BTECH
(SEM II) THEORY EXAMINATION 2021-22
ENGINEERING MATHEMATICS-II

Time:3 Hours | Total Marks:100

Notes-

- Attempt all sections and assume any missing data.
- Appropriate marks are allotted to each question, answer accordingly.

SECTION -A	Attempt all of following question in brief	Marks (10×2=20)	CO
Q.1(a)	Find the differential equation which represents the family of straight lines passing through the origins?		1
Q.1(b)	State the criterion for linearly independent solutions of the homogeneous linear nth order differential equation.		1
Q.1(c)	Evaluate: $\int_0^1 \frac{dx}{\sqrt{-\log x}}$.		2
Q.1(d)	Find the volume of the solid obtained by rotating the ellipse $x^2 + 9y^2 = 9$ about the x-axis.		2
Q.1(e)	Test the series $\sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{1}{n}$.		3
Q.1(f)	Find the constant term when $f(x) = 1 + \lvert x \rvert$ is expanded in Fourier series in the interval (-3, 3).		3
Q.1(g)	Show that $f(z) = z + 2\bar{z}$ is not analytic anywhere in the complex plane.		4
Q.1(h)	Find the image of $\lvert z - 2i \rvert = 2$ under the mapping $w = \frac{1}{z}$.		4
Q.1(i)	Expand $f(z) = e^{z/(z-2)}$ in a Laurent series about the point $z = 2$.		5
Q.1(j)	Discuss the nature of singularity of $\frac{\cot \pi z}{(z-a)^2}$ at $z = a$ and $z = \infty$.		5

SECTION -B	Attempt any three of the following questions	Marks (3×10=30)	CO
Q.2(a)	Solve: $\frac{d^2x}{dt^2} + \frac{dy}{dt} + 3x = e^{-t}$, $\frac{d^2y}{dt^2} - 4\frac{dx}{dt} + 3y = \sin 2t$.		1
Q.2(b)	Assuming $\Gamma n\, \Gamma(1-n) = \pi\, cosec\, n\pi$, $0 < n < 1$, show that $\int_0^{\infty} \frac{x^{p-1}}{1+x} dx = \frac{\pi}{\sin n\pi}$; $0 < p < 1$.		2
Q.2(c)	Test the series $\frac{x}{1.2} + \frac{x^2}{3.4} + \frac{x^3}{5.6} + \frac{x^4}{7.8} + \cdots\cdots$		3
Q.2(d)	If $f(z) = u + iv$ is an analytic function, find $f(z)$ in term of z if $u - v = \frac{e^y - \cos x + \sin x}{\cosh y - \cos x}$ when $f\left(\frac{\pi}{2}\right) = \frac{3-i}{2}$.		4
Q.2(e)	Evaluate by contour integration: $\int_0^{2\pi} e^{-\cos\theta} \cos(n\theta + \sin\theta)\, d\theta$; $n \epsilon I$.		5

BTECH
(SEM II) THEORY EXAMINATION 2021-22
ENGINEERING MATHEMATICS-II

SECTION -C	Attempt any one of the following questions	Marks (1×10=10)	CO
Q.3(a)	Use the variation of parameter method to solve the differential equation $(D^2-1)y = 2(1-e^{-2x})^{-1/2}$		1
Q.3(b)	Solve: $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\log(1+x)$.		1

SECTION -C	Attempt any one of the following questions	Marks (1×10=10)	CO
Q.4(a)	The arc of the cardioid $r = a(1+\cos\theta)$ included between $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ is rotated about the line $= \frac{\pi}{2}$. Find the area of surface generated.		2
Q.4(b)	Evaluate $\iiint xyz\ \sin(x+y+z)dx\,dy\,dz$, the integral being extended to all positive values of the variables subject to the condition $+y+z \le \frac{\pi}{2}$.		2

SECTION -C	Attempt any one of the following questions	Marks (1×10=10)	CO
Q.5(a)	Test for convergence of the series $\frac{a+x}{1!} + \frac{(a+2x)^2}{2!} + \frac{(a+3x)^3}{3!} + \cdots\cdots$		3
Q.5(b)	Obtain Fourier series for the function $f(x) = \begin{cases} 1+\frac{2x}{\pi}, & -\pi < x < 0 \\ 1-\frac{2x}{\pi}, & 0 < x < \pi \end{cases}$ Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} \cdots\cdots = \frac{\pi^2}{8}$.		3

SECTION -C	Attempt any one of the following questions	Marks (1×10=10)	CO
Q.6(a)	Prove that $w = \frac{z}{1-z}$ maps the upper half of the z-plane onto upper half of the w-plane. What is the image of the circle $\lvert z\rvert = 1$ under this transformation?		4
Q.6(b)	Find a bilinear transformation which maps the points $i, -i, 1$ of the $z-$plane into $0, 1, \infty$ of the $w-plane$ respectively.		4

SECTION -C	Attempt any one of the following questions	Marks (1×10=10)	CO
Q.7(a)	Evaluate $\oint_c \frac{e^z}{z(1-z)^3}dz$, *where c is* (i) $\lvert z\rvert = \frac{1}{2}$ (ii) $\lvert z-1\rvert = \frac{1}{2}$ (iii) $\lvert z\rvert = 2$.		5
Q.7(b)	Find the Taylor's and Laurent's series which represent the function $\frac{z^2-1}{(z+2)(z+3)}$ when (i) $\lvert z\rvert < 2$ (ii) $2 < \lvert z\rvert < 3$ (iii) $\lvert z\rvert > 3$.		5