$\delta(t) = \begin{cases} "\infty" & t = 0 \\ 0 & t \neq 0 \end{cases}$	$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$
$\int_{-\infty}^{+\infty} x(t)\delta(t)dt = x(0)$	$x[k]\delta[n-k] = x[n]\delta[n-k]$
$\delta(at) = \frac{1}{\lvert a\rvert}\delta(t)$	$\delta[kn] = \delta[n]$
$\delta(t) = \frac{d}{dt}u(t)$	$\delta[n] = u[n] - u[n-1]$
$u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau$	$u[n] = \sum_{m=-\infty}^{n} \delta[m]$
$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$	$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$
$u(0) = \frac{1}{2}$	$u[0] = 1$

6. Signal Energy and Power

① Energy:

$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$E_\infty = \lim_{N\to\infty} \sum_{-N}^{N} |x[n]|^2 = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

② Power:

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N} \sum_{-N}^{N} |x[n]|^2$$

Q1.3 Consider the continuous-time signal

$$x(t) = \delta(t+2) - \delta(t-2)$$

Calculate the value of E_∞ of the signal

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau$$

Solution:

$$y(t) = \int_{-\infty}^{t} \delta(\tau+2)d\tau - \int_{-\infty}^{t} \delta(\tau-2)d\tau$$

$$= u(t+2) - u(t-2)$$

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-2}^{2} 1 dt = 4$$

§1.2 Simple Calculation of Signals

1. Addition

Example 1.3: Determine $x_1(t) + x_2(t)$, giving

$x_1(t) = \begin{cases} 0 & t < 0 \\ \sin\pi t & t \geq 0 \end{cases}$ and $x_2(t) = -\sin\pi t$

Solution:

$$x_1(t) + x_2(t) = \begin{cases} 0 - \sin\pi t & t < 0 \\ \sin\pi t - \sin\pi t & t \geq 0 \end{cases}$$

$$= \begin{cases} -\sin\pi t & t < 0 \\ 0 & t \geq 0 \end{cases}$$

Example 1.4: Determine $x_1[n] + x_2[n]$, giving

$x_1[n] = \begin{cases} 3^n & \text{n} < 0 \\ n+1 & \text{n} \geq 0 \end{cases}$ and $x_2[n] = \begin{cases} 0 & \text{n} < -2 \\ 3^{-n} & \text{n} \geq -2 \end{cases}$

Solution:

$$x_1[n] + x_2[n] = \begin{cases} 3^n + 0 & \text{n} < -2 \\ 3^n + 3^{-n} & -2 \leq n < 0 \\ n+1+3^n & n \geq 0 \end{cases}$$

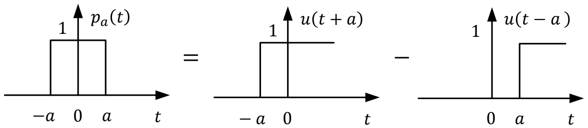
$$= \begin{cases} 3^n & \text{n} < -2 \\ 3^n + 3^{-n} & n = -1, -2 \\ n+1+3^n & n \geq 0 \end{cases}$$

Example 1.5: Construct $p_a(t)$ and $q_a(t)$ with the unit step functions.

Solution:

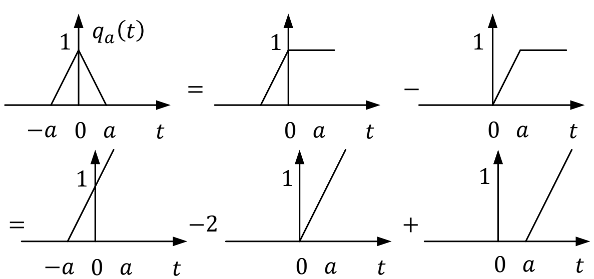
① $p_a(t) = \begin{cases} 1 & |t| < a \\ 0 & |t| > a \end{cases}$

$= u(t+a) - u(t-a)$



② $q_a(t) = \begin{cases} 1 - \frac{1}{a}|t| & |t| \leq a \\ 0 & |t| > a \end{cases}$

$= \left(1 + \frac{t}{a}\right) u(t+a) - 2\frac{t}{a} u(t) + \left(1 - \frac{t}{a}\right) u(t-a)$



2. Multiplication

Example 1.6: Determine $x_1(t) \cdot x_2(t)$ and $x_1[n] \cdot x_2[n]$, giving

a. $x_1(t) = \begin{cases} 0 & t < 0 \\ \sin\pi t & t \geq 0 \end{cases}$ and $x_2(t) = -\sin\pi t$

b. $x_1[n] = \begin{cases} 3^n & \text{n} < 0 \\ n+1 & \text{n} \geq 0 \end{cases}$ and $x_2[n] = \begin{cases} 0 & \text{n} < -2 \\ 3^{-n} & \text{n} \geq -2 \end{cases}$

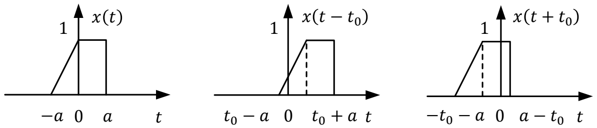
Solution:

a. $x_1(t) \cdot x_2(t) = \begin{cases} 0 & t < 0 \\ -\sin^2\pi t & t \geq 0 \end{cases}$

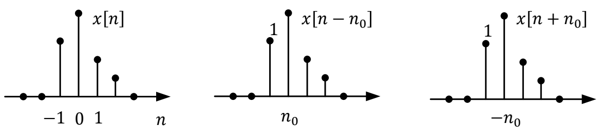
b. $x_1[n] \cdot x_2[n] = \begin{cases} 0 & \text{n} < -2 \\ 1 & -2 \leq n < 0 \\ (n+1)3^n & n \geq 0 \end{cases}$

3. Time Shifting

① $t_0 > 0$

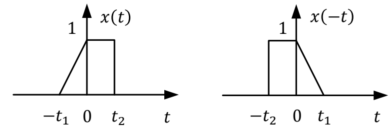


② $n_0 > 0$

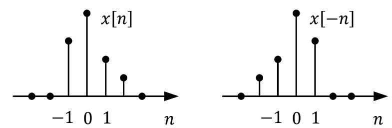


4. Time Reversal

①

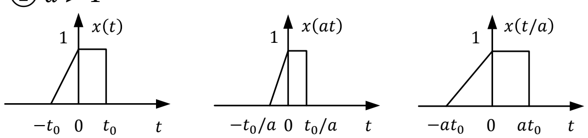


②

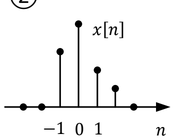


5. Time Scaling

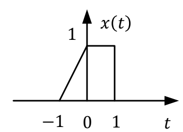
① $a > 1$

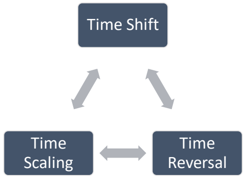


②



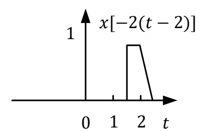
Example 1.7: Sketch $x(4-2t)$, giving $x(t)$ shown as follows.





Answer:

$x(4-2t) = x[2(2-t)] = x[-2(t-2)]$



6. Even-Odd Decomposition

① $x(t) = x_e(t) + x_o(t)$

$$\begin{cases} x_e(t) = \dfrac{1}{2}[x(t) + x(-t)] = x_e(-t) \\ x_o(t) = \dfrac{1}{2}[x(t) - x(-t)] = -x_o(-t) \end{cases}$$

If $x(t) = \begin{cases} 0 & t < 0 \\ x(t) & t > 0 \end{cases}$, $x(t)$ is called a causal signal

and satisfies

$$\begin{cases} x_e(t) = x_o(t) & t > 0 \\ x_e(t) = -x_o(t) & t < 0 \end{cases}$$

$x(t) = 2x_e(t) = 2x_o(t) \quad t > 0$

② $x[n] = x_e[n] + x_o[n]$

$$\begin{cases} x_e[n] = \dfrac{1}{2}\{x[n] + x[-n]\} = x_e[-n] \\ x_o[n] = \dfrac{1}{2}\{x[n] - x[-n]\} = -x_o[-n] \end{cases}$$

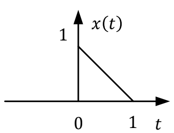
If $x[n] = \begin{cases} 0 & n < 0 \\ x[n] & n \geq 0 \end{cases}$, $x[n]$ is called a causal sequence

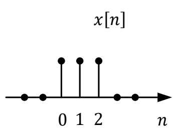
and satisfies

$$\begin{cases} x_e[n] = x_o[n] & n > 0 \\ x_e[n] = -x_o[n] & n < 0 \end{cases}$$

$x[n] = 2x_e[n] = 2x_o[n] \quad n > 0$

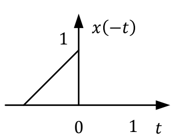
Example 1.8: Sketch the even and odd components of the signals given below.

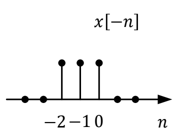




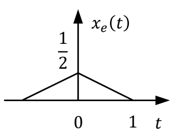
Solution:

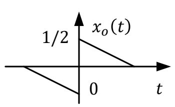
Firstly, determine $x(-t)$ and $x[-n]$





$x(t) = (1 - t)[u(t) - u(t-1)]$





$x[n] = \delta[n] + \delta[n-1] + \delta[n-2]$