Space Exploration Engineering Mid-term Report A2

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INTRODUCTION

The purpose of this assignment is to put into practice the Clohessy–Wiltshire equations with a simple rendez-vous case between a target and a chaser without means of propulsion. Namely, we will first study how to find the appropriate initial velocity of the chaser, given a desired configuration at a certain time T; then, we will look into the velocity at the time of rendez-vous as well as the overall trajectory of the spacecraft.

A2A: TWO-IMPULSE RENDEZ-VOUS MANEUVER

In this section, we consider a target—on a circular orbit around the Earth—and a chaser, whose coordinates are defined on a target-centered, target-fixed frame of reference. As such, we define the coordinates of the chaser in this frame of reference wrt. a certain time t as

$$\vec{X}(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} \tag{1}$$

The velocity of the chaser is thus defined as follows:

$$\vec{V}(t) = \begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{y}(t) \end{bmatrix} \tag{2}$$

The Clohessy–Wiltshire equations, applied to the system considered, can then be written under the form

$$\begin{bmatrix} \vec{X}(t) \\ \vec{V}(t) \end{bmatrix} = \begin{bmatrix} A(t) & B(t) \\ C(t) & D(t) \end{bmatrix} \begin{bmatrix} \vec{X}(0) \\ \vec{V}(0) \end{bmatrix} \tag{3}$$

Where

$$A(t) = \begin{bmatrix} 1 & 0 & 6(s\theta - \theta) \\ 0 & c\theta & 0 \\ 0 & 0 & 4 - 3c\theta \end{bmatrix}$$

$$B(t) = \begin{bmatrix} (4/\omega)s\theta - 3t & 0 & (2/\omega)(c\theta - 1) \\ 0 & (1/\omega)s\theta & 0 \\ (2/\omega)(1 - c\theta) & 0 & (1/\omega)s\theta \end{bmatrix}$$

$$C(t) = \begin{bmatrix} 0 & 0 & 6\omega(c\theta - 1) \\ 0 & -\omega s\theta & 0 \\ 0 & 0 & 3\omega s\theta \end{bmatrix}$$

$$D(t) = \begin{bmatrix} 4c\theta - 3 & 0 & -2s\theta \\ 0 & c\theta & 0 \\ 2s\theta & 0 & c\theta \end{bmatrix}$$

With ω the angular speed of the target in its circular rotation around the Earth, and θ defined as being equal to ωt. Additionally, $c\theta$ and $s\theta$ stand for $\cos\theta$ and $\sin\theta$ respectively.

A consequence of Equation 3 is:

$$\forall t \geq 0, \vec{X}(t) = A(t)\vec{X}(0) + B(t)\vec{V}(0) \tag{4}$$

Let us now consider a hypothetical time T where the chaser meets the target. In other terms,

$$\vec{X}(T) = \vec{0} \tag{5}$$

Using Equation 4, we derive the following:

$$A(T)\vec{X}(0) + B(T)\vec{V}(0) = \vec{0} \tag{6}$$

$$\vec{V}(0) = -B^{-1}(T)A(T)\vec{X}(0) \tag{7}$$

The above formula is valid as long as B is invertible, which requires that $8(1 - \cos(\omega T)) - 3\omega T \sin(\omega T) \neq 0$

Similarly, we can calculate the velocity of the chaser at the time T thanks to the formula:

$$\vec{V}(T) = C(T)\vec{X}(0) + D(T)\vec{V}(0) \tag{8}$$

A2B: STUDY OF THE TRAJECTORY

In order to study the trajectory of the chaser wrt. the target, we developed a program in C++ which calculates the coordinates of the chaser until contact, given initial information on the problem. The source code for the program can be found on [1].

First, let us consider a simple case such as the one proposed for the assignment, and described by the table below.

x_0	100 km
y_0	0 m
z_0	0 m
h	400 km
T	3600 s

TABLE I: Parameters for the first test

Given these initial conditions, the program computes all positions of the chaser until the time of rendez-vous. Figure 1 shows the trajectory followed by the spacecraft, only considering its x and z coordinates (y stays equal to zero for the whole trajectory because of the initial position of the chaser).

This ellipse-shaped trajectory is coherent with the model used and the case studied where only x_0 is not equal to zero.

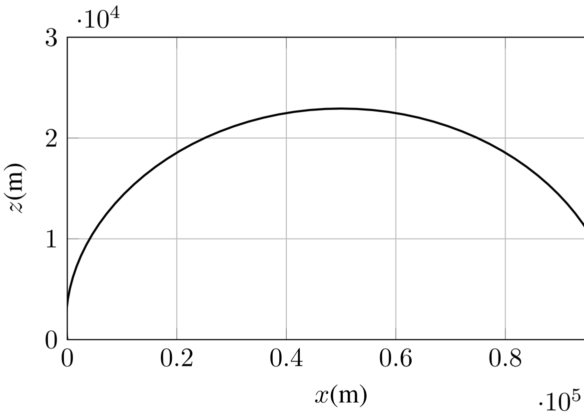

Fig. 1: Trajectory of the chaser from $t = 0$ to $t = T$

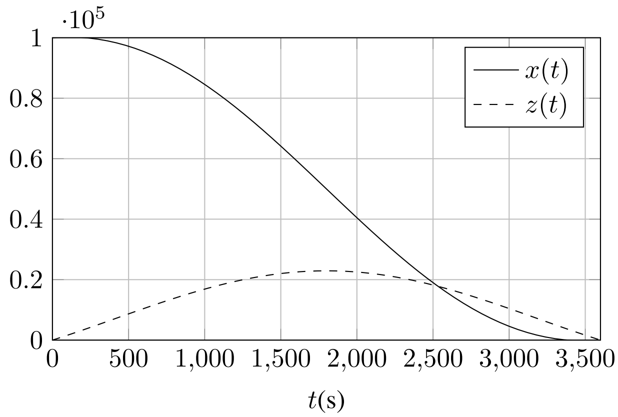

Fig. 2: x and z coordinates of the chaser as a function of time

We can also draw the evolution of both x and z coordinates as a function of time, as can be seen in Figure 2.

Both figures show evidence that the target does meet the chaser at $t = T$, as predicted by the equations.

The program also works for non-zero values of y_0 and z_0, as the plots in Figure 3 and Figure 4 showcase.

x_0	100 km
y_0	100 km
z_0	100 km
h	400 km
T	3600 s

TABLE II: Parameters for the second test

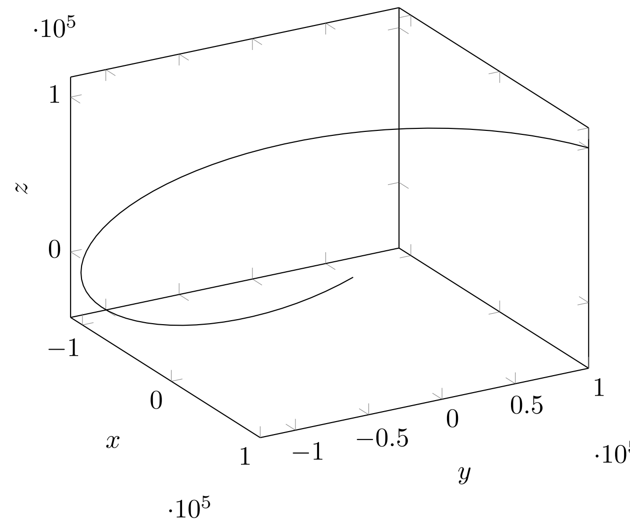

Fig. 3: 3D trajectory of the chaser from $t = 0$ to $t = T$

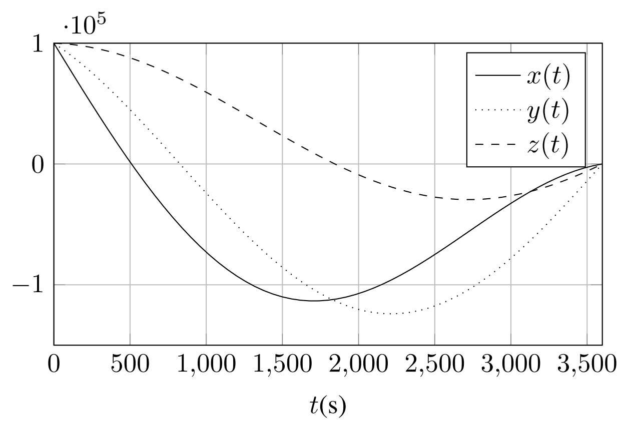

Fig. 4: x, y and z coordinates of the chaser as a function of time

References

[1] Romain Pessia. *Two-impulse rendez-vous maneuver calculator*. Github project on https://github.com/romainpessia/two-impulse-rendez-vous-maneuver-calculator, 2017.