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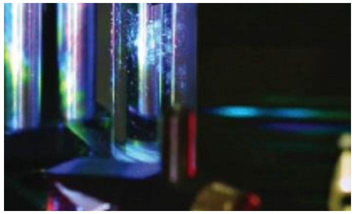

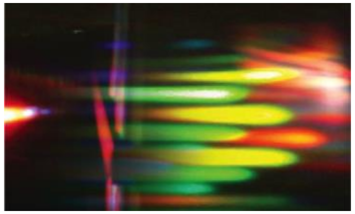

# COMP70058 Computer Vision

# Lecture 9 – Interest Point Detection


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for Robotic Surgery

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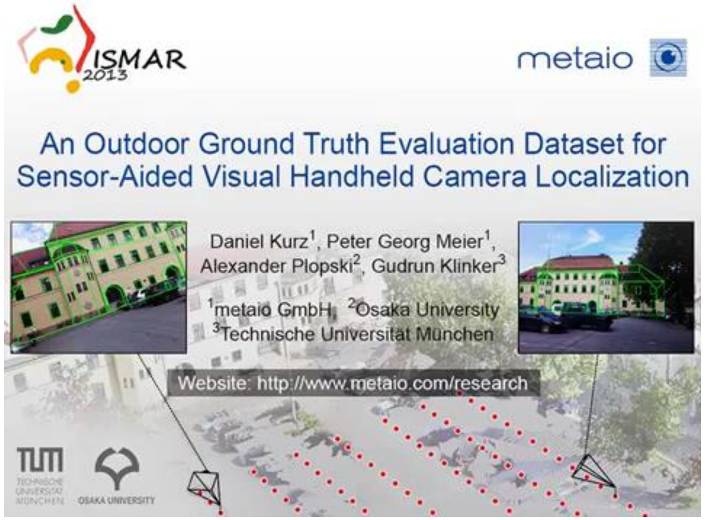
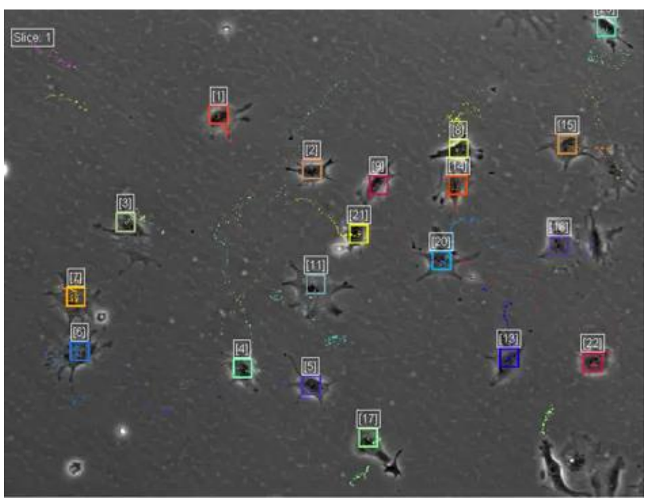
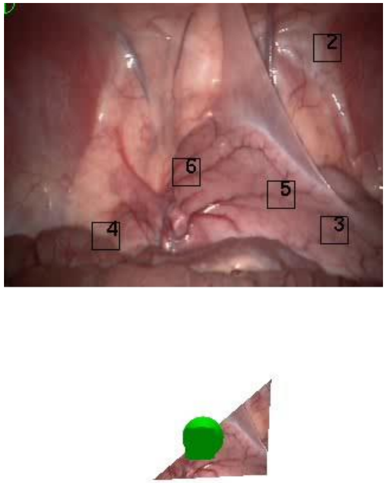
- Feature Detection
- Interest Points in Vision
- Corner Detection and Harris Corner Detector
- Shi-Tomasi Corner Detector
- Automatic Scale Selection

# Applications of Image Sequence Processing

- Image alignment and stitching
- Object recognition
- 3D reconstruction and modelling
- Motion tracking
- Indexing and content-based retrieval
- Robot mapping and navigation
- Gesture recognition



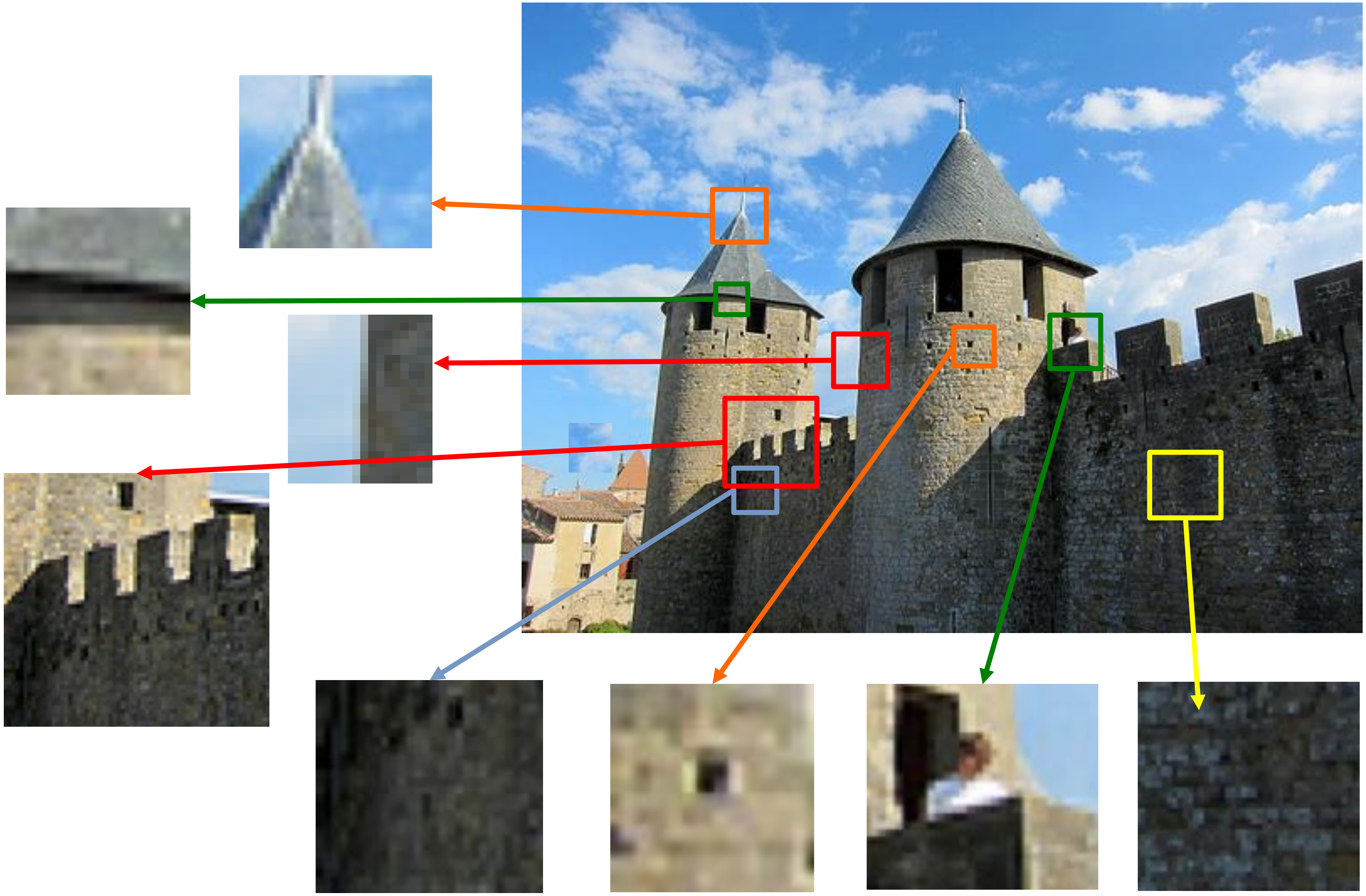



# Feature Detection

- What is a feature?
  - An interesting part of an image
- Why local features?
  - Locality – local features are associated with object features
  - Generalisability – local features are more generalisable for scene representation
  - Abundance – many of them, often with redundancy (numerically attractive)
  - Efficiency – fast to calculate, thus good for real-time performance
- Types of features
  - Edges
  - Corners (when there is high curvature in image gradient)
  - Blobs
  - Ridges

# What Features are Good for Tracking



# Interest Points in Vision

- Well defined position in images;
- Incorporate useful local image context (e.g., distinctive texture patch);
- Stable under local or global changes (e.g., illumination, contrast);
- Immune to geometrical changes (e.g., scale, translation, rotation);
- Generalisable to natural scenes and preferably with good mathematical representation.

| Edges | Corners | Blobs | Ridges |
|---|---|---|---|
| Sobel<br>Prewitt<br>Roberts<br>Robinson<br>Canny<br>Canny-Deriche | **Harris**<br>**Shi & Tomasi**<br>Yang & Chabat<br>Level Curvature<br>Susan<br>AST/FAST | LoG<br>DoG<br>DoH<br>(Determinant of Hessian)<br>MSER<br>(Maximally stable extremal regions)<br>PCBR<br>(Principal curvature based region) | Ridge sets<br>Valley sets<br>Relative critical sets |

# Interest Points in Vision

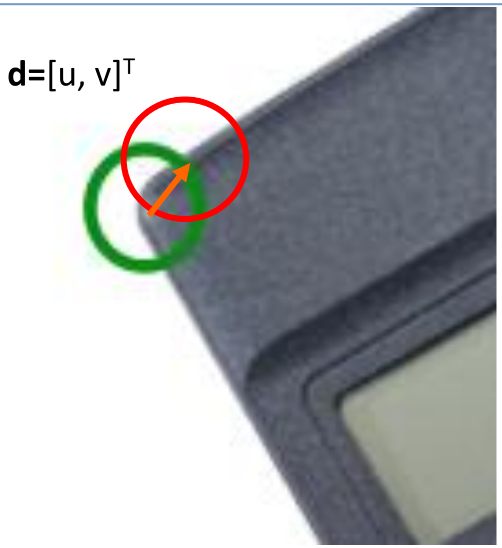
# Why Corners

- Movement in uniform region results in no change in pixel distribution within the window
- Movement along an edge also results in no change in pixel distribution within the window
- For corners, any movement will result in significant changes



# Harris Corner Detector

- Let's look at a small window W, when shifted by **d**=[u, v]$^T$
- The changes in intensity distribution due to **d** can be expressed as the squared difference of I(*x*+*u*, *y*+*v*)-I(*x*,*y*) within the window




$$\varepsilon(u,v) = \sum_{x,y \in W} \left( I(x+u, y+v) - I(x,y) \right)^2$$

For uniform patches, this will be near 0 and for distinctive patches such as corners, it will be large, therefore we are interested in maximising ε(u,v) while searching for corners

By Taylor series expansion $I(x+u, y+v) = I(x,y) + \frac{\partial I}{\partial x} u + \frac{\partial I}{\partial y} v + \ldots$

$$\varepsilon(u,v) \approx \sum_{x,y \in W} \left( I(x,y) + \frac{\partial I}{\partial x} u + \frac{\partial I}{\partial y} v - I(x,y) \right)^2, \text{ define } I_x \equiv \frac{\partial I}{\partial x} \text{ and } I_y \equiv \frac{\partial I}{\partial y} \text{ we have}$$

$$\varepsilon(u,v) \approx \sum_{x,y \in W} \left( uI_x + vI_y \right)^2 = \sum_{x,y \in W} \left( u^2 I_x^2 + 2uvI_xI_y + v^2 I_y^2 \right)$$

# Harris Corner Detector

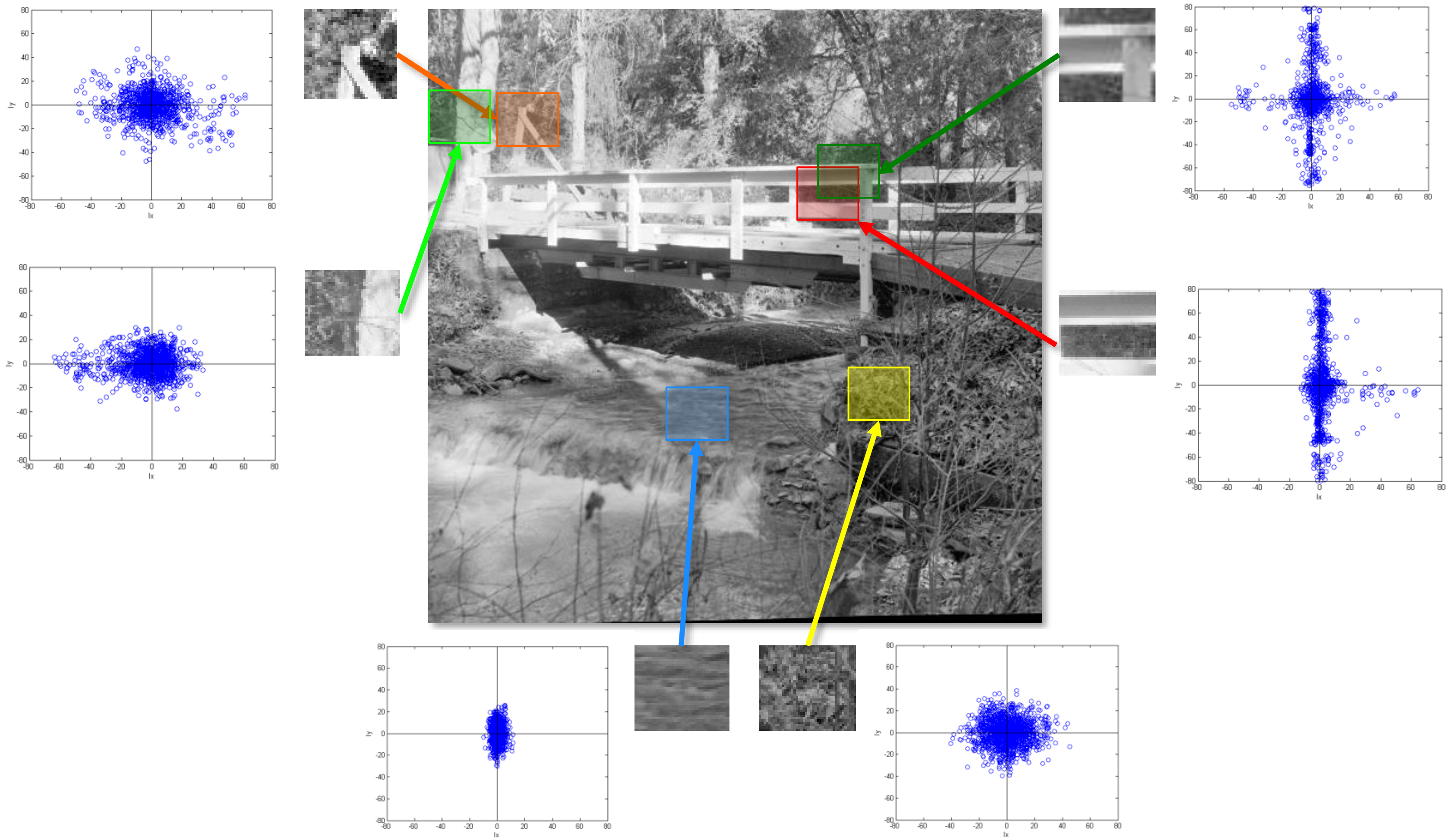
$$\varepsilon(u,v) \approx \sum_{x,y\in W} \left(uI_x + vI_y\right)^2 = \sum_{x,y\in W} \left(u^2 I_x^2 + 2uvI_x I_y + v^2 I_y^2\right)$$

- When expressed in matrix form, we have the following equivalent expression

$$\varepsilon(u,v) \approx [u,v] \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$\varepsilon(u,v) \approx \mathbf{d}^T \mathbf{M} \mathbf{d}$$

- We will get a large $E(u,v)$, where image derivatives are large.
- $M$ is a 2x2 matrix which can provide information about the directions of change.


80 60 40 20 0 -20 -40 -60 -80
Iy
-80 -60 -40 -20 0 20 40 60 80
Ix
80 60 40 20 0 -20 -40 -60 -80
Iy
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Ix
80 60 40 20 0 -20 -40 -60 -80
Iy
-80 -60 -40 -20 0 20 40 60 80
Ix
80 60 40 20 0 -20 -40 -60 -80
Iy
-80 -60 -40 -20 0 20 40 60 80
Ix
80 60 40 20 0 -20 -40 -60 -80
Iy
-80 -60 -40 -20 0 20 40 60 80
Ix
80 60 40 20 0 -20 -40 -60 -80
Iy
-80 -60 -40 -20 0 20 40 60 80
Ix

# Harris Corner Detector

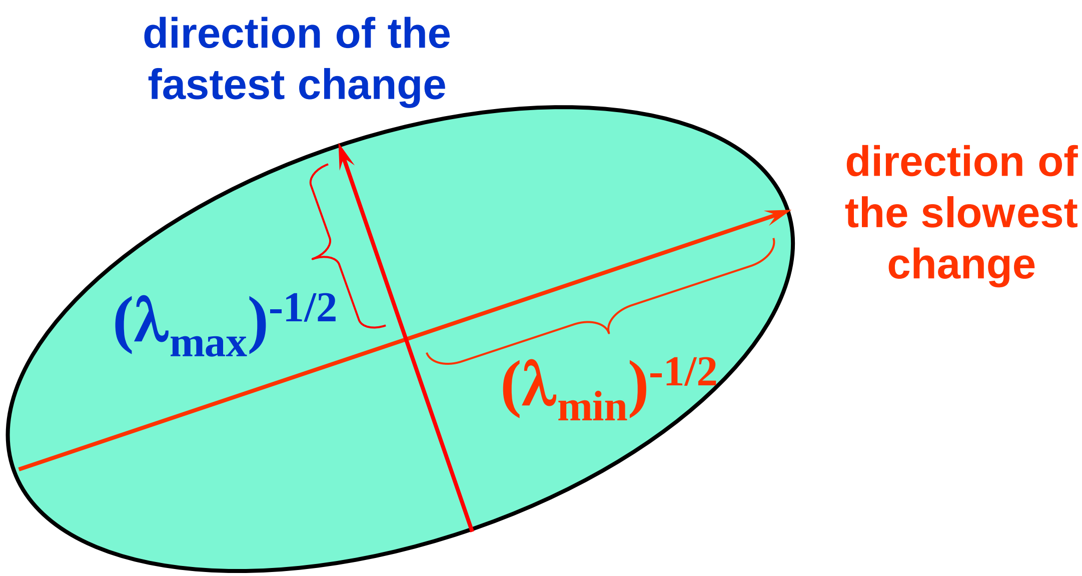
- Since $M$ is a real symmetric matrix, we can decompose $M$ as:

$$M = P \Lambda P^T$$

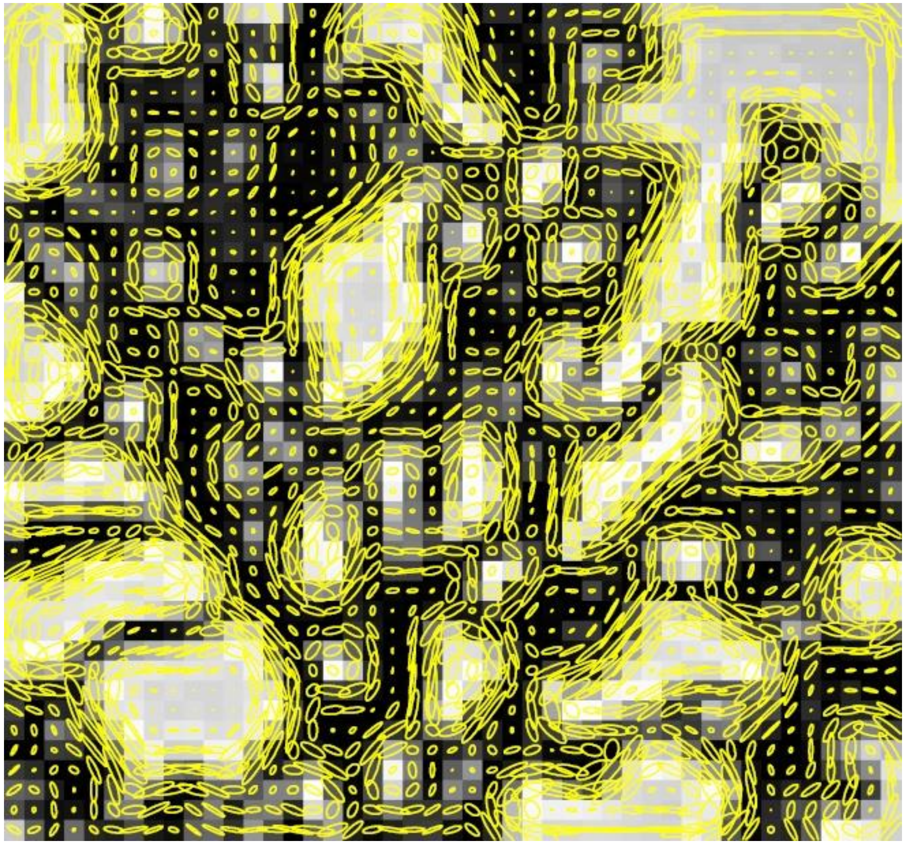
each column is an eigenvector

diagonal matrix with eigenvalues $\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$

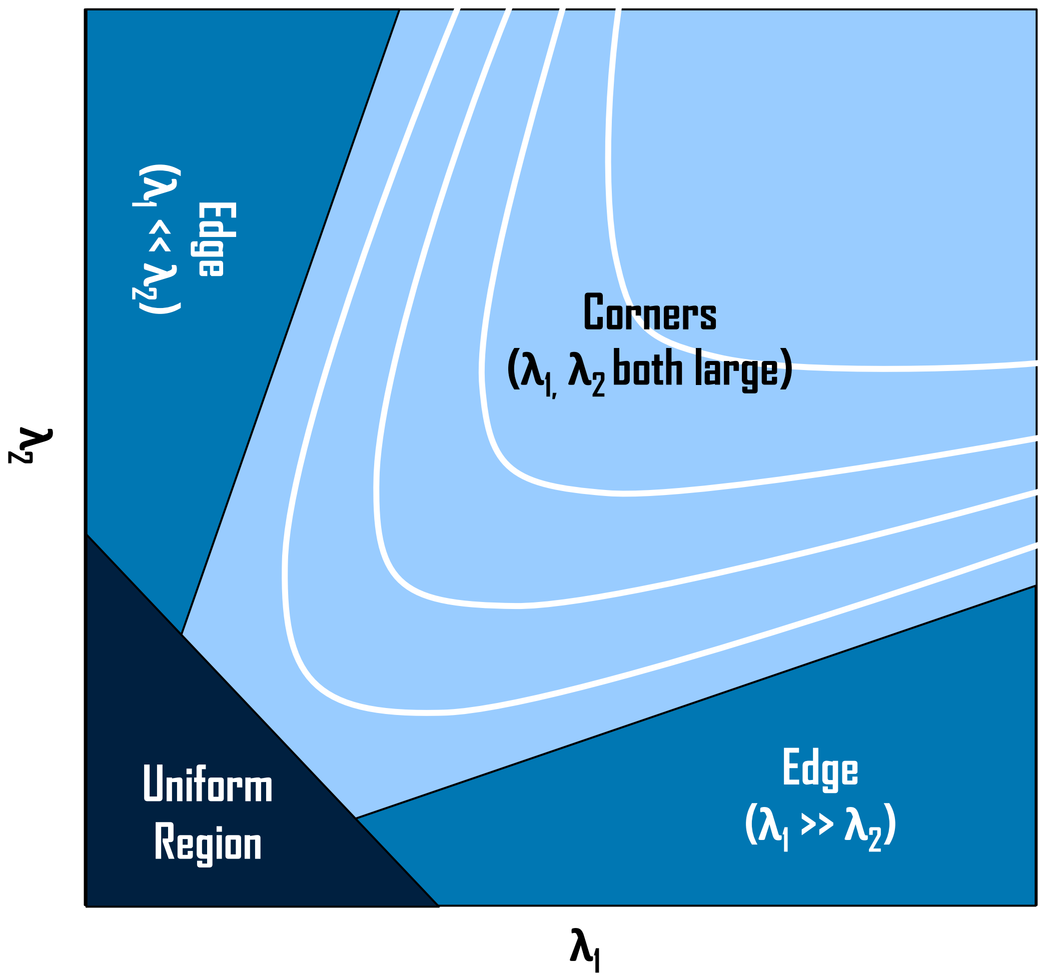
each row is an eigenvector



# Corner Response Map



# Corner Response Map



# Harris Corner Detector

$$\varepsilon(u,v) \approx \sum_{x,y \in W} \left(uI_x + vI_y\right)^2 = \sum_{x,y \in W} \left(u^2 I_x^2 + 2uvI_x I_y + v^2 I_y^2\right)$$

- When expressed in matrix form, we have the following equivalent expression

$$\varepsilon(u,v) \approx [u,v] \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$\varepsilon(u,v) \approx \mathbf{d}^T \mathbf{M} \mathbf{d}$$

- Measure of corner response (where λ1, λ2 are eigenvalues of M)

$$R = \det \mathbf{M} - k(trace\mathbf{M})^2$$

$$\det \mathbf{M} = \lambda_1 \lambda_2$$

Empirically determined constant

$$trace\mathbf{M} = \lambda_1 + \lambda_2$$

$$k \in [0.04, 0.06]$$

# Algorithm Design – Harris Corner Detection

- Compute x and y derivatives of image (can convolve image with derivatives of Gaussians);
- Compute products of derivatives at every pixel;
- Compute the sums of the products of derivatives;
- Construct the M matrix;
- Calculate the corner response value R;
- Threshold on value R, suppress non-maximum value.

$$I_x = G_x(x, y, \sigma) * f(x, y)$$
$$I_y = G_y(x, y, \sigma) * f(x, y)$$

$$\mathbf{M} = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}$$

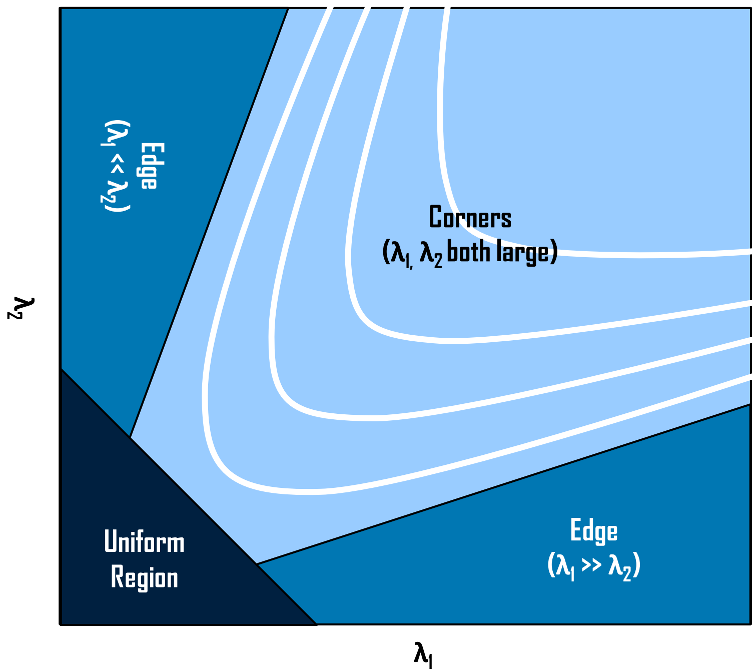
Empirically determined constant

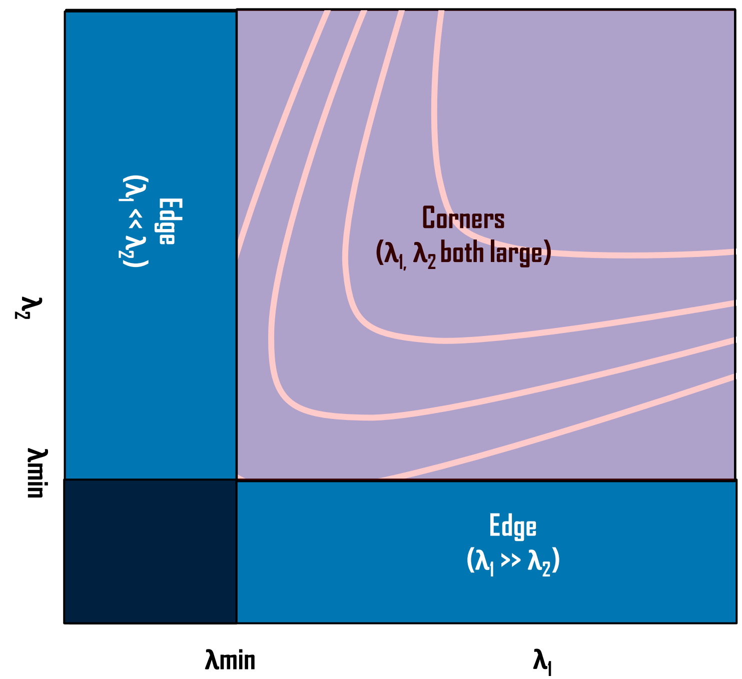
$$R = \det \mathbf{M} - k(trace\mathbf{M})^2$$

$$\det \mathbf{M} = \lambda_1 \lambda_2 \qquad trace\mathbf{M} = \lambda_1 + \lambda_2$$

# Harris Corner Detector in Action

# Shi-Tomasi Corner Detector




Empirically determined constant

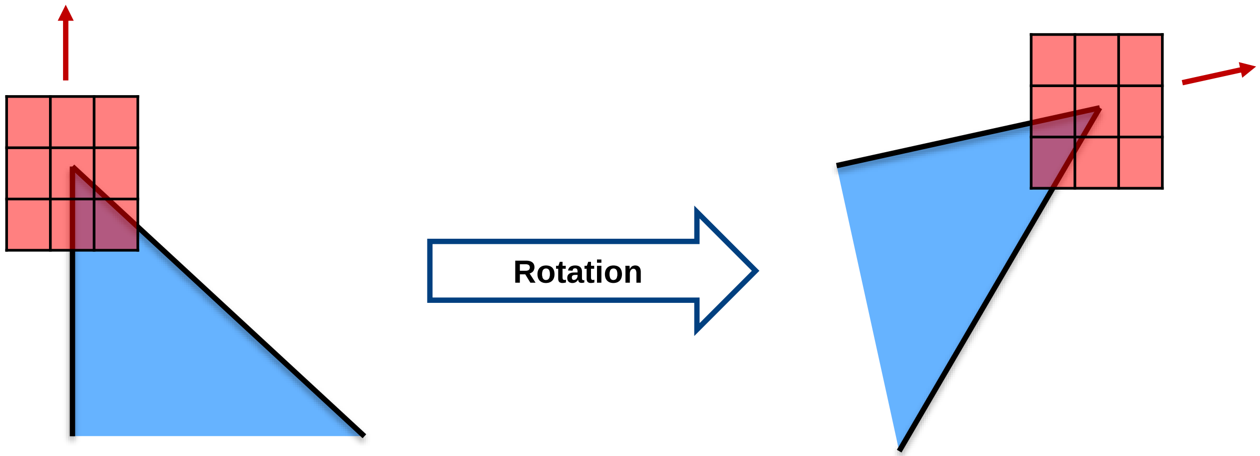
$$R = \det \mathbf{M} - k(trace\mathbf{M})^2$$

$$\det \mathbf{M} = \lambda_1 \lambda_2 \qquad trace\mathbf{M} = \lambda_1 + \lambda_2$$

$$R = \min(\lambda_1, \lambda_2)$$

# Corner Detector Invariance Properties

- If the corner has rotated, you can still get the same change of intensities when you shift the window. However, you need to shift along a different direction.
- The eigenvalues of $M$ do not change. But the eigenvectors change.
- Therefore, the ellipse representing the feature will rotate but its shape will not change.



# Corner Detector Invariance Properties

- When the image scale changes, the corner response will change.
- However, we can apply a corner detector at multiple scales and find the response at the most suitable scale.
- How do we normally get multi-scale images?
  - Gaussian smoothing with different $\sigma$
  - Sampling at different pixel resolutions

$\sigma = 1$ $\sigma = 3$ $\sigma = 5$ $\sigma = 7$

**Image convolved with Gaussian kernels of different $\sigma$ provide information at different scale.**

# Automatic Scale Selection

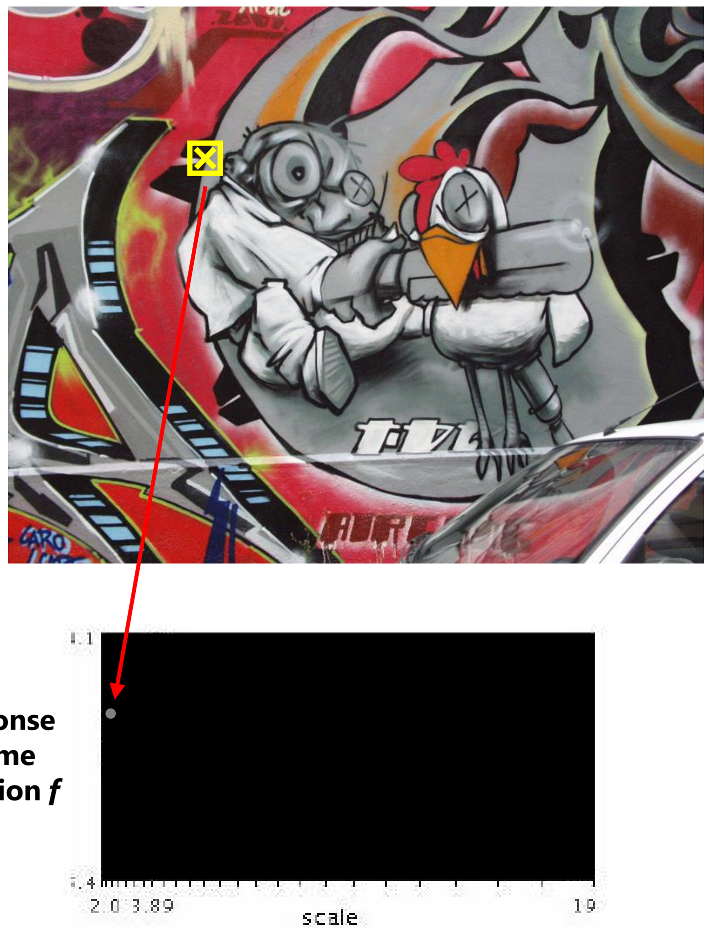
$$f(I_{i_1\ldots i_m}(x,\sigma)) = f(I_{i_1\ldots i_m}(x',\sigma'))$$

- We need to find the patch size for which the *f* response will be equal.

# Automatic Scale Selection

- Function responses for increasing scale (scale signature)

Response of some function ***f***



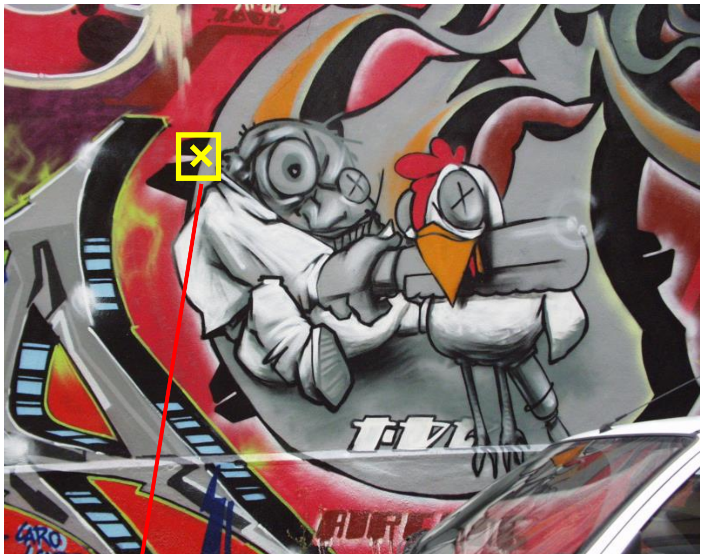

$$f(I_{i_1 \dots i_m}(x, \sigma))$$


$$f(I_{i_1 \dots i_m}(x', \sigma'))$$

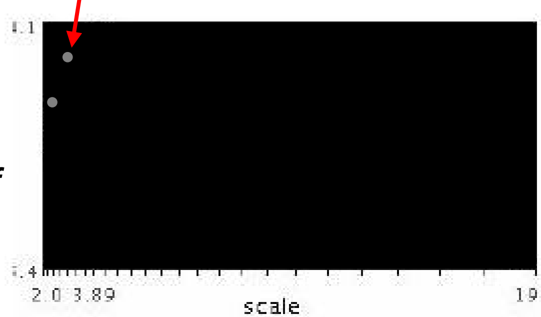
https://cs.brown.edu/courses/csci1430/

# Automatic Scale Selection

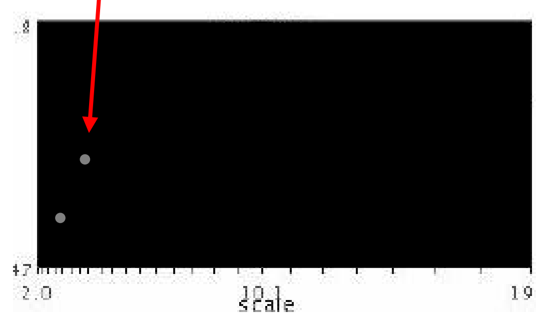
- Function responses for increasing scale (scale signature)



**Response of some function *f***




$$f(I_{i_1 \dots i_m}(x, \sigma))$$

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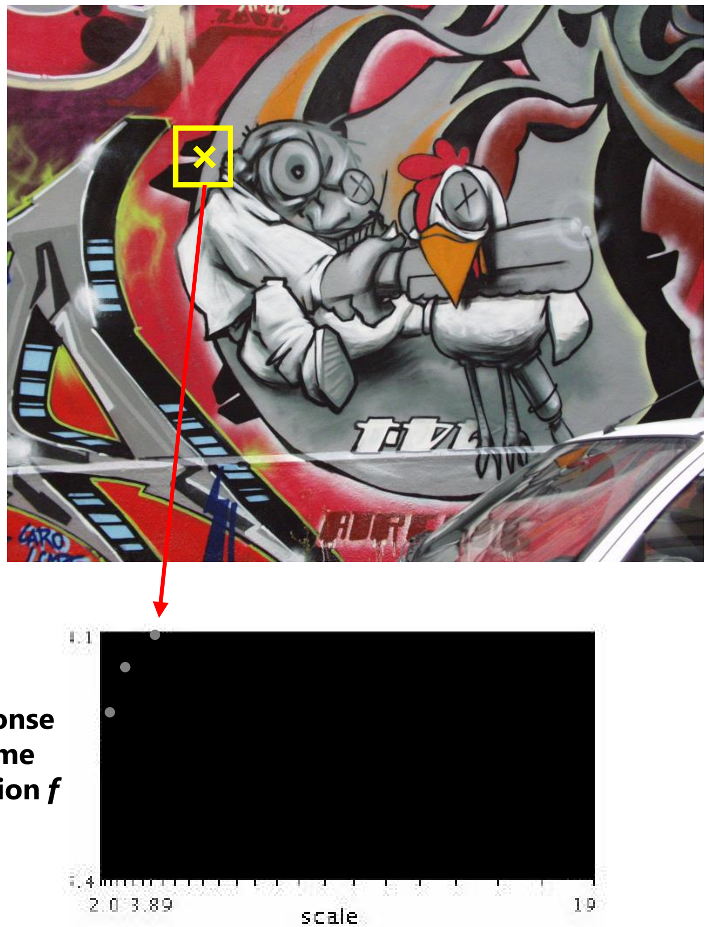


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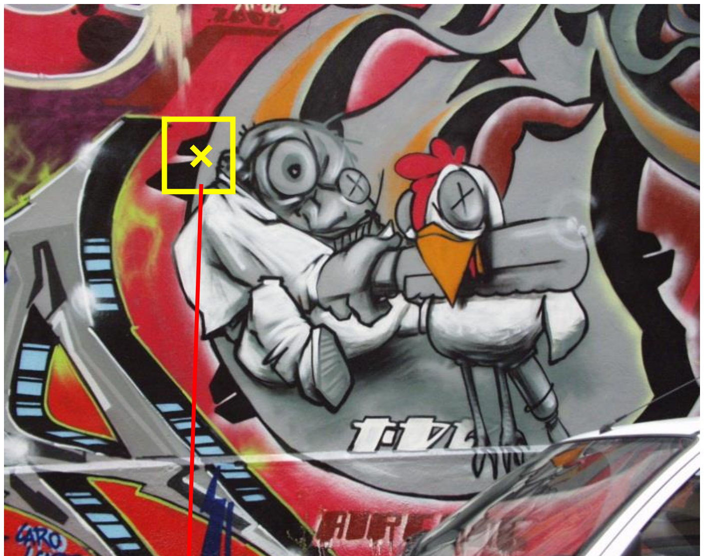

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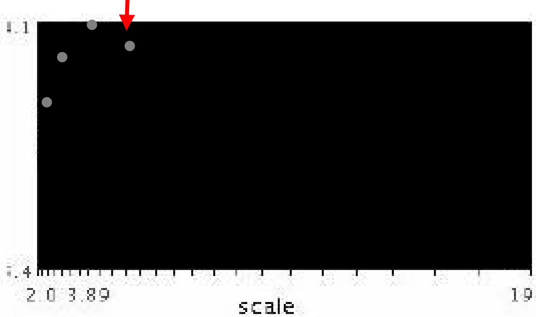
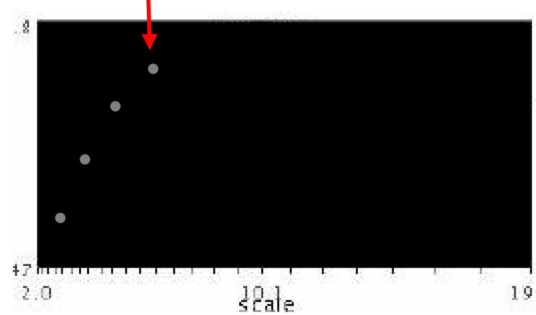



$f(I_{i_1 \dots i_m}(x, \sigma))$

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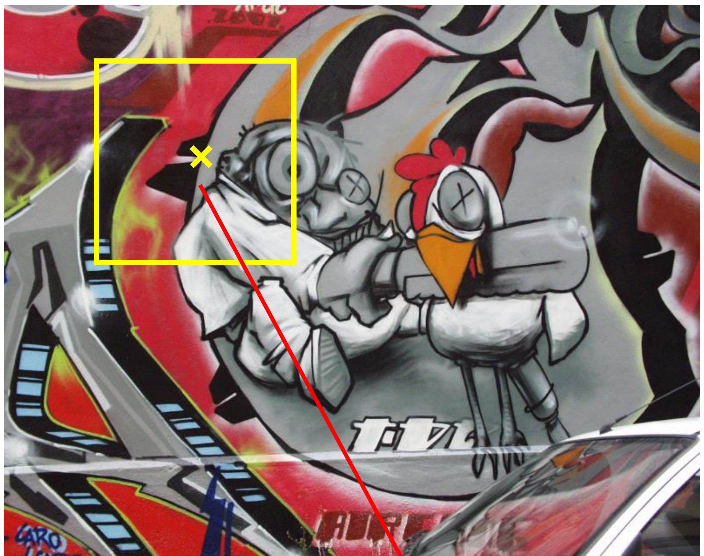
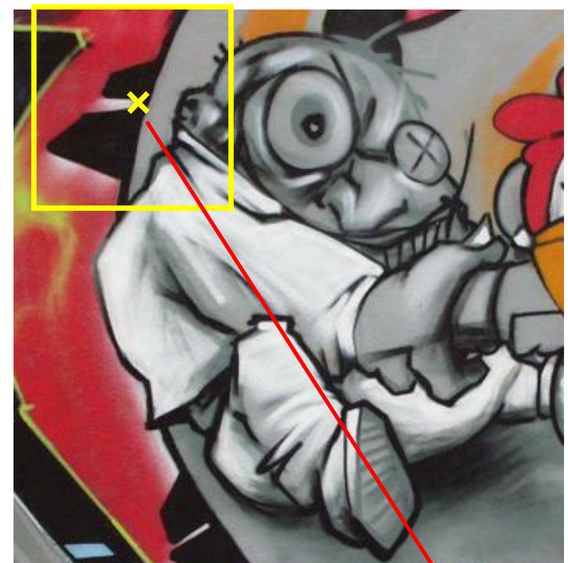
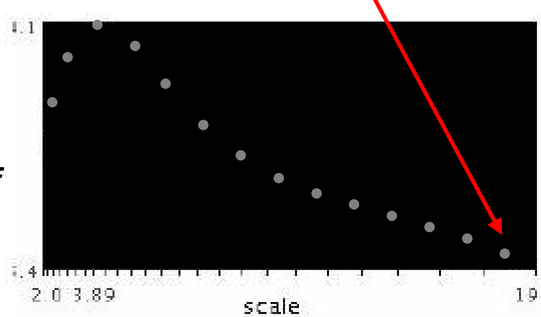
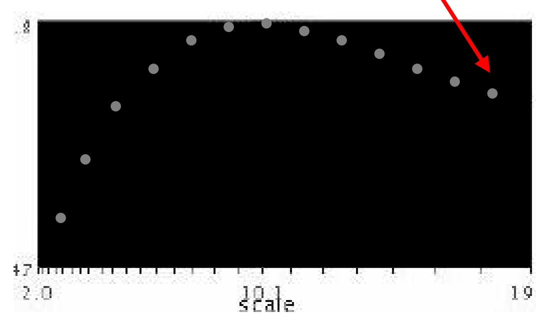




**Response of some function *f***




$$f(I_{i_1 \ldots i_m}(x,\sigma))$$




$$f(I_{i_1 \ldots i_m}(x',\sigma'))$$

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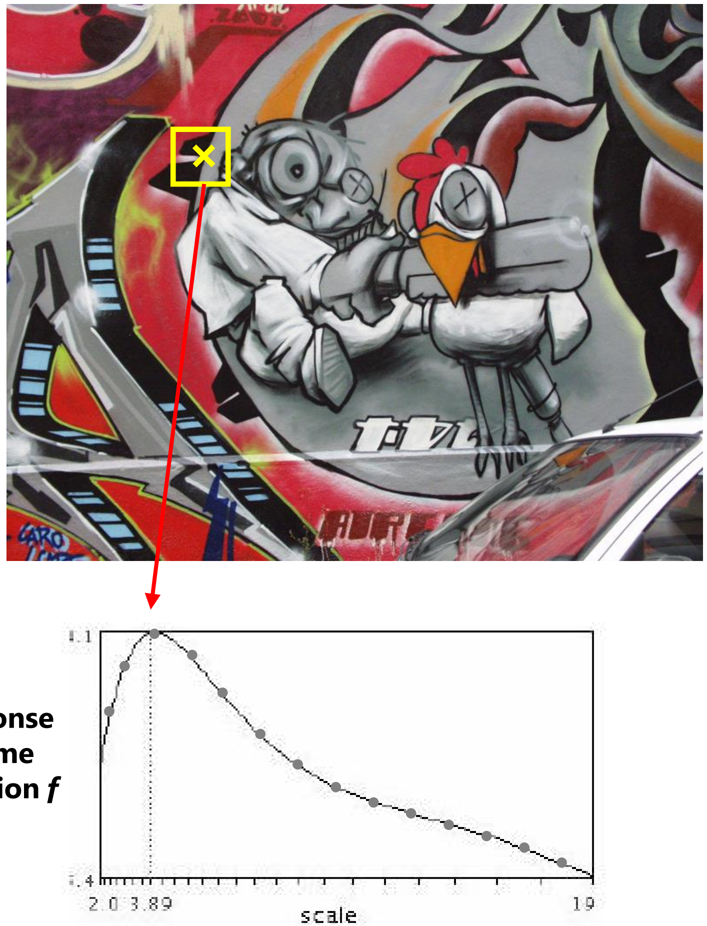
# Automatic Scale Selection

- Function responses for increasing scale (scale signature)




**Response of some function *f***

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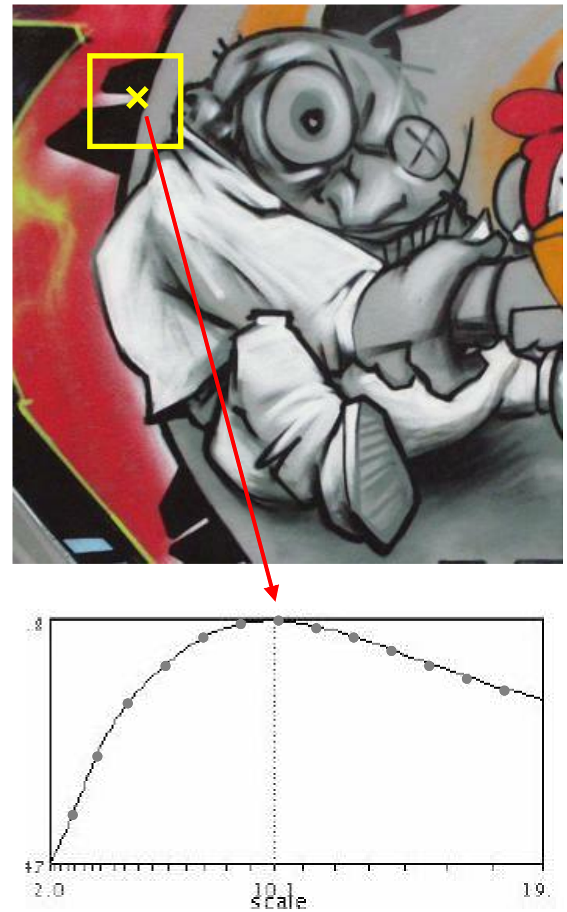


$$f(I_{i_1 \dots i_m}(x', \sigma'))$$

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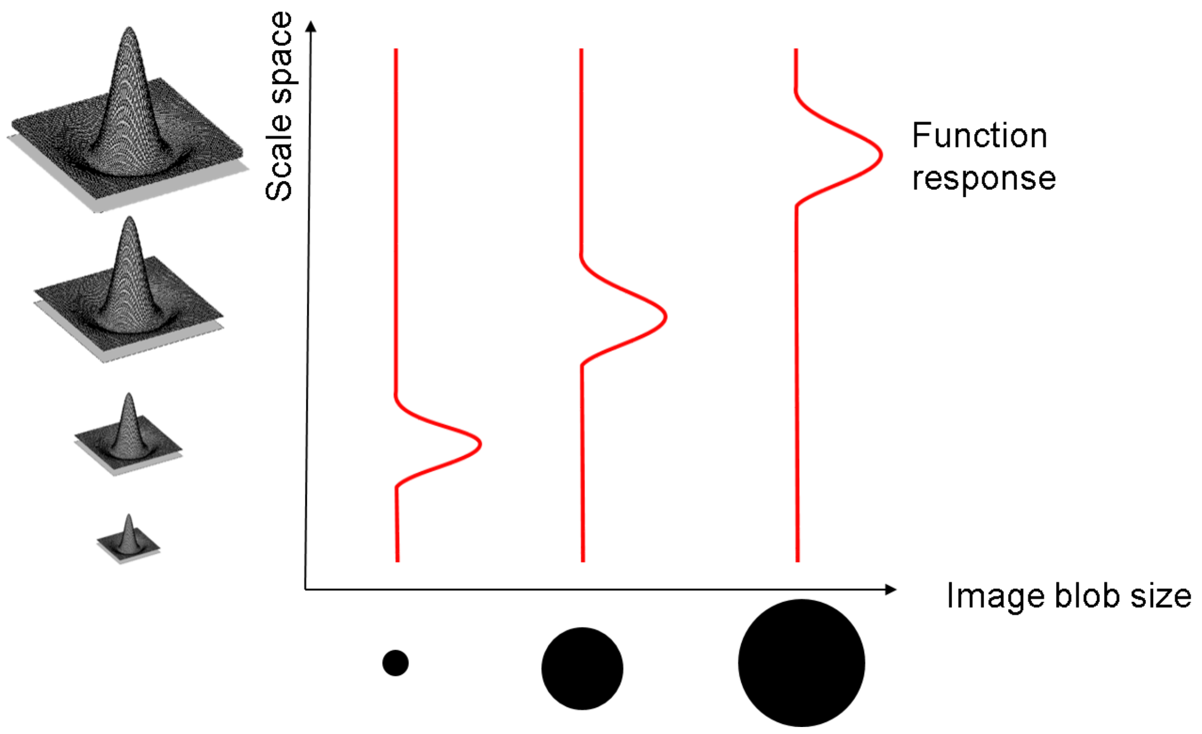
# Automatic Scale Selection

- The Laplacian of Gaussian (LoG) which is the second derivative of Gaussian is a suitable function to estimate the characteristic scale of image structures such as blobs and corners.
- When the size of the LoG kernel matches the size of an image structure the response attains an extremum.




*K. Mikolajczyk et al. Scale & affine invariant interest point detectors, IJCV 2004.*

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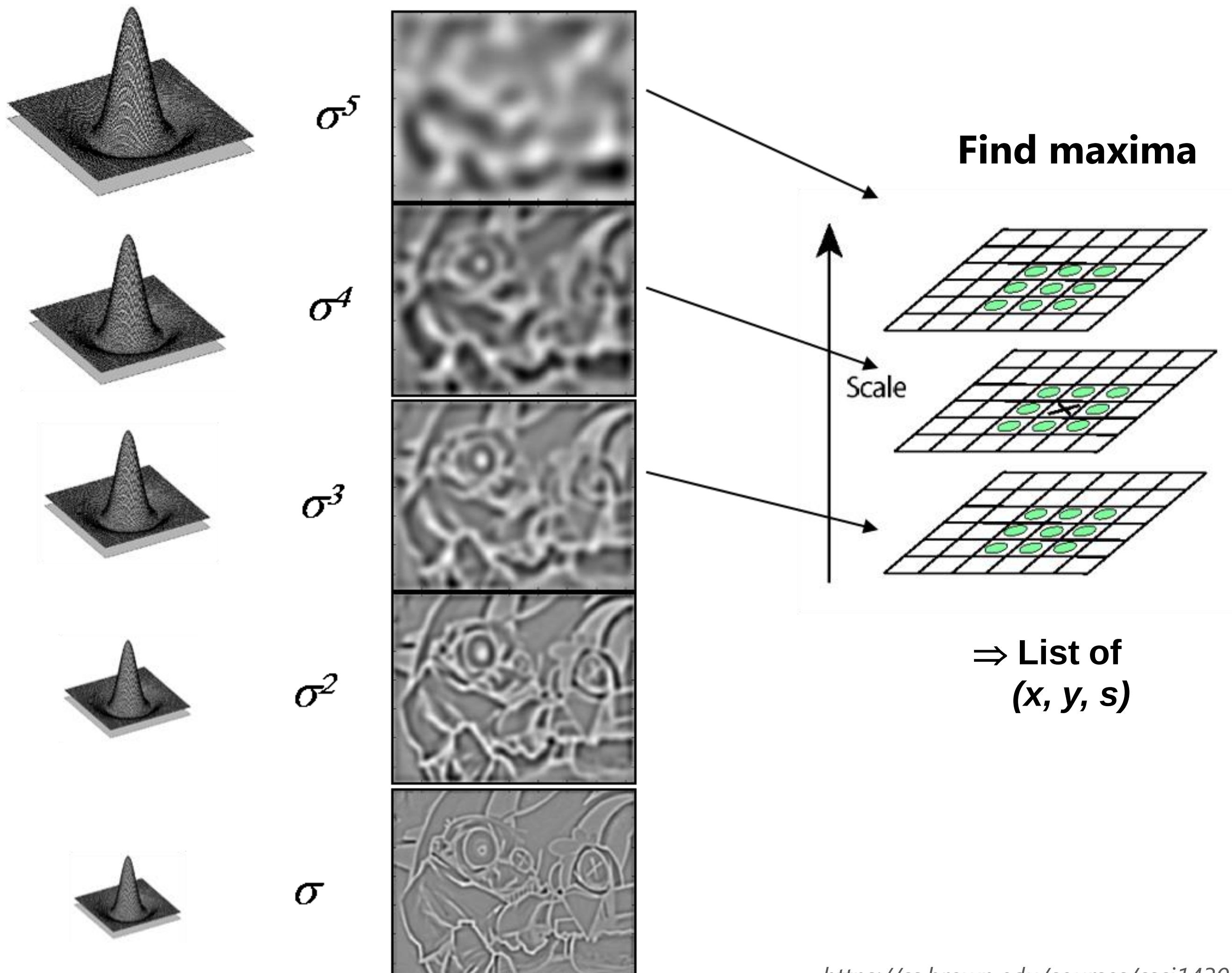
# Automatic Scale Selection

- Find local maxima in position-scale space of LoG

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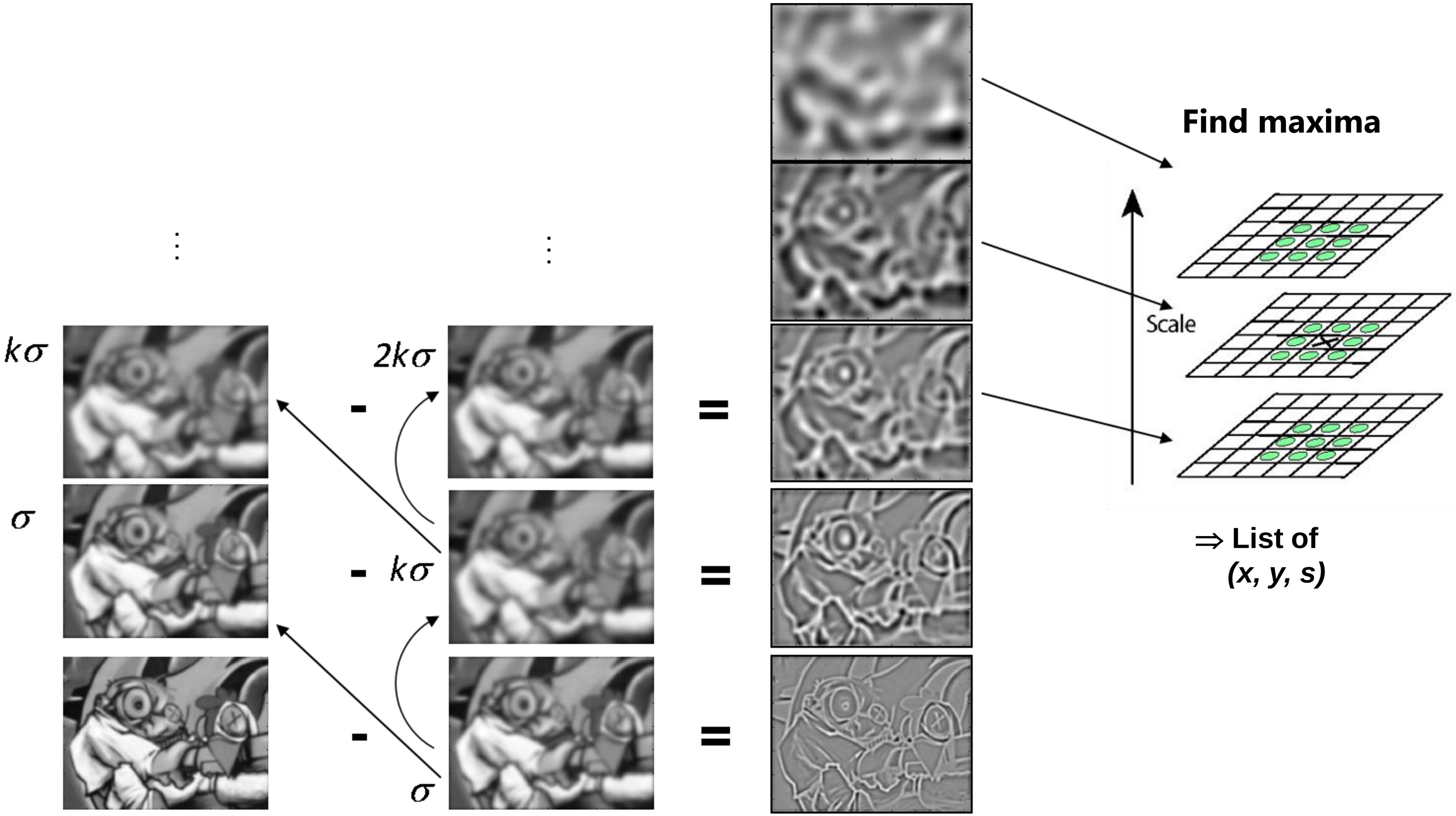
# Automatic Scale Selection

Approximate LoG with Difference-of-Gaussian (DoG).

1. Blur image with σ Gaussian kernel
2. Blur image with *k* σ Gaussian kernel
3. Subtract 2. from 1.

# Automatic Scale Selection

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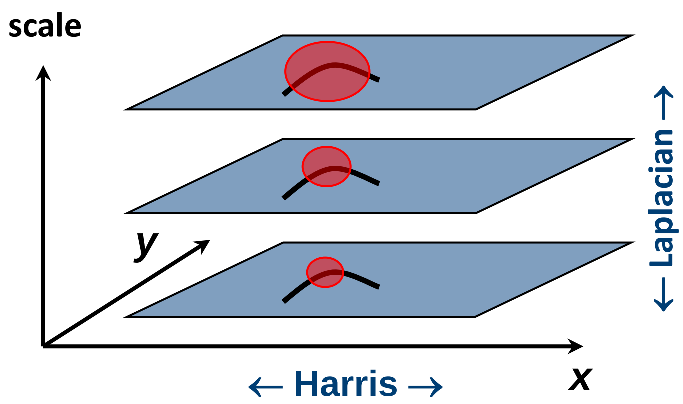
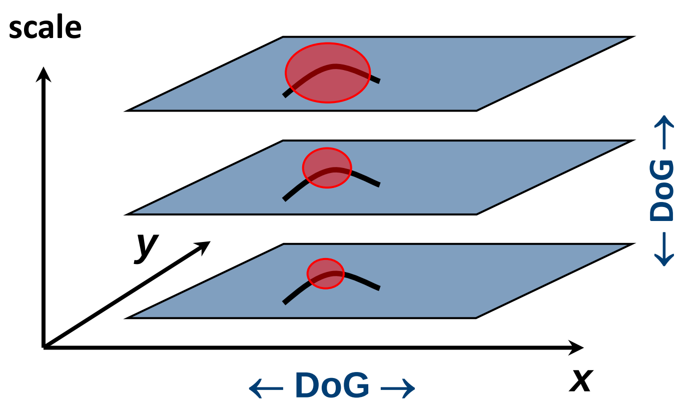
# Scale invariant detectors

- Harris-Laplace detector [1]
  - Find the optimal scale $\sigma$ using Laplacian response
  - Find the local maximum in space using Harris detector response




- SIFT [2]
  - Find the optimal scale $\sigma$ using DoG response
  - Find the local extremum in space using DoG response




*[1] K. Mikolajczyk et al. Scale & affine invariant interest point detectors, IJCV 2004.*
*[2] D. Lowe. Distinctive image features from scale-invariant keypoints, IJCV 2004.*

# Comparison of Keypoint Detectors

Table 7.1 Overview of feature detectors.

| Feature Detector | Corner | Blob | Region | Rotation invariant | Scale invariant | Affine invariant | Repeatability | Localization accuracy | Robustness | Efficiency |
|---|---|---|---|---|---|---|---|---|---|---|
| Harris | √ | | | √ | | | +++ | +++ | +++ | ++ |
| Hessian | | √ | | √ | | | ++ | ++ | ++ | + |
| SUSAN | √ | | | √ | | | ++ | ++ | ++ | +++ |
| Harris-Laplace | √ | (√) | | √ | √ | | +++ | +++ | ++ | + |
| Hessian-Laplace | (√) | √ | | √ | √ | | +++ | +++ | +++ | + |
| DoG | (√) | √ | | √ | √ | | ++ | ++ | ++ | ++ |
| SURF | (√) | √ | | √ | √ | | ++ | ++ | ++ | +++ |
| Harris-Affine | √ | (√) | | √ | √ | √ | +++ | +++ | ++ | ++ |
| Hessian-Affine | (√) | √ | | √ | √ | √ | +++ | +++ | +++ | ++ |
| Salient Regions | (√) | √ | | √ | √ | (√) | + | + | ++ | + |
| Edge-based | √ | | | √ | √ | √ | +++ | +++ | + | + |
| MSER | | | √ | √ | √ | √ | +++ | +++ | ++ | +++ |
| Intensity-based | | | √ | √ | √ | √ | ++ | ++ | ++ | ++ |
| Superpixels | | | √ | √ | (√) | (√) | + | + | + | + |

*Tuytelaars, T and Mikolajczyk, K (2008) Local invariant feature detectors: A survey Foundations and Trends in Computer Graphics and Vision, 3 (3). pp. 177-280.*

# Conclusions

- Feature Detection
- Interest Points in Vision
- Corner Detection and Harris Corner Detector
- Shi-Tomasi Corner Detector
- Automatic Scale Selection