Signal and System

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Abstract

These are notes on **Signal and System**.

Contents

1 Lecture1

1.1 What is a Signal

In this subsection, we will introduce some notions in **SS** course.

First, what is a signal? A signal is a **function**. Suppose **x** is a signal, then it can represent the following mapping relationship:

$$\begin{aligned} \mathbb{R}(\text{reals}) &\longrightarrow \mathbb{R}(\text{reals}) \\ \mathbb{Z}(\text{integers}) &\longrightarrow \mathbb{R}(\text{reals}) \\ \mathbb{Z}(\text{reals}) &\longrightarrow \mathbb{C}(\text{complexes}) \end{aligned} \tag{1}$$

In the above formula, we call the left **domain or input space**, we call the right **range or output sapce**, referring to the definition in the function,

At the same time, an element in domain is called **independent variable**, an element in range is called **dependent variable or the value of function**.

After we give the doamin and the range, we need a **rule** to map every element in the domain to the range, which says what we operate on the the independent variable. The rule is also called the **function relation**.

For example, considering the following euqation:

$$x(n) = \cos\frac{\pi}{4}n \tag{2}$$

we have the following relations:

- **domain**: $\mathbb{Z}$, a set of all integers;
- **independent variable**: n, integers;
- **range**: all real numbers in [-1,1];
- **value of function**: $x(n)$;
- **function relation**: $\boldsymbol{x}$, $\cos\frac{\pi}{4}[\cdot]$

1.2 DT Signal and CT Signal

Now let's introduce **DT** and **CT**.

For a signal **x**, if the domain is $\mathbb{Z}$ or the set of all integers, then the signal **x** is called **Discrete-Time** signal (**DT**). For example:

$$x(n) = \cos \frac{\pi}{4} n, \quad n \in \mathbb{Z} \tag{3}$$

For a signal $\boldsymbol{x}$, if the domain is $\mathbb{R}$ or the set of all reals, then the signal $\boldsymbol{x}$ is called **Continuous-Time** signal (**CT**). Also, you can call **CT signal** as **Analog signal**. For example:

$$x(t) = \mathrm{e}^{-t}, \quad t \in \mathbb{R} \tag{4}$$

Meanwhile:

- if we use n as the independent variable, that means the signal is **DT**;
- if we use t as the independent variable, that means the signal is **CT**;

these are the conventions.

Supplement: Does the signal have to be the function of **only time**? The answer is **No**. For example, the independent variables can be **space coordinates**.

1.3 Discrete-Time Signal and Digital Signal

Sometimes, we don't distinguish **digital signal** and **discrete-time signal**, but they still have obvious differences.

In terms of digital signal, it still meets the definition of function. However, everything (including independent variable in domain and dependent variable in range) are represented by digits.

In terms of discrete-time signal, the independent variable in domain is discrete, but the values of the function can be reals (such as infinite recurring decimal).

But for the computers, we can't have infinite precision, so we need to quantify the numbers, which will cause us to lose accurcy.

1.4 Examples and Conventions

Let's give some examples about **DT** and **CT** signal.

First, there is a continuous time example, a speech signal:

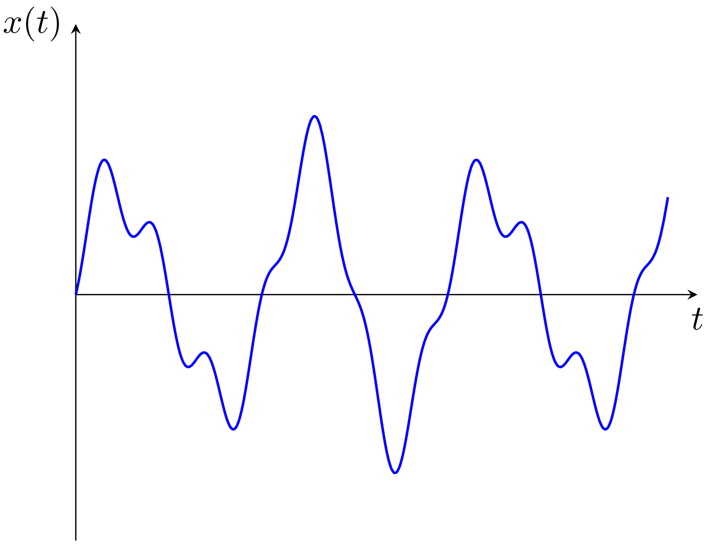

Figure 1: A Speech Signal

and there is an exp-deacy signal:

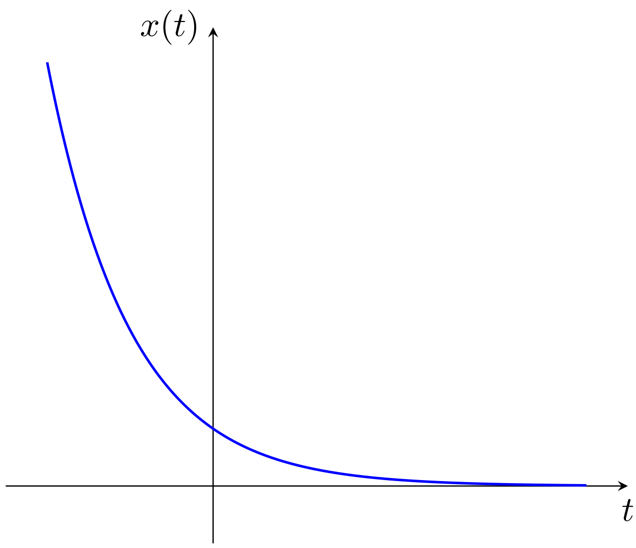

Figure 2: $x(t) = e^{-t}$

For the **CT** signal, we have the following conventions:

- x at time t: $x(t)$, refers to a specific value of the signal at time t;
- x without any arguments: $\boldsymbol{x}$, refers to the entire signal in $(-\infty, \infty)$;

However, in practice, when we define a signal, $x(t)$ also refers to the entire signal, not a specific value at time t.

Second, there is a discrete time signal:

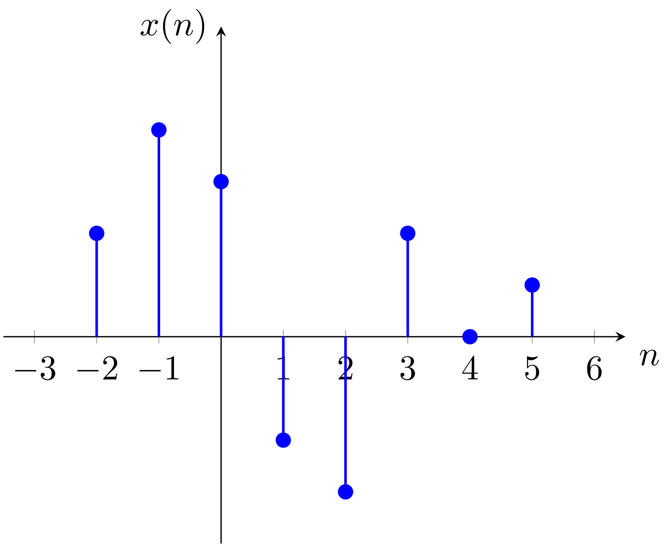

Figure 3: An Example for **DT** signal

The picture above is called **lollipops** or **stem plots**, which is designed specifically for the **DT** signal. We can find that, it's discrete on the domain.

1.5 Some Important DT signal

Now we introduce some important DT signal.

First, Kronecker Delta / DT Impulse.

In Lollipop language, it is represented as delta of n:

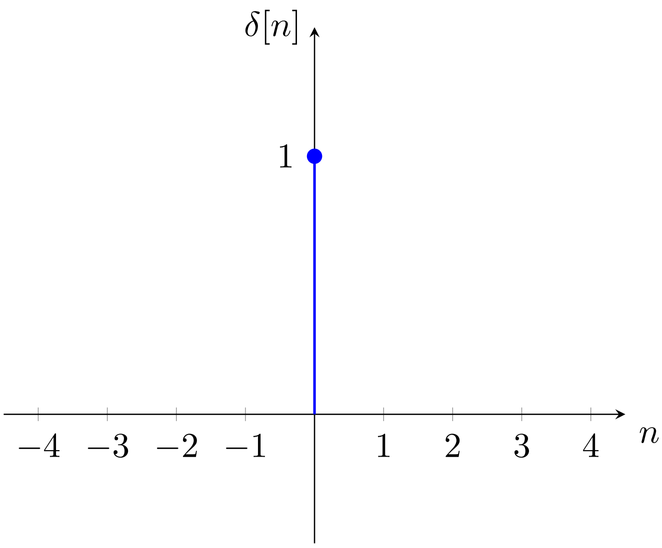

Figure 4: Kronecker Delata: $\delta[n]$

In mathmatical language, we can define $\delta[n]$:

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases} \tag{5}$$

From this example, you can find that, we use Square Brakets to enclose the variable n. And $\delta[n]$ is the most fundamental unit in DT signal.

Second, DT Unit Step.

In Lollipop language, the picture is as follows:

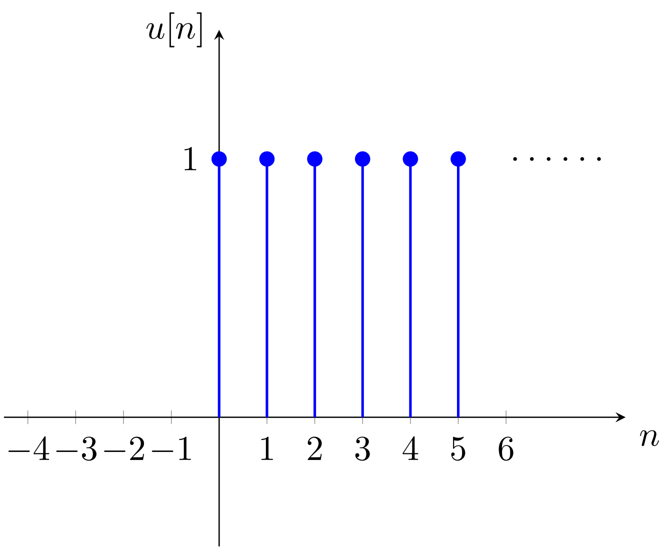

Figure 5: DT Unit Step: $u[n]$

In mathmatical language, we can define $u[n]$:

$$u[n] = \begin{cases} 1, & n \geqslant 0 \\ 0, & n \leq 0 \end{cases} \tag{6}$$

Also, we can define CT unit step $u(t)$:

$$u(t) = \begin{cases} 1, & t \geqslant 0 \\ 0, & t \leq 0 \end{cases} \tag{7}$$

and here is the picture:

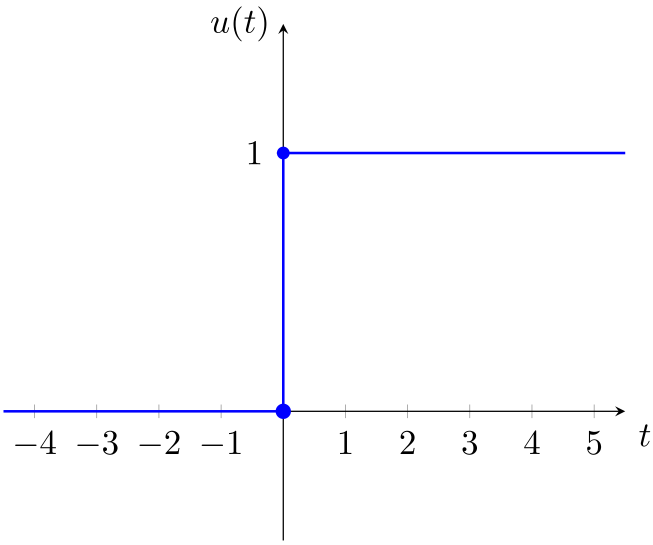

Figure 6: CT Unit Step: $u(t)$

From this example, you can find that, we use Circle Brakets to enclose the variable t.

1.6 Signal Addition

Before the official start of our work, let's introduce shifted impulses.

Look at the following pictures:

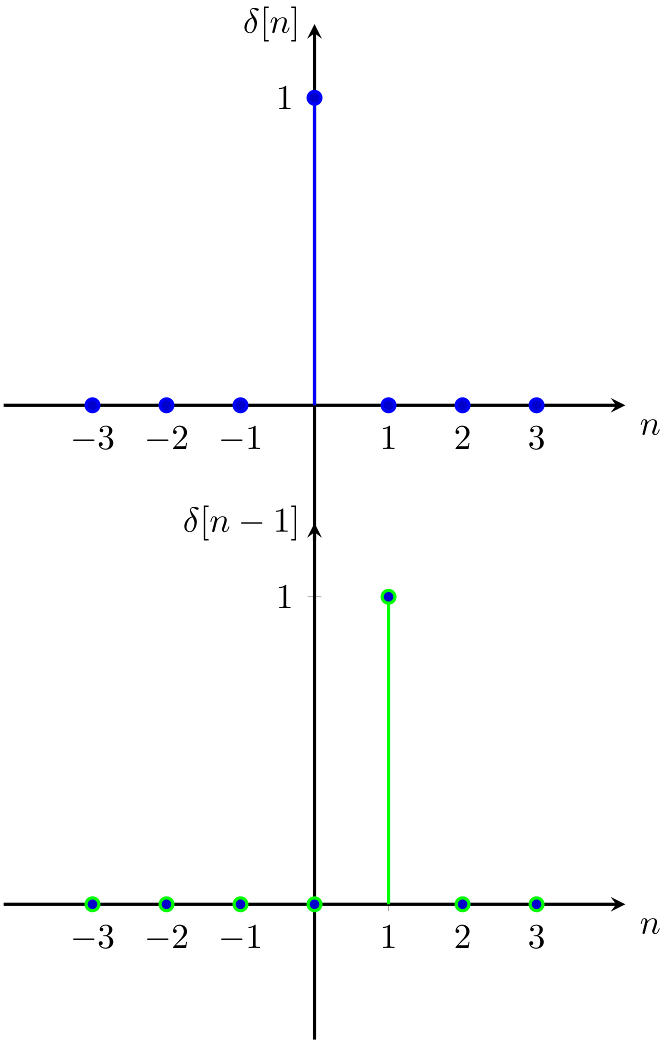

Figure 7: Shifted Impulses: $\delta[n]$ and $\delta[n-1]$

When we operate on the time variable n by adding or substracting it, the time axis remains unchanged, only the function graph moves left or right.

Now suppose we carry out the following operations:

$$x[n] = \delta[n] + \delta[n-1] \tag{8}$$

what will we get? We will get the following Lollipop:

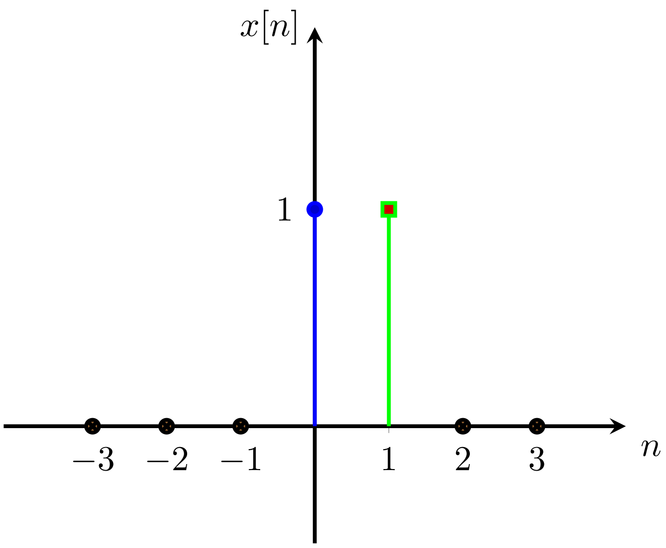

Figure 8: Signal Addition: $\delta[n] + \delta[n-1]$

So remember:

An impulse is not a scalar, it's a vector or tensor!

Signal addition means adding up the values of each signal at each time point!

1.7 Express $u[n]$ with $\delta[n]$

Now do you know how to use $\delta[n]$ to express $u[n]$?

Yes, we can give:

$$\begin{aligned} u[n] = \quad & \delta[n] \\ & +\delta[n-1] \\ & +\delta[n-2] \\ & +\delta[n-3] \\ & +\cdots \\ & +\delta[n-k] \end{aligned} \tag{9}$$

each shifted impulses is a vector. And we can give a more concise version:

$$u[n] = \sum_{k=0}^{\infty} \delta[n-k] \tag{10}$$

Here, it is necessary for us to emphasize the meaning of =: at every time point, the value of left signal is equal to value of the right signal.

Now let's take a look at the formula from another perspective:

$$u[n] = \sum_{k=0}^{\infty} \delta[n-k] \tag{11}$$

Make the following variable substitutions:

$$l = n - k \tag{12}$$

Then:

$$k = n - l \tag{13}$$

Substitute back, eliminate the variable k, change the upper and lower bounds of the summation:

$$u[n] = \sum_{l=-\infty}^{n} \delta[l] \tag{14}$$

How can we understand the equation above? First of all, we should know:

When we use the fixed variable n, we consider it as a signal, a function or a vector. (e.g. $f(x)$);

When we use another variable or a scalar, we might consider it as a value. (e.g. $f(10)$ or $f(k)$)

so the above equation is a definition, or represented as a signal.

So how can we calculate $u[10]$ using the equation? We do as follows:

$$u[10] = \cdots + \delta[-100] + \cdots + \delta[0] + \cdots + \delta[10] = 1 \tag{15}$$

and we can find, only $\delta[0] = 1$, other components are all 0.

We can regard the $\sum$ as a big net, only the net's bounds catch the 0 (the 0 is between $-\infty$ and n), then the result is 1:

$$u[n] = \begin{cases} 0, & n \leq 0 \quad (0 \notin (-\infty, n)) \\ 1, & n = 0 \\ 1, & n > 0 \quad (0 \in (-\infty, n)) \end{cases} \tag{16}$$

1.8 Integral and Derivative in DT

Now let's re-examine the formula:

$$u[n] = \sum_{l=-\infty}^{n} \delta[l] \tag{17}$$

we could say: In DT, the unit step is the cumulative sum of δ. In other words, it's the DT integral of δ.

Then, you can easily get: In DT, δ is the derivative of the unit step.

How can we verify the above idea? We can try to express $\delta[n]$ with $u[n]$.

Look at the following pictures:

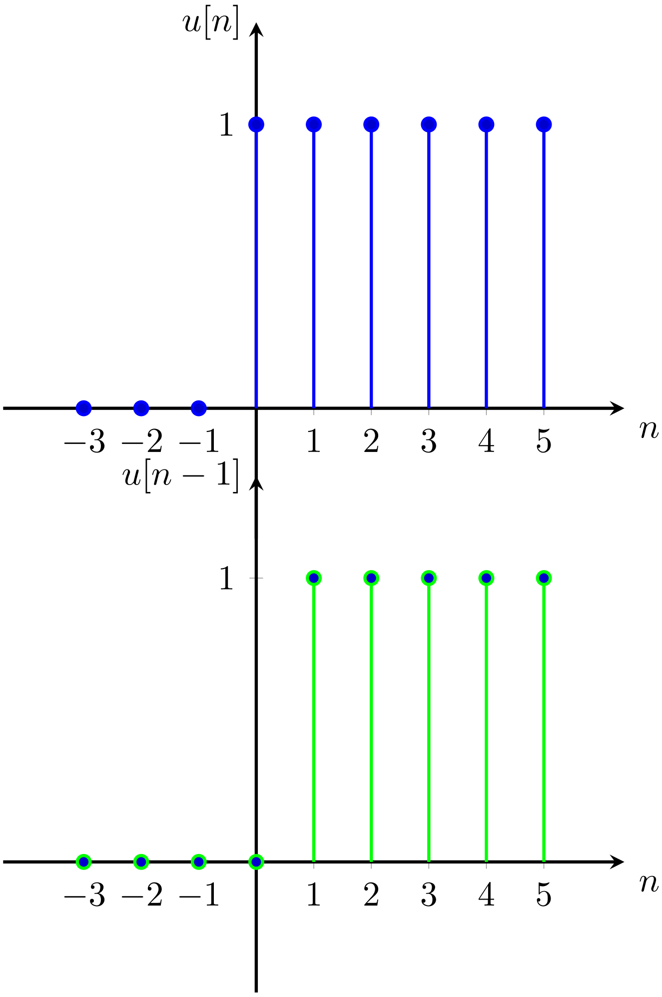

Figure 9: $u[n]$ and $u[n-1]$

So you can easily get:

$$\delta[n] = \frac{u[n] - u[n-1]}{1} \tag{18}$$

If we substitute 1 to Δ, we get:

$$\delta[n] = \frac{u[n] - u[n-1]}{\Delta} \tag{19}$$

Actually, it's the definition of the limit when we operate:

$$\Delta \to 0 \tag{20}$$

However, in DT, we cann't get a scalar smaller than 1, because we only use integers.

To sum up:

$$\delta[n] \underset{\frac{d}{dt} \text{ or } \nabla}{\overset{\int \text{ or } \sum}{\rightleftarrows}} u[n]$$

Figure 10: Relation of $\delta[n]$ and $u[n]$

In this picture, the signal can in DT or CT.

1.9 Why do We Need Impulses?

In the subsections above, we spent a considerable amount of time explaining Impulses. So why do we need it?

The answer is: **Any signal can be represented as a linear combination of shifted impulses.**

Let's give an example first:

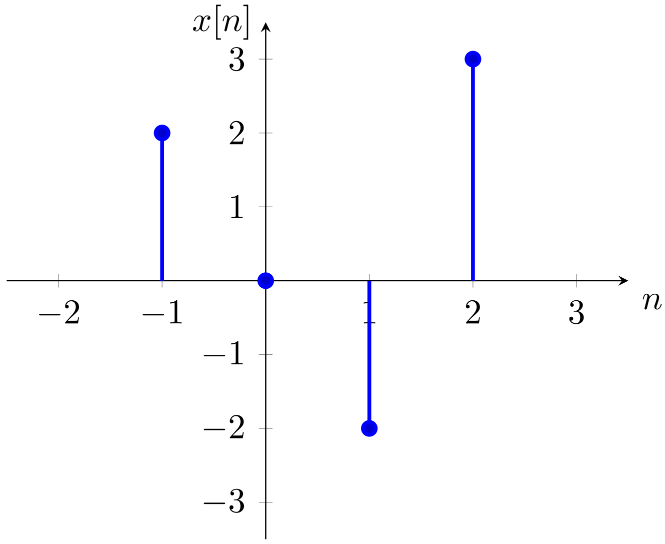

Figure 11: Example: $x[n]$ with Finite Components

Can you write $x[n]$ in terms of shifted $\delta[n]$?

Yes, the answer is:

$$\begin{aligned} x[n] =& 2\delta[n+1] \\ & -2\delta[n-1] \\ & +3\delta[n-2] \end{aligned} \tag{21}$$

Then what will happen if $x[n]$ has infinite components? For example:

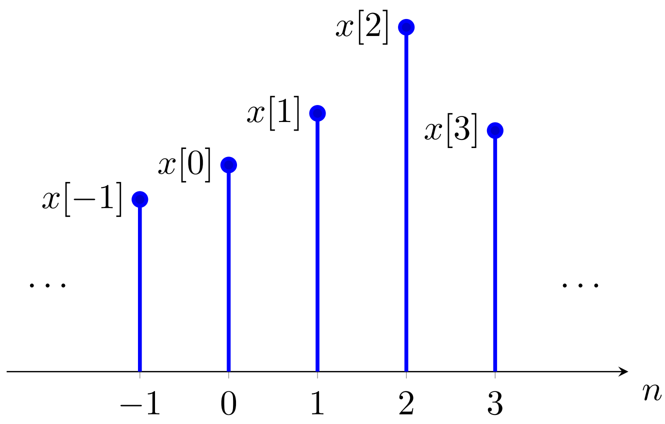

Figure 12: Example: $x[n]$ with infinite Components

By the same token, we have:

$$
\begin{aligned}
x[n] = \cdots + \quad & x[-1]\delta[n+1] \\
&+ x[0]\delta[n] \\
&+ x[1]\delta[n-1] \\
&+ x[2]\delta[n-2] \\
&+ x[3]\delta[n-3] \\
&+ \cdots
\end{aligned} \tag{22}
$$

And we can also give a more concise version:

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \tag{23}$$

Here, $x[k]$ is a scalar, represented as the value of $x[n]$ at time k. Both $x[n]$ and $\delta[n-k]$ are signals.

Now suppose we want to get $x[10]$, then we have:

$$x[10] = \sum_{k=-\infty}^{\infty} x[k]\delta[10-k] \tag{24}$$

You can easily find that, only $k = 10$ that $\delta[10-k] = 1$, otherwise is 0.

So we use the formula to filter the $x[k]$ out, and we can also say: $\delta[n]$ has a screening nature.

1.10 Introduction to System

Definition:

A system recives a signal x as an input, process it, and then outputs another signal y. As a convention we use H to represent the system:

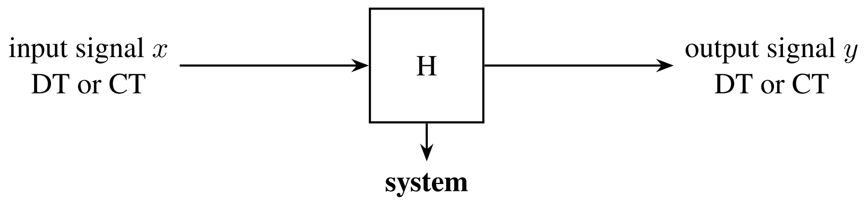

Figure 13: System, Input and Output

So what exactly is a system? In a word, systems are functions. So a system must have three elements of functions:

- Domain: X, Input spaces, a set of input signals;
- Range: Y, Output spaces, a set of output signals;
- Mapping relationship: H, a rule operating on a signal.

Give the relation pictures as follows:

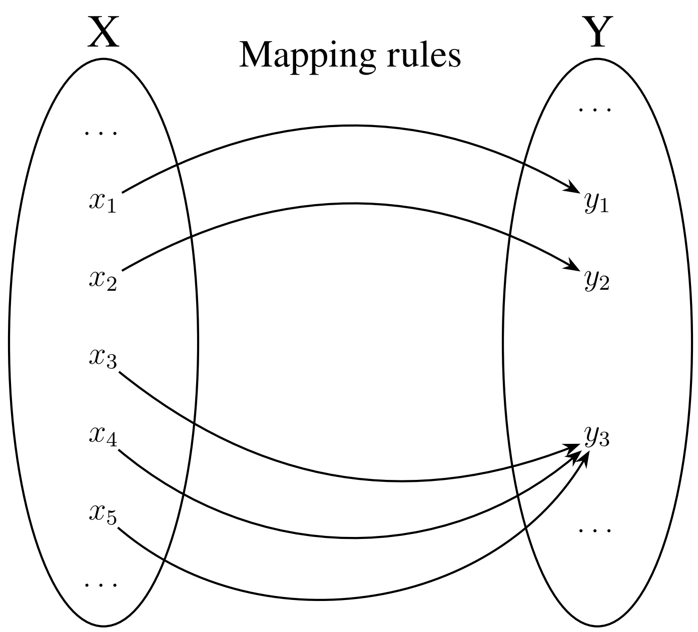

Figure 14: Input Space, Output Space and Mapping Rules

Note: Multiple xs are allowed to correspond to one y, one x is not allowed to correspond to multiple ys.

In most of the cases, we pay attention to: single input, single output.

2 Lecture2

2.1 System Properties–Linearity

What is system?

The nature of the system is independent of the input, and it associates the input with the output. For example:

$$y[n] = x[n] + x[n-1] \tag{25}$$

What is Linearity? This property contains two sub-properties:

- **Scaling property**:

  Suppose we have an arbitrary scalar α. Then if we expand input by α times, the output will also be expanded by α times.

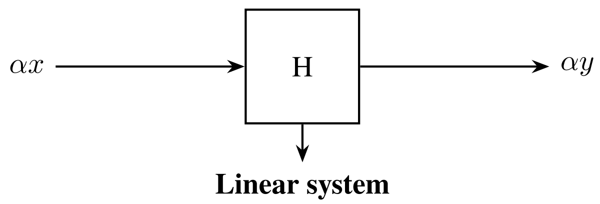

Figure 15: Scaling Property

- **Additivity**

  Suppose we have the following input-output conbination, all the inputs are arbitrary:

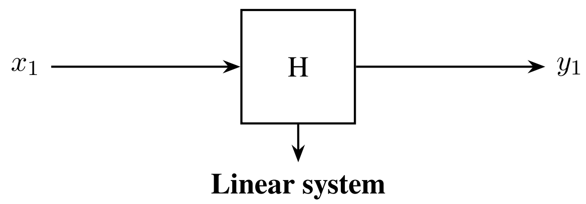

Figure 16: Combination1

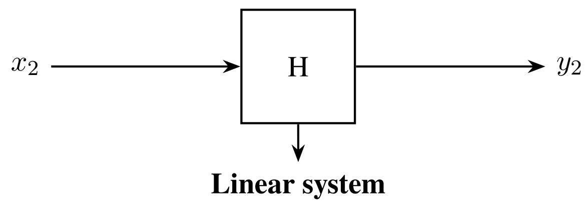

Figure 17: Combination2

Then additivity is manifested as :

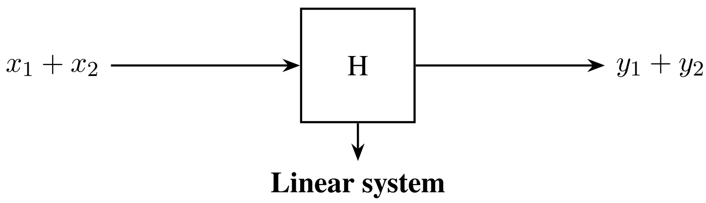

Figure 18: Additivity

2.2 Linearity Example1-2 Point Moving Average Filter

There is a 2-point moving average filter (DT):

$$y[n] = \frac{x[n-1] + x[n]}{2} \tag{26}$$

Does it have the linearity?

Proof.

Suppose there is an arbitrary input-output combination:

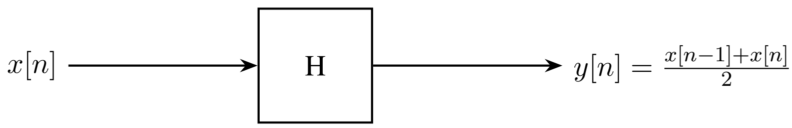

Figure 19: Example1

Now verify the scaling property.

Let $\hat{x}[n] = \alpha x[n], \quad \alpha \in \mathbb{C}$.

And the input-output relation is:

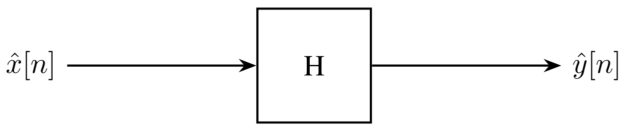

Figure 20: Hat Relation

From the definitio of the system, we know:

$$\hat{y}[n] = \frac{\hat{x}[n-1] + \hat{x}[n]}{2} \tag{27}$$

As we have the following relation:

$$\hat{x}[n] = \alpha x[n], \quad \alpha \in \mathbb{C} \tag{28}$$

So:

$$\begin{aligned} \hat{y}[n] =& \frac{\alpha\hat{x}[n-1] + \alpha\hat{x}[n]}{2} \\ =& \alpha\frac{\hat{x}[n-1] + \hat{x}[n]}{2} \\ =& \alpha y[n] \end{aligned} \tag{29}$$

Then let's verify the additivity.

Let $\hat{x}[n] = x_1[n] + x_2[n]$.

And the imput-output relation is:

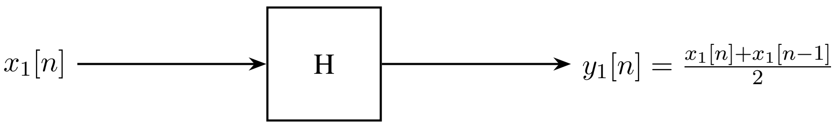

Figure 21: Relation1

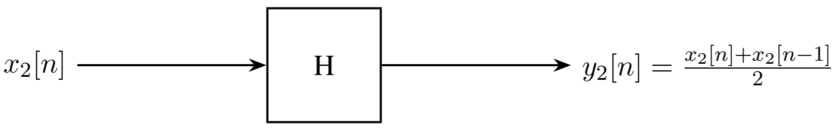

Figure 22: Relation2

From the definitio of the system, we know:

$$\hat{y}[n] = \frac{\hat{x}[n-1] + \hat{x}[n]}{2} \tag{30}$$

As we have the following relation:

$$\hat{x}[n] = x_1[n] + x_2[n] \tag{31}$$

So:

$$\begin{aligned} \hat{y}[n] =& \frac{\alpha\hat{x}[n-1] + \alpha\hat{x}[n]}{2} \\ =& \frac{x_1[n] + x_2[n] + x_1[n-1] + x_2[n-1]}{2} \\ =& y_1[n] + y_2[n] \end{aligned} \tag{32}$$

So this system satisfies the linearity.

End.

2.3 Linearity Example2-Median Filter

There is a median filter (DT):

$$y[n] = med\{x[n-1], x[n], x[n+1]\} \tag{33}$$

Does it have the linearity?

First, what is the median of a sequence of numbers? Consider there are three numbers, and we want to get the median.

If there are no equal numbers, the median is the number whose size is in the middle; If there are two or three equal numbers, the median is the average of equal numbers.

Then, we should have an intuitive understanding of this filter. Look at the following example:

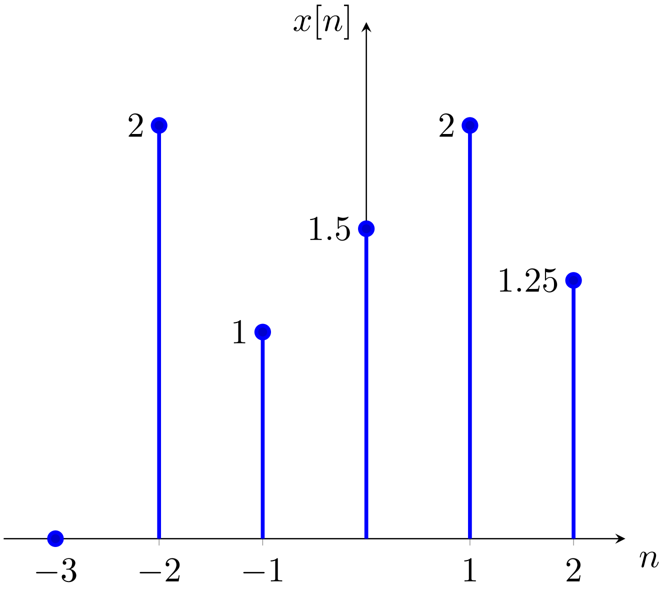

Figure 23: Median Filter Input: $x[n]$

How can we get $y[-2]$?

According to the definition, to get $y[-2]$, we need $x[-3] - 0$, $x[-2] - 2$ and $x[-1] - 1$. There are no equal numbers, so $y[-2]$ or the median is 1.

Similarly, we obtain the outputs at the other time points:

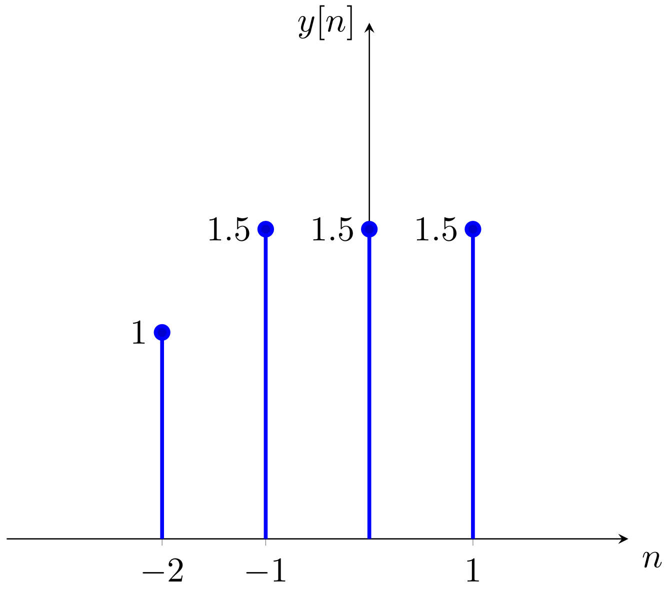

Figure 24: Median Filter Output: $y[n]$

Now, give the proof.

Proof.

First, we should verify whether it satisfies the scaling property.

Obviously, this property is satisfied. Because enlarging the same group of numbers by the same multiple doesn't change the order of size.

Then, does it satisfy the additivity? Intuition tells us that it's not satisfied.

If we want to prove that it's not satisfied, we only need to provide a counter example.

This example can be an impulse and a shifted impulse:

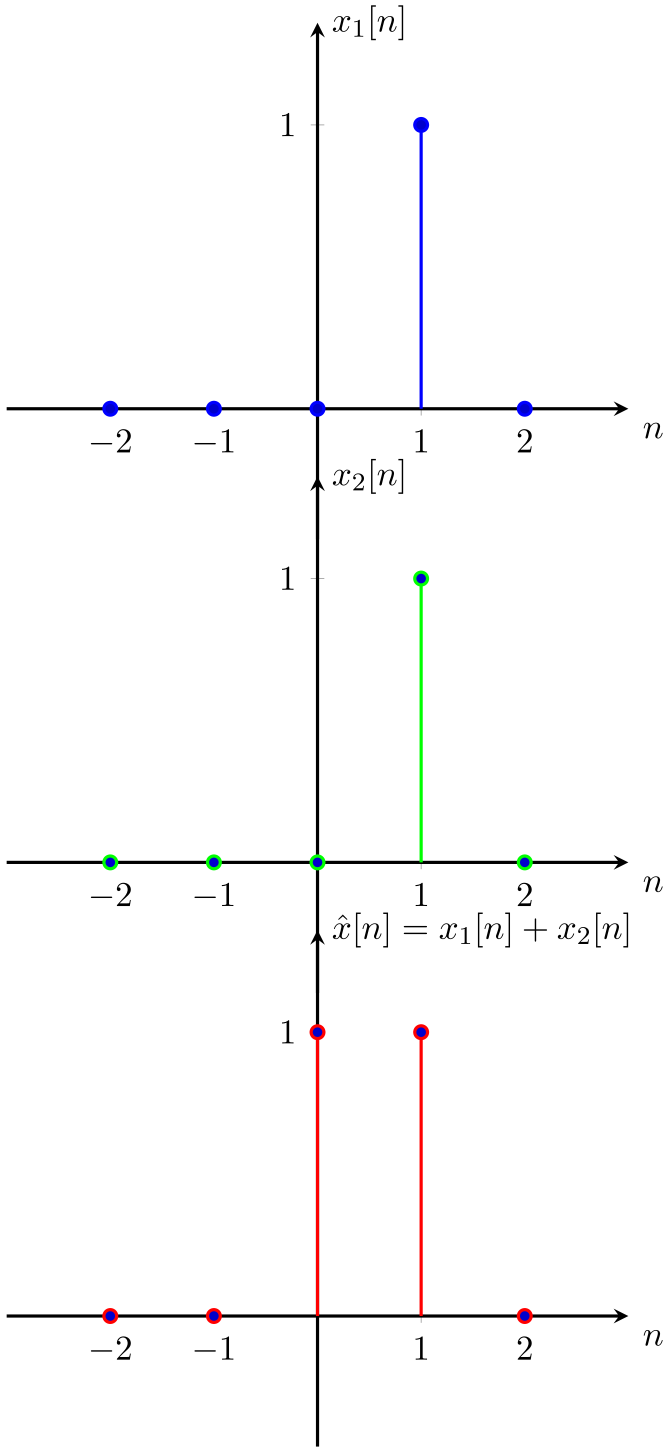

Figure 25: a Counter Example

Calculate their results after median filtering, you will find:

$$\hat{y}[n] \neq y_1[n] + y_2[n] \tag{34}$$

2.4 Time Invariance-TI

Look at the following input-output combination:

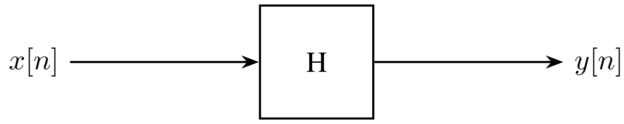

Figure 26: Combination1

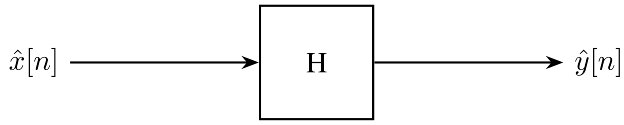

Figure 27: Combination2

For:

$$\text{every} \quad x \in \mathbb{X}, \quad \text{every} \quad N \in \mathbb{Z} \tag{35}$$

Let:

$$\hat{x}[n] = x[n-N] \tag{36}$$

If the system H satisfies the time invariance, then:

$$\hat{y}[n] = y[n-N] \tag{37}$$

Now can you give an example, satisfies the TI but doesn't satisfy the linearity? Yes, it's:

$$y[n] = x^2[n] \tag{38}$$

2.5 Impulse Response

If a system simulataneously satisfies the linearity and the TI, then we call it LTI System.

In the subsection **Why do We Need Impulses?**, we have known that: Any signal can be represented as a linear combination of shifted impulses.

In the system perspective, $\delta[n]$ is also very important, and if we give the LTI system an impulse as the input, the output is called the **Impulse Response**, and the corresponding symbol is $h[n]$:

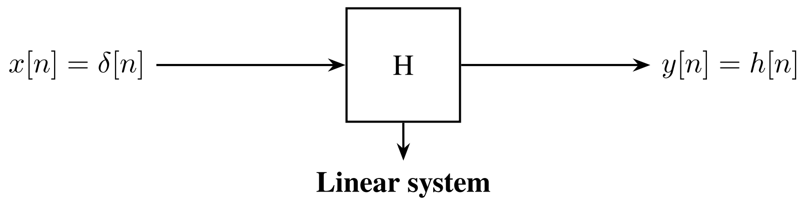

Figure 28: LTI System

FAB FACT

If I know the impulse response of an LTI system, then I will know the system response of any arbitraty input.

Proof.

Suppose there is an LTI system H.

Consider an arbitrary input-output combination:

$$\delta[n] \quad \xrightarrow{\text{LTI} \quad \text{system}} \quad h[n] \tag{39}$$

Give the impulse an arbitrary shift, according to the time-invariance:

$$\delta[n-k] \quad \xrightarrow{\text{LTI} \quad \text{system}} \quad h[n-k], \quad k \in Z \tag{40}$$

Give the shifted impulse an scaling scalar, according to the scaling property:

$$x[k]\delta[n-k] \quad \xrightarrow{\text{LTI} \quad \text{system}} \quad x[k]h[n-k] \tag{41}$$

According to the additivity, we have:

$$\sum_{k=-\infty}^{+\infty} x[k]\delta[n-k] \quad \xrightarrow{\text{LTI} \quad \text{system}} \quad \sum_{k=-\infty}^{+\infty} x[k]h[n-k] \tag{42}$$

Now you must have find that, the infinite series on the left can form an arbitrary signal, that is to say:

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k] \tag{43}$$

End.

To give you an intutive understanding, look at the following example.

There is an LTI system H, and we the impulse response is:

$$\delta[n] \quad \xrightarrow{\text{LTI} \quad \text{system}} \quad h[n] \tag{44}$$

And the following is the input:

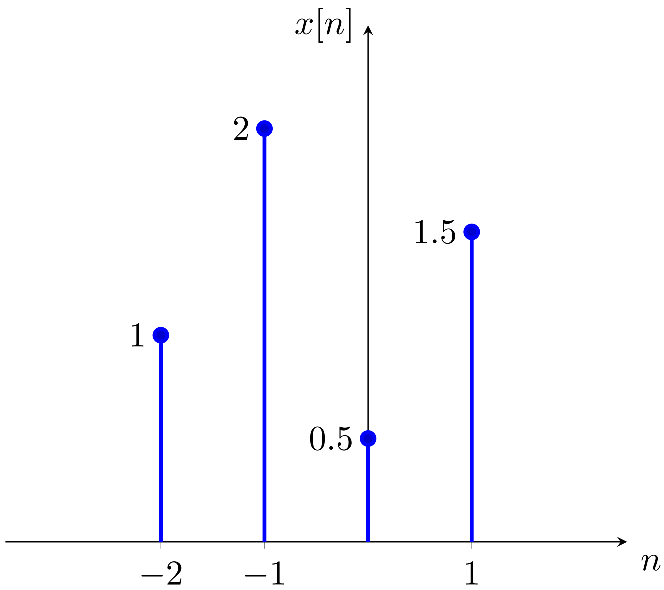

Figure 29: Median Filter Output: $y[n]$

What is the output?

Solution.

Decompose the input:

$$x[n] = \delta[n+2] + 2\delta[n+1] + 0.5\delta[n] + 1.5\delta[n-1] \tag{45}$$

An important point is: here every component is a signal, not a scalar.

As the system has additivity and scaling property, we can input each component seperately, then get the corresponding output:

$$y[n] = (1)h[n+2] + (2)h[n+1] + (0.5)h[n] + (1.5)h[n-1] \tag{46}$$

End.

2.6 Convolution

Considering an LTI system, if we have known the impulse response $h[n]$, then:

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] \tag{47}$$

We give this operation a name-**Convollution**, and the corresponding symbols is $*$:

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{+\infty} x[k]h[n-k] \\ &= x * h \end{aligned} \tag{48}$$

Now we can say: The output of an LTI system is the convolution of the input and the impulse response.

2.6.1 Is the convolution operation commutative?

Proof.

Let $l = n - k$, and change the summation variable of the above equation from k to l:

$$\begin{aligned} y[n] &= \sum_{l=-\infty}^{+\infty} x[n-l]h[l] \\ &= h * x \end{aligned} \tag{49}$$

So the convolution is commutative.

End.

2.6.2 Does the convolution operation have the associative law?

In other words, we want to prove:

$$(x * h_1) * h_2 = x * (h_1 * h_2) \tag{50}$$

Proof.

Let:

$$\begin{aligned} y[n] &= (x * h_1)[n] \\ &= \sum_{k=-\infty}^{+\infty} x[k]h_1[n-k] \end{aligned} \tag{51}$$

Then the left side of the equation (LSE) is:

$$\begin{aligned} \mathbf{LSE} &= (y * h_2)[n] \\ &= \sum_{m=-\infty}^{+\infty} \boxed{y[m]}\, h_2[n-m] \\ &= \sum_{m=-\infty}^{+\infty} \boxed{\sum_{k=-\infty}^{+\infty} x[k]h_1[m-k]}\, h_2[n-m] \\ &= \sum_{m=-\infty}^{+\infty} \sum_{k=-\infty}^{+\infty} x[k]h_1[m-k]h_2[n-m] \end{aligned} \tag{52}$$

In the above-mentioned summation process, we calculate the $y[m]$ first, so here m is considered as a scalar and the summation variable is k, $h[n-m]$ is also a scalar:

$$\mathbf{LSE} = \sum_{m=-\infty}^{+\infty} \boxed{\sum_{k=-\infty}^{+\infty} (x[k]h_1[m-k]h_2[n-m])} \tag{53}$$

We can also take out the box:

$$\mathbf{LSE} = \sum_{m=-\infty}^{+\infty} \sum_{k=-\infty}^{+\infty} x[k]h_1[m-k]h_2[n-m] \tag{54}$$

Suppose the summation converges absolutely, we can swap the order of the summation:

$$\mathbf{LSE} = \sum_{k=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} x[k]h_1[m-k]h_2[n-m] \tag{55}$$

Here we calculate the summation that uses m as a variable, so $x[k]$ and k is a scalar:

$$\textbf{LSE} = \sum_{k=-\infty}^{+\infty} x[k] \boxed{\sum_{m=-\infty}^{+\infty} h_1[m-k]h_2[n-m]} \tag{56}$$

Let's process $\sum_{m=-\infty}^{+\infty} h_1[m-k]h_2[n-m]$ first. Let $p = m-k$, so $m = p+k$:

$$\begin{aligned}\sum_{m=-\infty}^{+\infty} h_1[m-k]h_2[n-m] &= \sum_{p=-\infty}^{+\infty} h_1[p]h_2[n-(p+k)] \\ &= \sum_{p=-\infty}^{+\infty} h_1[p]h_2[(n-k)-p] \\ &= (h_1 * h_2)[n-k]\end{aligned} \tag{57}$$

So:

$$\begin{aligned}\textbf{LSE} &= \sum_{k=-\infty}^{+\infty} x[k] \boxed{\sum_{m=-\infty}^{+\infty} h_1[m-k]h_2[n-m]} \\ &= \sum_{k=-\infty}^{+\infty} x[k](h_1 * h_2)[n-k] \\ &= x * (h_2 * h_1)[n]\end{aligned} \tag{58}$$

End.

2.6.3 How can we understand $\sum_{k=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty} x[k]h_1[m-k]h_2[n-m]$?

Let:

$$T(m,k) = x[k]h_1[m-k]h_2[n-m] \tag{59}$$

Look at the following picture:

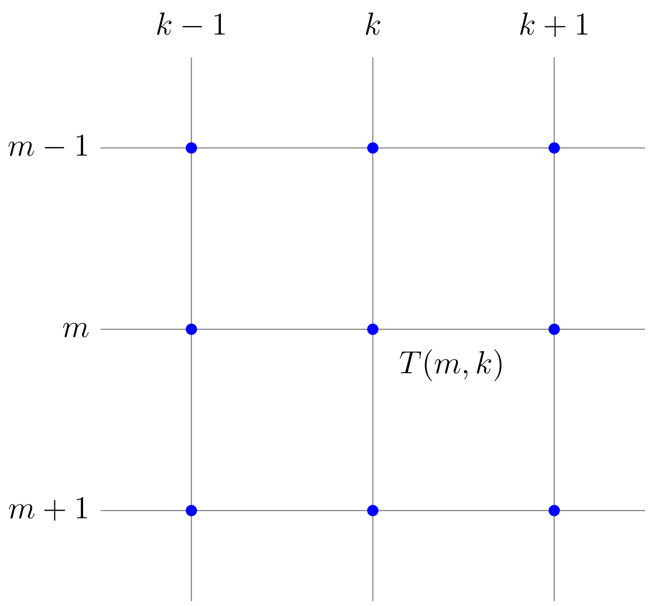

Figure 30: Binary Summation

So the summation becomes:

$$\sum_{k=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty} T(m,k) = \sum_{k=-\infty}^{+\infty}\boxed{\sum_{m=-\infty}^{+\infty} T(m,k)} \tag{60}$$

Actually the firsr summation sums all the elements at column k. After the summation finishes, we get a function about k, Then it sums using k.

2.6.4 A cascade of two LTI system.

Look at the following example:

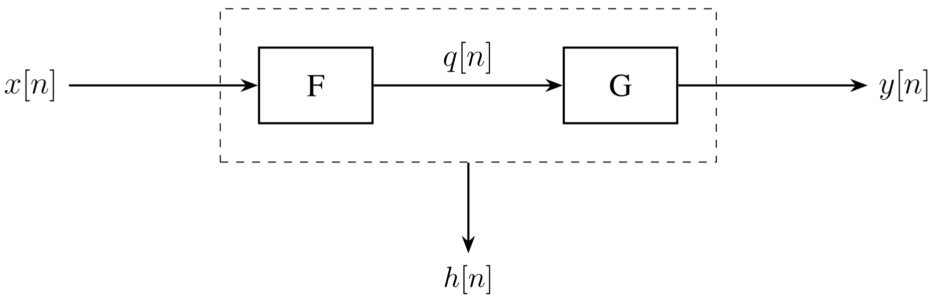

Figure 31: Cascade of two LTI systems, F and G

According to the direction of the signal flow, we have:

$$y[n] = (x * f) * g \tag{61}$$

As the convolution has the associative law, we have:

$$\begin{aligned} y[n] &= (x * f) * g \\ &= x * (f * g) \end{aligned} \tag{62}$$

So we have:

$$\begin{aligned} h &= f * g \\ &= g * f \end{aligned} \tag{63}$$

that is to say: we can change the order of F and G.

2.7 Convolution Interpretation

In this interpretation, you should remember one sentence:

Instantaneous actions have continuous consequences.

What dose $\delta[n]$ mean?

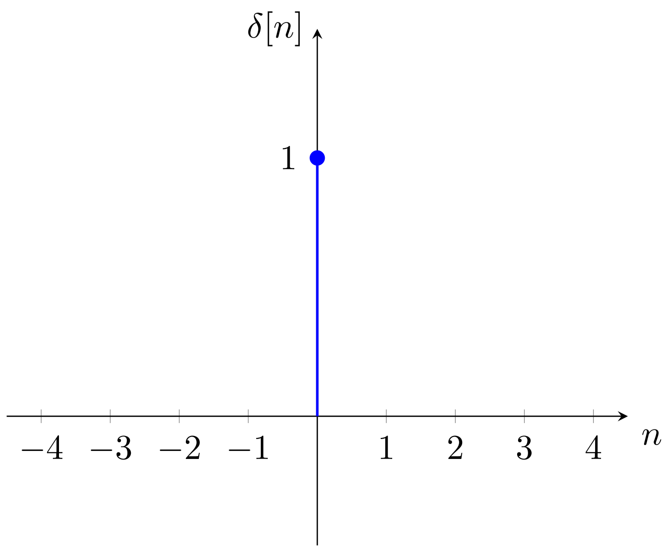

Figure 32: Delata: $\delta[n]$

It means, we give a very short impulse at time 0.

Suppose $\delta[n]$ is given to an LTI system as an input, the impulse response is as follows:

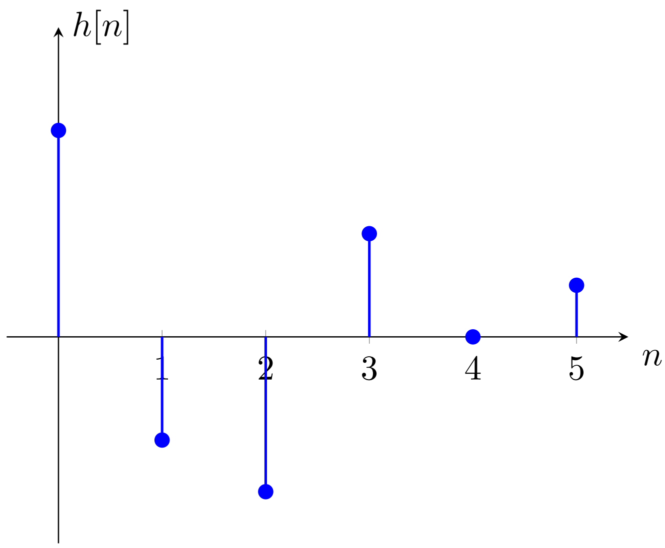

Figure 33: Impulse Response: $h[n]$

You can see, although we only give an impulse at time 0, this impulse remains in effect for the rest of the time.

If we give a shifted impulse $\delta[n-1]$ as an input, what will we get? Yes, according to the TI, we will get:

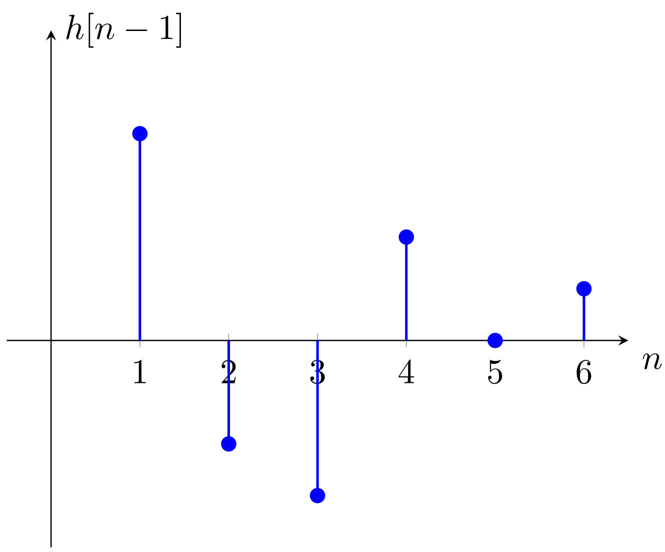

Figure 34: Shifted Impulse Response: $h[n-1]$

If we give an arbitrary input $x[n]$, we know $x[n]$ is a linear combination of different shifted impulses:

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k] \tag{64}$$

Here, you can understand $x[k]$ as a **weight** for different $\delta[n-k]$. Because when we calculate the sum,

$x[k]$ is seen as a scalar.

So each component in $x[n]$ or shifted impulses will have a lasting impact, and we should sum all the impact from each $\delta[n-k]$.

For example, now we want to get the output at 3 or $y[3]$, our input is:

$$x[n] = 3\delta[n] + 2\delta[n-1] + 3\delta[n-2] \tag{65}$$

we should sum all the components' impact at $y[3]$:

$$y[3] = 3h[3] + 2h[3-1] + 3h[3-2] \tag{66}$$