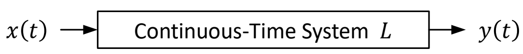
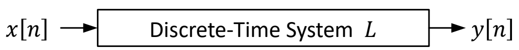
§1.3 Systems

1. Continuous-Time Systems and Discrete-Time Systems

① $y(t) = L[x(t)]$

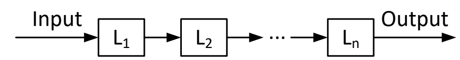


② $y[n] = L\{x[n]\}$

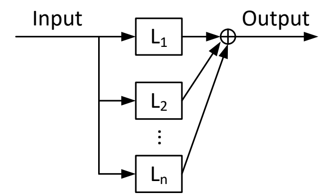


2. Interconnections of systems

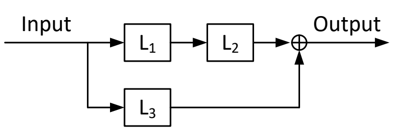
① Series (or cascade) interconnections



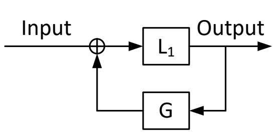
② Paralleled interconnections



③ Series-Paralleled interconnections



④ Feedback interconnections



3. Basic system properties

① Linearity

A linear system holds both the homogeneity and additivity.

a. Homogeneity

$L[ax(t)] = aL[x(t)]$ or $L\{ax[n]\} = aL\{x[n]\}$

b. Additivity

$L[x_1(t) + x_2(t)] = L[x_1(t)] + L[x_2(t)]$

$L\{x_1[n] + x_2[n]\} = L\{x_1[n]\} + L\{x_2[n]\}$

c. When a, b are arbitrary complex constants,

$L[ax_1(t) + bx_2(t)] = aL[x_1(t)] + bL[x_2(t)]$

$L\{ax_1[n] + bx_2[n]\} = aL\{x_1[n]\}+ bL\{x_2[n]\}$

② Time-Invariance

A system $y(t) = L[x(t)]$ or $y[n] = L\{x[n]\}$ is time-invariant, when the following statements hold for arbitrary t_0 and n_0

$$y(t - t_0) = L[x(t - t_0)]$$
$$y[n - n_0] = L\{x[n - n_0]\}$$

③ When a system is not only linear but also time-invariant, it is called a linear time-invariant (LTI) system.

For example, if $y(t) = S[x(t)]$ and system S is LTI,

$$\frac{y(t + \Delta t) - y(t)}{\Delta t} = S\left[\frac{x(t + \Delta t) - x(t)}{\Delta t}\right]$$

When $\Delta t \to 0$,

$$\frac{d}{dt} y(t) = S\left[\frac{d}{dt} x(t)\right]$$

Example 1.9: Determine whether or not system S is linear and time invariant.

a. $y(t) = S[x(t)]=\sin[x(t)]$

Solution:

Let $y_1(t) = \sin[x_1(t)]$ and $y_2(t) = \sin[x_2(t)]$
$y_1(t) + y_2(t) \neq \sin[x_1(t) + x_2(t)]$
$ay(t) \neq \sin[ax(t)]$

Consider $x_2(t) = x_1(t - t_0)$
$y_2(t) = \sin x_2(t) = \sin[x_1(t - t_0)]$
$y_1(t - t_0) = \sin\,[x_1(t - t_0)]$
$\therefore y_2(t) = y_1(t - t_0)$

System S is nonlinear and time invariant.

Example 1.9: Determine whether or not system S is Linear and time invariant.

b. $y[n] = S\{x[n]\} = n \cdot x[n]$

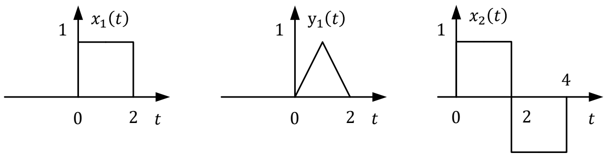
Solution:

Let $y_1[n] = ax_1[n]$ and $y_2[n] = \mathrm{n}x_2[n]$
$n\{ax_1[n] + bx_2[n]\} = a \cdot n \cdot x_1[n] + b \cdot n \cdot x_2[n]$
$n\{ax_1[n] + bx_2[n]\} = ay_1[n] + by_2[n]$

When $x_2[n] = x_1[n - n_0]$,
$y_2[n] = n \cdot x_1[n - n_0]$
$y_1[n - n_0] = (n - n_0)x_1[n - n_0]$
$\therefore y_2[n] \neq y_1[n - n_0]$

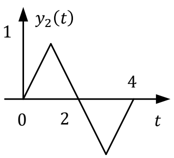
System S is linear and time varying.

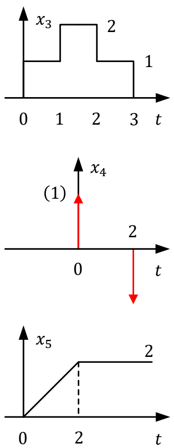
Example 1.10: Consider an LTI System whose response to the signal $x_1(t)$ is the signal $y_1(t)$. Determine and sketch carefully the response $y_2(t)$ of this system to the input $x_2(t)$. $x_1(t)$, $y_1(t)$ and $x_2(t)$ are depicted as follows.



Solution:

The system is know to be LTI.
$\because x_2(t) = x_1(t) - x_1(t - 2)$
$\therefore y_2(t) = y_1(t) - y_1(t - 2)$

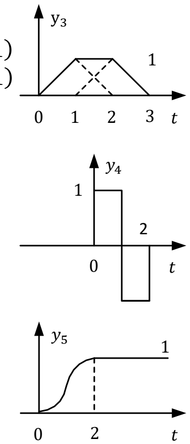




$$x_3(t) = x_1(t) + x_1(t-1)$$
$$y_3(t) = y_1(t) + y_1(t-1)$$

$$x_4(t) = \frac{d}{dt}x_1(t)$$
$$y_4(t) = \frac{d}{dt}y_1(t)$$

$$x_5(t) = \int_{-\infty}^{t} x_1(\tau)d\tau$$
$$y_5(t) = \int_{-\infty}^{t} y_1(\tau)d\tau$$



③ Systems with and without Memory

a. Memoryless

For example, $y(t) = Rx(t)$, $y[n] = (n-2)x[n]$

b. Memory

For example, $y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)d\tau$, $y[n] = \sum_{n=-\infty}^{\infty} x[n]$

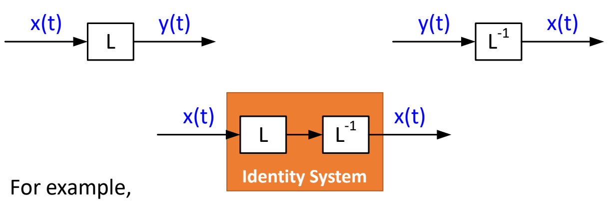
④ Causality

a. When $x(t) = 0\ (t < t_0)$, the causal linear system outputs $L[x(t)] = 0\ (t < t_0)$.

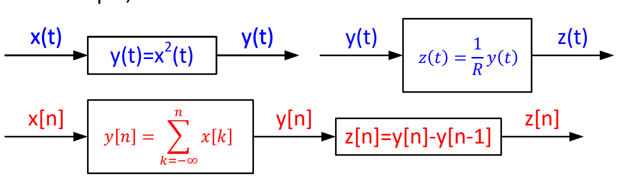
b. When $x(t) = 0\ (t < 0)$, the causal linear time-invariant system outputs $L[x(t)] = 0\ (t < 0)$.

c. All memoryless systems are causal.

⑤ Invertibility and Inverse Systems



For example,



⑥ Stability

a. When the input to a stable system is bounded, the output is also bounded.

For example, $y(t) = e^{x(t)}$ is stable.

It is because that when the input is bounded $|x(t)| \le B$,

the output $|y(t)| = \left|e^{x(t)}\right| \le e^{|x(t)|} \le e^{B} \triangleq B'$ is proved to be bounded.

b. If the output of the system is infinite even when a finite input applied to a system, this system is called an unstable system.

For example, $y(t) = tan[x(t)]$ is unstable, because when the input is bounded $x(t) = \frac{\pi}{2}$, the output is infinite.

Q1.4 Consider a discrete-time system with input $x[n]$ and output $y[n]$. The input-output relationship for this system is

$$y[n] = x[n]x[n-2]$$

a. Is the system memoryless?

b. Determine the output of the system when the input is $A\delta[n]$, where A is any real or complex number

c. Is the system invertible?

Solution:

a. No

b. $\{A\delta[n]\}\{A\delta[n-2]\} = A^2\delta[n]\delta[n-2] = 0$

c. No

Q1.5 Consider a discrete-time system with input $x[n]$ and output $y[n]$ related by

$$y[n] = \sum_{k=n-n_0}^{n+n_0} x[k]$$

where n_0 is a finite positive integer.

a. Is this system linear?

b. Is this system time-invariant?

c. Is this system stable?

Solution:

$$y[n-l] = \sum_{k=n-l-n_0}^{n-l+n_0} x[k] = \sum_{k=n-n_0}^{n+n_0} x[k-l]$$

If $|x[k]| < B$, $|y[n]| < (2n_0+1)B$

Q1.6 Consider a system S with input $x[n]$ and output $y[n]$ related by

$$y[n] = x[n]\{g[n] + g[n-1]\}$$

a. Is this system time invariant when $g[n] = 1$

b. Is this system time invariant when $g[n] = n$

c. Is this system time invariant when $g[n] = 1 + (-1)^n$

d. Is this system time invariant when $g[n] = e^{j2\pi n} + e^{j\pi n}$

Solution:

a. $y[n] = 2x[n]$

b. $y[n] = (2n-1)x[n]$

c. $y[n] = 2x[n]$

d. $y[n] = 2x[n]$

Homework			
1.14	**1.23**		
1.15	1.16	1.17	
1.24	1.25	1.26	
1.27	1.28	1.31	

① Do not wait until the last minute
② Express your own idea and original opinion
③ Keep in mind the zero-tolerance policy on plagiarism