



Asia Regional Finals–Manila Site  
2006 October 27

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PROBLEM F

## Period of an Infinite Binary Expansion

Let  $\{x\} = 0.a_1a_2a_3\dots$  be the binary representation of the fractional part of a rational number  $z$ . Suppose that  $\{x\}$  is periodic then, we can write

$$\{x\} = 0.a_1a_2\dots a_r(a_{r+1}a_{r+2}\dots a_{r+s})^\omega$$

for some integers  $r$  and  $s$  with  $r \geq 0$  and  $s > 0$ . Also,  $(a_{r+1}a_{r+2}\dots a_{r+s})^\omega$  denotes a nonterminating and repeating binary subsequence of  $\{x\}$ .

The subsequence  $x_1 = a_1a_2\dots a_r$  is called the *preperiod* of  $\{x\}$  and  $x_2 = a_{r+1}a_{r+2}\dots a_{r+s}$  is the *period* of  $\{x\}$ .

Suppose that  $|x_1|$  and  $|x_2|$  are chosen as small as possible then  $x_1$  is called the *least preperiod* and  $x_2$  is called the *least period* of  $\{x\}$ .

For example,  $x = \frac{1}{10} = 0.0001100110011(00110011)^\omega$  and 0001100110011 is a preperiod and 00110011 is a period of  $\frac{1}{10}$ .

However, we can write  $\frac{1}{10}$  also as  $\frac{1}{10} = 0.0(0011)^\omega$  and 0 is the least preperiod and 0011 is the least period of  $\frac{1}{10}$ .

The least period of  $\frac{1}{10}$  starts at the 2nd bit to the right of the binary point and the length of the least period is 4.

Write a program that finds the position of the first bit of the least period and the length of the least period where the preperiod is also the minimum of a positive rational number less than 1.

### Input

Each line is test case. It represents a rational number  $p/q$  where  $p$  and  $q$  are integers,  $p \geq 0$  and  $q > 0$ .

### Output

Each line corresponds to a single test case. It represents a pair where the first number is the position of the first bit of the least period and the second number is the length of the least period of the rational number.



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Sample Input

1/10  
1/5  
101/120  
121/1472

Sample Output

Case #1: 2,4  
Case #2: 1,4  
Case #3: 4,4  
Case #4: 7,11