HWRS 505: Vadose Zone Hydrology

Lecture 5

9/10/2024

Today:

1. Air-water system in capillary tubes
2. Model of a porous medium: Bundle of Capillary Tubes

Art of porous media flow

Optional, but strongly encouraged, Mini-project

Take a photo or a video (< 2 min) in your day-to-day life that you think best illustrates some cool phenomena of porous media flow.

I will create a dropbox on D2L for you to upload the photo or video (due on December 4th).

Depending on the quality of your picture or video, you can receive up to 5 bonus points in your final grade (out of 100 points).

Air-water system in capillary tubes

Review of Lecture 4

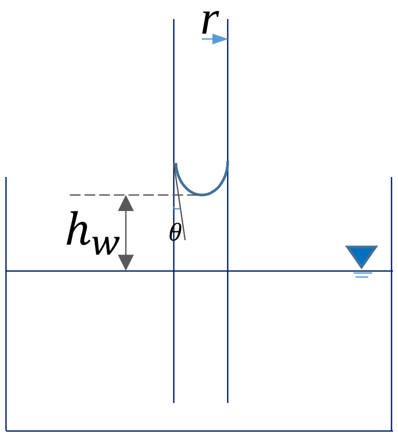
- Solute transport
  - Flux law for solute transport in porous media.
  - Derivation of advection-dispersion equation.
- Air-water system in a capillary tube
  - Interfacial tension
  - Wettability, contact angle
  - Capillary pressure, Young-Laplace Equation

Pressure jump across a fluid-fluid interface

Pressure jump across a fluid-fluid interface is determined by interfacial tension + geometry of the interface (radii of the curvature)

Air-water system in capillary tubes

Capillary rise in a Capillary tube



At the water table:

$$\left.\begin{aligned} P_a &= 0 \\ P_w &= 0 \\ z &= 0 \end{aligned}\right\} \Rightarrow H = 0$$

At the air-water interface in the tube:

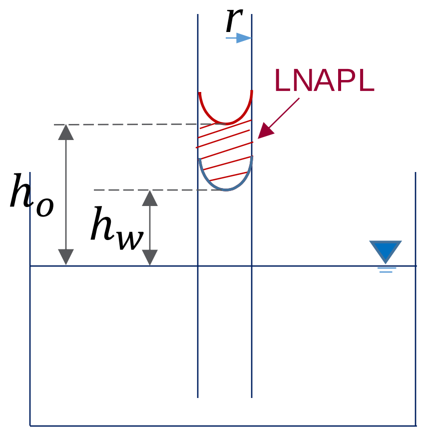
$$\left.\begin{aligned} P_a &= 0 \\ z &= h_w \\ H &= 0 \end{aligned}\right\} \Rightarrow \psi = -h_w \Rightarrow P_w = -\rho_w g h_w$$

$$\left.\begin{aligned} &p^{cap} = P_a - P_w = 0 - (-\rho_w g h_w) = \rho_w g h_w \\ &\text{Young-Laplace eqn: } p^{cap} = \frac{2\sigma\cos\theta}{r} \end{aligned}\right\} \Rightarrow$$

$$h_w = \frac{2\sigma\cos\theta}{\rho_w g r}$$

Air-water system in capillary tubes

Capillary rise in a Capillary tube in the presence of an LNAPL (Assuming zero contacts)



Note: w/o oil ($\theta = 0$)

$$h_w = \frac{2\sigma_{aw}}{\rho_w g r}$$

At the oil-water interface:

$$P_w = 0 - \rho_w g h_w = -\rho_w g h_w \quad \text{-----} \quad (1)$$

$$P_o = P_{ow}^{cap} + P_w = \frac{2\sigma_{ow}}{r} - \rho_w g h_w \quad \text{----} \quad (2)$$

continuing through the oil to the oil-air interface, the oil pressure

$$P_o = \frac{2\sigma_{ow}}{r} - \rho_w g h_w - \rho_o g (h_o - h_w) \quad \text{--} \quad (3)$$

At the oil-air interface:

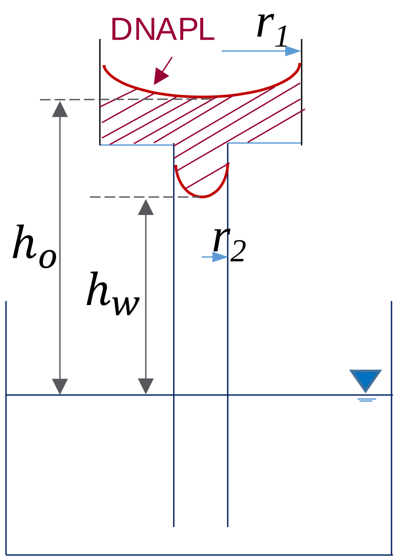
$$P_o = 0 - P_{oa}^{cap} = -\frac{2\sigma_{oa}}{r} \quad \text{------} \quad (4)$$

P_o in (3) and (4) are equal $\Rightarrow$

$$h_w = \left(\frac{2\sigma_{ow}}{r} + \frac{2\sigma_{oa}}{r} - \rho_o g h_o\right) / (\rho_w g - \rho_o g)$$

Air-water system in capillary tubes

Invasion of a nonwetting fluid into a pore (Assuming zero contacts)



At the air-oil interface:

$$P_o = P_a - P_{oa}^{cap} = -\frac{2\sigma_{oa}}{r_1}$$

continuing down to the oil-water interface, the pressure of oil is:

$$P_o = -\frac{2\sigma_{oa}}{r_1} + (h_o - h_w)\rho_o g$$

At the oil-water interface:

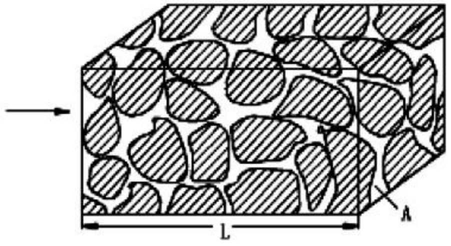
$$\begin{cases} P_{ow}^{cap} = P_o - P_w = -\frac{2\sigma_{oa}}{r_1} + (h_o - h_w)\rho_o g + \rho_w g h_w \\ P_{ow}^{cap} = \frac{2\sigma_{ow}}{r_2} \end{cases}$$

$$\Rightarrow h_o - h_w = \left[\frac{2\sigma_{ow}}{r_2} + \frac{2\sigma_{oa}}{r_1} - \rho_w g h_o\right] / (\rho_o g)$$

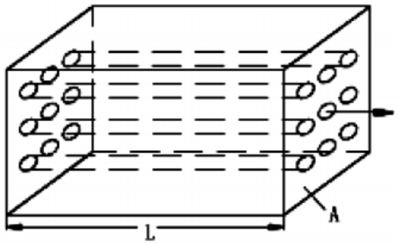
$$\Rightarrow h_w = \left[\frac{2\sigma_{ow}}{r_2} + \frac{2\sigma_{oa}}{r_1} - \rho_o g h_o\right] / \left[(\rho_w - \rho_o) g\right]$$

Air-water system in capillary tubes

Model of a porous medium as a Bundle of Capillary Tubes

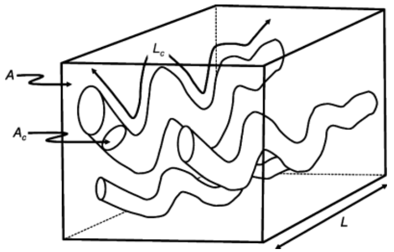


porous medium



Bundle of capillary tubes

Bear (1972)

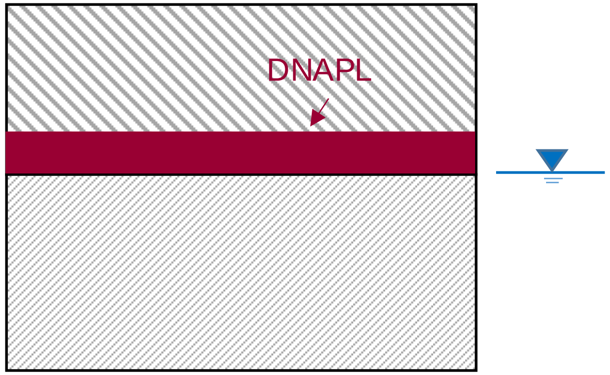


Bundle of tortuous capillary tubes

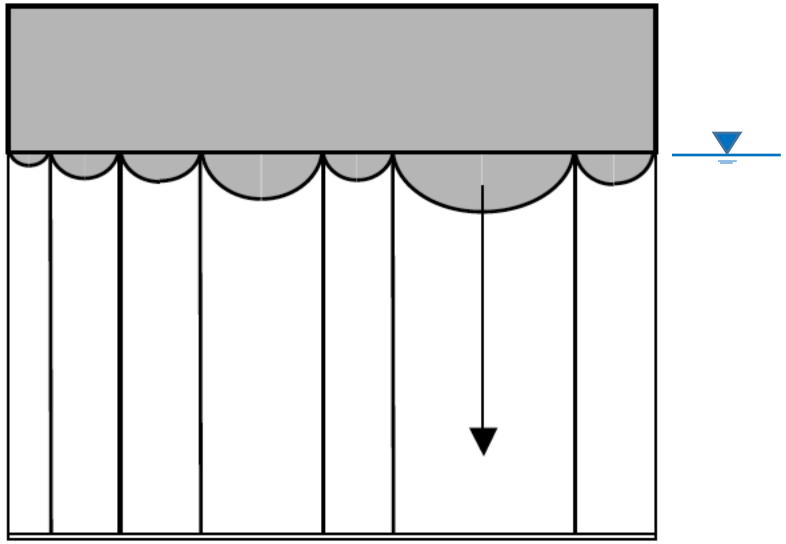
- ❖ Very simplified model, but its application has tremendously improved our understanding of fluid flow and transport phenomena in porous media. Some examples:
  - Permeability (already discussed)
  - Dispersion (already discussed)
  - Fluid invasion
  - Capillary transition zone
  - Soil water characteristic curve
  - Relative permeability

Air-water system in capillary tubes

Invasion of a nonwetting fluid into an aquifer



Representing the aquifer as a bundle of capillary tubes



1. Which is easier for DNAPL to invade?

   Coarse sand or fine-grained medium?

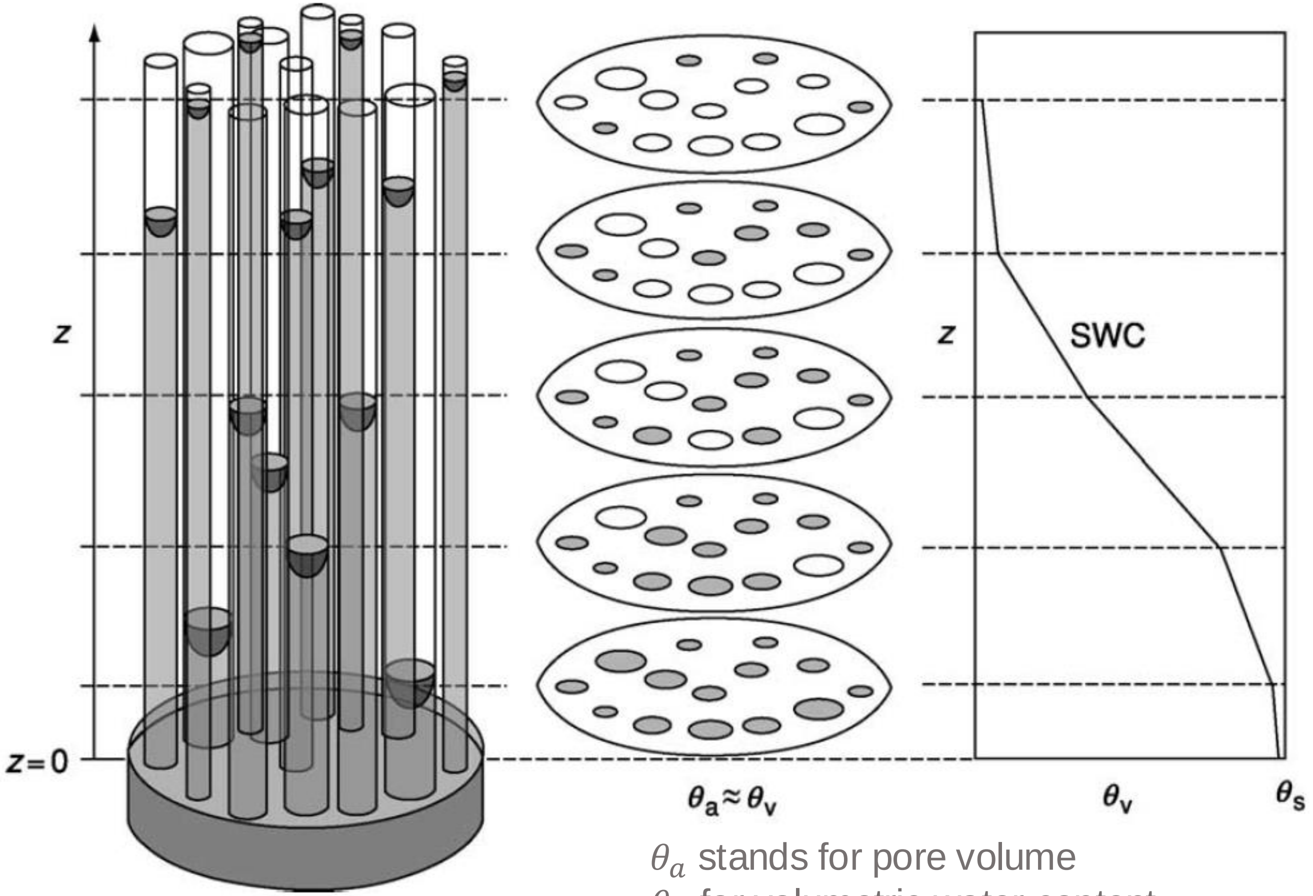
2. For some reason, if DNAPL modifies the wettability of the porous medium grain surfaces, e.g., the contact angle of water increases from 0º to something between 0º and 90º.

   What may happen to the DNAPL?

Air-water system in capillary tubes

Water retention (or capillary transition zone) in the vadose zone

Representing the soil as a
Bundle of Capillary Tubes



Tuller & Or (2004)

θ_a stands for pore volume
θ_V for volumetric water content
θ_s for volumetric water content at saturation