Paper Id: | 199103 | **Roll No:**

B. TECH.
(SEM I) THEORY EXAMINATION 2019-20
MATHEMATICS-I

Time: 3 Hours   *Total Marks: 100*

Note: **1.** Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1.   **Attempt *all* questions.**

Q. No.	Question	Marks	CO
a.	Show that vectors $(1, 6, 4)$, $(0, 2, 3)$ and $(0, 1, 2)$ are linearly independent.	2	1
b.	Define Lagrange's mean value theorem.	2	2
c.	If $u = x(1 - y), v = xy$, find $\frac{\partial(u,v)}{\partial(x,y)}$.	2	3
d.	Show that vector $\vec{V} = (x + 3y)\hat{\imath} + (y - 3z)\hat{\jmath} + (x - 2z)\hat{K}$ is solenoidal.	2	5
e.	Find the value of 'b' so that rank of $A = \begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & b \end{bmatrix}$ is 2.	2	1
f.	Evaluate $\int_0^2 \int_0^1 (x^2 + 3y^2)dydx$.	2	4
g.	Find grad $\emptyset$ at the point $(2, 1, 3)$ where $\emptyset = x^2 + yz$	2	5
h.	If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.	2	3
i.	Find $\frac{du}{dt}$ if $u = x^3 + y^3, x = a\cos t, y = b\sin t$.	2	3
j.	Find the area lying between the parabola $y = 4x - x^2$ and above the line $y = x$.	2	4

SECTION B

2.   **Attempt any *three* of the following:**

Q. No.	Question	Marks	CO
a.	Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 4 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$ and hence find A^{-1}.	10	1
b.	If $y = e^{m\cos^{-1}x}$, prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + m^2)y_n = 0$. Hence find y_n when $x = 0$.	10	2
c.	If $u^3 + v^3 + w^3 = x + y + z$, $u^2 + v^2 + w^2 = x^3 + y^3 + z^3$ and $u + v + w = x^2 + y^2 + z^2$, then show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{(x-y)(y-z)(z-x)}{(u-v)(v-w)(w-u)}$.	10	3
d.	Evaluate the integral by changing the order of integration: $I = \int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$.	10	4
e.	Verify Stoke's theorem for the vector field $\vec{F} = (x^2 - y^2)\hat{\imath} + 2xy\hat{\jmath}$ integrated round the rectangle in the plane $z = 0$ and bounded by the lines $x = 0, y = 0, x = a, y = b$.	10	5

SECTION C

3.   **Attempt any *one* part of the following:**

Q. No.	Question	Marks	CO
a.	For what values of λ and μ the system of linear equations: $$x + y + z = 6$$ $$x + 2y + 5z = 10$$ $$2x + 3y + \lambda z = \mu$$ has (i) a unique solution (ii) no solution (iii) infinite solution Also find the solution for $\lambda = 2$ and $\mu = 8$.	10	1
b.	Find the rank of the matrix $A = \begin{bmatrix} 1 & 3 & 4 & 2 \\ 2 & -1 & 3 & 2 \\ 3 & -5 & 2 & 2 \\ 6 & -3 & 8 & 6 \end{bmatrix}$ by reducing it to normal form.	10	1

4.   **Attempt any *one* part of the following:**

Q. No.	Question	Marks	CO
a.	Verify the Cauchy's mean value theorem for the function e^x and e^{-x} in the interval $[a, b]$. Also show that 'c' is the arithmetic mean between a and b.	10	2
b.	Trace the curve $r^2 = a^2 \cos 2\theta$.	10	2

5.   **Attempt any *one* part of the following:**

Q. No.	Question	Marks	CO
a.	If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$, prove that $\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z} = 0$.	10	3
b.	Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.	10	3

6.   **Attempt any *one* part of the following:**

Q. No.	Question	Marks	CO
a.	Evaluate $\iint (x + y)^2 dx\, dy$, where R is the parallelogram in the xy-plane with vertices $(1,0), (3,1), (2,2), (0,1)$ using the transformation $u = x + y,\ v = x - 2y$.	10	4
b.	Find the volume of the region bounded by the surface $y = x^2,\ x = y^2$ and the planes $z = 0,\ z = 3$.	10	4

7.   **Attempt any *one* part of the following:**

Q. No.	Question	Marks	CO
a.	Verify the divergence theorem for $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{K}$ taken over the rectangular parallelepiped $0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le z \le 1$.	10	5
b.	Find the directional derivative of $\emptyset(x,y,z) = x^2yz + 4xz^2$ at $(1, -2, 1)$ in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$. Find also the greatest rate of increase of $\emptyset$.	10	5