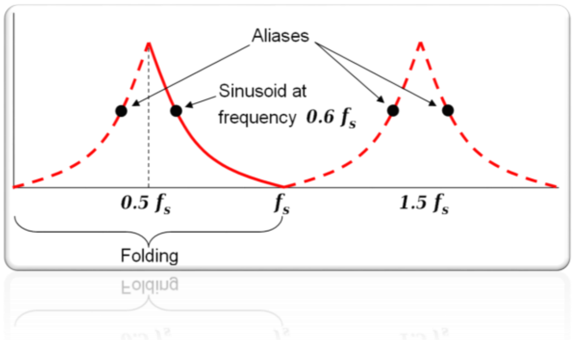
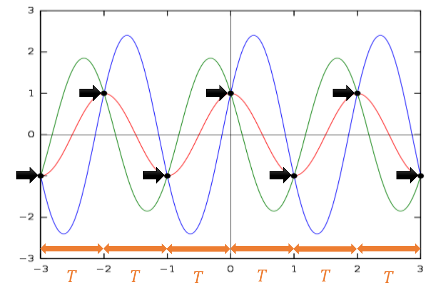
Chapter 7 Sampling



§7.1 Sampling Theorem

In general, with no additional conditions, a signal cannot be uniquely specified by a sequence of equally spaced samples.

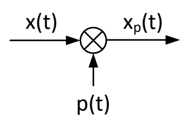
$$x_1(t) \neq x_2(t) \neq x_3(t)$$
$$x_1(kT) = x_2(kT) = x_3(kT)$$



1. Impulse-Train Sampling

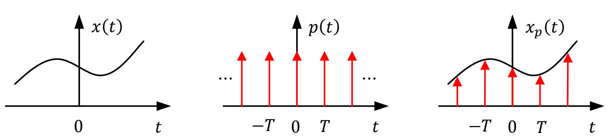
$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

T—Sampling period



$$x_p(t) = \sum_{n=-\infty}^{\infty} x(t)\delta(t - nT) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT)$$

$x(nT)$ —Sample values of $x(t)$

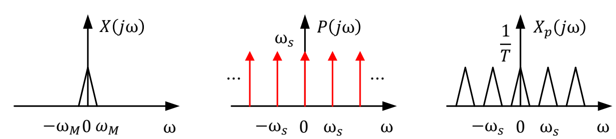


$$P(j\omega) = F\left\{\sum_{k=-\infty}^{\infty} \frac{1}{T} e^{-jk\omega_s t}\right\} = \omega_s \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s)$$

$\omega_s = \frac{2\pi}{T}$ —Sample Frequency

$$X_p(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X[j(\omega - k\omega_s)]$$

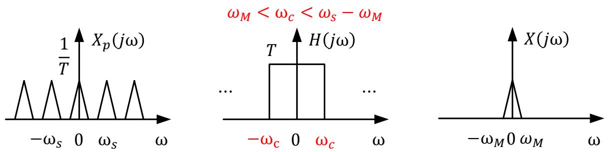
When $\omega_M < (\omega_s - \omega_M)$, or equivalently, $\omega_s > 2\omega_M$, there is no overlap between the shifted replicas of $X(j\omega)$

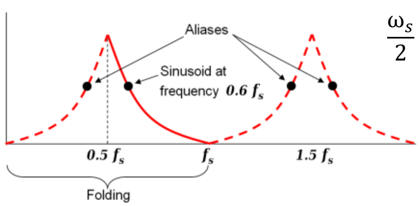


Sampling Theorem:

①Let $x(t)$ be a band-limited signal with $X(j\omega) = 0$ for $|\omega| > \omega_M$. Then $x(t)$ is uniquely determined by its samples $x(nT)$ for $n \in Z$, if $\omega_s > 2\omega_M$, where $\omega_s = \frac{2\pi}{T}$.

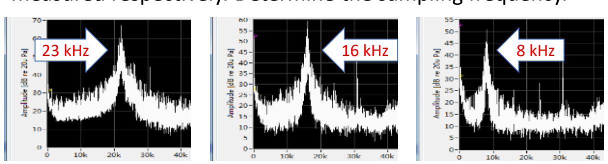
②We can reconstruct $x(t)$ by generating a periodic impulse train in which successive impulses have amplitudes that are successive sample value. This impulse train is then processed through an ideal low-pass filter with gain T and cutoff frequency greater than ω_M, and less than $\omega_s - \omega_M$.





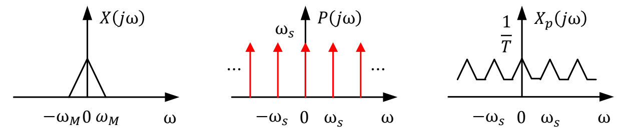
$\frac{\omega_s}{2}$ — Mirror Frequency

Example 7.1 Three input frequencies to an unknown system are 25 kHz, 32 kHz and 40 kHz. With the output frequencies measured respectively. Determine the sampling frequency.

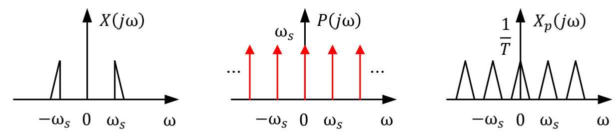


$\omega_s \gg 2\omega_M$ —Over Sampling

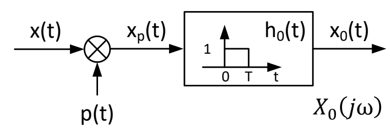
$\omega_s < 2\omega_M$ —Under Sampling



Bandpass Sampling



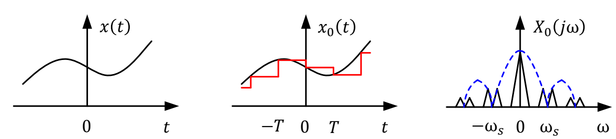
2. Sampling with a Zero-Order Hold



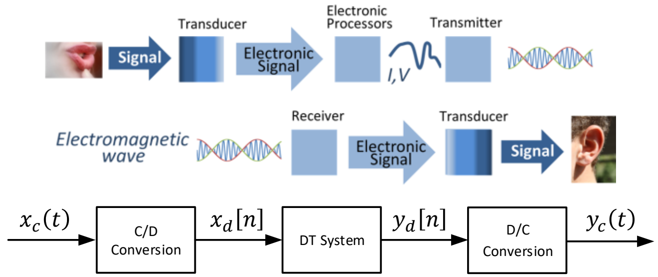
$$x_0(t) = x_p(t) * h_0(t)$$

$$X_0(j\omega) = \frac{1}{2\pi} X_p(j\omega) * H_0(j\omega)$$

$$X_0(j\omega) = \frac{1}{\pi T} \sum_{k=-\infty}^{\infty} \left\{ e^{-j\omega\frac{T}{2}} \left[\frac{\sin\left(\frac{\omega T}{2}\right)}{\omega} \right] X[j(\omega - k\omega_s)] \right\}$$



3. Discrete-Time Processing of Continuous-Time Signals



$x_d[n] = x_c(nT)$ $y_d[n] = y_c(nT)$

$$y_c(nT) = x_d[n] * h[n] = \sum_{k=-\infty}^{+\infty} h[k]x_c(nT - kT)$$

Homework			
7.3	7.9		
7.1	7.2	7.6	

① Do not wait until the last minute
② Express your own idea and original opinion
③ Keep in mind the zero-tolerance policy on plagiarism