Signal and System

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Abstract

These are notes on **Signal and System**.

Contents

1 Lecture1

1.1 What is a Signal

In this subsection, we will introduce some notions in **SS** course.

First, what is a signal? A signal is a **function**. Suppose **x** is a signal, then it can represent the following mapping relationship:

$$\begin{aligned} \mathbb{R}(\text{reals}) &\longrightarrow \mathbb{R}(\text{reals}) \\ \mathbb{Z}(\text{integers}) &\longrightarrow \mathbb{R}(\text{reals}) \\ \mathbb{Z}(\text{reals}) &\longrightarrow \mathbb{C}(\text{complexes}) \end{aligned} \tag{1}$$

In the above formula, we call the left **domain or input space**, we call the right **range or output sapce**, referring to the definition in the function,

At the same time, an element in domain is called **independent variable**, an element in range is called **dependent variable or the value of function**.

After we give the doamin and the range, we need a **rule** to map every element in the domain to the range, which says what we operate on the the independent variable. The rule is also called the **function relation**.

For example, considering the following euqation:

$$x(n) = \cos\frac{\pi}{4}n \tag{2}$$

we have the following relations:

- **domain**: $\mathbb{Z}$, a set of all integers;
- **independent variable**: n, integers;
- **range**: all real numbers in [-1,1];
- **value of function**: $x(n)$;
- **function relation**: $\boldsymbol{x}$, $\cos\frac{\pi}{4}[\cdot]$

1.2 DT and CT

Now let's introduce **DT** and **CT**.

For a signal $\mathbf{x}$, if the domain is $\mathbb{Z}$ or the set of all integers, then the signal $\mathbf{x}$ is called **Discrete-Time** signal (**DT**). For example:

$$x(n) = \cos\frac{\pi}{4}n, \quad n \in \mathbb{Z} \tag{3}$$

For a signal $\boldsymbol{x}$, if the domain is $\mathbb{R}$ or the set of all reals, then the signal $\boldsymbol{x}$ is called **Continuous-Time** signal (**CT**). Also, you can call **CT signal** as **Analog signal**. For example:

$$x(t) = \mathrm{e}^{-t}, \quad t \in \mathbb{R} \tag{4}$$

Meanwhile:

- if we use n as the independent variable, that means the signal is **DT**;
- if we use t as the independent variable, that means the signal is **CT**;

these are the conventions.

Supplement: Does the signal have to be the function of **only time**? The answer is **No**. For example, the independent variables can be **space coordinates**.

1.3 Discrete-Time Signal and Digital Signal

Sometimes, we don't distinguish **digital signal** and **discrete-time signal**, but they still have obvious differences.

In terms of digital signal, it still meets the definition of function. However, everything (including independent variable in domain and dependent variable in range) are represented by digits.

In terms of discrete-time signal, the independent variable in domain is discrete, but the values of the function can be reals (such as infinite recurring decimal).

But for the computers, we can't have infinite precision, so we need to quantify the numbers, which will cause us to lose accurcy.

1.4 Examples and Conventions

Let's give somen examples about **DT** and **CT**.

First, there is a continuous time example, a speech signal:

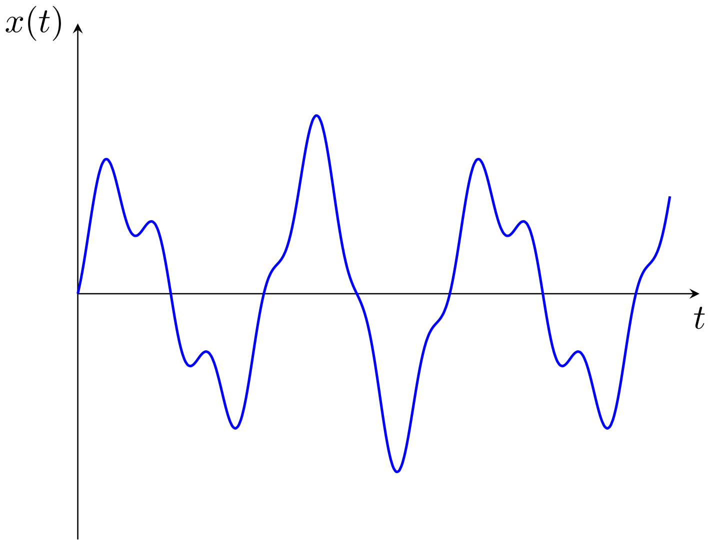

Figure 1: A Speech Signal

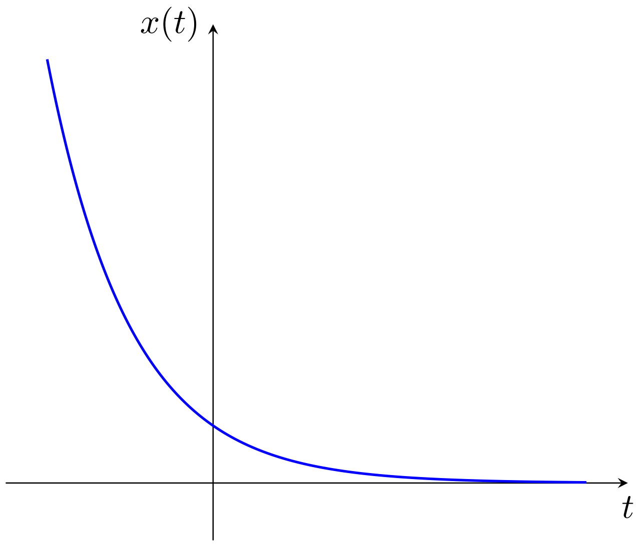

Figure 2: $x(t) = e^{-t}$

For the **CT** signal, we have the following conventions:

- x at time t: $x(t)$, refers to a specific value of the signal at time t;
- x without any arguments: $\boldsymbol{x}$, refers to the entire signal in $(-\infty, \infty)$;

However, in practice, when we define a signal, $x(t)$ also refers to the entire signal, not a specific value.

Second, there is a discrete time signal:

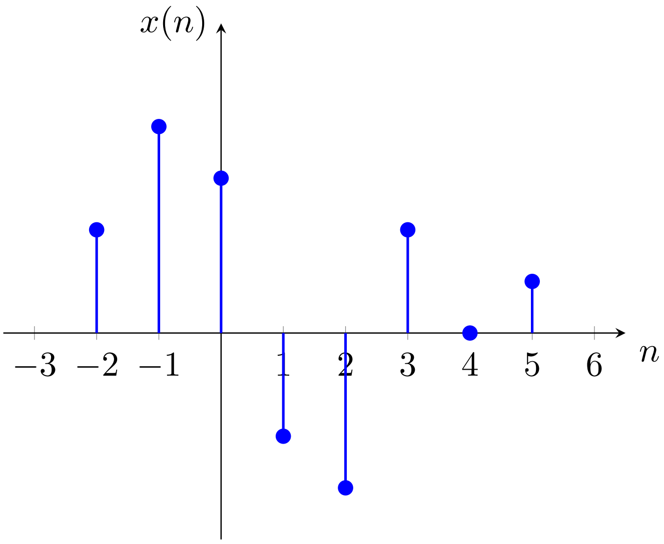

Figure 3: An Example for **DT** signal

The picture above is called **lollipops** or **stem plots**, which is designed specifically for the **DT** signal. We can find that, it's discrete on the domain.