

Digital Signal Processing

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Abstract

These are notes on **DSP**.

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1 Introduction

1.1 Overview

Some prerequisite:

In order to study this course better, you need to have some mathematical maturity and a foundation of calculus.

The textbook we use is **Discrete-Time Signal Processing, 3rd edition** written by A. V. Oppenheim.

1.2 Some Suggestions

In the first subsection, let's give the following suggestions.

No reading, No learning.

After we enter a high level of learning, we mainly depend on reading, because no one will tell us the key points. The reason for setting up a series of courses is to make our learning more targeted.

No writing, No reading.

What is effective reading? You must write something in the paper, such as the math notations. Only in that way will your brain not skip many details.

No data, No truth.

In the vast majority of scientific fields, we must apply theories and methods to real data rather than relying on simulation.

No Analytic, No understanding.

If you say, "You have well understood something", you must provide a detailed analysis using symbolic language.

No programming, No cognition.

Finally, if you wish to have an intuitive cognition of something, the typical approach is to use a series of visualization methods (such as pictures, tables...) to present the images in your mind.

1.3 Digital vs Analog

There are the following differences between the two.

First, digital means **programmable**. Analog rely on hardware, digital are true software, which is very flexible and easy to change.

Second, in the digital world we use bit to describe precision. And digital has a high and controllable precision.

Third, storing, digital has a high storage density and compressible.

Finally, digital's cost is low, because it's manufactured on a silicon wafer.

1.4 Course Arrangement

The digital signal processing course has the following chapters.

Chapter1: Preliminary for Digital.

In this chapter, we will introduce Discrete Signal and System from the following two perspectives:

- Time domain;
- Frequency domain.

Chapter2: How to obtain digital signal.

In this chapter, we will introduce:

- A/D: Using sampling to help us move from the real word (continuous) to digital word.
- D/A: Return journey.

Chapter3: How to process digital signal.

In this chapter, we have two basic tools for recognizing signals:

- Fourier Transform.
- Z transform.

Also we will introduce Linear Filtering;

- Representation: Discrete Convolution.
- Implementation: Time domain \longleftrightarrow Frequency domain.
- Architecture: from software \longrightarrow chip.

Chapter4: How to improve performance.

We have the following pursuit:

- Faster: FFT.
- More accurate
 - Quantization Noise.
 - Finite word length.

Chapter5: How to design linear filters.

We will introduce the following classic filters:

- FIR.
- IIR.
- Hilbert transform.

Chapter6: How to extend linear filters.

We will introduce:

- Real signal \longrightarrow Complex signal.
- Multirate filters.

Chapter7: How to apply linear filters.

1.5 History of DSP

Now let's briefly review the history of DSP.

1730, Fourier, Fourier Transform.

In the process of studying the heat conduction, Fourier discovered that:

any complex motion \longleftrightarrow linear combination of simple harmonic motion

and that is the Fourier Transform.

1900, Marconi, Electromagnetic waves are simple harmonics

This discovery extends Fourier Transform from thermal to electromagnetic.

1930, Bode, Control Science.

In the field of control science, the first step is perception. Perception is sampling to get data, and extract useful information from the data.

1940, Word War 2.

During this period, two important discoveries were made:

- Wiener Filtering: the foundation of Optimal Filtering (Stochastic Process required).
- Nyquist-Shannon Sampling Theorem.

1950, The Apollo Moon-Landing Project.

This plan gave rise to Kalman Filtering, another methods of Optimal Filtering.

1960, Cooley and Tukey, FFT.

This is the origin of signal processing.

1980, DSP (processor).

A special CPU designed for signal processing task.

2 Discrete Time Signal and System

2.1 Conventions

Considering the continuous signal $x(t)$, we can find it has two elements:

- Independent Variable: It can be time or space, here is time t .
- Dependent Variable.

Therefore, we can draw a conclusion: Signal is a function.

However, a mathematical function often has analytical expression, for example:

$$x(t) = 3t^2 \quad (1)$$

But signals often don't have explicit analytical expressions, so we need to process the signal to get information.

Now let's introduce continuous-time signal and discrete-time signal:

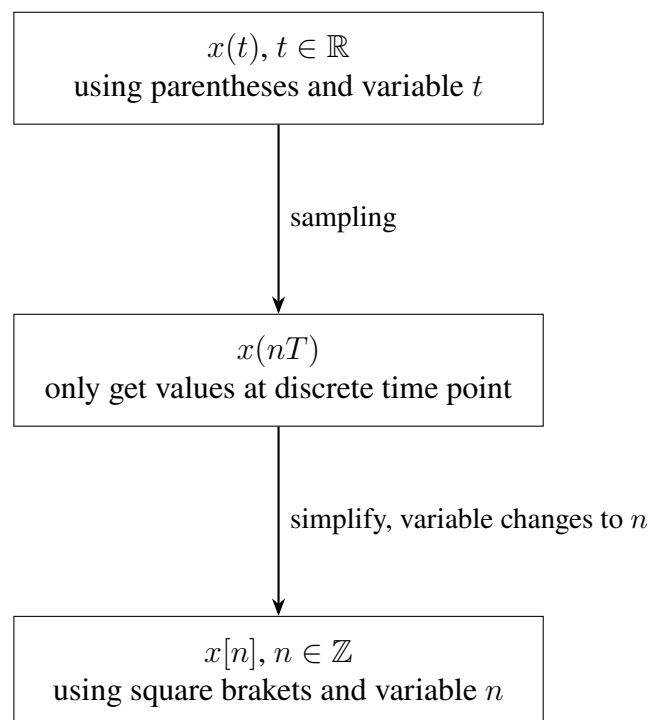


Figure 1: Signals

2.2 Special Signals

Here we introduce some signals.

First, Impulse signal. Definition:

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases} \quad (2)$$

The most important point about this signal is every signal $x[n]$ can be decomposed into the linear combination of shifted impulse $\delta[n - k]$:

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k] \underbrace{\delta[n - k]}_{\text{fundamental element}}, k \in \mathbb{Z} \quad (3)$$

And this formula leads to an important idea:

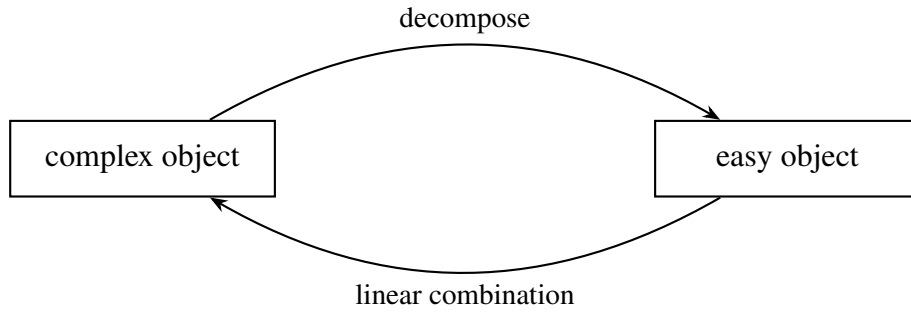


Figure 2: An important Idea

This idea tells us: Complex objects can be decomposed into the linear combination of many easy objects.

Second, Unit step. Definition:

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases} \quad (4)$$

So what is the relation of $u[n]$ and $\delta[n]$? We can easily get:

$$\begin{aligned} \delta[n] &= u[n] - u[n - 1] \\ u[n] &= \sum_{k=-\infty}^n \delta[k] \end{aligned} \quad (5)$$

Finally, we should introduce an important function:

$$x_{\omega}(t) = e^{j\omega t} \quad (6)$$

The subscript of x is ω , means the frequency.

Why do we introduce it? Because if we perform a linear transform on this function, the frequency doesn't change.

And we can give the discrete-time version:

$$x_{\omega}[n] = e^{j\omega n} \quad (7)$$

As $n \in \mathbb{Z}$, you can get an interesting property:

$$\begin{aligned} x_{\omega+2\pi}[n] &= e^{j(\omega+2\pi)n} \\ &= e^{j\omega n} \end{aligned} \quad (8)$$

This means, in the frequency domain, this function has periodicity, and the period is 2π .

What frequency does 2π here correspond to in the physical world? You will get the answer in the following chapter.

2.3 Discrete-Time System

We use the following diagram to represent the system and its function:

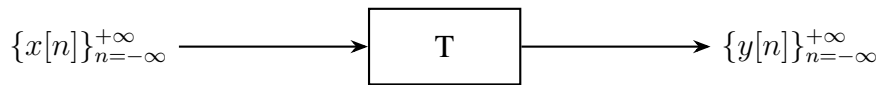


Figure 3: $y[n] = T(\{x[n]\})$

Why does we use $\{x[n]\}_{n=-\infty}^{+\infty}$ and $\{y[n]\}_{n=-\infty}^{+\infty}$ to represent the input and output? Because the system is divided into:

- Memory: $y[n]$ (output at time point n) depends on current and past inputs.
- Memoryless: $y[n]$ (output at time point n) only depends on current time point input $x[n]$.

So when we express the function of a system using symbols, a more rigorous way of writing it is :

$$y[n] = T(\{x[n]\}) \quad (9)$$

This means the system T operates on a sequence of inputs, not only one input at time n . But for simplicity, we write it:

$$y[n] = T(x[n]) \quad (10)$$

Next, we introduce some typical systems.

Delay Device:

$$y[n] = x[n - n_d] \quad (11)$$

n_d is a scalar.

How should we understand the function of the system? We can give an example.

Suppose n_d is 2, and we want to get the output at time 3, that is $y[3]$. According to the formula, we have:

$$\begin{aligned} y[3] &= x[3 - n_d] \\ &= x[1] \end{aligned} \quad (12)$$

This means, if we want to get the current output, we need to use the past input. So this system is **memory**.

Integrator:

$$y[n] = \sum_{k=-\infty}^n x[k] \quad (13)$$

This formula leads to an important idea:

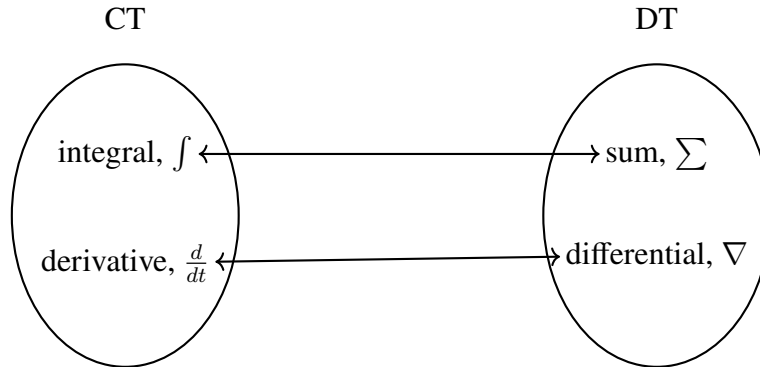


Figure 4: Continuous and Discrete Time Operations

Obviously, Integrator is **memory**.

Square Device:

$$y[x] = x^2[n] \quad (14)$$

Obviously, Integrator is **memoryless**.

2.4 Linearity

Linearity is an important property of systems, and also the key to our research.

Let's give the definition from the surface. If a system T is a linear system, then:

- Additivity: $T(x[n] + y[n]) = T(x[n]) + T(y[n])$.
- Scaling property: $T(\alpha x[n]) = \alpha T(x[n])$.

(we don't know it's "memory" or "memoryless").

This definition just gives us the surface input-output relation, but the system is still a black box.

Now let's analyze the internal structure of the system using [An important Idea](#).

Suppose T is a linear system, look at the following operations:

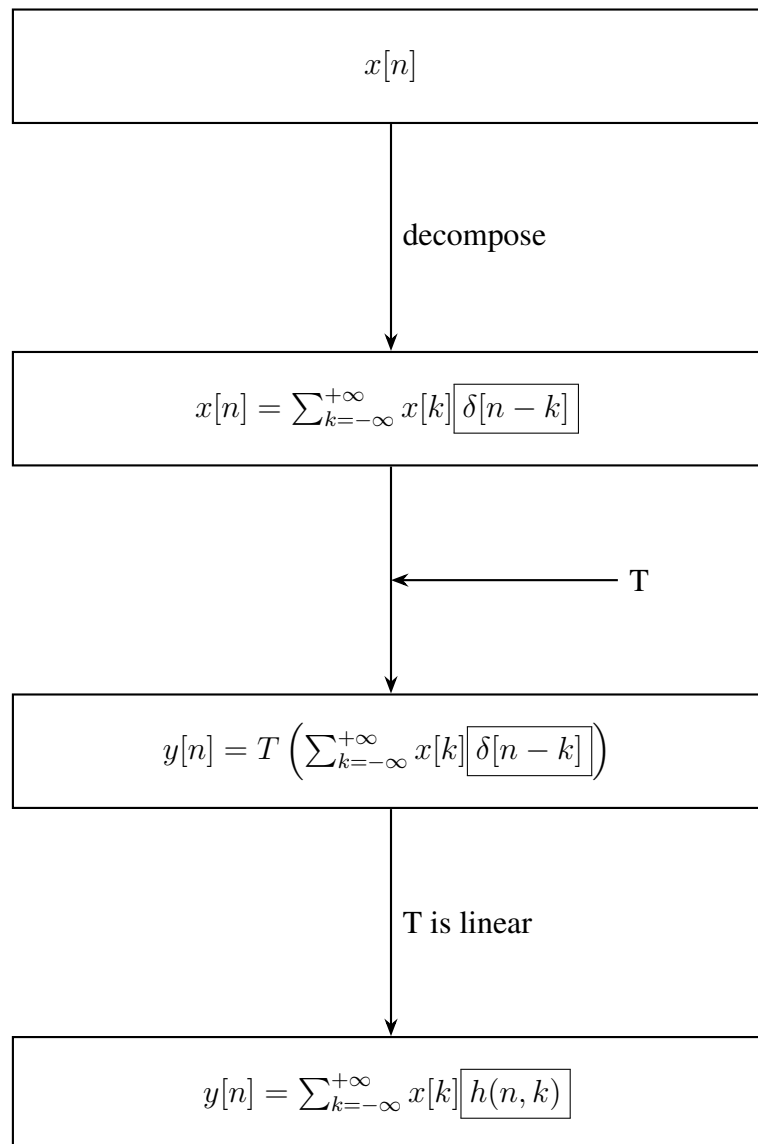


Figure 5: System Process

So we realize the following function:

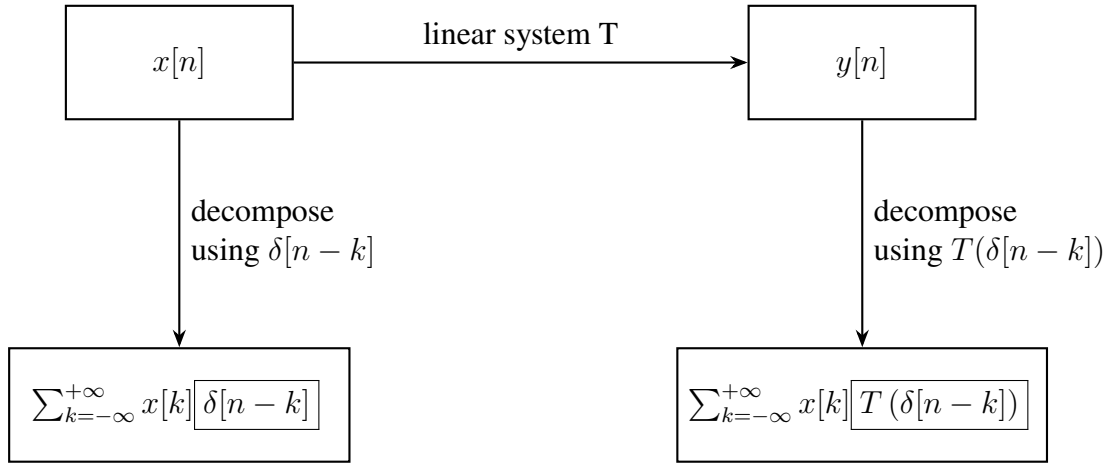


Figure 6: Summary of Linear System Derivation

That is to say, we decompose the input $x[n]$ into the linear combination of impulses. And as the system T is linear, the output can also be decomposed into the linear combination, and the elements is impulse response:

$$h(n, k) = T(\delta[n - k]) \quad (15)$$

2.5 Matrix Perspective

Matrix is also a linear transform.

Suppose we have the following matrix T :

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad (16)$$

We use this matrix to operate on the following column vector:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (17)$$

The results are:

$$\begin{bmatrix} \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix} & \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix} & \begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_1 \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix} + x_2 \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix} + x_3 \begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix} \quad (18)$$

How can we using the idea above to explain this?

Actually, we perform the following operations:

$$\begin{aligned} T \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} &= T \left(x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right) \\ &= x_1 T \left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right) + x_2 T \left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right) + x_3 T \left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right) \end{aligned} \quad (19)$$

We have the similar "impulse":

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (20)$$

And we have the similar "impulse response":

$$T \left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix}, \quad T \left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix}, \quad T \left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix} \quad (21)$$

From this example: "impulse response" is a nature of the system itself, has nothing to do with the input.