Northwestern University
Deptartment of EECS

EE 495: Game Theory for Networked Systems — Spring 2022

Midterm Exam

Problem 1 (35 points) For each one of the statements below, state whether it is true or false. If the answer is true, prove it. If the answer is false, give a counterexample. Explanations and counterexamples are required for full credit. (7 points each)

(1) If a static game has a unique strictly dominant strategy equilibrium, then this game *must* also have a unique Nash equilibrium.

(2) In a Wardrop equilibrium, each player is infinitesimally small, hence their strategic behavior cannot steer system towards inefficiency and all Wardrop Equilibria maximize social welfare.

(3) In the following simultaneous static prisoner's dilemma game, there exists a correlated equilibrium where players play (C, C) with probability 1.

	D	C
D	$-2, -2$	$0, -3$
C	$-3, 0$	$-1, -1$

(4) Let G be any two-player supermodular game and let (x_1, x_2) and $(\tilde{x}_1, \tilde{x}_2)$ be two distinct Nash equilibria of this game, where in each case the first component indicates the action of player 1 and the second component indicates the action of player 2. Furthermore, assume starting from any of these equilibria, unilateral deviation will strictly decrease that player's payoff. If $x_1 > \tilde{x}_1$, it must be that $x_2 \geq \tilde{x}_2$.

(5) For any finite game, a weakly dominated action cannot be used with positive probability in a correlated equilibrium.

Problem 2 (10 points)*(Mixed Strategy Equilibrium)* Consider a variant of the meeting up for lunch game discussed in class, where the payoff matrix is given as below. Find all (mixed and pure strategy) Nash equilibria.

		Alice	
		TE	Sargent
Bob	TE	(2,1)	(0,0)
	Sargent	(0,0)	(3,4)

Problem 3 (15 points) *(Potential Game)*

1. Consider the following game

	A	
	L	R
B U	(1,-2)	(10,5)
D	(3,100)	(4,99)

   Show that is is an exact potential game by constructing the potential function.

2. Consider the parameterized version of the game

	A	
	L	R
B U	(a,b)	(c,d)
D	(e,f)	(g,h)

   Write down the relations between the parameters, such that this game and the one in the previous part would share the same exact potential potential function.

Problem 4 (25 points) *(Patent Race for a New Market)*

Consider a patent race game, where the players are 2 firms: Alps and Bees, which we denote by A and B, respectively. Both firms simultaneously choose a spending budget on research $x_i \geq 0$ ($i = A, B$). Innovation occurs at time $T(x_i)$, which is a function of the spending, where the derivative of $T(x)$ satisfies $T'(x) < 0$ (i.e., more budget leads to faster innovation). The first firm to develop the innovation can file a patent for it worth V dollars (assume no discounting), while any firm developing the innovation later gets no value. The total cost spent by firm i to develop the innovation is x_i, and if both players innovate simultaneously they share its value equally.

1. Formulate the situation as a static game by specifying the payoff functions π_i for firms $i = A, B$.
2. Show that in all pure strategy Nash equilibria one of the players does not invest.
3. Does this game have a pure strategy equilibrium when there are three players, each choosing $x_i \geq 0$, $i = A, B, C$. If there is a tie, the value of the patent is split equally among those who tie.

Problem 5 (15 points) *(Iterated Elimination of Strictly Dominated Strategies in Cournot Competition)*

Consider a market in which the price charged for quantity Q (total quantity in the market) of some good is given by $P(Q) = 1 - Q$. Assume that the cost of producing a unit of this good is 0. In this problem, we use subscript to index the firm and superscript to count the number of rounds of eliminations of strictly dominated strategies.

1. Assume that there are two firms in the market. Starting from the original strategy space $S_i^0 = [0, \infty]$, each firm carries out iterated elimination of the strictly dominated strategies. Argue that for $i = 1, 2$, after one round of elimination, we have the set S_i^1 surviving the elimination process can be written as $S_i^1 = [0, 1/2]$.

2. Still for two firms, construct the sets of strategies S_1^k, S_2^k for any $k \geq 1$ and conclude that S_i^∞ is a singleton, i.e. it has only one element. How many Nash Equilibria does this game have?

3. Now assume that there are three firms. Show that S_1^∞ is not a singleton.