Lab L3 Generation of Random Variables

Shaoyong Guo
Politecnico di Torino
s296966
s296966@studenti.polito.it

I. Lab Requirements

Simulating is good method to study how it works for the processing of a system. It is necessary to use some random data to imitate the input of a system. According to Law Of Large Number(LLN), the mean of the data obtained from a large number of trials should tend to become closer to the expected value as more trials are performed. Previous researchers have provided many theoretical probabilistic models based on LLNs for data theory. Simulators need generators of random variable with specific distributions. In our simulation, obtaining random data through an Inverse-transform technique of classical math probabilities models is an efficient way. There are lots of probabilistic models for random generation. In our lab, there are some classical probabilities models used to study the generation of random data, Rayleigh distribution, Lognormal distribution, Beta distribution, Chi square distribution and Rice distribution.

II. Approach

A. Rayleigh distribution

(1) The probability density function(PDF) of the Rayleigh distribution is:

$$pdf(x) = \frac{xe^{-x^2/2\sigma^2}}{\sigma^2}$$

(2) The cumulative distribution function (CDF) on the support of X is:

$$F(X) = P(X \leq x) = 1 - e^{-x^2/2\sigma^2}$$

(3) Inverse-transform for the cumulative distribution function:

$$X = \sigma * \sqrt{-2 * log(1-p)}$$

B. Lognormal distribution

(1) The probability density function(PDF) of the Lognormal distribution is:

$$pdf(x) = \frac{e^{-(log(x)-u)^2/2\sigma^2}}{x\sigma\sqrt{2\pi}}$$

(2) The cumulative distribution function (CDF) on the support of X is:

$$F(X) = P(X \leq x) = 0.5 * [1 + erf(\frac{log(x)-u}{\sigma\sqrt{2}})]$$

(3) Inverse-transform for the cumulative distribution function:

$$X = e^{\mu+\sigma\sqrt{2}\psi^{-1}(2p-1)}$$

C. Beta distribution

(1) The probability density function(PDF) of the Beta distribution is:

$$pdf(x) = \frac{x^{(\alpha-1)}(1-x)^{(\beta-1)}}{B(\alpha,\beta)}$$

(2) The cumulative distribution function (CDF) on the support of X is:

$$K(X;n,p) = P(X \leq x) = I_{1-p}(n-k, k+1)$$

(3) Inverse-transform for the cumulative distribution function:

$$X = (1-(1-\mu)^{1/b})^{1/a}$$

D. Chi square distribution

(1) The probability density function(PDF) of the Chi square distribution is:

$$pdf(x) = \frac{1}{2^{k/2}\Gamma(k/2)} x^{k/(2-1)} e^{-x/2}$$

(2) The cumulative distribution function (CDF) on the support of X is:

$$F(X) = P(X \leq x) = \frac{1}{\Gamma(k/2)}\gamma(\frac{k}{2}, \frac{x}{2})$$

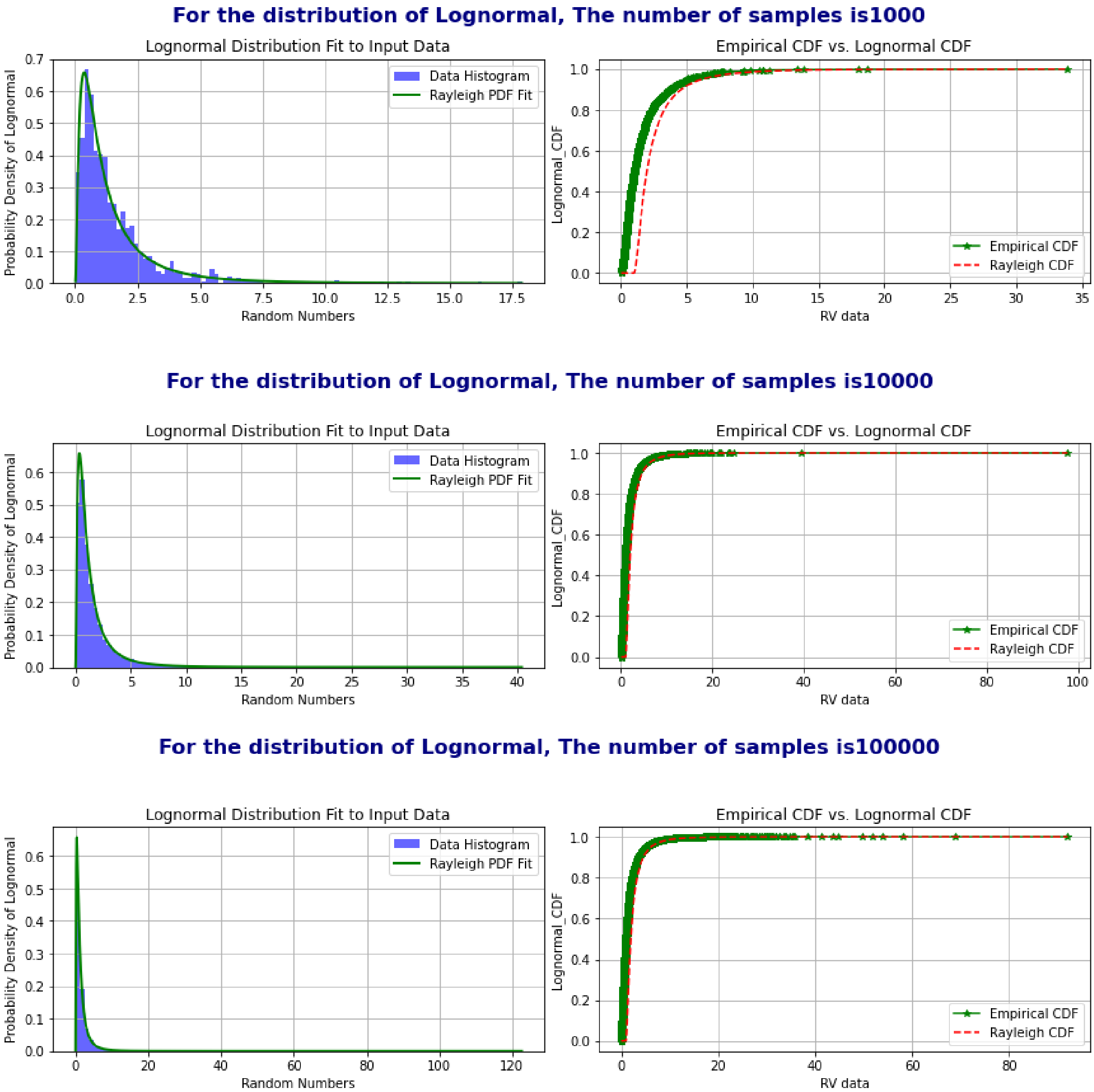
E. Rice distribution

(1) The probability density function(PDF) of the Rice distribution is:

$$pdf(x) = \frac{x}{\sigma^2} e(\frac{-(x^2+v^2)}{2\sigma^2}) I_0(\frac{xv}{\sigma^2})$$

(2) The cumulative distribution function (CDF) on the support of X is:

$$F(X) = P(X \leq x) = 1 - Q_1(\frac{v}{\sigma}, \frac{x}{\sigma})$$



Gambar 1. Differences in appearance and semantic segmentation output between GTA5 and Cityscapes.

III. EXPERIMENTS

A. Experiment Design

In the experiment, the first step is to generate a series of numerical data by inverting a CDF with a specific probability. This involves two parameters: a random uniform distribution as the input probability to the CDF, and the output of inverse cumulative function as the random data generator.

Next, it will test and evaluate the accuracy of the random data. I used a histogram to show the distribution of the random data generated in the previous step. All of the random data was divided equally into 100 parts and used as input for the histogram. I then use a line graph to display the distribution of the original probability distribution on the same panel. Another uniform function generates a series of random input data from 0 to the maximum value, which serves as input for the PDF of the original probability function. Finally, the fit of the histogram and line graph can be validated.

IV. CONCLUSION

In the lab, I have generated 1000, 10000 and 100000 levels of random data using the inverse cumulative function technique. As shown in the example lognormal distribution in Figure 1, better fitting results are obtained as the number of samples increases. It can be seen that the random numbers generated using the inverse cumulative function technique meet the mathematical predictions.