

**B. TECH.**  
**(SEM II) THEORY EXAMINATION 2018-19**  
**MATHEMATICS-II**

Time: 3 Hours

Total Marks: 100

Note: Attempt all Sections. If require any missing data; then choose suitably.

**SECTION A**

1. Attempt all questions in brief.

2 x 10 = 20

QNo.	Question	Marks	CO
a.	Find the P.I of $\frac{d^2y}{dx^2} + 4y = \sin 2x$	2	1
b.	Solve simultaneous equations $\frac{dx}{dt} = 3y$ , $\frac{dy}{dt} = 3x$	2	1
c.	Find the volume of solid generated by revolving the circle $x^2 + y^2 = 25$ about y-axis.	2	2
d.	Evaluate $\Gamma\left(-\frac{5}{2}\right)$ . where $\Gamma$ is gamma function	2	2
e.	Find the Fourier constant $a_1$ of $f(x) = x^2$ , $-\pi \leq x \leq \pi$	2	3
f.	Discuss the convergence of sequence $a_n = \frac{2n}{n^2+1}$ .	2	3
g.	Show that complex function $f(z) = z^3$ is analytic.	2	4
h.	Define Conformal mapping.	2	4
i.	Evaluate $\int_0^{1+i} (x^2 - iy) dz$ along the path $y = x$ .	2	5
j.	Find residue of $f(z) = \frac{\cos z}{z(z+5)}$ at $z = 0$	2	5

**SECTION B**

2. Attempt any three of the following:

QNo.	Question	Marks	CO
a.	Use Frobenius method to solve $9x(1-x)\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 0$	10	1
b.	Apply Dirichlet integral to find the volume of an octant of the sphere $x^2 + y^2 + z^2 = 25$ .	10	2
c.	Find half range sine series of $f(x) = \begin{cases} x & 0 < x < 2 \\ 4-x & 2 < x < 4 \end{cases}$	10	3
d.	Show that $u = x^4 - 6x^2y^2 + y^4$ is harmonic function. Find complex function $f(z)$ whose $u$ is a real part.	10	4
e.	Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in regions (i) $1 <  z  < 2$ (ii) $2 <  z $	10	5

**SECTION C**

3. Attempt any one part of the following:

QNo.	Question	Marks	CO
a.	Solve $\frac{d^2y}{dx^2} + y = \tan x$ by method of variation of parameter.	10	1

b.	Solve $x^2 \frac{d^2y}{dx^2} - 2(x^2 + x) \frac{dy}{dx} + (x^2 + 2x + 2)y = 0$ by Normal Form.	10	1
----	--	----	---

**4. Attempt any one part of the following:**

QNo.	Question	Marks	CO
a.	Prove that $\beta(m, n) = \frac{\Gamma m \Gamma n}{\Gamma(m+n)}$ where $\Gamma$ is gamma function	10	2
b.	Use Beta and Gamma function to solve $\int_0^\infty \frac{1}{1+x^4} dx \int_0^{\frac{\pi}{2}} \sqrt{\cot \theta} d\theta$	10	2

**5. Attempt any one part of the following:**

QNo.	Question	Marks	CO
a.	Find the Fourier series of $f(x) = x \sin x$ , $-\pi \leq x \leq \pi$	10	3
b.	State D' Alembert's test. Test the series $1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} \dots \dots + \frac{x^n}{n^2+1} + \dots \dots \dots$	10	3

**6. Attempt any one part of the following:**

QNo.	Question	Marks	CO
a.	Let $f(z) = \frac{x^2 y^5 (x+iy)}{x^4 + y^{10}}$ when $z \neq 0$ , $f(z) = 0$ when $z = 0$ . Prove that Cauchy Riemann satisfies at $z = 0$ but function is not differentiable at $z = 0$ .	10	4
b.	Find Mobius transformation that maps points $z = 0, -i, 2i$ into the points $w = 5i, \infty, -\frac{i}{3}$ respectively.	10	4

**7. Attempt any one part of the following:**

QNo.	Question	Marks	CO
a.	Using Cauchy Integral formula evaluate $\int_c \frac{\sin z}{(z^2 + 25)^2} dz$ where $c$ is circle $ z  = 8$	10	5
b.	Apply residue theorem to evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$	10	5