ECE495 Game Theory and Networked Systems

Lecture 1: Introduction to Game Theory

1 Introduction

Definition 1. *Game theory is the study of understanding self-interested, rational agents, individuals, in a setting of strategic interdependence.*

In other words, game theory is the study of how individual actors behave in a situation where they may both compete and collaborate for goals. These individuals are assumed to have their own best interest at the forefront of their decision making, and their actions are assumed to affect the outcome of the game for other participants. When selecting game theory as a model, these assumptions must hold. For instance, consider the problem of swarm robotics. It is well-established that the members of the swarm are independent agents; however, they are never in competition, so a more straightforward optimization approach will suffice. In order for game theory to be a good choice of model, the agents should have separate and potentially conflicting goals. Additionally, the preference of each agent for certain outcomes must be rational.

2 Why We Study Game Theory?

We study game theory, because we are interested in the behavior of the systems which it models. Stock trading, corporate maneuvering, and even interpersonal behavior can often be explained or modeled using game theory. While humans are not rational agents in all situations, behavioral models involve greater complexity and cost than game theory, and may or may not offer an improvement in results. Furthermore, other entities, such as corporations and nation-states, can and often do behave in rational fashions, making game theory ideal for the analysis and prediction of their behaviors.

Additionally, game theory has applications to the design of systems. The same techniques used to analyze and predict the behavior of actors in existing systems can be applied in the design of new ones, in order to ensure that collective behavior of actors does not cause undue stress on the system or cause it to operate in an unforeseen and undesirable manner.

Game theory has especial applications in economics, and is heavily associated with that field in the existing literature. The reasons behind this are largely historical; however, it is clear that in a free marketplace the actions of independent actors will often revert to those predicted by game theory, which makes such models useful in that space.

3 Utility

Utility is a basic means of representing an individual's preferences over consumption of goods and services. In other words, it is satisfaction of users as a result of their preferences. Preferences start from choices made by individuals among possible consumption bundles, a set of goods and services.

Suppose X is a set of possible goods and services available in the system, market. Let $A \subseteq X$ and $B \subseteq X$ denote some consumption bundles, different possible preferences of a user. We can define preference relations over A and B. *Strict relation*, $A \succ B$, means A is preferred over B. In other words, the user likes A better than B. *Indifference relation*, $A \sim B$, means A and B are about to be same for the user. If the user asked to choice between A or B, she may flip a coin to decide, it will not affect her satisfaction. Lastly, strict relation can be relaxed, as $A \succeq B$, which means A is at least as good as B.

Assumption: Individual preferences are rational, which eventually means that these preferences can be mapped to real numbers.

Definition 2. *A rational set of preferences is a set that is both complete and transitive.*

Definition 3. *A complete ordering is an ordering with the following property: for any two elements of the set, either one is more preferred, the other is more preferred, or both are equally preferred. (There is no confusion.) Mathematically, suppose X is a set of preferences, $\forall x, y \in X$; either $x \succeq y$, or $y \succeq x$, or both ($x \sim y$). This is also often called the totality property of an ordering.*

Definition 4. *A transitive ordering is an ordering with the following property: Suppose X is a set of preferences, $\forall x, y, z \in X$; if $x \succeq y$ and $y \succeq z$, then $x \succeq z$.*

The problem with this assumption may be observed by an example with set of items in a restaurant. Let a customer be asked to choose among two alternatives: Chicken and Shrimp, and she picked Shrimp (meaning that $s \succeq c$). Then, the customer may choose Chicken over Beef (meaning that $c \succeq b$). When the customer is asked to choose between Shrimp and Beef; we expect that, if the customer is rational, then she will pick Shrimp, since $s \succeq b$ by transitivity property. However, this may not always happen, because of dependencies of preferences on what is presented, and change of options.

Despite the problem above, we assume rational preferences throughout this course (as does most of economic theory and game theory). The main reason for this is that

- Game Theory tries to get qualitative understanding of how incentives govern behavior. This can be reached only when all the agents in the system are rational. If we want to study with irrational agents, we need to perturb our system, which is studied in Behavioral Economics.

Given a rational set of preferences, a basic result in economics it that these can be represented via a utility function.

Definition 5. *The utility function of an agent i, $U_i(A)$, is a mapping from the set of potential outcomes of a situation to the real numbers, generally with more preferred outcomes receiving higher utility values.*

We emphasize the following aspect of utility functions:

1. Utilities are not real. Utilities are extracted based on behavior of agents. As long as, $A \succeq B \Rightarrow U(A) > U(B)$ is true, our utility measure is consistent with behavior.

In general, rather than using an ordinal system of preferences, we will use a cardinal system of utilities, as this is easier to work with. When preferences are rational, this mapping preserves ordering; when they are irrational, it is generally not well defined.

We assume the following properties for utility function:

1. **Monotonicity:** The utility function is a monotonically non decreasing function. That is, an agent will always be at least as happy with an additional item as he is currently. Implicit in this is the assumption that it is free to discard items, i.e. the assumption of free disposal. If it were not, the agent might not be happy to acquire particular ones.

2. **Concavity:** Utility function is assumed to be a concave function, unless stated otherwise. This is also known as the law of diminishing returns.

A pictorial description of a utility function, according to the assumptions above, can be given in Fig. 1. From Fig. 1, x_2 has more items than x_1, so $U_i(x_2) \geq U_i(x_1)$ holds, due to the monotonicity property. The red line from $U_i(x_1)$ to $U_i(x_2)$ is never greater than the utility function, due to the concavity property.

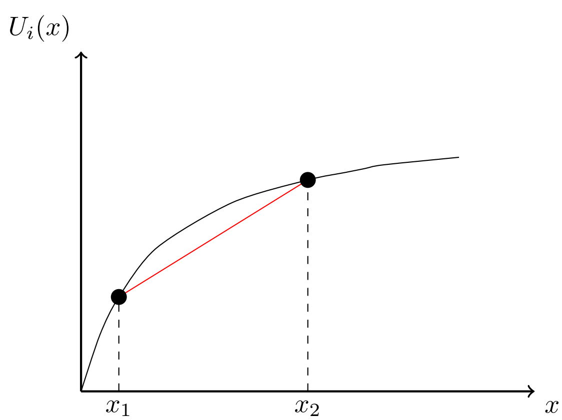

Figure 1: A pictorial description of a utility function