

Logic

- Q1
- i) $(p \vee q) \vee \neg(p \wedge q)$
 - ii) $p \rightarrow (q \vee (\neg \neg r))$
 - iii) ~~$((p \vee q) \vee \neg(p \wedge q)) \leftrightarrow r$~~ $p \rightarrow q$
 - iv) $r \rightarrow ((p \wedge q) \leftrightarrow (\neg s \vee \neg t))$
 - v) $\neg(p \rightarrow q) \wedge r$

correspondence

- Q2 ex)
- i) p : everyone's ordering pizza
 q : everyone's having coffee
 - ii) p : Umberto is here
 q : Angela arrived on time
 r : Elena decide to come
 - iii) p : agent knows either optimized or lights on
 q : robot swing arm
 - iv) r : she's the one withdrew them
 p : Jacob read The Age ...
 q : Jacob read Invisible ...
 s : Jolene return The Age
 t : Jolene return Invisible
 - v) p : you arrive on time
 q : Kate arrive on time
 r : she's not coming

for any evaluation v

- Q2 $A \models B \rightarrow \perp$, so ~~(if)~~ if $h_v(A)$, then $h_v(B \rightarrow \perp)$ true
 so when $h_v(A) = t$, $h_v(B)$ must be f
 that is $h_v(B) = f$ or $h_v(\perp) = t$
 \hookrightarrow if $h_v(B) = f$, so map A to t , B to f
 when $h_v(A) = f$, \otimes

$B \rightarrow \perp \models A$, so if $h_v(B \rightarrow \perp) = t$, then $h_v(A) = t$
 $h_v(B \rightarrow \perp) = t$ is $h_v(B) = f$,
 so when $h_v(B) = f$, $h_v(A)$ must be t

In total we have 4 possible h_v , $h_{v_1} \sim h_{v_4}$

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$h_{v_1}(A) = h_{v_1}(B) = t$, by \otimes , not possible

$h_{v_2}(A) = h_{v_2}(B) = f$, by \otimes not possible

only $h_{v_3}(A) : A \mapsto t, B \mapsto f$,

$h_{v_4} : A \mapsto f, B \mapsto t$ are possible

thus for any possible V , $h_v(A) \neq h_v(B)$

\Rightarrow : so $A \leftrightarrow \neg B$ is ~~always~~ ~~not~~ evaluate as true

$h_v(\neg(A \leftrightarrow \neg B)) \equiv f$, not satisfiable

\Leftarrow : by contrapositive at least one of

~~if not $h_v(A) \neq h_v(B)$~~ h_{v_1}, h_{v_2} above are possible

then when evaluate V such that $h_v(A) = h_v(B)$

$h_v(\neg(A \leftrightarrow \neg B))$ can be evaluate as true

ii) $h_v(A_1 \vee A_2) \neq h_v(B_1 \vee B_2), \forall v$

~~$h_v(A_1) \neq h_v(A_2) \neq h_v(B_1) \neq h_v(B_2)$~~ $\neq h_v(B_1) \vee h_v(B_2) \neq$

~~A_1, A_2, B_1, B_2~~

~~v_1, v_2, v_3, v_4~~ ~~t, t, f, f~~ possible when A_1 true

~~v_2, v_3~~

~~A_1, A_2, B_1, B_2~~ $\neg B_1 \wedge \neg B_2$

~~t, f, f, t~~ $t \wedge t = t$

~~t, f, f, t~~ by antisatisfied, A_1 true

~~t, f, f, t~~ only ff is possible for B_1, B_2

and this make $A_1 \models \neg B_1 \wedge \neg B_2$ valid

Q3

p, q, r			$\neg(((q \rightarrow \neg r) \wedge (\neg p \vee q) \wedge \neg(\neg p \wedge (p \rightarrow \neg r))))$										
\Rightarrow	f	f	f	t	f	t	t	f	t	t	f	t	t
\Rightarrow	f	f	t	f	f	t	f	t	t	f	t	f	f
\Rightarrow	f	t	f	f	t	t	t	t	t	t	t	f	t
	f	t	t	t	t	f	f	f	t	t	t	t	f
	t	f	f	t	f	t	t	f	f	f	f	t	t
\Rightarrow	t	f	t	f	f	t	f	t	f	f	f	f	f
\Rightarrow	t	t	f	(f)	t	t	t	t	f	t	t	t	t
	t	t	t	t	t	f	f	f	f	t	t	t	f

no valid ttf, tft, ftf, ftt

Q4) a) both b) both c) neither d) neither
 e) neither f) CNF g) neither

ii) a) r is pure, so remove $\{p, q, r\}$
 now q is pure negative, only $\{\{p, s\}, \{p, \neg s\}\}$ left
 p is pure $\{\}$

b) $\{r\}$ is unit, so
 $\{\{p\}, \{q, s\}, \{\neg s, \neg p\}, \{\neg p, \neg q\}\}$
 now $\{p\}$ unit, so
 $\{\{q, s\}, \{\neg s\}, \{\neg q\}\}$
 $\{\neg s\}$ unit, $\{\{q\}, \{\neg q\}\} \Rightarrow \{\{\}\}$

Q5) i) $p \vee q$

ii) $\neg q \vee r \vee \neg s$

iii) $(\neg p \vee s) \vee \neg q \equiv (\neg p \vee \neg q) \wedge (s \vee \neg q)$

iv) $p \vee \neg r$

v) $s \vee p$

$\{\{p, q\}, \{\neg q, r, \neg s\}, \{\neg p, \neg q\}, \{s, \neg q\}, \{p, \neg r\}, \{s, p\}\}$

consider $p, \neg p$, if p true

$\{\{\neg q, r, \neg s\}, \{\neg q\}, \{s, \neg q\}\}$

q is pure so $\{\}$ valid

Q6 i) Yes ii) Yes

iii) Yes iv) syntax error

v) Yes vi) no, $f(x, x)$ not output true false

vii) Yes

Q7 i) $C = \{ \}$ $P_1 = \{ \text{painter, Paris, London} \}$ $P_2 = \{ \text{color} \}$

$\exists X \forall Y \text{ painter}(X) \text{ color}(Y) \text{ Paris}(X)$

$\rightarrow \neg \exists Z \forall Y \text{ painter}(Z) \text{ color}(Y) \text{ London}(Z)$

$\text{painter}(X) := X \text{ is painter}$

$\text{color}(Z, Y) \text{ color}(X, Y) := X \text{ use } Y$

ii) $C = \{ \text{Charlie} \}$ $P_2 = \{ \text{love} \}$

$\forall X \exists Y (\neg X = \text{Charlie}) \wedge (\forall Z \text{ love}(Y, Z) \rightarrow \text{love}(X, Y))$

$\wedge \neg \text{love}(\text{Charlie}, W) \text{ love}(X, Y) := X \text{ loves } Y$

iii) $C = \{ \}$ $P_1 = \{ \text{student} \}$ $P_2 = \{ \text{submit, grade} \}$

$\forall X \forall Y \text{ student}(X) \text{ coursework}(Y)$

$\text{submit}(X, Y) \vee \text{grade}(X, Y)$

$\text{submit}(X, Y) := X \text{ submit } Y \text{ on time}$

$\text{grade}(X, Y) := X \text{ receive grade for } Y$

iv) $C = \{ \text{Nadia.B.} \}$ $P_2 = \{ \text{know, compose, study} \}$

$\text{know}(X, Y) := X \text{ knows } Y$

$\text{compose}(X, Y) := X \text{ compose } Y$

$\text{study}(X, Y) := X \text{ study with } Y$

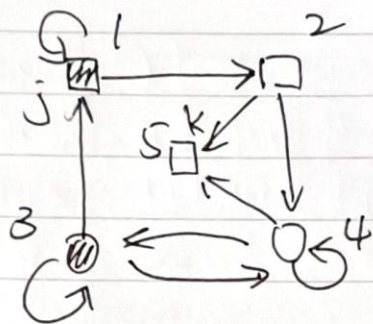
$\forall X (\forall Y \text{ study}(X, \text{Nadia}) \vee \text{know}(X, \text{Nadia}) \wedge \text{compose}(X, \text{Nadia}, Y))$

$\rightarrow \text{know}(X, Y) \wedge$

$\wedge \neg \exists Z \text{ compose}(W, Z) \wedge \text{study}(W, \text{Nadia})$

$\rightarrow \text{know}(X, Z)$

Q8 i)



- i) see object 2, 4, 5
 for 2, take $Y=5$, $a(2,5) \checkmark$
 for 4, take $Y=5$, $a(4,5) \checkmark$
 for 5, no such Y X so false

ii) ~~consider negation~~

$$\exists X \exists Y \exists Z (a(X, Z) \wedge a(Y, Z) \wedge \neg (a(X, Y) \vee a(Y, X)))$$

$$\equiv \exists X \exists Y (\exists Z (a(X, Z) \wedge a(Y, Z)) \wedge \neg a(X, Y) \wedge \neg a(Y, X))$$

$Z=1$ $a(1,1) \wedge a(3,1) \checkmark$ $X=1, Y=3$
 check $a(1,3) \vee a(3,1) \checkmark$

$Z=2$

$Z=3$ $a(3,3) \wedge a(4,3) \checkmark$ $X=3, Y=4$
 check $a(3,4) \vee a(4,3) \checkmark$

$Z=4$ $a(3,4) \wedge a(2,4) \checkmark$ $X=2, Y=3$
 check $a(2,3) \vee a(3,2) \times$ False
 counter example

iii) only need to check $X=1, 3$

for $X=1$, for $\exists Y \exists Z (a(Y, Z) \wedge a(Z, X) \wedge \neg a(Y, X))$

only possible Z 's are 1, 3 by $a(Z, X)$

~~but no Y s.t. $a(Y, Z)$~~

take $Y=4, Z=3$, $a(4,3) \wedge a(3,1) \wedge \neg a(4,1)$

so $\exists Y, Z$, false

iv) possible $X = 1, 3, 4$ by $a(X, X)$.

consider negation:

$$\textcircled{1} \forall X (\neg a(X, X) \vee \forall Y \neg \exists Z (a(X, Y) \vee a(Z, X) \rightarrow a(Z, k)))$$

$\neg a(X, X)$ is true $\text{for } X = 3, 4$

so consider $X = 1, 3, 4$

See if $\forall Y \neg \exists Z (a(X, Y) \vee a(Z, X) \rightarrow a(Z, k))$ ~~(1)~~ $\textcircled{2}$

for $X = 1$ As $X \neq k$

consider negation again

$$\textcircled{3} \exists Y \exists Z (a(X, Y) \vee a(Z, X) \rightarrow a(Z, k)) \text{ for } X = 1, 3, 4$$

Y can be choose by X to make $a(X, Y)$ false

for $X = 1, Y = 2$ so a Z

to make $a(X, Y) \vee a(Z, X)$ false

so the implication is true

for $X = 1$, choose $Y = Z = 5$

$X = 3$, choose $Y = Z = 5$

$X = 4$, $Y = Z = 1$ \square

by this we see ~~(3)~~ true

so ~~(2)~~ is false for all $X = 1, 3, 4$

so statement ~~(1)~~ is false

so the statement is true