

## Short-time Traffic Flow Prediction with ARIMA-GARCH Model

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**Abstract**—Short-time traffic flow prediction is a significant interest in transportation study, and it is essential in congestion control and traffic network management. In this paper, we propose an Autoregressive Integrated Moving Average with Generalized Autoregressive Conditional Heteroscedasticity (ARIMA-GARCH) model for traffic flow prediction. The model combines linear ARIMA model with nonlinear GARCH model, so it can capture both the conditional mean and conditional heteroscedasticity of traffic flow series. The model is calibrated, validated and used for prediction based on PeMS single loop detector data. The performance of the hybrid model is compared with that of standard ARIMA model. The results show that the introduction of conditional heteroscedasticity cannot bring satisfactory improvement to prediction accuracy, in some cases the general GARCH(1,1) model may even deteriorate the performance. Thus for ordinary traffic flow prediction, the standard ARIMA model is sufficient.

### I. INTRODUCTION

WITH the fast pace of urbanization, the influence of transportation inefficiency is becoming more and more significant. Traffic congestion is one of the major headaches in metropolitan area nowadays. In major cities of China, congestion is relatively common. And in order to slow down the deterioration of traffic, a number of advanced systems are developed and deployed, including intelligent traffic control system and traffic guidance system, etc. Traffic prediction plays a very important role in supporting demand forecasts needed by those traffic management systems. Without a precise and efficient prediction of the near future, the systems can only work reactively to solve problems that have already happened rather than work proactively to prevent them from happening. In a word, a good prediction is the premise of the Intelligent Transportation Systems (ITS). Since transportation systems are usually large-scale time-variant complex systems consisting of various nonlinear and stochastic dynamics at multiple levels, traffic prediction remains a challenging problem [1]-[6]. And during the last two decades, new methods of traffic prediction are consistently concerned by researchers all over the world.

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A number of methods from different areas are modified and implemented in traffic prediction, the principles behind those methods can be significantly different. Auto Regressive Moving Average (ARMA) model is a kind of most widely used regression analysis method which aims to determine the regression type relationship between the historical data and the future data [7]-[8]. The model was discussed in details by George E. P. Box and Gwilym M. Jenkins in 1970, and it opened a new road to the analysis of time series. So far, the model as well as its derivatives has been widely applied to model many types of time series, including traffic flow series. And they become indispensable tools for short-time prediction [9]-[11].

However, in such ARMA model, the variance is assumed to be constant. Yet in reality, the variance is never constant. Volatility Clustering is frequently used in economic area to describe similar phenomenon, which reflects non-constant variance. To deal with the conditional heteroscedasticity of time series, the ARCH and Generalized ARCH (GARCH) model are introduced by Engle and Bollerslev respectively. GARCH is enormously influential and successful in capturing some of the most important empirical features of time series, and is widely used in research and applications. The most distinctive character of GARCH is its conditional variance, which means the variance varies over time [12].

The ARIMA or GARCH model alone is widely applied in time series modeling and prediction research, especially in financial time series analysis [13]-[14]. The hybrid ARIMA-GARCH model is developed more recently and receives much interest [15]-[16]. ARIMA-GARCH model is implemented in many areas other than financial analysis for prediction and forecast purposes. It is used to predict electricity prices [17] and the rain attenuation level in Earth-to-Satellite links [18], both of them achieve good results. Some studies also apply the ARIMA-GARCH model to model and predict the computer network traffic, the results show that the model can capture prominent traffic characteristics and has better prediction accuracy [19]-[20]. In transportation study area, the hybrid model is adopted by researchers to model and forecast univariate traffic speed series [21], both the evolution of traffic speed levels and the evolution of traffic speed volatility are captured.

The paper is organized as follows. Section II presents an overview of the ARIMA-GARCH model. Section III introduces the daily profile, datasets and performance indexes we used to conduct our research, the fitting procedure for the hybrid model is also presented. In section IV, the ARIMA-GARCH model is used to capture the conditional

heteroscedasticity of the datasets. And we use our model to predict the real freeway traffic flow data. The prediction results are presented and compared with those of standard ARIMA model. Section V discusses the results and concludes the paper.

## II. MODELING METHODOLOGY

The model combines the linear ARIMA and conditional heteroscedasticity GARCH to form a nonlinear hybrid time series model.

Let  $X_t$  be the stationary time series we want to study, and it can be modeled as an ARMA process as [7]-[8]

$$\phi(B)X_t = \theta(B)\varepsilon_t \quad (1)$$

Where  $B$  is the backshift operator  $BX_t = X_{t-1}$ , and

$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p$ ,  $p$  is the order of AR polynomial.

$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q$ ,  $q$  is the order of MA polynomial.

ARIMA model is a generalization of standard ARMA model, it uses differencing technique to handle non-stationarity. If the time series  $X_t$  we want to study is non-stationary, by proper differencing, we can get a stationary series to feed in the standard ARMA model as

$$\phi(B)(1-B)^d X_t = \theta(B)\varepsilon_t \quad (2)$$

Where  $d$  is the order of differencing.

The ARMA/ARIMA model proposed above is a linear time series model which assumes constant variance of  $\varepsilon_t$ . However related researches have shown the existence of conditional variance, therefore these models cannot capture such characteristic well. Moreover, if the hypothesis of constant error variance does not hold, the least squares algorithm used to estimate the parameters of ARIMA will be biased. A solution to the problem consists in modeling heteroscedasticity as a nonlinear relationship between consecutive errors. The GARCH model, which has time-varying variance, can help to better characterize the statistical features of traffic flow series. ARIMA-GARCH combines ARIMA model with GARCH model to form a nonlinear time series model. The general GARCH ( $p, q$ ) model for the conditional variance of  $\varepsilon_t$  is defined as [12]

$$\sigma_t^2 = E_{t-1}(\varepsilon_t^2) \text{ and } \varepsilon_t = \sigma_t e_t, e_t \sim N(0,1) \quad (3)$$

GARCH model characterizes the conditional variance of prediction error  $\varepsilon_t$  by imposing alternative parameters to capture serial dependence on the past sequence of observations as

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 \quad (4)$$

With constraints

$$\sum_{i=1}^q \alpha_i + \sum_{j=1}^p \beta_j < 1 \quad \alpha_0 > 0$$

$$\alpha_i > 0, i=1,2,3,\dots,q \quad \beta_j > 0, j=1,2,3,\dots,p \quad (5)$$

In this paper, we only study the prediction of one-step ahead traffic flow value based on historical data collected from single loop detector. Whenever a new measured flow data point is available, it is added to the historical data database and will be used for the next step forecast.

## III. PRELIMINARY

### A. Daily Profile

The daily profile refers to the M-shape curve of daily traffic flow, and traffic flow data collected from the same site during a consecutive period of time usually shares similar daily profile. After removing the profile, the residual time series we obtain usually behaves like a stochastic process, and is more stationary than the original flow data. Some research finds that prediction based on residual time series rather than the original traffic flow series has the potential to achieve better prediction performance. So in this paper we mainly study the prediction based on residual time series.

A very basic method – simple average method is used to get the daily profile. Suppose the sampled traffic flow data used to calculate daily profile has  $N$  consecutive working days, and there are  $n$  sampling data point per day. For instance, if the sample interval is 3 minutes, then  $n = 480$ . The traffic flow vectors can be written as

$$Y_1(t) = [y_1(1), y_1(2), \dots, y_1(n)], \dots, Y_N(t) = [y_N(1), y_N(2), \dots, y_N(n)] \quad (6)$$

The profile can be calculated with simple average method as

$$\bar{Y}_{profile}(t) = \left[ \frac{1}{N} \sum_{i=1}^N y_i(1), \dots, \frac{1}{N} \sum_{i=1}^N y_i(n) \right] \quad (7)$$

### B. Data Resource

In the paper, we choose the open-access Performance Measurement System (PeMS) traffic flow datasets to study [22]. The objective of PeMS project is to collect real-time freeway data from freeways in California to measure traffic performance. The raw data is obtained every 30 seconds, and we further aggregate the raw data to 3 min, 5 min, 10 min and 15 min interval.

The datasets used in this paper is collected at Detector 1006210 (NB 99 Milgeo Ave). The detector is located at north bound freeway SR99, Ripon city, San Joaquin County, California, Latitude 37.74675, Longitude -121.133. The road has 3 lanes, and the sampling time period is from October 1st 2009 to November 30th 2009. Since the pattern of traffic flow at holidays is different from that of working days, in the paper we only focus on traffic flow data at working days.

### C. Performance Indexes

To study the performance of our hybrid traffic flow prediction model, some performance indexes should be selected to measure the discrepancy between the observed value and the predicted value. In this paper, we choose the

following three error indexes, which are widely used both in theory and practice.

- 1) Mean Absolute Error (MAE)

$$MAE = \frac{1}{n} \sum_{t=1}^n |\hat{y}(t) - y(t)| \quad (8)$$

where  $\hat{y}(t) = \{\hat{y}(1), \hat{y}(2), \dots, \hat{y}(n)\}$  denotes the predicted traffic flow set we want to study.

- 2) Mean Squared Error (MSE)

$$MSE = \frac{1}{n} \sum_{t=1}^n [\hat{y}(t) - y(t)]^2 \quad (9)$$

- 3) Mean Relative Error (MRE)

$$MRE = \frac{1}{n} \sum_{t=1}^n \left| \frac{\hat{y}(t) - y(t)}{y(t)} \right| \quad (10)$$

#### D. Fitting Procedure

In this paper, we adopt the follow procedure to build the model for prediction.

Step 1: Pre-processing the measured traffic flow series to get a nearly stationary residual series. We subtract the daily profile from the measured data.

Step 2: Fitting ARIMA model with the residual series, which includes determining the order of  $p$ ,  $q$  and  $d$  and estimating parameters of  $\phi_1, \phi_2, \dots, \phi_p$  and  $\theta_1, \theta_2, \dots, \theta_q$ . When the differencing order  $d=0$ , the model is simplified to the ARMA form, and it is found that first order differencing ( $d=1$ ) of the residual series is adequate. Autocorrelation function (ACF) and its partial function (PACF) are typically used to determining the order of  $p$ ,  $q$ . In this paper, a choice of  $p=2$  and  $q=2$  is selected for all the datasets to better make the comparison. And we also test ARIMA ( $d=1$ ) and ARMA ( $d=0$ ) respectively.

Step 3: Fitting GARCH model with the prediction error series  $\varepsilon_t$ , which includes determining the order of  $p$ ,  $q$  and estimating corresponding parameters. However, identifying the order of  $p$ ,  $q$  is difficult both in theory and practice. So typically  $p=1$  and  $q=1$  is used as standard order.

Step 4: Validating the model to determining whether it is accurate enough and sufficient for prediction. Typically we would test whether ARCH effect exists in the normalized

error of the fitted model. Ljung-Box test is implemented for this purpose, and the fitting process may be iterated until no significant heteroscedasticity exists in the normalized error series. For series without heteroscedasticity, the model can be used for prediction.

Step 5: Implementing the model to get one-step ahead prediction. Since the residual series is used for study, the predicted residual values should plus the daily profile to get the predicted traffic flow values.

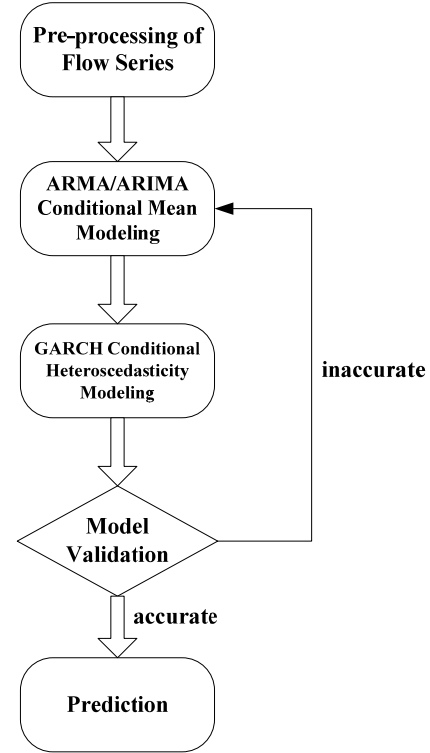


Fig. 1. The procedure of fitting ARIMA-GARCH model for prediction

#### IV. PERFORMANCE EVALUATION

##### A. Heteroscedasticity Test

To build an ARIMA-GARCH model we need to first carry out traffic flow prediction with ARIMA model and get the

TABLE I  
LJUNG-BBOX TEST OF PREDICTION ERROR  $\varepsilon_t$  WITH SIGNIFICANCE LEVEL  $\alpha = 0.05$

Lags		1	2	3	4	5	6	7	8	9	10
$\chi^2_{1-\alpha,h}$		3.84	5.99	7.81	9.49	11.07	12.59	14.07	15.51	16.92	18.31
3 min	$Q$	<b>64.28</b>	<b>177.18</b>	<b>277.59</b>	<b>326.59</b>	<b>336.95</b>	<b>338.81</b>	<b>347.11</b>	<b>350.09</b>	<b>350.15</b>	<b>358.61</b>
	p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5 min	$Q$	<b>108.56</b>	<b>187.10</b>	<b>197.22</b>	<b>208.51</b>	<b>211.24</b>	<b>217.78</b>	<b>217.95</b>	<b>218.37</b>	<b>219.79</b>	<b>222.84</b>
	p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10 min	$Q$	<b>69.31</b>	<b>88.72</b>	<b>95.92</b>	<b>95.93</b>	<b>101.31</b>	<b>108.69</b>	<b>113.51</b>	<b>114.36</b>	<b>114.51</b>	<b>114.80</b>
	p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
15 min	$Q$	<b>7.78</b>	<b>22.02</b>	<b>23.56</b>	<b>31.84</b>	<b>32.21</b>	<b>41.18</b>	<b>45.22</b>	<b>47.85</b>	<b>48.25</b>	<b>57.92</b>
	p-value	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

The bold values in the table indicate  $Q > \chi^2_{1-\alpha,h}$ , and the larger the difference, the more significant the heteroscedasticity is.

TABLE II  
LJUNG-BOX TEST OF NORMALIZED ERROR  $\varepsilon_t / \sigma_t$  AND  $[\varepsilon_t / \sigma_t]^2$  WITH SIGNIFICANCE LEVEL  $\alpha = 0.05$

Lags		1	2	3	4	5	6	7	8	9	10
$\chi^2_{1-\alpha,h}$		3.84	5.99	7.81	9.49	11.07	12.59	14.07	15.51	16.92	18.31
3 min	$Q$	<b>5.60</b>	<b>7.79</b>	<b>7.87</b>	<b>9.77</b>	<b>11.87</b>	12.24	12.76	14.02	<b>19.01</b>	<b>21.59</b>
$\varepsilon_t / \sigma_t$	p-value	0.02	0.02	0.05	0.04	0.04	0.06	0.08	0.08	0.03	0.02
3 min	$Q$	<b>15.02</b>	<b>37.18</b>	<b>37.39</b>	<b>38.05</b>	<b>38.81</b>	<b>39.03</b>	<b>39.20</b>	<b>40.49</b>	<b>43.01</b>	<b>44.78</b>
$[\varepsilon_t / \sigma_t]^2$	p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5 min	$Q$	<b>8.70</b>	<b>9.02</b>	<b>10.05</b>	<b>10.83</b>	<b>16.95</b>	<b>20.07</b>	<b>21.05</b>	<b>21.15</b>	<b>21.43</b>	<b>21.45</b>
$\varepsilon_t / \sigma_t$	p-value	0.00	0.01	0.02	0.03	0.00	0.00	0.00	0.01	0.01	0.02
5 min	$Q$	<b>6.55</b>	<b>11.10</b>	<b>11.28</b>	<b>11.29</b>	<b>13.87</b>	<b>14.55</b>	<b>15.38</b>	<b>15.59</b>	15.89	16.01
$[\varepsilon_t / \sigma_t]^2$	p-value	0.01	0.00	0.01	0.02	0.02	0.02	0.03	0.05	0.07	0.10
10 min	$Q$	2.95	2.97	4.45	4.49	5.01	7.81	13.00	13.15	13.58	14.09
$\varepsilon_t / \sigma_t$	p-value	0.09	0.23	0.22	0.34	0.41	0.25	0.07	0.11	0.14	0.17
10 min	$Q$	0.34	0.67	1.00	1.09	1.09	1.74	1.78	1.92	1.92	1.95
$[\varepsilon_t / \sigma_t]^2$	p-value	0.56	0.72	0.80	0.90	0.96	0.94	0.97	0.98	0.99	1.00
15 min	$Q$	0.74	1.48	4.10	5.29	5.31	7.57	11.76	12.00	12.05	12.29
$\varepsilon_t / \sigma_t$	p-value	0.39	0.48	0.25	0.26	0.38	0.27	0.11	0.15	0.21	0.27
15 min	$Q$	0.07	0.13	0.49	0.55	0.65	0.66	1.87	1.89	1.93	2.00
$[\varepsilon_t / \sigma_t]^2$	p-value	0.79	0.94	0.92	0.97	0.99	1.00	0.97	0.98	0.99	1.00

The bold values in the table indicate  $Q > \chi^2_{1-\alpha,h}$ , and the larger the difference, the more significant the heteroscedasticity is.

prediction error  $\varepsilon_t$  of the fitted ARIMA model. Before building the GARCH model, testing for heteroscedasticity is carried out by Ljung-Box test to determine whether heteroscedasticity exists in the time series. It is a portmanteau test which tests the overall randomness of the series based on a number of lags, rather than tests randomness at each distinct lag. Here lags indicate the lags of the autocorrelation function. The null hypothesis  $H_0$  states that there is no ARCH or GARCH heteroscedasticity. For significance level  $\alpha$ , the critical region for rejection of the hypothesis of randomness is  $Q > \chi^2_{1-\alpha,h}$ , where  $Q$  is the Ljung-Box  $Q$ -statistic, and  $\chi^2_{1-\alpha,h}$  is the  $\alpha$ -quantile of the chi-square distribution with  $h$  degrees of freedom.

In Table I, we notice that  $Q > \chi^2_{1-\alpha,h}$  for all the four datasets with  $\text{Lags} \leq 10$ , which indicates that heteroscedasticity exists and building a GARCH model on the error series is justifiable. GARCH (1,1) is the most widely and successfully used model, so in this paper we model the variance of the  $\varepsilon_t$  using GARCH (1,1). The modeled conditional standard deviation and the prediction error  $\varepsilon_t$  are shown in Fig.2 and Fig.3.

Ljung-Box test of the normalized error  $\varepsilon_t / \sigma_t$  and the squared normalized error  $[\varepsilon_t / \sigma_t]^2$  can validate the performance of GARCH (1,1) model, as shown in Table II. The results indicate that ARIMA-GARCH model can significantly reduce the heteroscedasticity in the error series and better capture the statistical characters of traffic flow data.

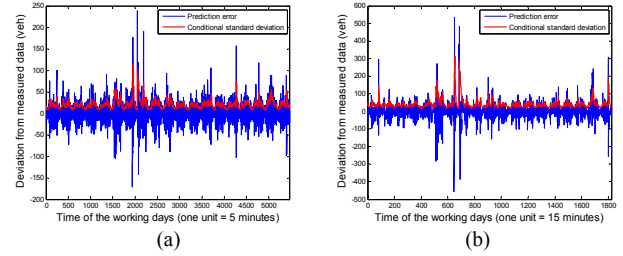


Fig.2. The prediction error of ARIMA-GARCH model during November 2009, together with the predicted conditional standard deviation of the error.  
(a) 5 min aggregation time, (b) 15 min aggregation time

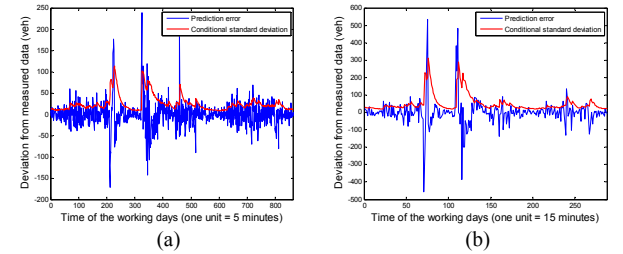


Fig.3. The prediction error of ARIMA-GARCH model from 11-10-2009 to 11-12-2009 when the flow fluctuated more violently, together with the predicted conditional standard deviation of the error.  
(a) 5 min aggregation time, (b) 15 min aggregation time

### B. Comparison of Prediction Results

After successfully building the ARIMA-GARCH model, we predict with it to study its performance. To make a clear comparison, the standard ARIMA model of the same order is built and used for prediction with the same datasets.

We use PeMS single loop detector data to build the models and make prediction. Among the two months' traffic flow data, the October 2009 data is used for fitting the models and

calculating daily profile, while the November 2009 data is used for prediction. Four types of aggregation time length, 3 min, 5 min, 10 min and 15 min, are all studied.

In our study, we adopt EViews software to build both ARIMA and ARIMA-GARCH models and predict with them. EViews (Econometric Views) is a statistical software mainly used for time series oriented econometric analysis. It is a powerful tool for time series estimation and forecasting, thus can meet our demand. The prediction results are listed in Table III to Table VI.

TABLE III  
THE PERFORMANCES OF DIFFERENT PREDICTION MODELS, WHEN THE AGGREGATION TIME LENGTH IS 3 MIN.

Method	MAE	MRE (%)	MSE
ARMA(2,2)	11.35	11.21	251.98
ARIMA(2,1,2)	11.40	11.24	254.26
ARMA(2,2)-GARCH(1,1)	11.38	11.26	255.30
ARIMA(2,1,2)-GARCH(1,1)	11.41	11.28	255.49

TABLE IV  
THE PERFORMANCES OF DIFFERENT PREDICTION MODELS, WHEN THE AGGREGATION TIME LENGTH IS 5 MIN.

Method	MAE	MRE (%)	MSE
ARMA(2,2)	15.20	8.96	484.56
ARIMA(2,1,2)	15.29	9.01	488.64
ARMA(2,2)-GARCH(1,1)	15.22	8.97	489.31
ARIMA(2,1,2)-GARCH(1,1)	15.31	9.02	494.70

TABLE V  
THE PERFORMANCES OF DIFFERENT PREDICTION MODELS, WHEN THE AGGREGATION TIME LENGTH IS 10 MIN.

Method	MAE	MRE (%)	MSE
ARMA(2,2)	23.75	6.94	1355.5
ARIMA(2,1,2)	24.07	7.09	1370.8
ARMA(2,2)-GARCH(1,1)	23.58	6.88	1333.5
ARIMA(2,1,2)-GARCH(1,1)	23.91	7.03	1369.6

TABLE VI  
THE PERFORMANCES OF DIFFERENT PREDICTION MODELS, WHEN THE AGGREGATION TIME LENGTH IS 15 MIN.

Method	MAE	MRE (%)	MSE
ARMA(2,2)	30.86	6.00	2539.2
ARIMA(2,1,2)	31.42	6.20	2583.6
ARMA(2,2)-GARCH(1,1)	30.66	5.94	2506.8
ARIMA(2,1,2)-GARCH(1,1)	30.67	5.95	2516.5

The prediction results show that the four types of models perform similarly at a same aggregation time length. As expected, the MRE index for all the models become larger when the aggregation time length decreases from 15 min to 3 min. For our specific datasets and prediction method that base on residual data, first order differencing is unnecessary since ARMA model outperforms ARIMA model. Moreover,

ARIMA-GARCH model has similar prediction accuracy as standard ARIMA model. For 10 min and 15 min aggregation time, the performance is slightly better than that of ARIMA model, yet for 3 min and 5 min aggregation time, the performance is slightly worse. The results indicate that although conditional heteroscedasticity is considered and modeled by GARCH, it is not adequate to significantly improve the prediction performance. Since a general GARCH(1,1) model is used, it is definitely not accurate for all the cases. So it is possible that GARCH(1,1) cannot capture the more violently fluctuating conditional variance of the traffic flow series of 3 min and 5 min aggregation time, thus counterproductively deteriorate the performance. Overall, the more complex ARIMA-GARCH model cannot provide a satisfactory result.

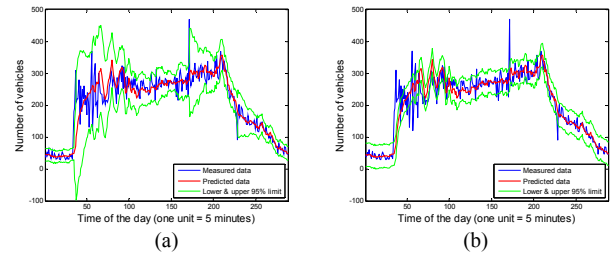


Fig.4. An illustration of the prediction performance and corresponding confidence interval when the aggregation time length is 5 min.  
(a) ARIMA-GARCH, (b) ARIMA

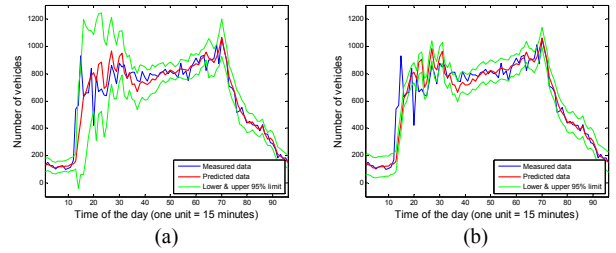


Fig.5. An illustration of the prediction performance and corresponding confidence interval when the aggregation time length is 15 min.  
(a) ARIMA-GARCH, (b) ARIMA

In addition to predict the one-step ahead traffic flow value, the ARIMA-GARCH model can also predict the one-step ahead conditional variance for the prediction error  $\varepsilon_t$ , thus the corresponding confidence interval can be calculated. In standard ARIMA model, it is assumed that  $\varepsilon_t$  follows Gaussian distribution  $N(0, \sigma^2)$ , while in GARCH model,  $\varepsilon_t$  is supposed to follow conditional Gaussian distribution  $N(0, \sigma_t^2)$ . According to the basic theory of Gaussian distribution, the 95% confidence interval limit can be calculated, as is presented in Fig.4 and Fig.5. In standard ARIMA model, the space between upper and lower limit is constant. While in the contrary, it is time-variant in ARIMA-GARCH model. When the traffic flow fluctuates more violently, a relatively wide confidence interval is proposed and thus a low predictability. In Fig.4 and Fig.5, we choose the traffic flow data of 11-11-2009, in which the

traffic flow has more severe volatility, and we notice that ARIMA-GARCH model proposes larger 95% confidence interval during fluctuating morning peak. We furthermore count the hit rate, which means the proportion of measured data points that fall within the 95% limit, and find that the time-variant confidence interval is more accurate, yet the improvement is also not significant. For 5 min aggregation time (5472 prediction points) the hit rates are 94.70% (ARIMA-GARCH) and 92.73% (ARIMA) respectively, and for 15 min aggregation time (1824 prediction points) the hit rates are 93.26% (ARIMA-GARCH) and 92.54% (ARIMA) respectively.

## V. CONCLUSION

ARIMA-GARCH is a popular hybrid model which can both capture the linear conditional mean and nonlinear conditional variance of time series, and it has been widely used in economic research such as the prediction of stock price. In this paper, we implement the ARIMA-GARCH model to predict traffic flow, using only individual loop detector information from PeMS. Moreover, the same datasets are fed to standard ARIMA model to make a comparison.

We reach the conclusions that

- 1) According to Ljung-Box test, the prediction error  $\varepsilon_t$  of the fitted ARIMA model has the property of conditional heteroscedasticity, thus applying GARCH model is justifiable. Moreover, after GARCH modeling, the normalized error is less autocorrelated and the conditional heteroscedasticity is reduced.
- 2) Compared with ARIMA model, the prediction performance of ARIMA-GARCH model is relatively similar. The general GARCH(1,1) model combined with the ARIMA of the same order cannot always improve the prediction accuracy. For traffic flow prediction methods using individual loop detector data, hybrid prediction models may not superior to basic models in general.
- 3) Although the improvement on prediction performance brought by the hybrid model is limited, yet since ARIMA-GARCH model predict the conditional mean and the conditional variance at the same time, it can provide a series of time-variant confidence interval. And the time-variant confidence interval is more accurate than the consistent confidence interval provided by the standard ARIMA model.

Above all, ARIMA describes the evolution of traffic flow, while GARCH describes the evolution of traffic flow volatility, so the hybrid model capture the characteristics of traffic flow more comprehensively. Yet, due to the relatively complicated modeling procedure and the less convincing performance of ARIMA-GARCH model, for ordinary traffic flow prediction, the standard ARIMA model is sufficient.

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