x_a = \sum_{\substack{b\in B\\ s.t.\ f2_b=l}} y_b \qquad \forall l\in L \qquad (2)$$

$$y_b \ = \ 0\ s.t. l2_b = 0 \qquad \forall b\in B \qquad (3)$$

$$\sum_{\substack{a\in A\\ s.t.\ f1_a=v}} x_a \le maxID \qquad \forall v\in v \qquad (4)$$

$$\sum_{\substack{a\in A\\ s.t.\ t1_a=l}} x_a \le initialLVC(1+increase\times s_l) \qquad \forall l\in L \qquad (5)$$

$$\sum_{\substack{b\in B\\ s.t\ \ t2_b=c}} lc_b = num \qquad \forall c\in C \qquad (6)$$

$$x_a, y_a \ge 0 \qquad \forall a\in A, \forall b\in B \qquad (7)$$

Note: s.t. is an abbreviation for "such that".

Constraints Descriptions

Constraints (1) says the total amount of vaccines going to a CCD from its neighbouring LVC's must be equivalent to the population of the CCD, only if that CCD is assigned to the LVC.

Constraint (2) governs the demand of vaccines at an LVC, as the vaccines flowing out of it in a week must be equivalent to those flowing into it. It also set the vaccines at the LVC to be 0 if it is to be closed.

Constraint (3) addresses the fact that the citizens of CCD's are only able to obtain the vaccine from a neighbouring LVC (as a distance of 0km in the data represents a non-neighbouring LVC).

Constraints (4) and (5) says that the total vaccines imported to an ID and the total vaccines administered at an LVC, does not exceed its capacity throughout the operation. The capacity of an ID is constant, however the capacity of an LVC depends on whether its upgraded or not, for which the capacity of the LVC to be either 13000 doses (without upgrade) and 50% more for those that are upgraded.

Constraints (6) assigns each CCD to only access one of its neighbouring LVC.

Constraint (7) is a general constraint, defining the non-binary variables to be greater than or equal to 0 (this is an automatic constraint in Gurobi).

FORMULATION FOR ERADICATION PROCESS

Sets

O	Options {Option 1, Option 2, Option 3, Option 4}
C	CCDs {CCD0, …, CCD24}

Data

$cost_{c,o}$	The cost of choosing option $o \in O$ at CCD $c \in C$ ($)
$prob_o$	The probability of eradicating virus for option $o \in O$
$lprob_o$	The log (base 10) of the probability of eradicating virus for option $o \in O$
num	The number of options that are to be chosen for each CCD $c \in C$
$budget$	The set-out budget for this operation ($), this is tested for values ≤ $6285000 as that cost from Communication 9 was deemed too much for pacific paradise.

Decision Variable

$x_{c,o} \in \{0,1\}$	Binary variable that represents 1 if option $o \in O$ is picked for CCD $c \in C$, 0 if not.

Objective Function

Maximise The Sum of the Log of the probabilities:

$$\max \sum_{c \in C} \sum_{o \in O} lprob_o x_{c,0}$$

- From log laws, when $\log(a) + \log(b) = \log(ab)$. Hence, by adding all the log probabilities, it is found that:

  $$\sum_{c \in C} \sum_{o \in O} lprob_o x_{c,0} = \log_{10}\left(\prod_{c \in C} \sum_{o \in O} prob_o x_{c,0}\right)$$

  As log base 10 functions are increasing functions, by finding the maximum of $log\left(\prod_{c \in C} \sum_{o \in O} prob_o x_{c,0}\right)$, the objective function is essentially finding the max possible:

  $$\prod_{c \in C} \sum_{o \in O} prob_o x_{c,0}$$

  Which is the objective of this operation, to maximise the product of all probabilities for the chosen options at CCDs (the total probability of eradication across Pacific paradise, assuming independence).

Constraints

$$\sum_{o \in O} x_{c,o} = num \qquad \forall c \in C \qquad (1)$$

$$\sum_{c \in C} \sum_{o \in O} cost_{c,o} x_{c,o} \leq budget \qquad (2)$$

<u>Constraints Descriptions</u>

Constraint (1) says that only one option must be assigned to the respective CCD.

Constraint (2) states that the total cost of the operation to be smaller than or equal to the given budget.

Note: As the maximum budget to be tested is $6,285,000 and for a feasible solution, the budget must be at least the sum of the cheapest option for each CCD (found to be 1,496,000 choosing the 0.95 option), the budgets were tested in increments:

$$\frac{6285000 - 1496000}{50} = \$106{,}580$$

Ranging from $1,496,000 to $6,285,000.

Also note, the cost for maximum probability of eradication may not exactly equal the budget set out, rather the budget is a limitation on how much it would end up costing.