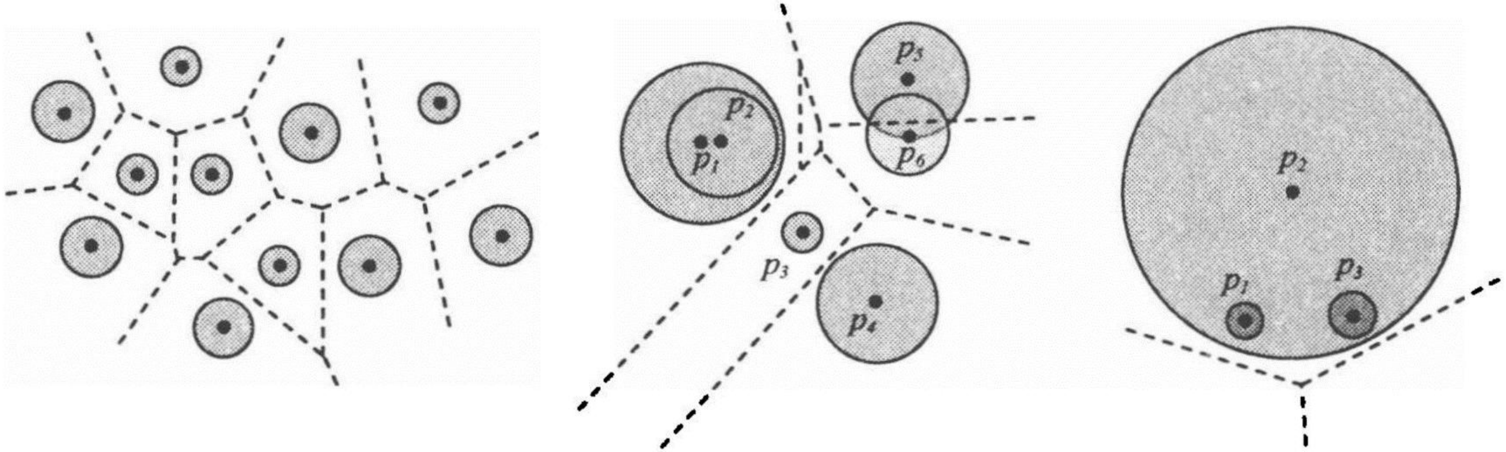
General Voronoi

Energy Diagram

Euclidean distance ➡ Energy distance

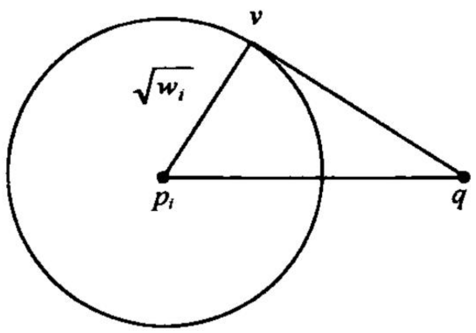
$$d(q, p_i) = |qp_i|^2 - \omega_i$$



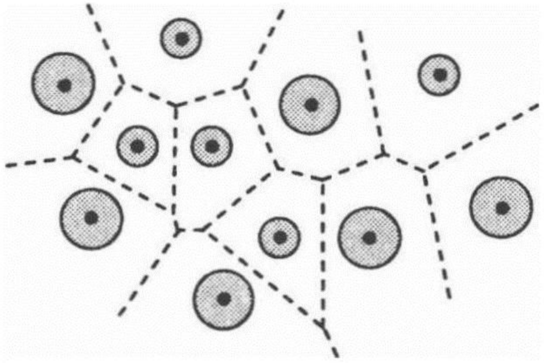
Energy Diagram

Weighted circle $C(p_i, r_i)$: the center is p_i and the radius $r_i = \sqrt{\omega_i}$.

- If $d(q, p_i) > 0$, q is outside of $C(p_i, r_i)$;
- If $d(q, p_i) = 0$, q is on $C(p_i, r_i)$;
- If $d(q, p_i) < 0$, q is inside of $C(p_i, r_i)$;
- If ω_i<0, we always have $d(q, p_i) > 0$ and q is outside of the imaginary circle $C(p_i, r_i)$.



$$d(q, p_i) = |qp_i|^2 - \omega_i$$

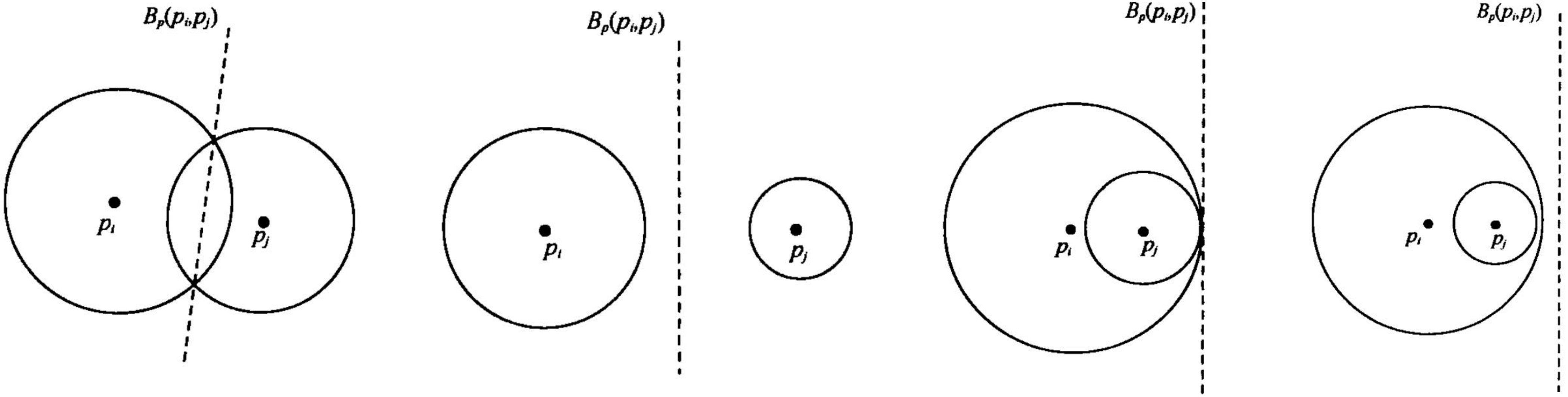


Energy Diagram

Energy Area: $VR(p_i) = \{q|d(q,p_i) \leq d(q,p_j), i \neq \text{j}\}$

Energy Edge: $B(p_i,p_j) = \{q|d(q,p_i) = d(q,p_j), i \neq \text{j}\}$

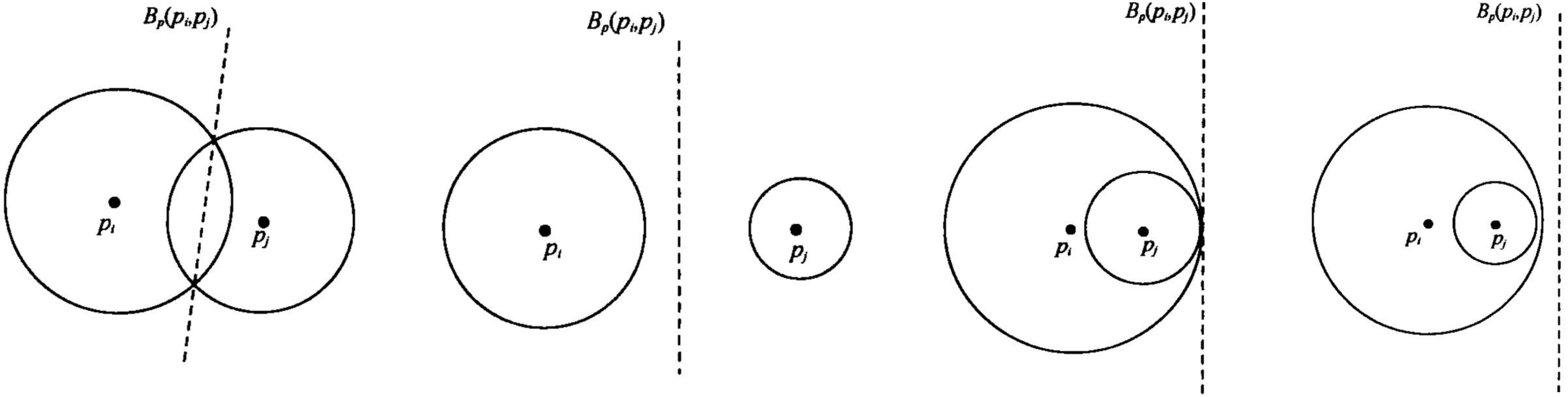
Examples of energy edge:



Energy Diagram

$B(p_i, p_j)$ is a line.

$B(p_i, p_j)$ is perpendicular to $p_i p_j$.

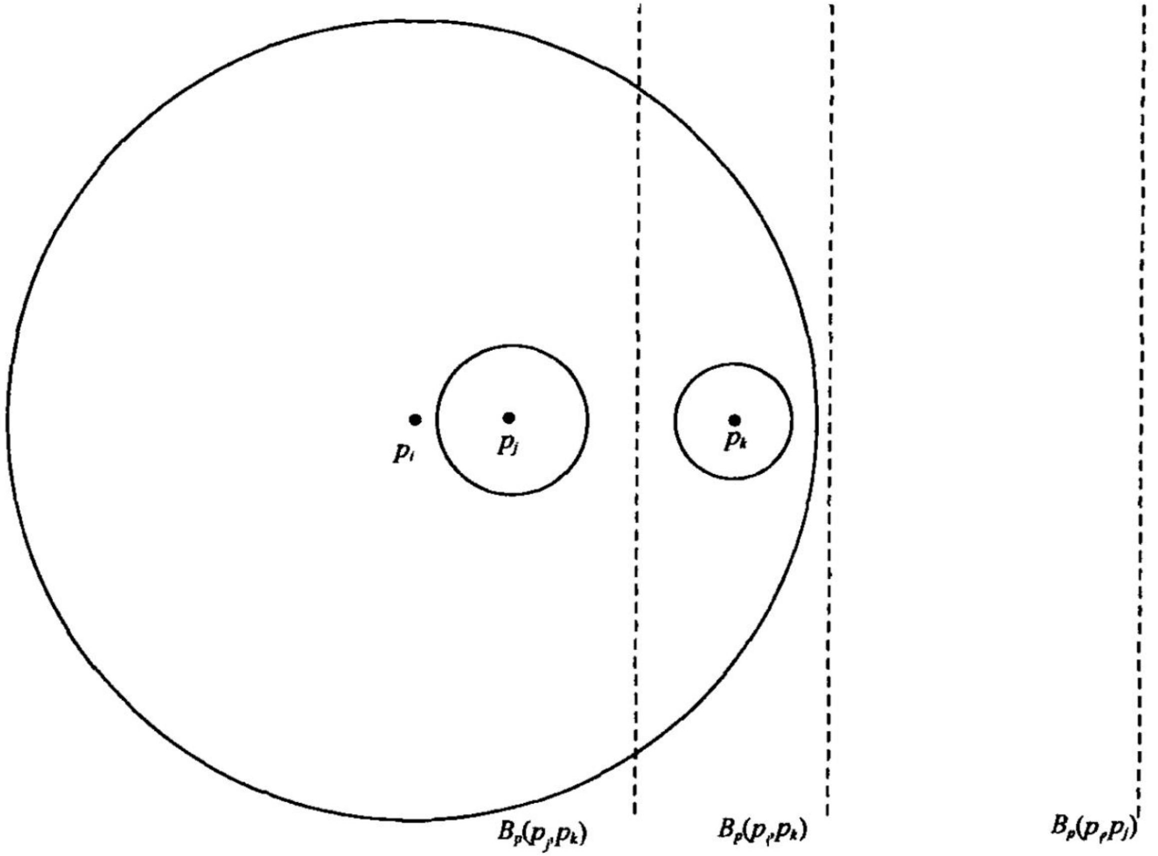


Energy Diagram

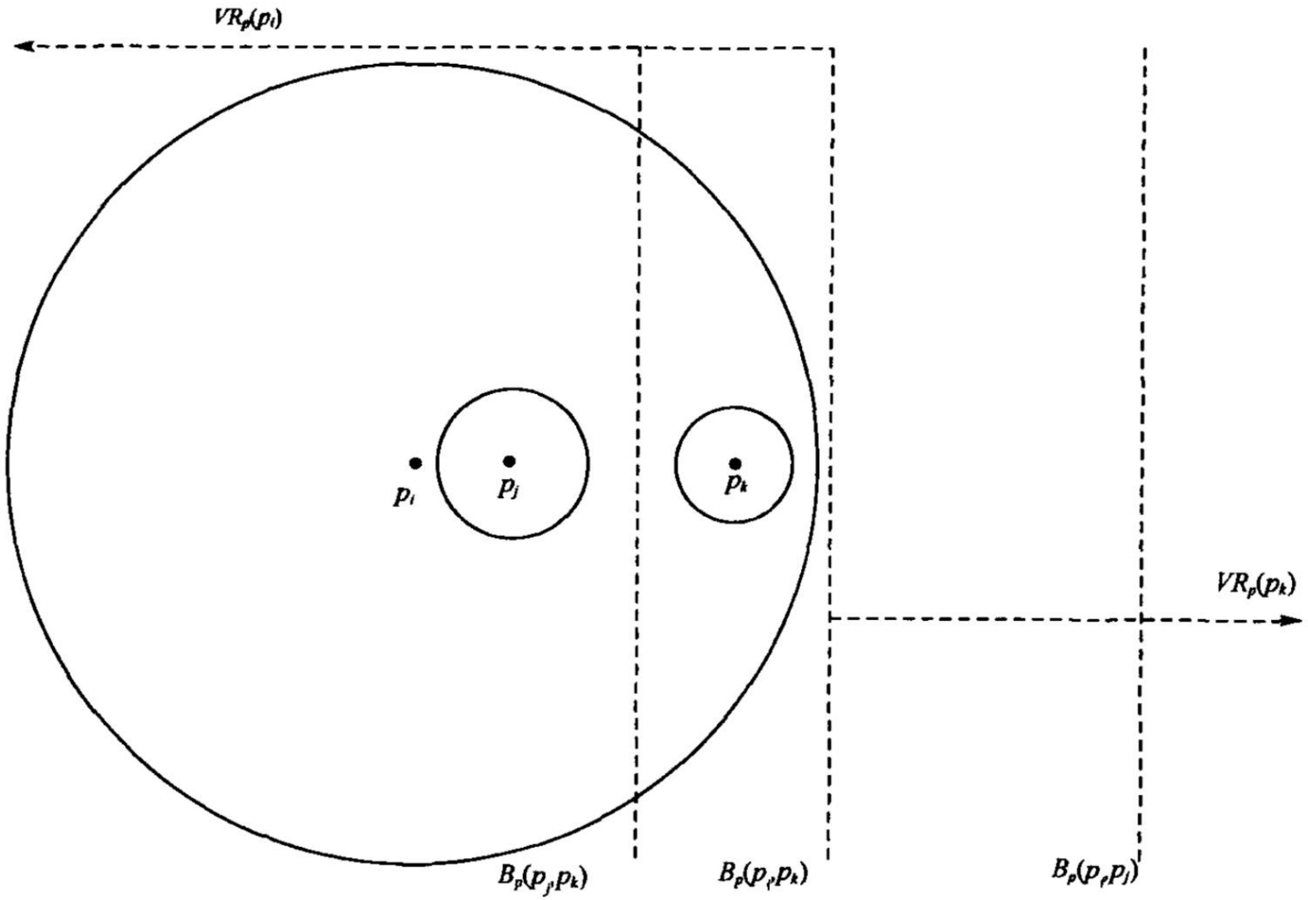
- For any pint on $B(p_i, p_j)$, $d(q, p_i) = d(q, p_j)$, $|qp_i|^2 - \omega_i = |qp_j|^2 - \omega_j$.
  The energy diagram does not change if $\omega_i + \Delta\omega = \omega_j + \Delta\omega$.

- $VR(p_i)$ is also the intersection of several half planes.

Energy Diagram

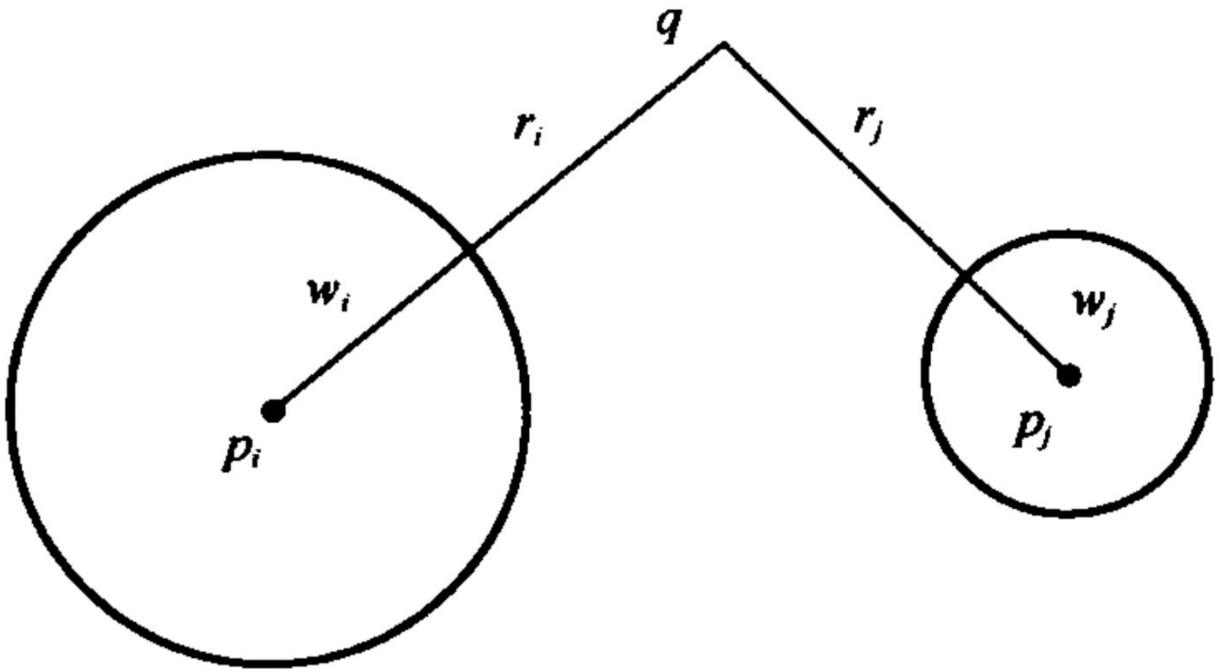


Energy Diagram



Addition Weighted Voronoi Diagram

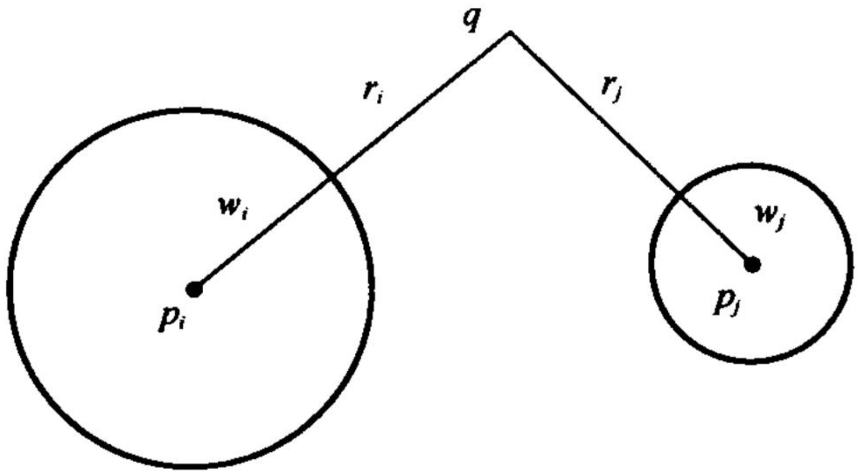
$$d(q, p_i) = |qp_i| - \omega_i$$



Addition Weighted Voronoi Diagram

Weighted circle $C(p_i, r_i)$: the center is p_i and the radius is ω_i.

- If $d(q, p_i) > 0$, q is outside of $C(p_i, \omega_i)$;
- If $d(q, p_i) = 0$, q is on $C(p_i, \omega_i)$;
- If $d(q, p_i) < 0$, q is inside of $C(p_i, \omega_i)$;



Addition Weighted Voronoi Diagram

For two seeds p_i, p_j, the half plane of p_i can be denoted as

$$h(p_i, p_j) = \{q|\ |qp_i| - \omega_i \le |qp_j| - \omega_j, j \ne i, p_i, p_j \in P\}$$

$$\Longrightarrow \quad h(p_i, p_j) = \{q|\ |qp_i| - |qp_j| \le \omega_i - \omega_j, j \ne i, p_i, p_j \in P\}$$

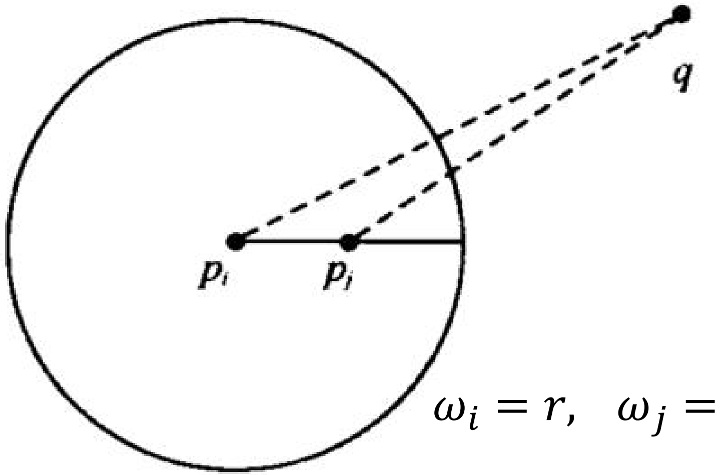
Addition Weighted Voronoi Diagram

$$h(p_i, p_j) = \{q \mid |qp_i| - |qp_j| \le \omega_i - \omega_j, j \ne i, p_i, p_j \in P\}$$

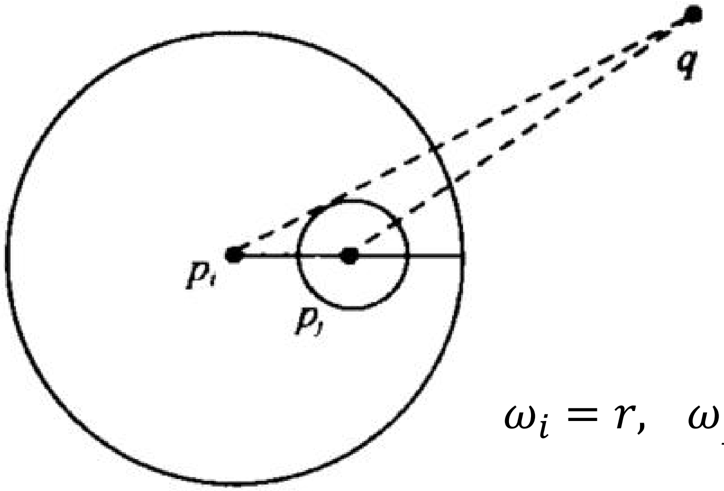
1. $\mathbf{0 < |p_i p_j| < \omega_i - \omega_j}$

For Δqp_ip_j, we get $|qp_i| - |qp_j| < |p_ip_j|$. Then, $|qp_i| - |qp_j| < \omega_i - \omega_j$.

Then, $h(p_i, p_j)$ is the whole plane, and $h(p_j, p_i)$ is empty.



$\omega_i = r, \quad \omega_j = 0$



$\omega_i = r, \quad \omega_j = r/4$

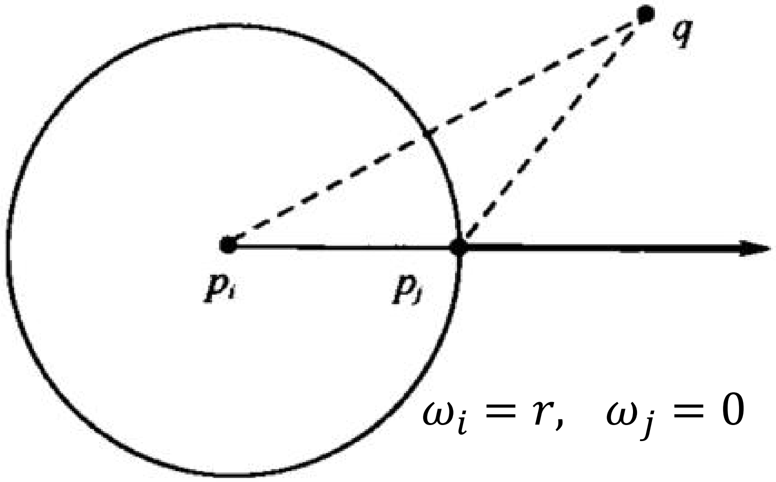
Addition Weighted Voronoi Diagram

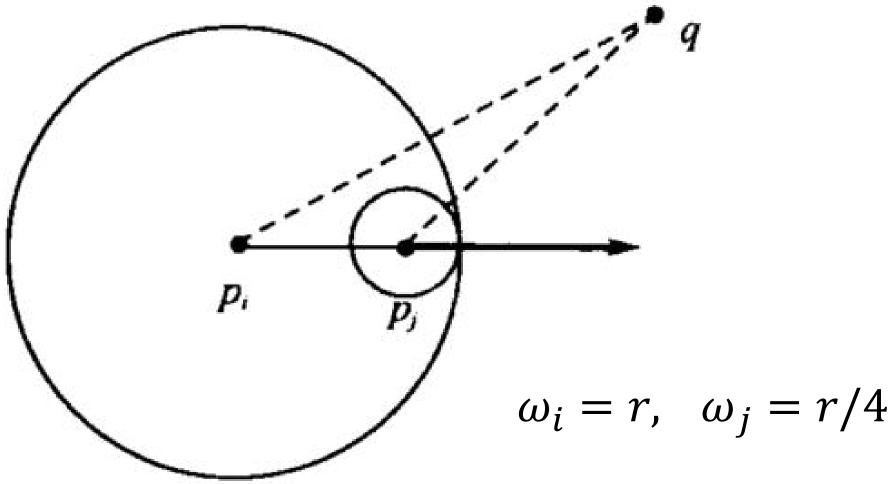
2. $|p_ip_j| = \omega_i - \omega_j$

For the ray p_ip_j removing the line segment p_ip_j, $|qp_i| - |qp_j| = \omega_i - \omega_j$.

For other areas, we get $|qp_i| - |qp_j| < |p_ip_j|$. Then, $|qp_i| - |qp_j| < \omega_i - \omega_j$.

Then, $h(p_i, p_j)$ is the whole plane, and $h(p_j, p_i)$ is the ray p_ip_j removing the line segment p_ip_i (bold in the figure).



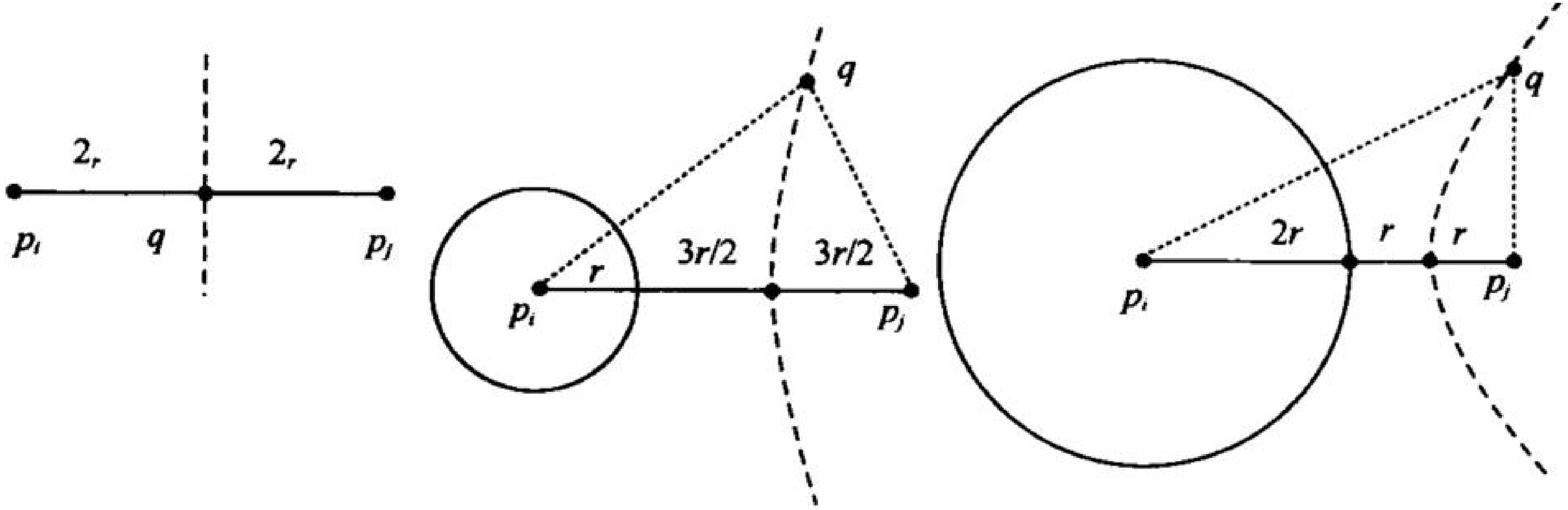


Addition Weighted Voronoi Diagram

3. $|p_i p_j| > \omega_i - \omega_j$

When $\omega_i = \omega_j$, the bisector is perpendicular bisector.

When $\omega_i - \omega_j > 0$, the bisector is one hyperbola curve. For the point on the hyperbola curve, the difference of the distance to p_i *and* p_j is the constant $\omega_i - \omega_j$.



$\omega_i = \omega_j$ $\qquad$ $\omega_i = r, \quad \omega_j = 0$ $\qquad$ $\omega_i = 2r, \quad \omega_j = 0$

Addition Weighted Voronoi Diagram

Examples