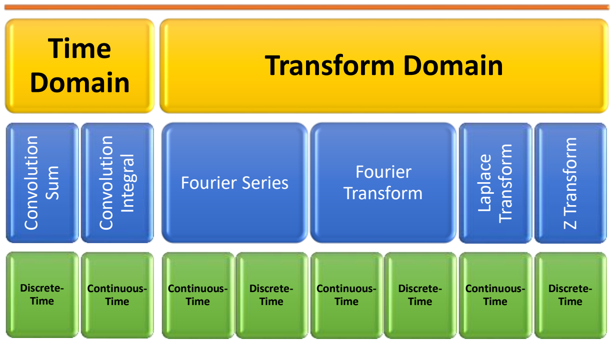
Chapter 1 Signals and Systems



§1.0 Course Specification

1. Aim of the course

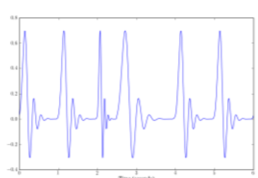
① Develop the necessary mathematical tools, such as the *Fourier transform*, *Laplace transform* and *Z-transform*

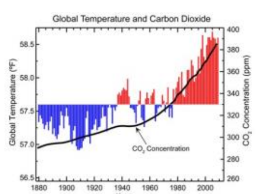
② Analyze and design *linear time-invariant systems*

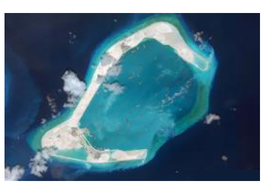
③ Build experience in applying these mathematical tools to the solution of realistic signal processing systems

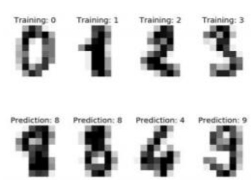
http://open.163.com/special/opencourse/signals.html

Signals are represented mathematically as functions of one or more independent variables.

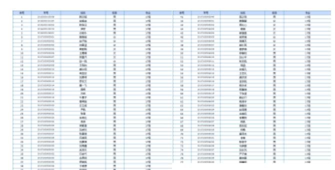


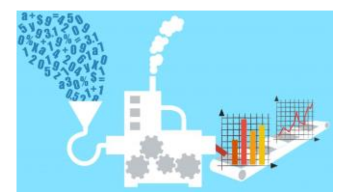


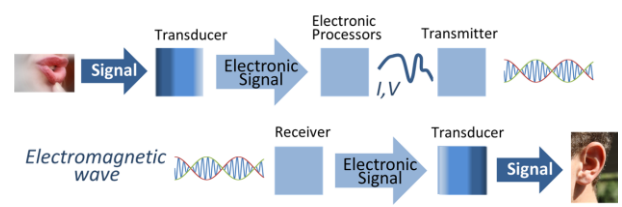




Data, Information and Signals

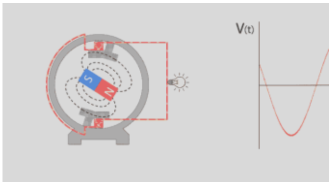




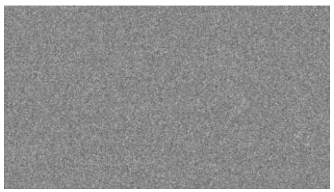


§1.1 Classification of Signals

1. Deterministic Signals and Stochastic Signals



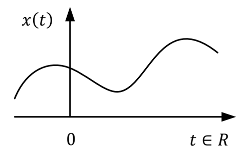
A signal is said to be deterministic if there is no uncertainty with respect to its value at any instant of time.

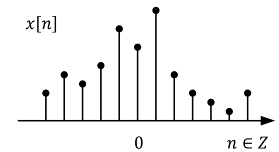


A signal is said to be stochastic if there is uncertainty with respect to its value at some instant of time.

§1.1 Classification of Signals

2. Continuous-Time Signals and Discrete-Time Signals





① Waveform and Sequence

② Analog Signals and Digital Signals

3. Real Signals and Complex Signals

① Real Signals:

$$\begin{cases} x^*(t) = x(t) \\ x^*[n] = x[n] \end{cases}$$

② Complex signals:

$$\begin{cases} x(t) = x_R(t) + jx_I(t) \\ x[n] = x_R[n] + jx_I[n] \end{cases}$$

For example,

$$\begin{cases} x(t) = e^{s_0 t} \\ x[n] = {z_0}^n \end{cases}$$

a. real signals when s_0 and z_0 are real constants

b. complex signals when s_0 and z_0 are complex constants

Complex Exponentials

First let

$$f(x) = \cos x + j\sin x$$

Consider the derivative of $f(x)e^{-jx}$

$$\frac{d}{dx}(\cos x + j\sin x)e^{-jx} = \frac{d}{dx}\cos x e^{-jx} + j\frac{d}{dx}\sin x e^{-jx}$$

$$= -\sin x e^{-jx} + \cos x\left(-je^{-jx}\right) + j\cos x e^{-jx} + \sin x e^{-jx}$$

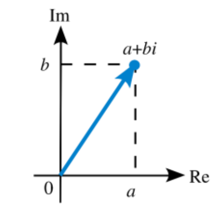
$$= 0$$

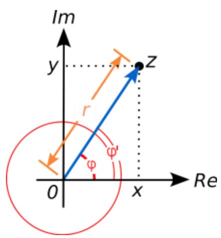
So that $f(x)e^{-jx}$ is a constant function.

$$f(x)e^{-jx} = f(0)e^{-j0} = 1$$

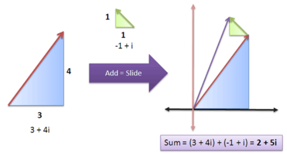
$$(\cos x + j\sin x)e^{-jx} = 1 \xrightarrow{\text{yields}} e^{jx} = \cos x + j\sin x$$

Complex Numbers

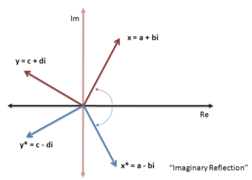




Complex Addition



Complex Conjugates



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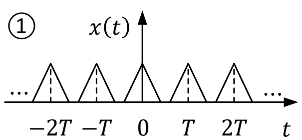

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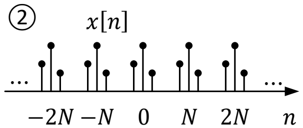

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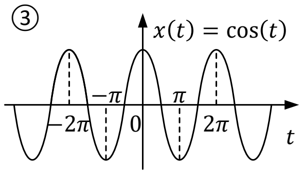

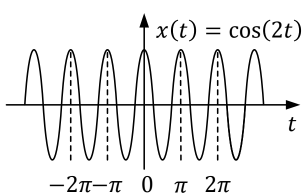

4. Periodic Signals and Aperiodic Signals

$x(t)$ and $x[n]$ are called periodic signals if

$$x(t) = x(t + kT) \qquad (\exists T \neq 0, \forall k \in Z)$$

and

$$x[n] = x[n + mN] \quad (\exists N \neq 0, \forall m \in Z).$$

Otherwise, $x(t)$ and $x[n]$ will be referred to as aperiodic signals.

The fundamental periods are defined as

$$\min\{T | T > 0\} = T_0$$

and

$$\min\{N | N > 0\} = N_0,$$

respectively.

④ Continuous-Time Complex Exponent Signals:

$$e^{j\omega_0 t} = cos\omega_0 t + jsin\omega_0 t$$

$e^{j\omega_0 t} = e^{j\omega_0 (t+T)} \Rightarrow e^{j\omega_0 T} = 1 \Rightarrow \omega_0 T = 2\pi k \ (k \in Z)$

Therefore, $e^{j\omega_0 t}$ is a periodic signal, of which the period is

$T = \dfrac{2\pi}{\omega_0} k$

and the fundamental period is

$T_0 = \min\{T | T > 0\} = \dfrac{2\pi}{|\omega_0|} \ (\omega_0 \neq 0)$

Euler's Relation: $\begin{cases} \cos x = \frac{1}{2}\left(e^{jx} + e^{-jx}\right) \\ \sin x = \frac{1}{2j}\left(e^{jx} - e^{-jx}\right) \end{cases}$ $x = \omega_0 t$ or $\omega_0 n$

⑤ Discrete-Time Complex Exponent Signals:

$$e^{j\omega_0 n} = cos\omega_0 n + jsin\omega_0 n$$

a. Because $e^{j(\omega_0+2\pi k)n} = e^{j\omega_0 n}e^{j2\pi kn} = e^{j\omega_0 n}$,
we need only to consider $0 \le \omega_0 < 2\pi$ or $-\pi \le \omega_0 < \pi$.

b. In order for $e^{j\omega_0 n} = e^{j\omega_0 (n+N)} \Rightarrow e^{j\omega_0 N} = 1$,
$\omega_0 N = 2\pi m \ (m \in Z)$
Therefore, $\frac{\omega_0}{2\pi} = \frac{m}{N}$ must be a rational number to make $e^{j\omega_0 n}$ a periodic signal.

Q1.1 Determine the fundamental period of the signal

$$x(t) = 2\cos(10t+1) - \sin(4t-1)$$

and

$$x[n] = 1 + e^{j\frac{4}{7}\pi n} - e^{j\frac{2}{5}\pi n}$$

Solution:

a. The fundamental period of $\cos(10t+1)$ is $\frac{\pi}{5}$
   The fundamental period of $\sin(4t-1)$ is $\frac{\pi}{2}$
   Therefore, the fundamental period of $x(t)$ is π
b. The fundamental period of 1 is arbitrary
   The fundamental period of $e^{j\frac{4}{7}\pi n}$ is 7
   The fundamental period of $e^{j\frac{2}{5}\pi n}$ is 5
   Therefore, the fundamental period of $x[n]$ is 35

Example 1.1: Determine the fundamental periods of $x_1[n] = e^{j\left(\frac{4}{3}\pi n+2\right)}$ and $x_2[n] = e^{j\frac{n}{4}}$.

Solution:

a. $x_1[n] = e^{j\left(\frac{4}{3}\pi n+2\right)} = e^{j\frac{4}{3}\pi n}e^{j2}$

$\omega_0 = \frac{4\pi}{3}$

$\frac{2\pi}{\omega_0} = \frac{3}{2} \in Q$

$x_1[n]$ is a periodic signal and the fundamental period is 3.

$$\boxed{x_3[n] = e^{j(a\pi n+b)} \quad \frac{2\pi}{\omega_0} = \frac{2}{a}}$$

b. $x_2[n] = e^{j\frac{n}{4}}$

$\omega_0 = \frac{1}{4}$

$\frac{2\pi}{\omega_0} = 8\pi \notin Q$

$x_2[n]$ is an aperiodic signal.

$$\boxed{x_4[n] = e^{j(an+b)} \quad \frac{2\pi}{\omega_0} = \frac{2\pi}{a}}$$