Traffic Flow Prediction for Road Transportation Networks With Limited Traffic Data

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***Abstract*—Obtaining accurate information about current and near-term future traffic flows of all links in a traffic network has a wide range of applications, including traffic forecasting, vehicle navigation devices, vehicle routing, and congestion management. A major problem in getting traffic flow information in real time is that the vast majority of links is not equipped with traffic sensors. Another problem is that factors affecting traffic flows, such as accidents, public events, and road closures, are often unforeseen, suggesting that traffic flow forecasting is a challenging task. In this paper, we first use a dynamic traffic simulator to generate flows in all links using available traffic information, estimated demand, and historical traffic data available from links equipped with sensors. We implement an optimization methodology to adjust the origin-to-destination matrices driving the simulator. We then use the real-time and estimated traffic data to predict the traffic flows on each link up to 30 min ahead. The prediction algorithm is based on an autoregressive model that adapts itself to unpredictable events. As a case study, we predict the flows of a traffic network in San Francisco, CA, USA, using a macroscopic traffic flow simulator. We use Monte Carlo simulations to evaluate our methodology. Our simulations demonstrate the accuracy of the proposed approach. The traffic flow prediction errors vary from an average of 2% for 5-min prediction windows to 12% for 30-min windows even in the presence of unpredictable events.**

***Index Terms*—Historical time traffic flows, least squares method, optimization, traffic flow prediction.**

I. Introduction

TRAFFIC flow prediction is considered as a challenging problem in transportation planning and car navigation systems. Traffic flows in a network can be estimated using historical traffic flow data. However, traffic flow prediction cannot solely rely on past traffic data due to the following reasons: 1) On-road traffic events such as accidents, road closure, etc., affect the traffic flows in the network, and their effect cannot be predicted *a priori*; 2) off-road events can have a major impact on traffic flows and may not be included in the usual historical traffic flow data; and 3) traffic data are not available for all links in a traffic network simply because most links are not equipped with traffic sensors.

Traffic flow disruptions that affect the estimates of link traffic flows can be categorized as predictable and unpredictable. Predictable disruptions include traffic signals, stop signs, public transit services, scheduled sport events, music concerts, road constructions/repairs, etc. Unpredictable disruptions include automobile accidents, breakdowns, and emergency road closures. The impact of disruption on traffic flow depends on the location, the duration of the disruption, and the demand during the time of the disruption. Studies regarding the impact of these types of disruption on traffic flows include [1]–[3].

The problem that arises is whether we can predict traffic flow ahead of time given the historical traffic information, information about scheduled events, and real-time traffic data where available. In principle, due to unpredictable disruptions, long-term predictions may not be accurate enough for reliable practical use. However, short-term traffic prediction, if properly done, may reach an accuracy level that is useful for several applications when compared with no prediction or inaccurate prediction.

There have been many studies in the literature regarding short-term traffic flow prediction. Short-term forecasting models include nonlinear models such as neural network models [4]–[8] and linear models such as Kalman filters [9]–[13] and autoregressive integrated moving average (ARIMA) models [14]–[17]. ARIMA models are linear estimators based on the past values of the modeled time series [18]. The nature of data and the type of application determine the modeling method used for traffic prediction. Schmitt and Jula [19] investigated the limitations of linear models that are commonly used and observed that "near-future travel times can be better predicted by a combined predictor." A combined predictor is a linear combination of a historical mean predictor and a current real-time predictor. Guo *et al.* [20] compared different modeling approaches for short-term traffic prediction and concluded that using a prediction error feedback approach improves the prediction accuracy under normal and abnormal conditions. In another study, Smith *et al.* [21] compared parametric (seasonal ARIMA) and nonparametric (data-driven regression) models and showed that "traffic condition data are characteristically stochastic, as opposed to chaotic." Moreover, it was argued that seasonal ARIMA models have better performance than nonparametric regression models. Furthermore, experimental studies showed that ARIMA models outperform heuristic forecast benchmarks [15]. The performance of ARIMA models can be improved by considering temporal–spatial correlations. Multivariate models are introduced to take into account these

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correlations. Kamarianakis and Prastacos [22] and Min and Wynter [23] proposed the space–time autoregressive integrated moving-average model to satisfy interrelations between links.

The given traffic prediction models become inaccurate under partially missing data. Missing data indicates the unavailability of traffic data for a certain period of time in part of a transportation network due to sensor malfunction or noise-contaminated data. This problem frequently occurs in transportation networks [24]–[26]. Many studies address the issue of traffic prediction with partially missing traffic data. For instance, van Lint *et al.* [4] presented a neural network for travel time prediction under missing traffic data. Sun *et al.* [27] introduced a Bayesian method to forecast traffic flows where a certain period of historical data is missing for some links of the transportation network. The missing portion of historical traffic data is approximated by using a Gaussian mixture model. Moreover, other statistical and probabilistic methods are used to address the missing traffic data problem [28]–[34]. These approaches are not applicable to large transportation networks where historical traffic data are not available for the majority of the arterial links simply because of the lack of sensor measurements. Therefore, the problem of short-term traffic flow prediction based on completely unavailable traffic data in some links due to lack of sensors in the network is an open challenging problem.

In this paper, we propose a methodology that predicts the traffic flows for all the links in a transportation network over a short time horizon. It consists of two steps: 1) traffic flow data completion and 2) short-term traffic flow prediction. In the first step, we use dynamic origin–destination (OD) matrix estimation with the help of a macroscopic simulator to generate traffic flow data at all links based on demand, historical data, and the limited real-time data available using an online optimization methodology. In the second step, we use the traffic flow data at all links generated from the first step to recursively predict future flows by adapting to changes in real-time data due to unpredictable changes.

As a case study, we focus on a road network in the downtown region of San Francisco, California, which we refer to as the Downtown SF subnetwork. A Monte Carlo experimental method is used to evaluate the proposed algorithm under a wide range of uncertainties.

This paper is organized as follows: In Section II, we present the methodology for generating traffic flow data at links that have no sensors. In Section III, we present the short-term traffic flow prediction algorithm. We demonstrate our approach using the Downtown SF subnetwork in Section IV. We present our conclusions in Section V.

II. Traffic Flow Data Completion

Here, we consider a traffic network that contains links with no available measurements. To address the issue of lacking traffic information for the majority of links in a transportation network, we define link-to-link dividing ratio (LLDR) as the ratio of traffic flow that propagates from a specific link to the adjacent links. These LLDRs vary for different links and also depend on the time of the day. A transportation network consists of several elements such as links, nodes, zones, etc.

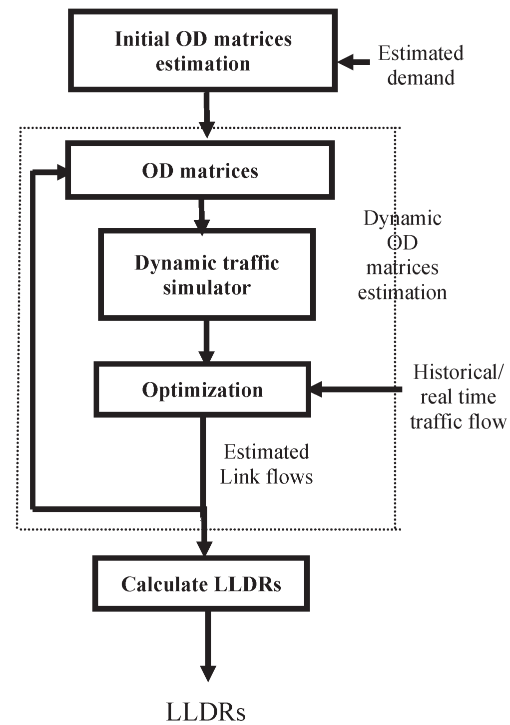

Fig. 1. Traffic flow completion methodology.

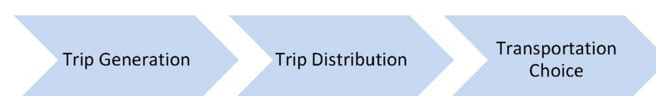

Fig. 2. Three-stage model.

Nodes are connected by links, and the links represent streets or freeways. Zones are places that a considerable number of people visit such as schools, stadiums, commercial buildings, and so on. One zone is defined for each residential district. The OD matrix determines the number of trips within zones in each time interval. Each OD matrix is assigned for one transportation choice. We consider two types of transportation choices, namely, personal cars and buses. There are many studies regarding OD matrix estimation. In previous efforts, the OD matrices are estimated using least squares techniques to minimize the difference between the measured link flows and the estimated ones [35]–[37]. Some other studies focus on dynamic OD matrix estimation [38]–[40].

In the proposed methodology, traffic flow data completion is performed in two steps: the initial OD matrix estimation and the dynamic OD matrix estimation. Let V_{ij}^n be the ith row and the jth column element of an OD matrix. It determines the total number of trips from zone i to zone j in the time interval $n \in \{1, \ldots, T\}$. Fig. 1 shows the block diagram of the proposed methodology for traffic flow data completion. Dynamic OD matrix estimation consists of two parts: the dynamic traffic simulator and the optimization algorithm.

A. *Initial OD Matrix Estimation*

The purpose of the initial OD matrix estimation is to estimate the OD matrices based on estimated demand data from the

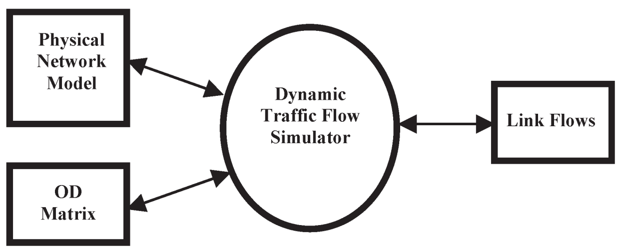

Fig. 3. Dynamic traffic flow generation.

region. For instance, students go to schools at 8 A.M. and leave schools at 2 P.M. We can roughly estimate the demand for each school in the morning based on the number of students who use the car and bus for commuting to school. There are two main methods in transportation forecasting [41]: 1—the gravity model; 2—the activity-based model. The activity-based model is on the microscopic level and requires much more information than the gravity model. Since, in our case, the data are limited, we use the gravity model to estimate the initial OD matrices, as explained below and shown in Fig. 2.

The first stage in the model, which is referred to as trip generation, is the aggregated travel demand for each zone. Each zone has a certain production, i.e., the number of trips that begin at that zone, and a certain attraction, i.e., the number of trips that culminate at that zone. The information (i.e., production and attraction) can be obtained through surveys and information from the region. For instance, in a big sport game, the attraction includes thousands of cars and possibly hundreds of buses heading to the parking lots adjacent to the stadium before the start of the game. The structure is the stadium, and one zone is assigned for each parking lot. The production/attraction ratio of each zone (parking lot) for this specific structure (stadium) can be estimated based on the capacity of the parking lots. Moreover, each residential district is considered as one structure within one zone in the center of the district. Let us denote trip production of zone i and trip attraction of zone j in time interval n, with P_i^n and A_j^n, respectively. P_i^n determines the total number of trips originating from zone i, and A_j^n is the total number of trips ending in zone j.The production and attraction of each zone can be generated as

$$P_i^n = \sum_{q \in Q} \eta_q^n(i) P_q^n \qquad \forall i \in Z, \forall n \tag{1}$$

$$A_j^n = \sum_{q \in Q} \xi_q^n(j) A_q^n \qquad \forall j \in Z, \forall n \tag{2}$$

where A_q^n and P_q^n are the total attraction and production for structure q in time interval n. Structure q is a subset of Q, which contains all the places in the region of study that people commute in. The attraction/production of each structure is distributed to the adjacent zones. Each structure has a production/attraction ratio for each zone. $\eta_q^n(i)$ and $\xi_q^n(j)$ represent production/attraction ratios for zones i and j, respectively. Parameter Z corresponds to the set of all zones in the network.

The second stage of the model, which is referred to as trip distribution, determines the number of trips that traverse from one zone to another. The number of trips can be estimated as follows:

$$V_{ij}^n = \vartheta_{ij}^n . P_i^n . A_j^n . e^{\psi^n d_{ij}} \qquad \forall i, j \in Z, \forall n \tag{3}$$

where V_{ij}^n is the number of trips from zone i to zone j in time interval n. The correlation between two zones can be illustrated in different formats such as exponential, linear, and so on [41]. In this paper, we choose the exponential format since it provides the best fit for the selected region, which is an urban region [42]. d_{ij} is defined as the shortest distance between zones i and j. Parameter ψ^n is a negative constant, and ϑ_{ij}^n is a scaling factor to adjust the total number of trips from zone i to zone j in time interval n to satisfy the following constraint equations [42]:

$$\sum_{1 \le j \le Z} V_{ij}^n = P_i^n \qquad \forall i \in Z, \forall n \tag{4}$$

$$\sum_{1 \le i \le Z} V_{ij}^n = A_j^n \qquad \forall j \in Z, \forall n. \tag{5}$$

The next step in Fig. 2, which is referred to as the transportation choice stage, determines the proportion of V_{ij}^n that use a specific transportation mode such as bicycles, single- or multipassenger cars, or public transit vehicles. The proportion of V_{ij}^n that uses single car and public transit are used to generate OD matrices of the trips at different intervals of time. For instance, buses represent X% of the total number of vehicles, and their proportion is denoted by $V_{ij,b}^n$, where $V_{ij,b}^n = 0.01 * X * V_{ij}^n$, and the rest (100-X)% are cars ($V_{ij,c}^n = V_{ij}^n - V_{ij,b}^n$) for a pair of zones (i, j). The ratios of buses and vehicles are variable depending on the location of zones. Buses have a higher ratio in the downtown region than they have in a local residential area.

B. Dynamic OD Matrix Estimation

The initial OD matrices that drive the dynamic traffic simulator are adjusted based on an optimization procedure presented below, which minimizes the error between measured flows (where available) and estimated ones as well as travel times. It is assumed that trips will follow routes that minimize travel time.

1) Dynamic Traffic Simulator: The physical network model consists of various objects such as links, nodes, zones, etc. The OD matrix represents the aggregated number of trips from one zone to another. Traffic flows on each link can be obtained by feeding demand to a traffic flow simulator, while taking into account physical network characteristics. Fig. 3 shows the diagram for dynamic traffic flow generation.

The proposed methodology evaluates the propagation of flows in the network by calculating the LLDRs based on the link flows generated by the dynamic traffic flow simulator and the optimization algorithm described below.

2) Optimization: Historical time traffic flows, derived from historical traffic flow data, are used as one of the inputs to the optimization algorithm. They are used in both the dynamic OD matrix estimation and short-term traffic prediction models. Historical time traffic flows vary seasonally, weekday to

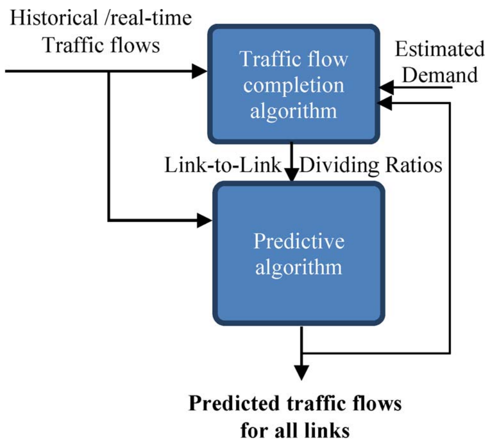

Fig. 4. Overall, block diagram.

weekend, and during a specific day. For instance, major events that occur adjacent to a road network and draw hundreds or possibly thousands of attendees can have huge impacts on network flows.

In this paper, traffic flows and information about major events are taken into account for M consecutive months. Given the major event data such as date and capacity of the venue, historical time traffic flows can be also derived for each specific event. Days are classified into three general categories: 1) regular day; 2) holiday; and 3) major events. For regular days, the average hourly traffic flow for link l (AHTF_l) is expressed as

$$\mathrm{AHTF}_l(h,d,m)=\frac{\sum_{w\in W}\mathrm{HTF}_l(h,d,w,m)}{\sum_{w\in W}1}\qquad \begin{array}{c}\forall h\in H\\ d\in D\\ m\in M\\ l\in L\end{array} \tag{6}$$

where HTF_l is the hourly traffic flow for link l, h indicates the specific hour of the day, d is the day of the week, w represents the week, and m is the month of the year. For the second and third categories, (6) is also used only by taking into account historical traffic flows corresponding to the specific event or holiday. Due to the important role of recent data, a weighting factor is introduced to have a better historical time traffic flow evaluation. The weighted-average hourly traffic flow for link l, (WAHTF_l) is calculated as

$$\mathrm{WAHTF}_l(h,d)=\sum_{m=1}^{|M|}\frac{(|M|-m+1)^2}{\sum_{k=1}^{|M|}(|M|-k+1)^2}e^{-\zeta m}\mathrm{AHTF}_l(h,d,m)\qquad \begin{array}{c}\forall h\in H\\ d\in D\end{array} \tag{7}$$

where ζ is a constant positive coefficient, and the number of members in a set is denoted by $|\cdot|$. For the sake of simplicity, in the rest of this paper, WAHTF_l is denoted by $\bar{v}_{l,n}$, where n is the time interval.

A physical transportation network is defined on a set of links and nodes, i.e., $g=(N,L)$, where N and L represent nodes and links in the network, respectively. Let S denote all OD pairs, and θ is the set of all routes connecting OD pair $s\in S$. Each route $r\in\theta$ consists of one or more links from L. The cost of route $r\in\theta$ for OD pair s, which is denoted as $c_{s,r,t}$, depends on the traffic flow of the route. We define cost as the travel time of loaded route in the network. We also denote $\bar{v}_{l,n}$ as the historical time traffic flow on link l, and $v_{l,n}$ represents the estimated flow of link l in time interval n, where n is an integer, and $n\in\{1,\ldots,T\}$ and $t\in[0,T]$, where t indicates time, and T is the length of the total interval. Since traffic sensors are only available for a portion of links in the network, we use the results from the three-stage model that covers all links in the network as the initial solution for the optimization algorithm. Let $v_{s,r,t}$ be the generated flow (number of vehicles per time slot) of route r for the OD pair s through the three-stage model. Therefore, the flow of link l is calculated as

$$v_{l,n}=\sum_{s\in S}\sum_{r\in\theta}\sum_{t\in T}\rho_{s,r,n}^{l,t}v_{s,r,t}\qquad \begin{array}{c}\forall l\in L\\ n\in\{1,\ldots,T\}\end{array} \tag{8}$$

where $\rho_{s,r,n}^{l,t}$ is a decision variable equal to 1 if link l is on route r connecting OD pair s during time t in time slot n and 0 otherwise. Note that $c_{s,r,t}$ is the function of $\rho_{s,r,n}^{l,t}v_{s,r,t}$ [41]. Moreover, $v_{l,n}$ must satisfy the flow conservation law; thus, we have

$$v_{l,n}=\sum_{k\in A(l)}R_{l,k,n}v_{k,n}\qquad \begin{array}{c}\forall l\in L\\ n\in\{1,\ldots,T\}\end{array} \tag{9}$$

where $v_{k,n}$ represents the flow of links feeding link l, and $R_{l,k,n}$ is defined as the LLDR of link k to link. We define $\varphi^n(R)$ as a set of $v_{l,n}$ and the corresponding $v_{s,r,t}$ as well as $R_{l,n}$ that satisfies (8) and (9).

Note that $A(l)=\{j\in L|\mathrm{head}(j)=\mathrm{tail}(l)\}$. This contains all the links that are feeding link l. We refer to the optimization algorithm formulated below as the master problem. Thus

$$\min\quad g(V)$$

w.r.t.

$$v_{s,r,t}=\alpha_1\sum_{n\in\{1,\ldots,T\}}\sum_{l\in L}\left[\frac{\bar{v}_{l,n}-\sum_{s\in S}\sum_{r\in\theta}\sum_{t\in T}\rho_{s,r,n}^{l,t}v_{s,r,t}}{\bar{v}_{l,n}}\right]^2 + \alpha_2\sum_{s\in S}\sum_{r\in\theta}\sum_{t\in T}c_{s,r,t}v_{s,r,t} \tag{10}$$

$$\text{subject to}\qquad v_{s,r,t}\geq 0. \tag{11}$$

The objective function (10) minimizes the normalized variation between the historical time link flows and the simulated ones as well as the total cost (travel time) of the network. Constraint (11) imposes a nonnegative value for link flows. Coefficients $0\leq\alpha_1,\ \alpha_2\leq 1$ weigh the relative importance of average historical count data and total travel time. The optimization (10) and (11) aim to minimize the total travel time in addition to minimizing the error between the measured flows (where available) and estimated ones by taking into account the fact that, in general, drivers select traffic routes that minimize travel time. The objective is to guide the solution using new efficient routes with the minimum total network cost. The column generation algorithm is one of the most common algorithms to find the efficient routes [43]. It starts with the subset of solution $(\theta_1 \subset \theta)$ of (10), which can be presented as

$$\min \quad g(V)$$

w.r.t.

$$v_{s,r,t} = \alpha_1 \sum_{n\in\{1,\ldots,T\}} \sum_{l\in L} \left[\frac{\bar{v}_{l,n} - \sum_{s\in S} \sum_{r\in\theta_1} \sum_{t\in T} \rho_{s,r,n}^{l,t} v_{s,r,t}}{\bar{v}_{l,n}} \right]^2 + \alpha_2 \sum_{s\in S} \sum_{r\in\theta_1} \sum_{t\in T} c_{s,r,t} v_{s,r,t} \tag{12}$$

$$\text{subject to} \qquad v_{s,r,t} \geq 0. \tag{13}$$

Using information from the region as well as sensor data, we would like to estimate link flows for the specified time window on a specific link. Using an observation interval (such as a six-month period), we are able to calculate the average flow for a particular time of the day using (7).

Solution Methodology: We start with a subset of solution $\theta_1 \subset \theta$ (restricted master problem) using a column generation algorithm. The restricted master problem is a way to expedite the process of finding the lowest cost route. The restricted master problem is defined as a subset of the master problem (θ_1). The initial solution of the restricted master problem is determined by the three-stage model, as explained in detail in the previous section. Initially, the column generation algorithm begins with the restricted master problem with only a small subset of routes (initial solution). Then, it adds new eligible routes to the previous routes. The restricted master problem is defined as

$$\min \quad g(V)$$

w.r.t.

$$v_{s,r,t} = \alpha_1 \sum_{n\in\{1,\ldots,T\}} \sum_{l\in L} \left[\frac{\bar{v}_{l,n} - \sum_{s\in S} \sum_{r\in\theta_1} \sum_{t\in T} \rho_{s,r,n}^{l,t} v_{s,r,t}}{\bar{v}_{l,n}} \right]^2 + \alpha_2 \sum_{s\in S} \sum_{r\in\theta_1} \sum_{t\in T} c_{s,r,t} v_{s,r,t} \tag{14}$$

$$\text{subject to} \qquad v_{s,r,t} \geq 0 \tag{15}$$

where $\theta_1 \subset \theta$. The initial solution θ_1 is derived by feeding the initial OD matrices into a dynamic traffic simulator. The column generation algorithm operates as follows.

Step 1: Apply the initial solution.
Step 2: Calculate the total cost.
Step 3: Add new eligible routes.
Step 4: Calculate new total cost, go to step 3.

We take a derivative with respect to $v_{s,r,t}$ to find new eligible routes, i.e.,

$$g_{m,p,w} = \frac{\partial g(V)}{\partial v_{m,p,w}} = 2\alpha_1 \sum_{t\in[0,T]} \sum_{l\in L} \left[\frac{\bar{v}_{l,n} - \sum_{s\in S} \sum_{r\in\theta} \sum_{t\in T} \rho_{s,r,n}^{l,t} v_{s,r,t}}{\bar{v}_{l,n}} \right] \times \rho_{m,p,n}^{l,w} + \alpha_2 c_{m,p,w}. \tag{16}$$

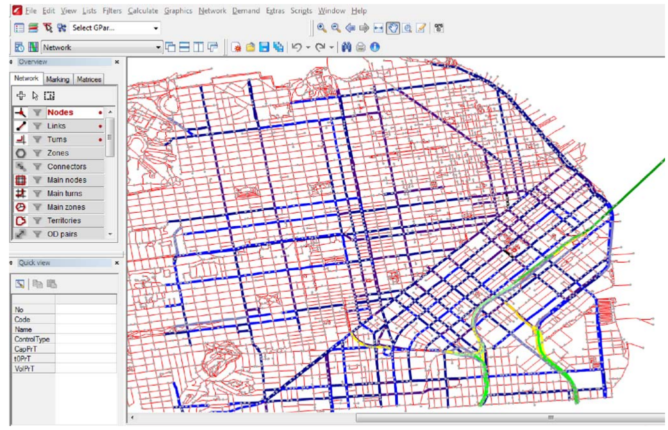

Fig. 5. Case study traffic network.

The routes with the negative value of $g_{m,p,w}$ are eligible routes and are added to the previous routes. As a result, the problem of finding a new eligible route to add to the previous routes has changed to the dynamic shortest route problem with new link costs. Finding the minimum cost flows in this case is NP-complete. In the static case, the computation time for finding the shortest path in a graph with N nodes is $O(n)$; however, in the dynamic case, 2^{N-1} routes must be evaluated from node 1 to N. Thus, the computational time exponentially grows [44]. The LLDRs can be calculated based on the estimated link flows using (9).

III. Traffic Flow Prediction

Here, the estimated LLDRs as well as current and historical traffic flows are used by the traffic prediction algorithm to make predictions, as explained in this section. Fig. 4 shows the overall structure of the prediction approach.

The approach to be presented in detail below is summarized as follows: First, an autoregressive model of the link flows is introduced that takes into account the uncertain nature of traffic and historical traffic data including the most recent ones as well as historical time traffic flows in its coefficients. Due to the nonstationary nature of traffic flows, we detrend the traffic flow data by subtracting the corresponding long-term historical flow means $\bar{v}_{l,t}$, which are calculated in Section II, from the real-time flows $v_{l,t}$ to obtain a stationary process. Then, we train the autoregressive model using the most recent data to calculate the model parameters. Finally, we apply the least squares method with the LLDR constraints to expand the short-term prediction results to the entire network.

Let $v_{l,t}$ represent the flow for the specific link l at time t that is assumed to be generated by the autoregressive model [15], i.e.,

$$v_{l,t} = \beta_0 + \sum_{m=1}^{M} \beta_m v_{l,t-m} + \varepsilon_{l,t} \qquad \begin{array}{c}\forall l \in L \\ \forall t\end{array} \tag{17}$$

where the coefficients $\beta\text{s}, \text{s} = 0, \ldots, \text{M}$ are the parameters of the autoregressive model, and $\varepsilon_{l,t}$ forms a white noise process that is the innovation process uncorrelated with $v_{l,t}$. The order M of the model is found by applying the Akaike information criterion [45]. Historical time traffic flow means, which are calculated in previous section, are denoted by $\bar{v}_{l,t} = \boldsymbol{E}\{v_{l,t}\}$. Using the autoregressive model (17), $\bar{v}_{l,t}$ is represented by

$$\bar{v}_{l,t+1} = \beta_0 + \sum_{m=0}^{M-1} \beta_m \bar{v}_{l,t-m} \qquad \begin{array}{c}\forall l \in L\\ \forall t.\end{array} \tag{18}$$

Then, these historical means are subtracted from real-time traffic flows. We define the error process $e_{l,t}$ as

$$e_{l,t} = v_{l,t} - \bar{v}_{l,t} \qquad \begin{array}{c}\forall l \in L\\ \forall t.\end{array} \tag{19}$$

Then, from (17) and (18), we have

$$e_{l,t} = \sum_{m=1}^{M} \beta_m e_{l,t-m} + \varepsilon_{l,t} \qquad \begin{array}{c}\forall l \in L\\ \forall t.\end{array} \tag{20}$$

The uncertainty in the system is modeled by a white noise process; therefore, $e_{l,t}$ is a stationary mean-zero process for all the links in the network. Now, we represent this model in the matrix form as

$$\boldsymbol{e}_{l,t} = A_l \boldsymbol{e}_{l,t-1} + \boldsymbol{\epsilon}_{l,t} \qquad \begin{array}{c}\forall l \in L\\ \forall t\end{array} \tag{21}$$

where $\boldsymbol{e}_{l,t}$, A, and $\boldsymbol{\epsilon}_{l,t}$ are defined as

$$\boldsymbol{e}_{l,t} = \boldsymbol{v}_{l,t} - \bar{\boldsymbol{v}}_{l,t}, \quad \boldsymbol{\epsilon}_{l,t} = \begin{bmatrix} \varepsilon_{l,t} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$A_l = \begin{bmatrix} \beta_1 & \beta_2 & \cdots & \cdots & \beta_M \\ 1 & 0 & \cdots & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \qquad \begin{array}{c}\forall l \in L\\ \forall t\end{array}$$

$$\boldsymbol{v}_{l,t} = \begin{bmatrix} v_{l,t} \\ v_{l,t-1} \\ \vdots \\ v_{l,t-M+1} \end{bmatrix} \quad \bar{\boldsymbol{v}}_{l,t} = \begin{bmatrix} \bar{v}_{l,t} \\ \bar{v}_{l,t-1} \\ \vdots \\ \bar{v}_{l,t-M+1} \end{bmatrix}. \tag{22}$$

Using the matrix form, it enables us to expand the prediction algorithm to more than one time step in the future. Let $\hat{\boldsymbol{e}}_{l,t}$ be the estimated value of $\boldsymbol{e}_{l,t}$. The following estimator minimizes the estimated value of $\boldsymbol{e}_{l,t+1}$ given the flow data at times t, $t-1, t-2, \ldots$ [46]:

$$\begin{aligned}\hat{\boldsymbol{e}}_{l,t+1|t} &= E\{\boldsymbol{e}_{l,t+1}|\boldsymbol{v}_{l,t}\} \qquad \forall l \in L,\ \forall t\\ &= A_l E\{\boldsymbol{e}_{l,t}|\boldsymbol{v}_{l,t}\} + E\{\boldsymbol{\epsilon}_{l,t}\}\end{aligned} \tag{23}$$

where $\hat{\boldsymbol{e}}_{l,t+1|t}$ is the estimate of $\boldsymbol{e}_{l,t+1}$ given the flow data at times $t, t-1, t-2, \ldots$. Since $\boldsymbol{e}_{l,t+1}$ is calculated from (19), we have $E\{\boldsymbol{e}_{l,t}|\boldsymbol{v}_{l,t}\} = E\{\boldsymbol{e}_{l,t}|\boldsymbol{e}_{l,t}\}$. Considering the fact that the real-time $\boldsymbol{e}_{l,t}$ is known at time t and $\boldsymbol{\epsilon}_t$ has zero mean and is uncorrelated with $v_{l,t}$, the predicted value of traffic flow error becomes

$$\hat{\boldsymbol{e}}_{l,t+1|t} = A_l \boldsymbol{e}_{l,t} \qquad \begin{array}{c}\forall l \in L\\ \forall t.\end{array} \tag{24}$$

In general, the predictions need to be made for more than one time step ahead in the future. Since the real-time information is available for up to time t, $\hat{\boldsymbol{e}}_{l,t+1|t}, \hat{\boldsymbol{e}}_{l,t+2|t}, \ldots, \hat{\boldsymbol{e}}_{l,t+p|t}$ are informationally equivalent, and

$$\begin{aligned}\hat{\boldsymbol{e}}_{l,t+p|t} &= E\{\boldsymbol{e}_{l,t+p}|\boldsymbol{e}_{l,t}\}\\ &= E\{\boldsymbol{e}_{l,t+p}|\hat{\boldsymbol{e}}_{l,t+1|t}\} = \cdots \qquad \begin{array}{c}\forall l \in L\\ \forall t\end{array}\\ &= E\{\boldsymbol{e}_{l,t+p}|\hat{\boldsymbol{e}}_{l,t+p-1|t}\}\end{aligned} \tag{25}$$

$$\hat{\boldsymbol{e}}_{l,t+p|t} = A_l^p \boldsymbol{e}_{l,t} \qquad \begin{array}{c}\forall l \in L\\ \forall t.\end{array} \tag{26}$$

The value $\hat{\boldsymbol{e}}_{l,t+p|t}$ is estimated after calculating $\beta_1, \ldots, \beta_M$. The truncated error process $e_{l,k}, k \in \{t-T+1, t-T, \ldots, t\}$ is used to obtain the sample autocovariance functions

$$C_l(j) = \frac{1}{T} \sum_{i=t-T}^{t-j} e_{l,i+j} e_{l,i} \qquad \begin{array}{c}\forall l \in L\\ \forall j \in \{0, 1, \ldots, T\}\end{array} \tag{27}$$

where T is the length of the short-term historical data used for fitting the model. The least squares method is used to estimate the $\beta's$ parameters. Considering the expanded predicted value $\hat{v}_{l,t+1} = \sum_{m=0}^{M-1} \beta_m \tilde{v}_{l,t-m}$, the mean square residuals are defined as [47]

$$\begin{aligned}R_l(\beta) &= \frac{1}{T} \sum_{t=1}^{T} W_t (e_{l,t} - \hat{e}_{l,t})^2\\ &= \frac{1}{T} \sum_{t=1}^{T} W_t \left(e_{l,t} - \sum_{m=1}^{M} \beta_m e_{l,t-m} \right)^2 \qquad \forall l \in L\end{aligned} \tag{28}$$

where $W's$ are the positive normalizing weights. The objective is to minimize the residuals with respect to $\{\beta_m; m = 1, 2, \ldots, M\}$ by assuming $e_{l,0} = e_{l,-1} = \cdots = e_{l,-M+1} = 0$. The coefficients (β_m) are calculated by setting the gradient of the residual function to zero. The model contains M parameters; hence, there are M equations as

$$\frac{\partial R_l}{\partial \beta_i} = -2 \sum_{t=1}^{T} W_t e_{l,t-i} \left(e_{l,t} - \sum_{m=1}^{M} \beta_m e_{l,t-m} \right) = 0 \qquad \forall l \in L. \tag{29}$$

The coefficients $(\hat{\beta}_m)$ are obtained by solving the linear equation

$$\begin{bmatrix} C_l(0) & C_l(1) & \cdots & C_l(M-1) \\ C_l(1) & C_l(0) & \cdots & C_l(M-2) \\ \vdots & \vdots & \ddots & \vdots \\ C_l(M-1) & C_l(M-1) & \cdots & C_l(0) \end{bmatrix} \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \vdots \\ \hat{\beta}_M \end{bmatrix} = \begin{bmatrix} C_l(1) \\ C_l(2) \\ \vdots \\ C_l(M) \end{bmatrix} \qquad \forall l \in L. \tag{30}$$

After the calculation of $\hat{\boldsymbol{e}}_{l,t+p|t}$, $\boldsymbol{v}_{l,t+p|t}$, the prediction of link flows at time $t+p$ is obtained by

$$\boldsymbol{v}_{l,t+p|t} = \bar{\boldsymbol{v}}_{l,t+p} + \hat{\boldsymbol{e}}_{l,t+p|t} \qquad \begin{matrix} \forall l \in L \\ \forall t. \end{matrix} \tag{31}$$

The short-term traffic flow prediction model is recalibrated once the new traffic data are received. However, the results from the short-term traffic prediction model include the critical coverage gaps where the real-time data are not available, which can be a large portion of an urban transportation network. Therefore, we take into account both historical data and prediction results to minimize travel times with respect to the predicted traffic flows $(\hat{v}_{l,t+p|t})$ as

$$\text{minimize} \quad \left[\gamma_1 \sum_{l \in L_1} \left[\frac{v_{l,t+p|t} - \hat{v}_{l,t+p|t}}{v_{l,t+p|t}}\right]^2 + \gamma_2 \sum_{l \in L} \left[\frac{\bar{v}_{l,t+p} - \hat{v}_{l,t+p|t}}{\bar{v}_{l,t+p}}\right]^2\right] \tag{32}$$

$$\text{s.t.} \qquad \hat{v}_{l,t+p} \in \varphi^{t+p}(R) \tag{33}$$

where $0 < \gamma_2 < \gamma_1 < 1, v_{l,t+p|t}$ is the result from the short-term traffic prediction model. $\bar{v}_{l,t+p}$ is the historical time traffic flow at time $t+p$ for the link l, *and* $\bar{v}_{l,t+p|t}$ is the final traffic flow prediction for link flows based on the historical data as well as the prediction model. It is worth noting that we take $\gamma_1 > \gamma_2$ because of the relative importance of the role of the results from the prediction model in computing link flows. L_1 is defined as the number of links with available real-time data and, L denotes the number of all the links in the transportation network. The objective is to find $\bar{v}_{l,t+p|t}$ for all the links in the network including links where data are unavailable. Constraint (33) yields link flows satisfying the LLDRs derived from the traffic flow completion model. The output of the given model is the set of link flows and updated LLDRs. The LLDRs are calculated from link flows using (8).

IV. Computational Experiment

The San Francisco, California downtown region is chosen as the case study to perform experiments and demonstrate the effectiveness of the developed algorithm. Traffic flow and event data as well as incident data are provided by UC Berkeley PATH [48]. Clearly, the quality and quantity of data have an impact on the quality of the final solution. We use a macroscopic traffic simulator based on the commercial software VISUM for the traffic assignment step [49]. The inputs of VISUM are the OD matrices, and the outputs are the estimated link flows. The OD matrices are adjusted based on the available traffic volumes and the initial OD matrices using the optimization formulation introduced in Section II. Fig. 5 shows the traffic network in the VISUM format.

The traffic network under study contains more than 20 000 links, and real-time traffic data are only available for about 16% of the links in the network. The initial traffic flow estimation model is used to generate traffic flows based on the information

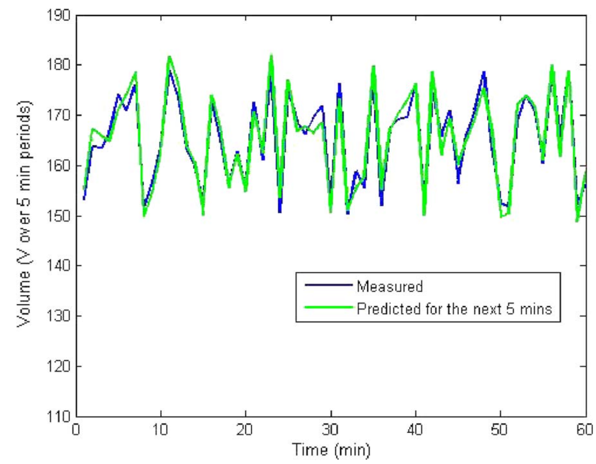

Fig. 6. Time-series analysis (scenario 1).

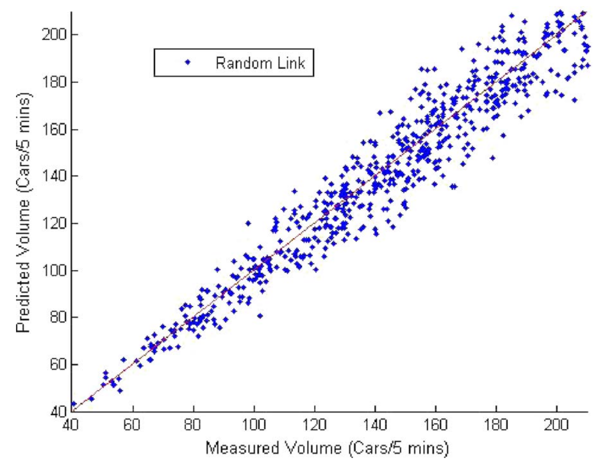

Fig. 7. Prediction scatterplot (scenario 2).

from the big venues (e.g., AT&T Park stadium), which provides a benchmark for the traffic flow completion model. Traffic data are received every 1 min for those 16% links, and no data are available for the rest of the links.

Three scenarios are used to demonstrate the accuracy of the proposed model. The first one presents a time-series scenario. At each step, traffic flows are predicted for the next 5 min. For instance, traffic flows for time $t+5$ are predicted based on the traffic data up to time t, and for time $t+6$ up to time $t+1$. The second scenario is defined as the comparison of the predicted and measured volumes for the 30-min horizon prediction. Finally, the third scenario indicates the prediction of flows on the targeted links for the next 5-, 10-, 15-, 20-, 25-, and 30-min time horizon. Note that the targeted links are chosen to be in the category of nonavailable data links. Therefore, the assumption is that no historical and real-time traffic data are available; however, in fact, both are available to verify the proposed model results. Fig. 6 demonstrates the time-series scenario for the next 5 min. Fig. 7 shows the scatterplot for the second scenario, and Table I presents the root-mean-square percentage error (RMSPE) for the third scenario, which is the prediction of targeted links every 5 min for the next half hour.

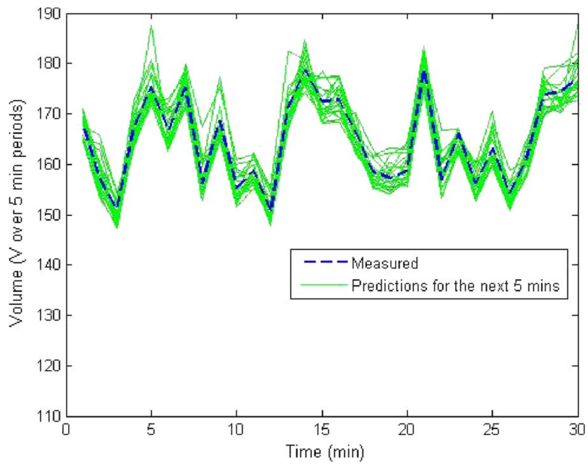

Fig. 8. Monte Carlo experiments results for 5-min-ahead predictions over a 30-min period.

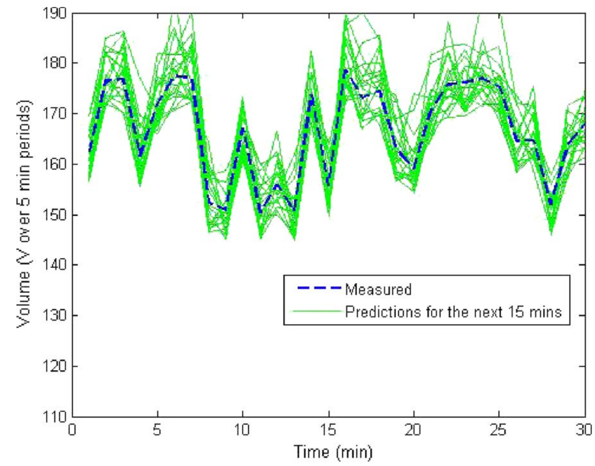

Fig. 9. Monte Carlo experiment results for 15-min-ahead predictions over a 30-min period.

TABLE I
RMSPE (SCENARIO 3)

Time (min)	RMSPE (%) [11 am]	RMSPE (%) [3 pm]	RMSPE (%) [8 pm]
5	1.98	2.4	1.62
10	6.3	6.56	4.86
15	7.02	7.04	7.38
20	7.34	10.72	8.1
25	8.28	11.36	9.54
30	9.9	11.84	10.14

The first column represents the prediction timing, i.e., 5, 10, 15, ... min from present time. Columns 2–4 demonstrate the RMSPE for the targeted links at 11 A.M., 3 P.M., and 8 P.M. on weekdays, respectively, defined as

$$\text{RMSPE} = \sqrt{\frac{1}{N}\sum_{l=1}^{N}\left[\frac{\hat{v}_l - v_l}{v_l}\right]^2} \tag{34}$$

where $\bar{v}_l$ is the predicted traffic flow, and v_l indicates the measured one for link l. Results in Figs. 6 and 7 and Table I present the accuracy of the proposed methodology to predict traffic flows in the urban region with the complex characteristics. Fig. 7 also suggests that the proposed predictor is unbiased.

In addition to the given experiments, which were performed under normal conditions, Monte Carlo experiments are carried out to evaluate the effects of random uncertainties due to the stochastic nature of traffic data. The most possible source of error in the proposed predictive algorithm is that the estimated LLDRs from the historical traffic data might not be valid for some links of the network at the current time due to accidents or social events. To measure the sensitivity of the model with respect to these variations, Monte Carlo experiments are performed. The method is as follows.

1) At each experiment, the LLDRs are calculated.
2) 50% of the LLDRs are randomly selected.
3) Selected LLDRs are multiplied by a random variable Γ.
4) The new LLDR set is fed to the predictive algorithm, and the results are recorded.

These four steps are repeated 1000 times for each data sample. The coefficient Γ is assumed to be a Gaussian random variable $\Gamma \sim N(1, 0.1)$. The variance $\sigma^2 = 0.1$ is selected based on the observations from Downtown SF traffic data ($\sigma = 0.316$). In the cases that the new LLDR is bigger than 1, the fixed value of 1 is chosen for the specific LLDR. Figs. 8 and 9 show the error for a specific data sample for 5-min and 15-min time horizon prediction. In this sample, the standard deviation of the Monte Carlo results is $\sigma = 0.09$, which indicates that the sensitivity of the predictive algorithm is about 28% with respect to the LLDR variations. The sensitivity of traffic flow predictions to the error in LLDR calculations suggests that the prediction/simulation model is robust, and the noise does not escalate through different steps of the model.

V. CONCLUSION

In this paper, we have estimated traffic flows in all links in a traffic network where traffic data are unavailable and used the information to predict short-term traffic flow for the entire transportation network. A large network in the San Francisco area was used to demonstrate the efficiency and accuracy of the methodology. Monte Carlo simulations were used to account for random effects and uncertainties. The results demonstrate accurate predictions of traffic flow rates up to 30 min ahead of time under normal operations. In the case of events, the prediction algorithm adapts to the changes and modifies its prediction outputs with good accuracy.

One of the limitations of this paper is the lack of adequate number of data during normal and incident traffic conditions to perform additional tests. Future work will involve the collection of additional real-time data as they become available due to emerging traffic sensor technologies, the use of vehicles as probes, and vehicle-to-infrastructure communications. Such data may be used to improve the accuracy of our approach as well as validate it under different traffic scenarios and different networks.

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