

Interactive Computer Graphics: Lecture 10

Ray tracing



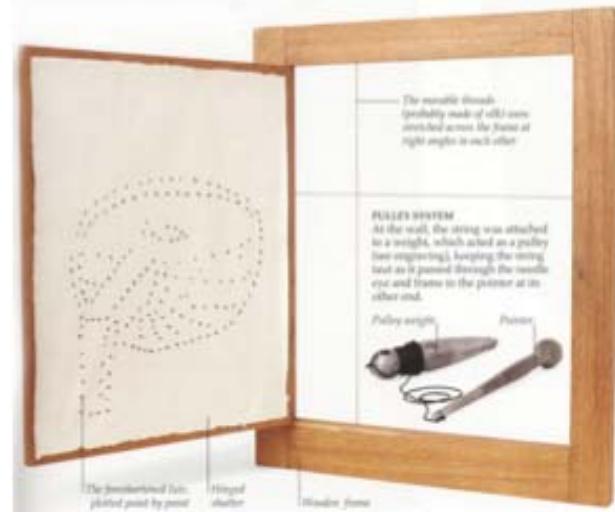
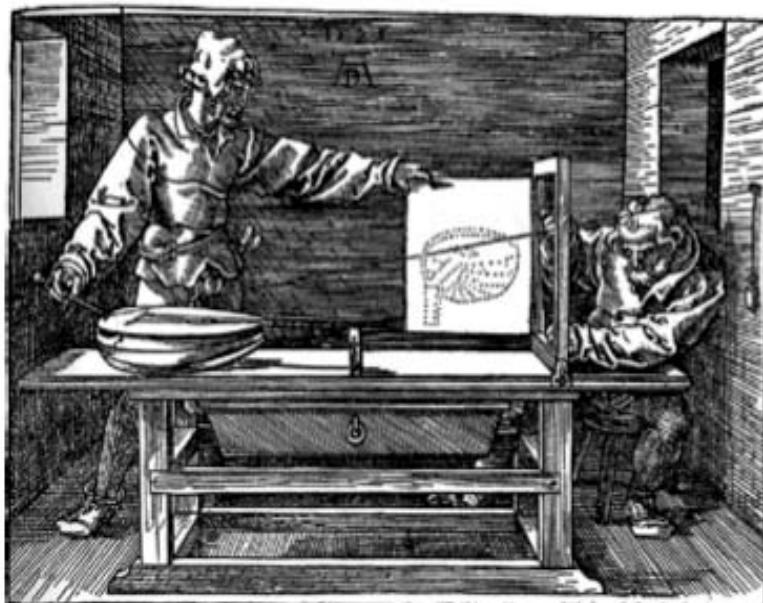


Direct and Global Illumination

- **Direct illumination**: A surface point receives light directly from all light sources in the scene.
 - Computed by the direct illumination model.
- **Global illumination**: A surface point receives light after the light rays interact with other objects in the scene.
 - Points may be in shadow.
 - Rays may refract through transparent material.
 - Computed by reflection and transmission rays.

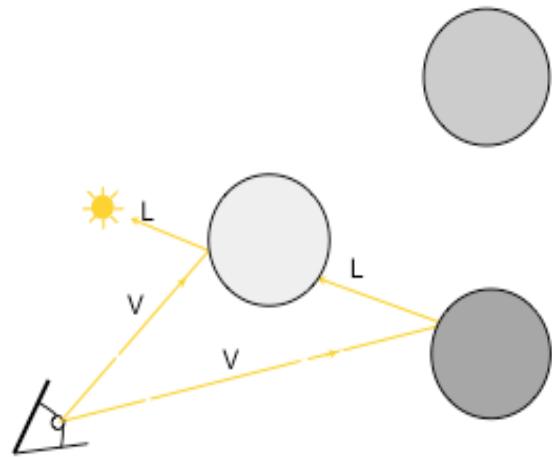
Albrecht Dürer's Ray Casting Machine

- Albrecht Dürer, 16th century



Arthur Appel, 1968

- On calculating the illusion of reality, 1968
- Cast one ray per pixel (ray casting).
 - For each intersection, trace one ray to the light to check for shadows
 - Only a local illumination model
- Developed for pen-plotters



Ray casting

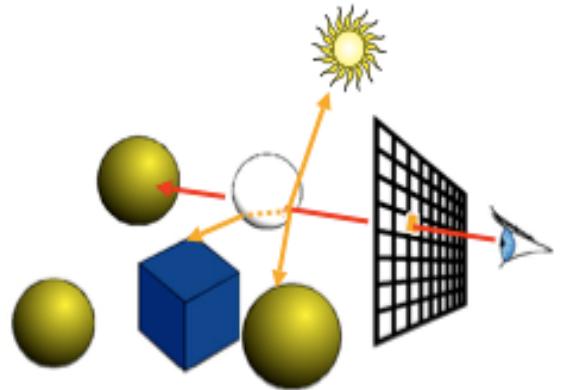
cast ray

Intersect all objects

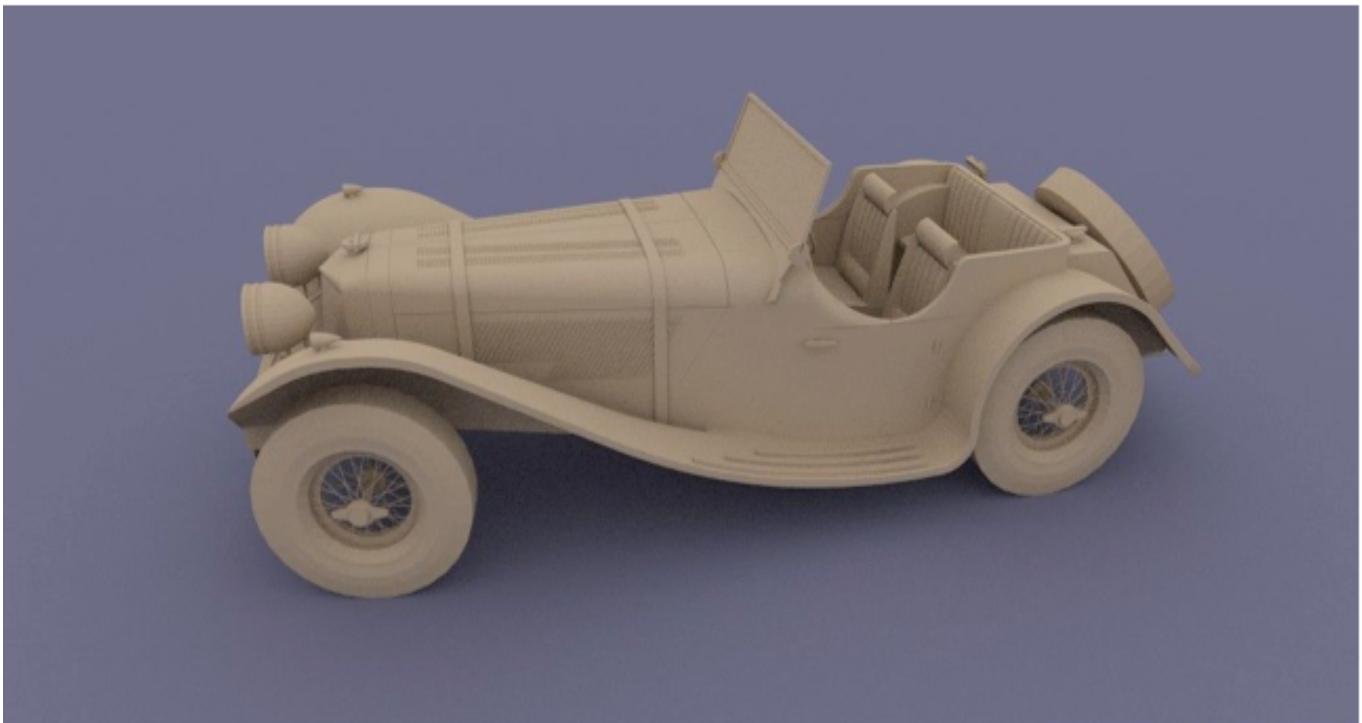
color = ambient term

For every light cast shadow ray

col += local shading term

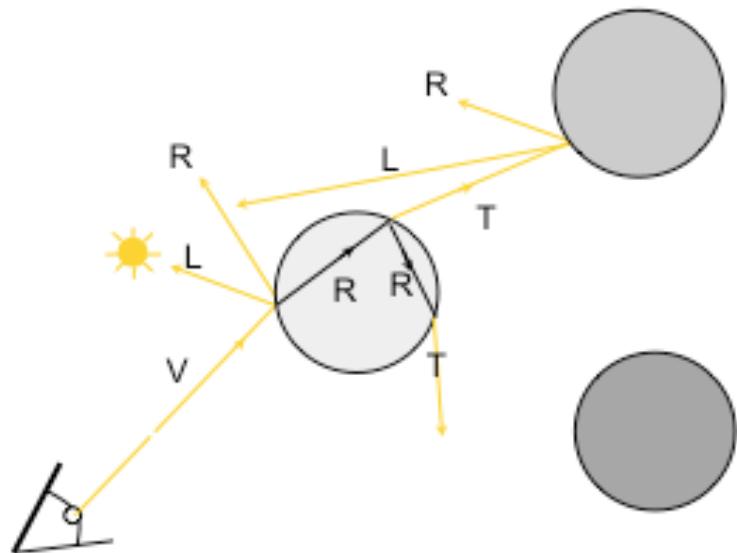


Ray casting

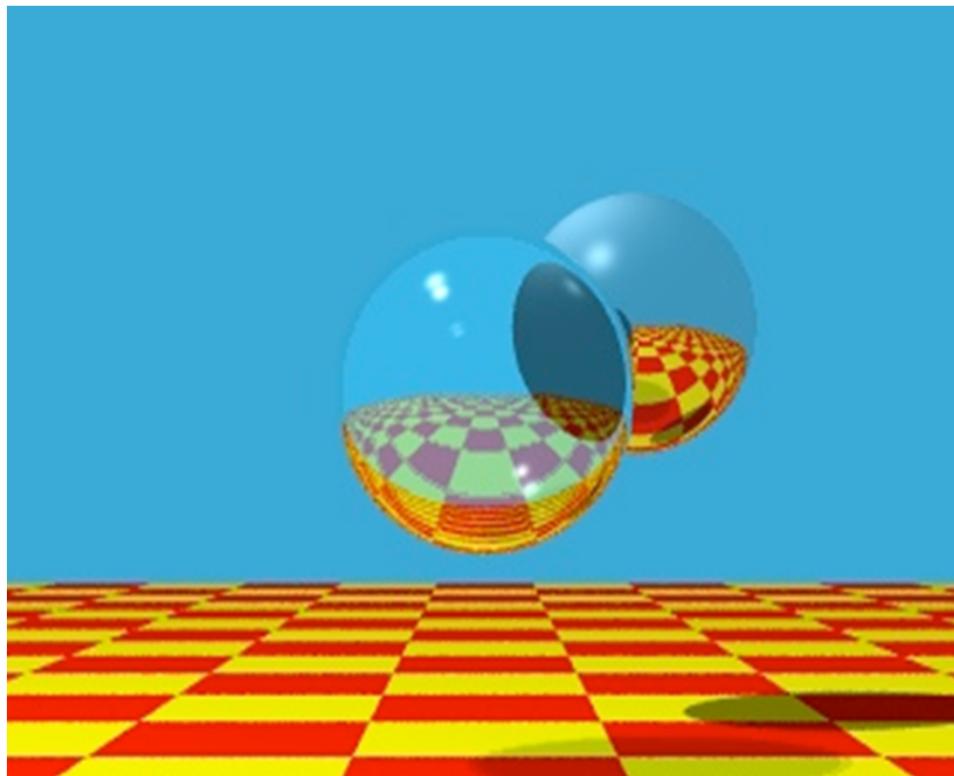


Turner Whitted, 1980

- An Improved Illumination Model for Shaded Display, 1980
- First global illumination model:
 - An object's color is influenced by lights and other objects in the scene
 - Simulates specular reflection and refractive transmission



Turner Whitted, 1980



Recursive ray casting

```
trace ray
```

```
    Intersect all objects
```

```
    color = ambient term
```

```
    For every light
```

```
        cast shadow ray
```

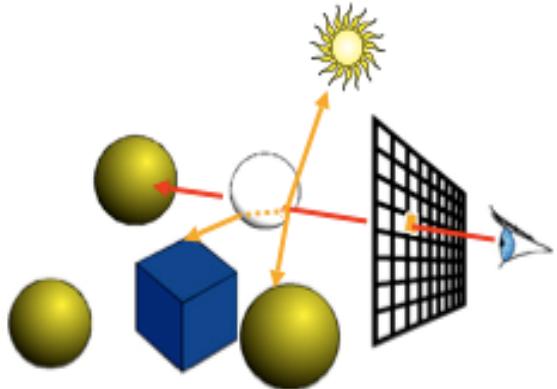
```
        col += local shading term
```

```
    If mirror
```

```
        col += k_refl * trace reflected ray
```

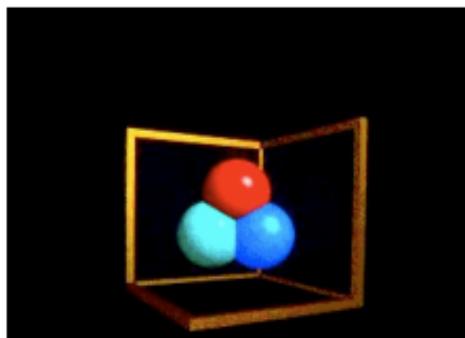
```
    If transparent
```

```
        col += k_trans * trace transmitted ray
```



Does it ever end?

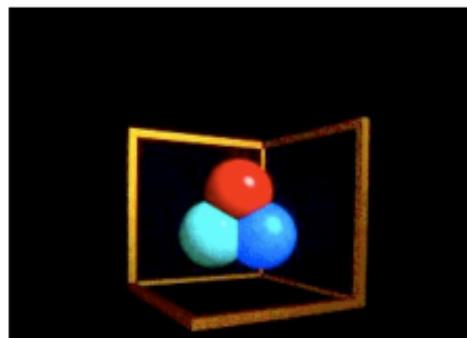
- Stopping criteria:
 - Recursion depth: Stop after a number of bounces
 - Ray contribution: Stop if reflected / transmitted contribution becomes too small



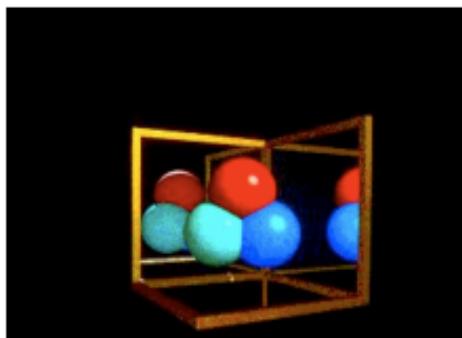
0 recursion

Does it ever end?

- Stopping criteria:
 - Recursion depth: Stop after a number of bounces
 - Ray contribution: Stop if reflected / transmitted contribution becomes too small



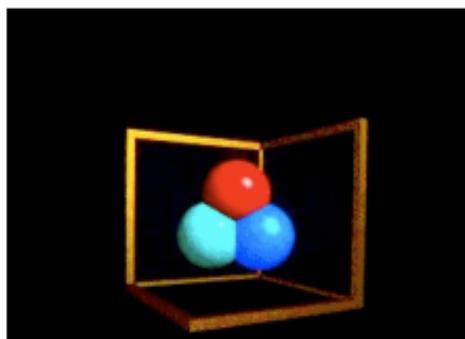
0 recursion



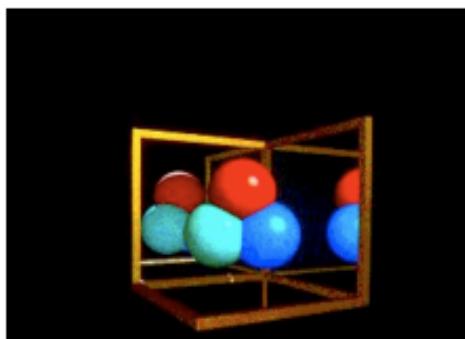
1 recursion

Does it ever end?

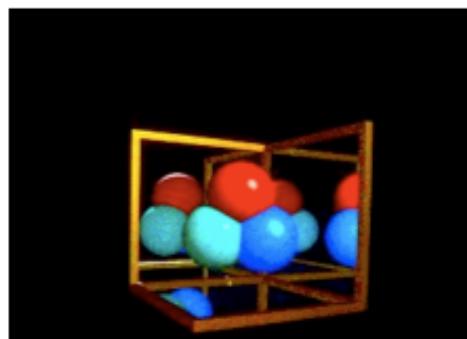
- Stopping criteria:
 - Recursion depth: Stop after a number of bounces
 - Ray contribution: Stop if reflected / transmitted contribution becomes too small



0 recursion



1 recursion



2 recursions

Ray tracing: Primary rays

- For each ray we need to test which objects are intersecting the ray:
 - If the object has an intersection with the ray we calculate the distance between viewpoint and intersection
 - If the ray has more than one intersection, the smallest distance identifies the visible surface.
- Primary rays are rays from the view point to the nearest intersection point
- Local illumination is computed as before:

$$L = k_a + (k_d(\mathbf{n} \cdot \mathbf{l}) + k_s(\mathbf{v} \cdot \mathbf{r})^q)I_s$$

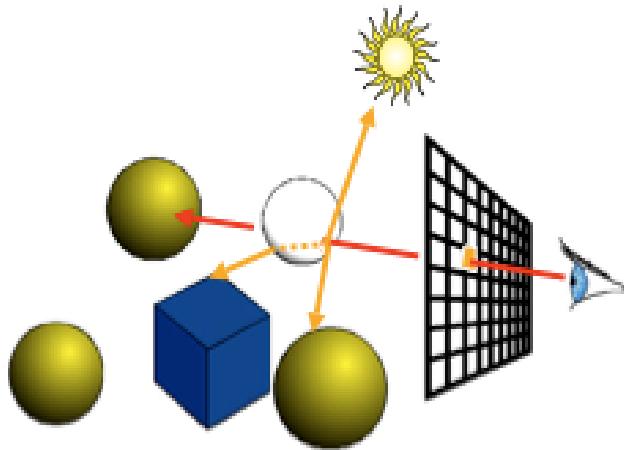
Ray tracing: Secondary rays

- Secondary rays are rays originating at the intersection points
- Secondary rays are caused by
 - rays reflected off the intersection point in the direction of reflection
 - rays transmitted through transparent materials in the direction of refraction
 - shadow rays

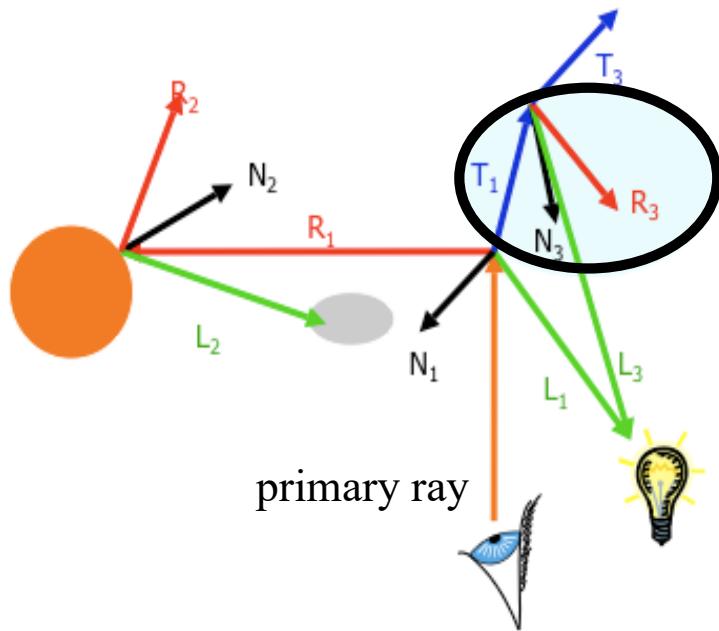
Recursive ray tracing: Putting it all together

- Illumination can be expressed as

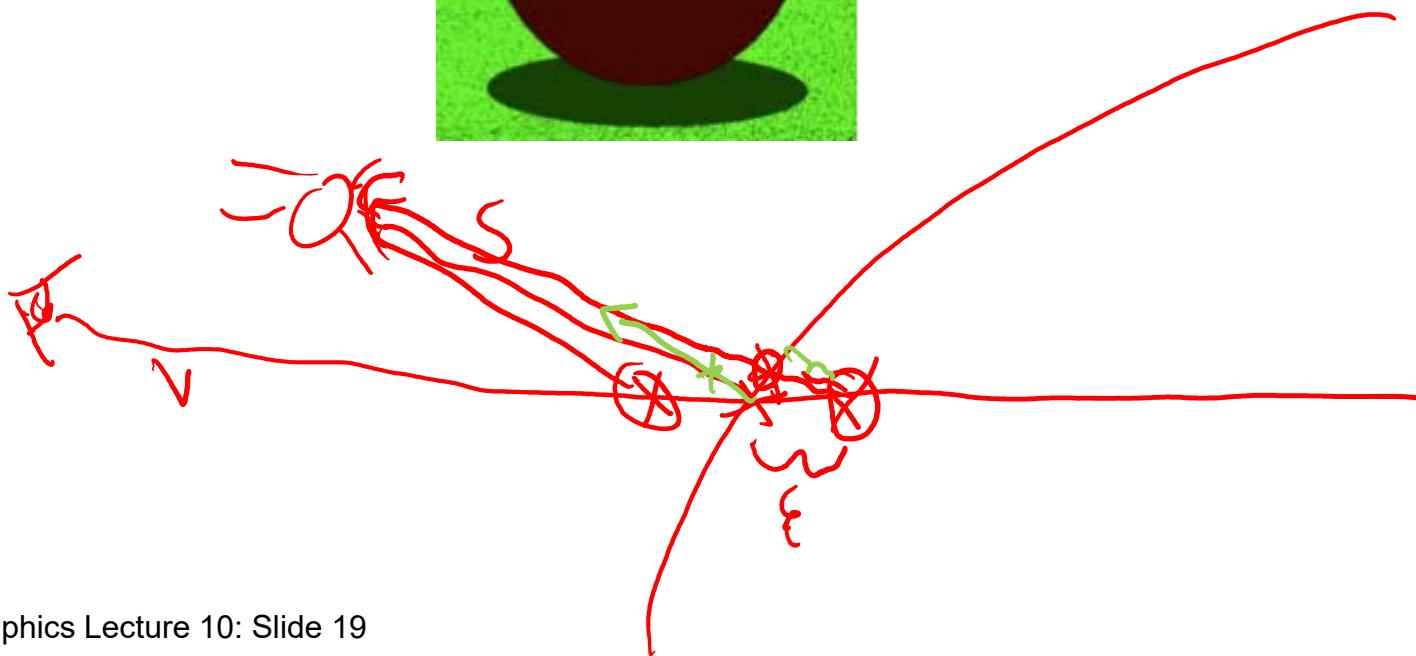
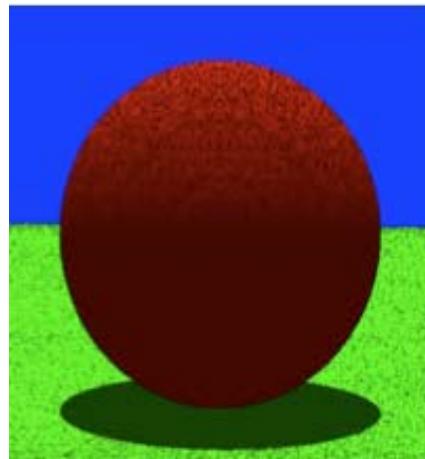
$$L = k_a + (k_d(\mathbf{n} \cdot \mathbf{l}) + k_s(\mathbf{v} \cdot \mathbf{r})^q)I_s + k_{reflected}L_{reflected} + k_{refracted}L_{refracted}$$



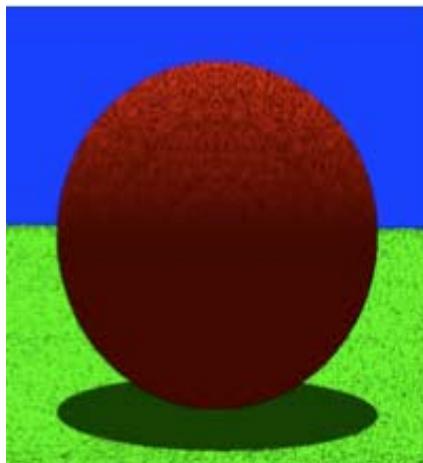
Recursive Ray Tracing: Ray Tree



Precision Problems



Precision Problems



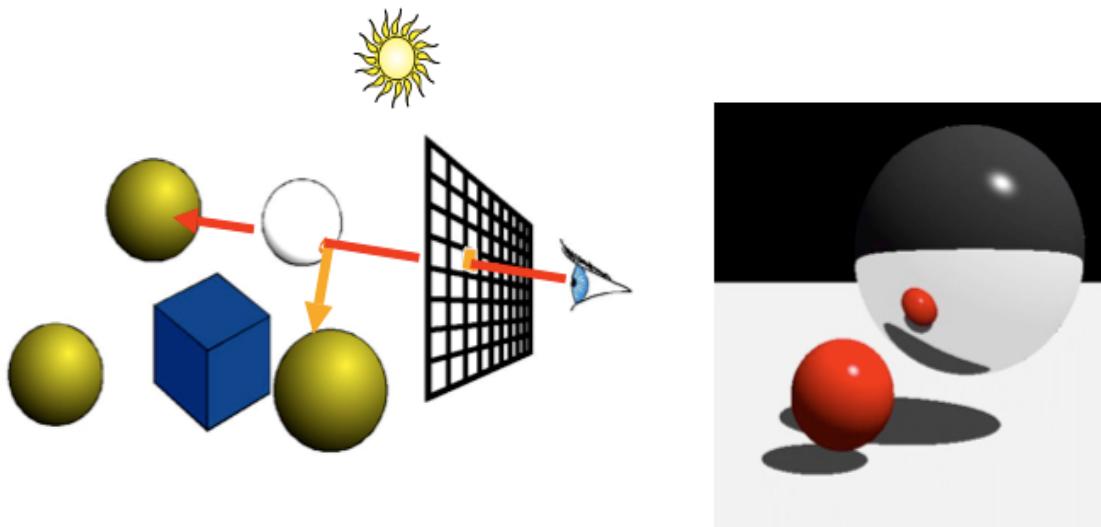
- In ray tracing, the origin of (secondary) rays is often below the surface of objects
 - Theoretically, the intersection point should be on the surface
 - Practically, calculation imprecision creeps in, and the origin of the new ray is slightly beneath the surface
- Result: the surface area is shadowing itself or the ray continues on the wrong side

ε to the rescue ...

- Check if t is within some epsilon tolerance:
 - if $\text{abs}(\mu) < \varepsilon$
 - point is on the surface
 - else
 - point is inside/outside
 - Choose the ε tolerance empirically
- Move the intersection point by epsilon along the surface normal so it is outside of the object
- Check if point is inside/outside surface by checking the sign of the implicit (sphere etc.) equation

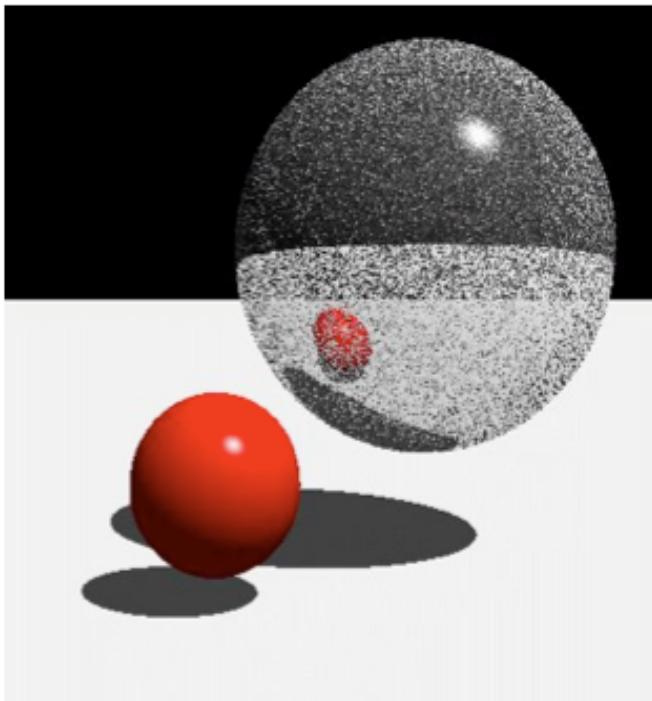
Mirror reflection

- Compute mirror contribution
- Cast ray in direction symmetric wrt. normal
- Multiply by reflection coefficient (color)

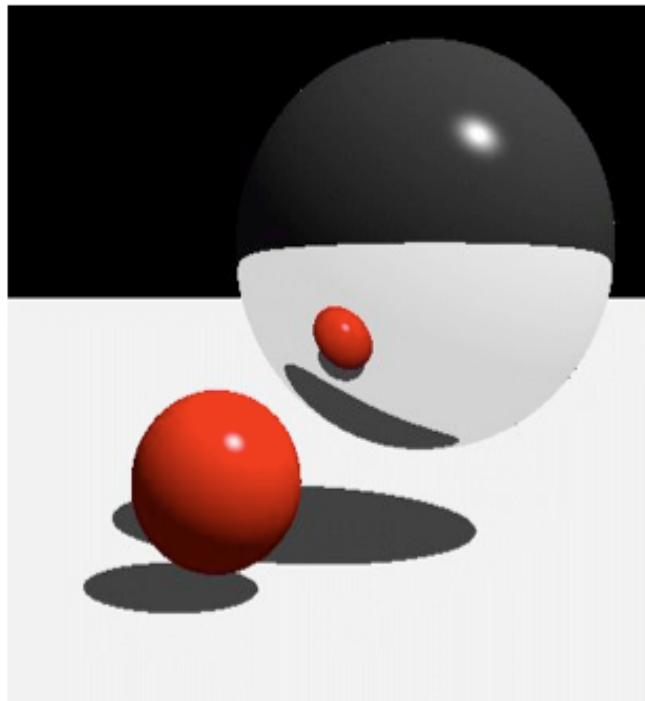


Mirror reflection

- Don't forget to add epsilon to the ray

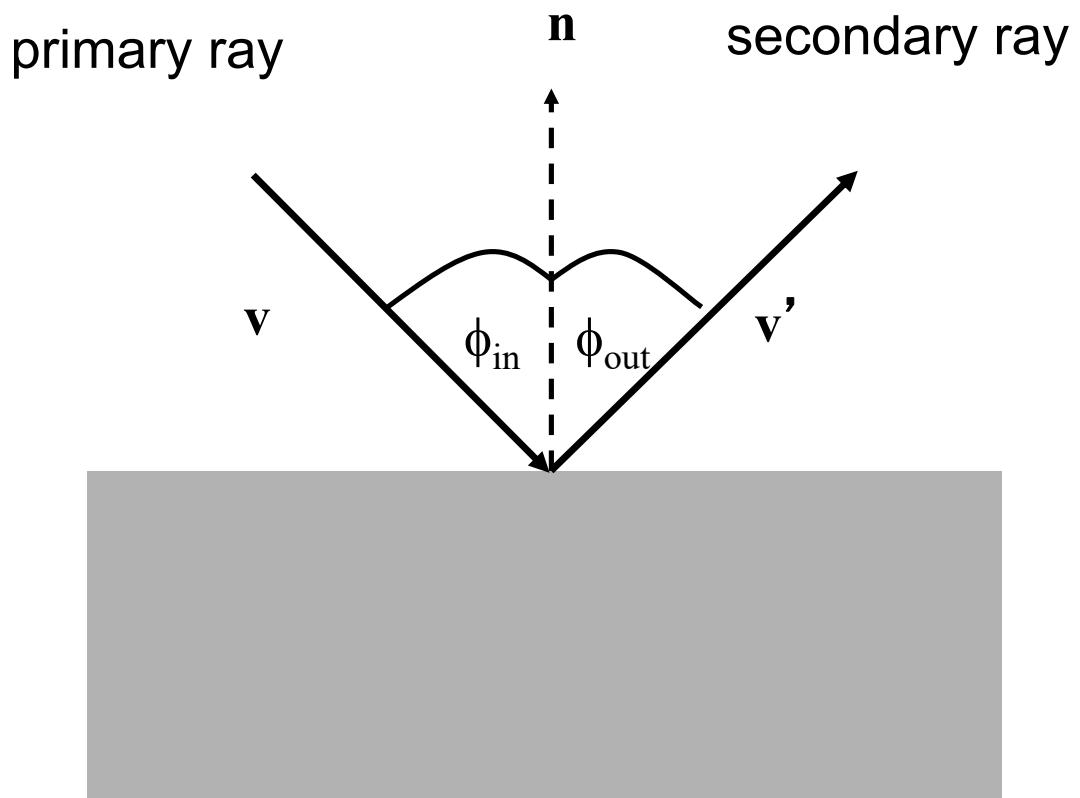


Without epsilon



With epsilon

Mirror reflection



Mirror reflection

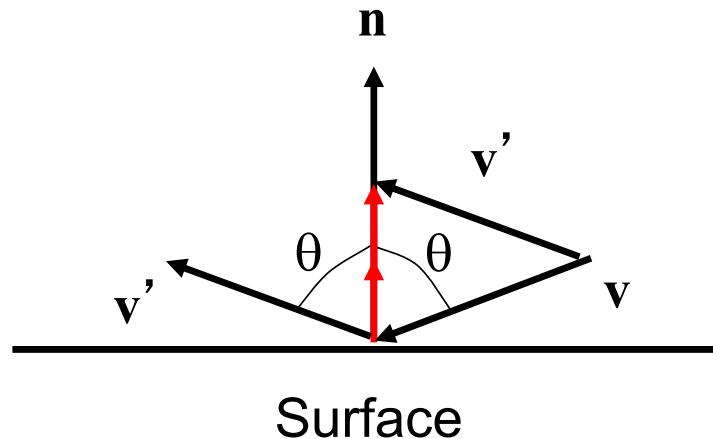
- To calculate illumination as a result of reflections
 - calculate the direction of the secondary ray at the intersection of the primary ray with the object.

given that

- \mathbf{n} is the unit surface normal
- \mathbf{v} is the direction of the primary ray
- \mathbf{v}' is the direction of the secondary ray as a result of reflections

$$\mathbf{v}' = \mathbf{v} - (2\mathbf{v} \cdot \mathbf{n})\mathbf{n}$$

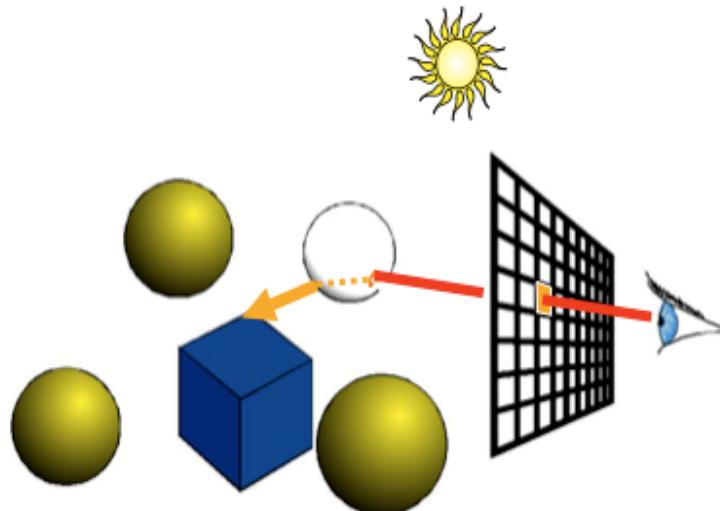
Mirror reflection



$$\mathbf{v}' = \mathbf{v} - (2\mathbf{v} \cdot \mathbf{n})\mathbf{n}$$

Transparency

- Compute transmitted contribution
- Cast ray in refracted direction
- Multiply by transparency coefficient

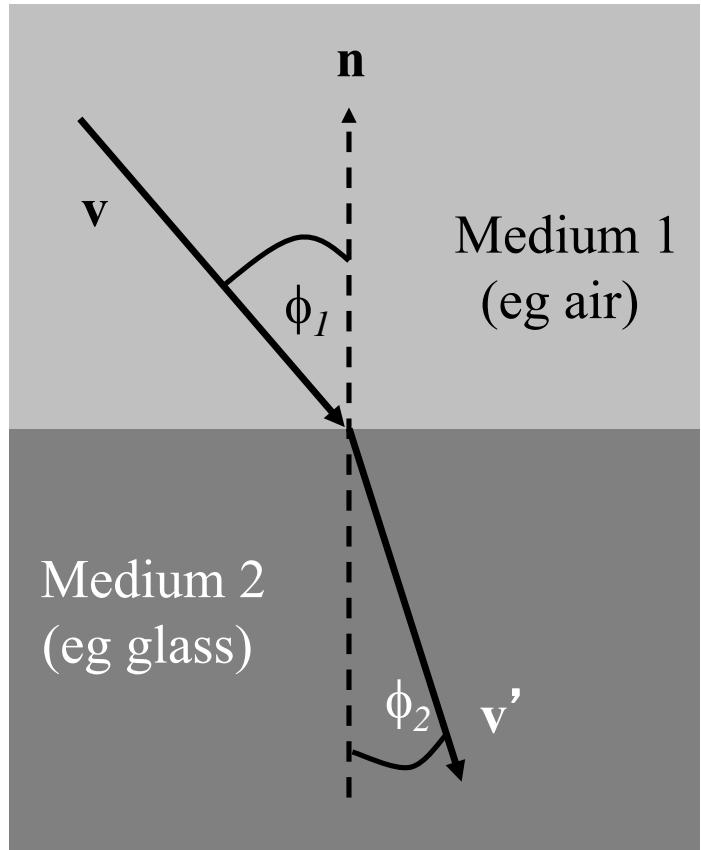


Refraction

- The angle of the refracted ray can be determined by Snell's law:

$$\eta_1 \sin(\phi_1) = \eta_2 \sin(\phi_2)$$

- η_1 is a constant for medium 1
- η_2 is a constant for medium 2
- ϕ_1 is the angle between the incident ray and the surface normal
- ϕ_2 is the angle between the refracted ray and the surface normal



Refraction

- In vector notation Snell's law can be written:

$$k_1(v \cdot n) = k_2(v' \cdot n)$$

- The direction of the refracted ray is

$$\mathbf{v}' = \frac{\eta_1}{\eta_2} \left(\sqrt{\left(\mathbf{n} \cdot \mathbf{v}\right)^2 + \left(\frac{\eta_2}{\eta_1}\right)^2} - 1 - \mathbf{n} \cdot \mathbf{v} \right] \cdot \mathbf{n} + \mathbf{v} \right)$$

Refraction

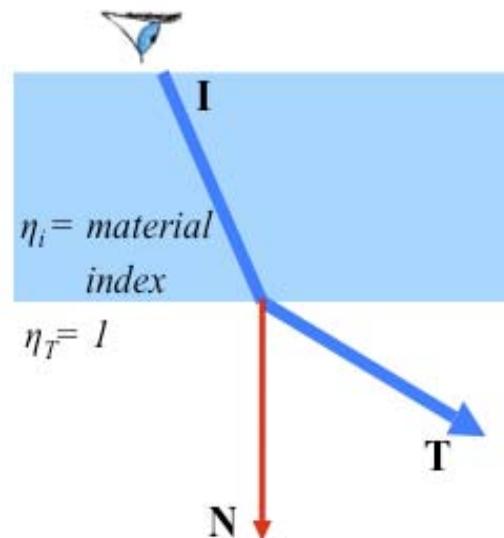
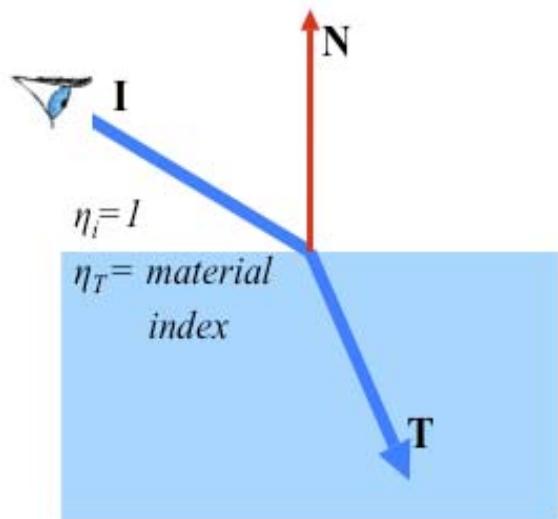
- This equation only has a solution if

$$(\mathbf{n} \cdot \mathbf{v})^2 > 1 - \left(\frac{\eta_2}{\eta_1} \right)^2$$

- This illustrates the physical phenomenon of the limiting angle:
 - if light passes from one medium to another medium whose index of refraction is low, the angle of the refracted ray is greater than the angle of the incident ray
 - if the angle of the incident ray is large, the angle of the refracted ray is larger than 90°
→ the ray is reflected rather than refracted

Refraction

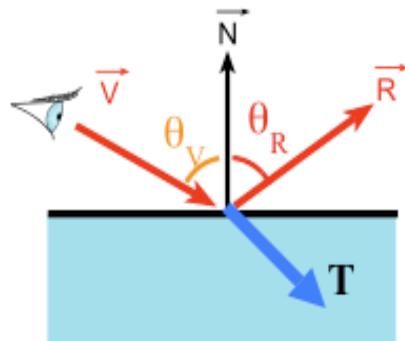
- Make sure you know whether you are entering or leaving the transmissive material



Amount of reflection and refraction

- Traditional (hacky) ray tracing
 - Constant coefficient reflection
 - Component per component multiplication
- Better: Mix reflected and refracted light according to the Fresnel factor.

$$L = k_{fresnel} L_{reflected} + (1 - k_{fresnel}) L_{refracted}$$

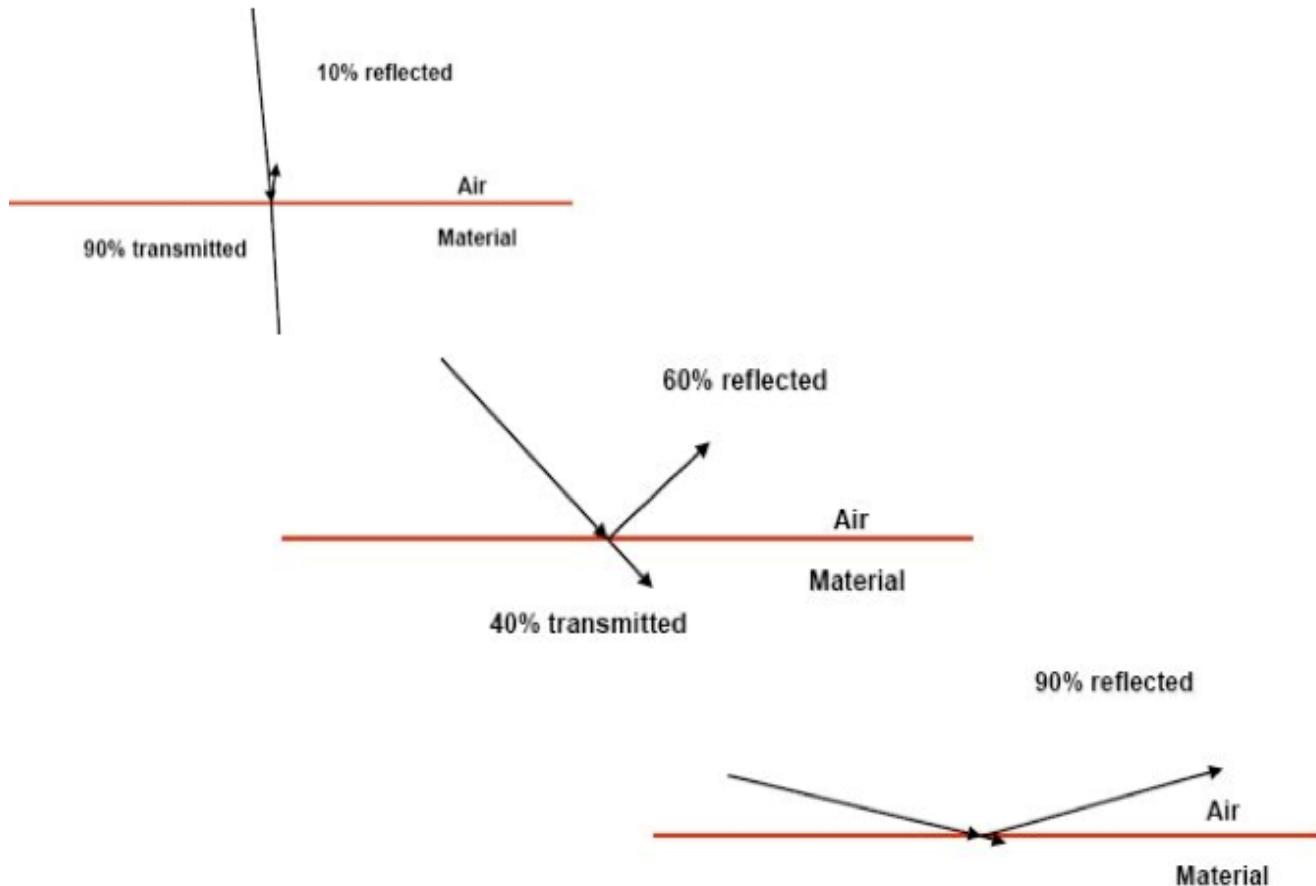


Fresnel factor

- More reflection at grazing angle



Fresnel factor



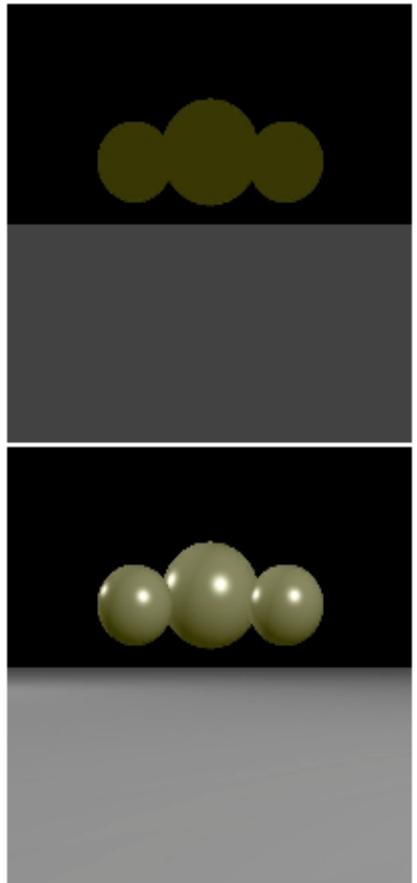
Schlick's Approximation

- Schlick' s approximation

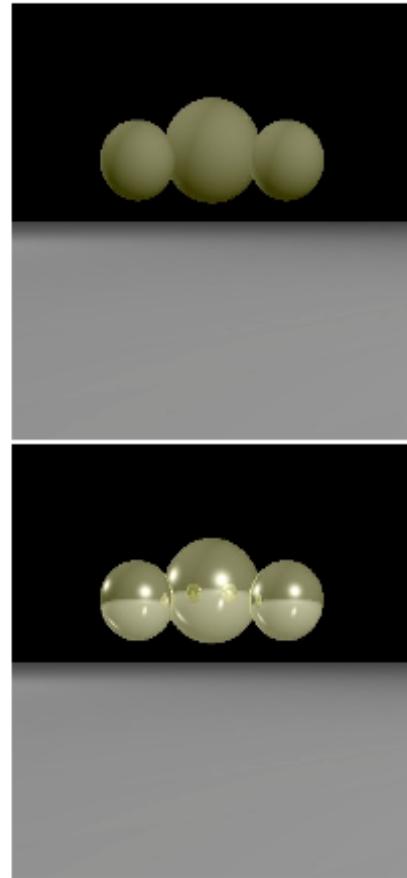
$$k_{fresnel}(\theta) = k_{fresnel}(0) + (1 - k_{fresnel}(0))(1 - (\mathbf{n} \cdot \mathbf{l}))^5$$

- $k_{fresnel}(0)$ = Fresnel factor at zero degrees
- Choose $k_{fresnel}(0) = 0.8$, this will look like stainless steel
- Fresnel factor at zero degrees has low value ~ 0.05 for dielectric materials like plastic.

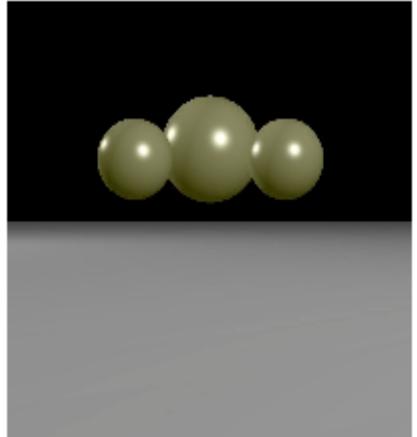
Example



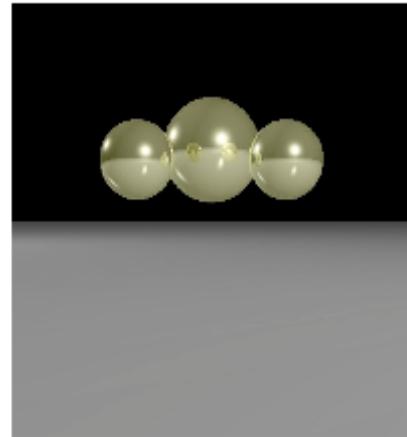
Ambient



+ Diffuse

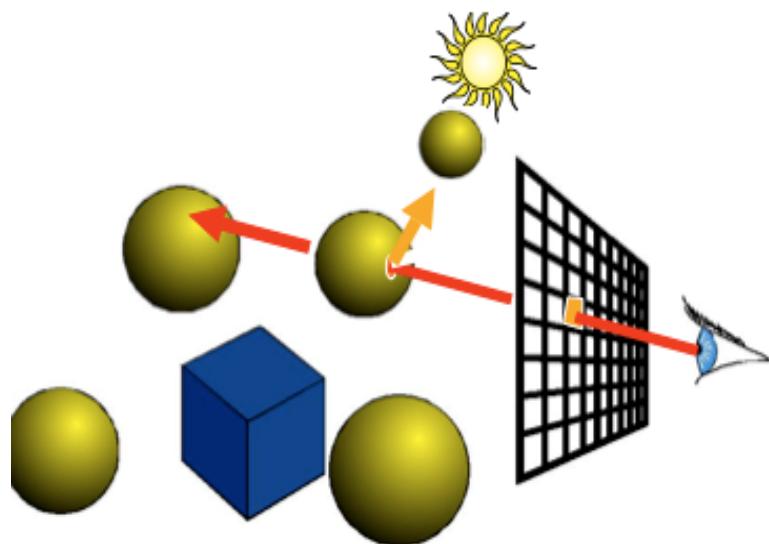


+ Specular

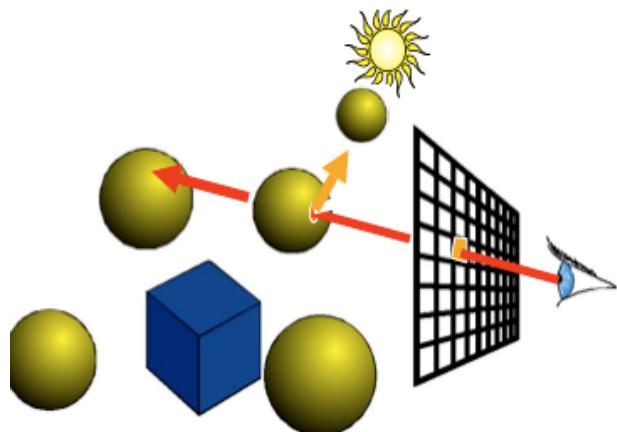


+ $I_{\text{reflected}}$

How do we add shadows?



How do we add shadows?

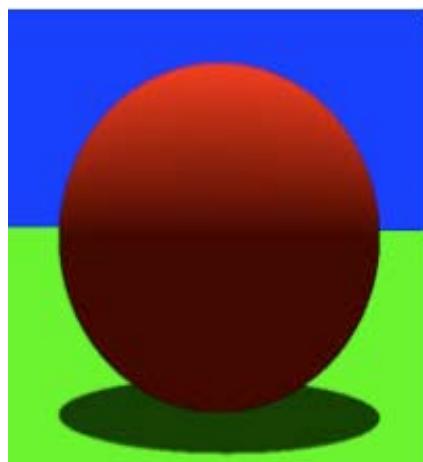
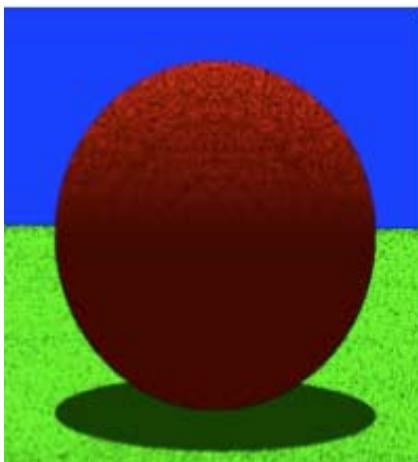


$$L = k_a + s(k_d(\mathbf{n} \cdot \mathbf{l}) + k_s(\mathbf{v} \cdot \mathbf{r})^q)I_s + k_{reflected}L_{reflected} + k_{refracted}L_{refracted}$$

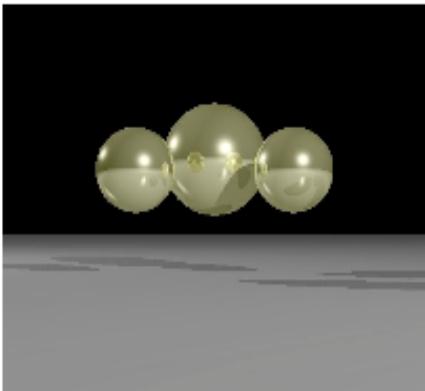
$$s = \begin{cases} 0 & \text{if light source is obscured} \\ 1 & \text{if light source is not obscured} \end{cases}$$

Shadows: Problems?

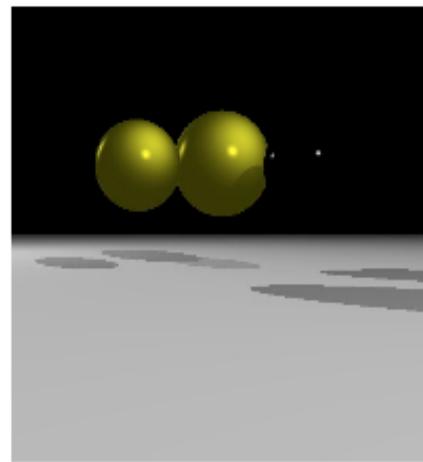
- Make sure to avoid self-shadowing



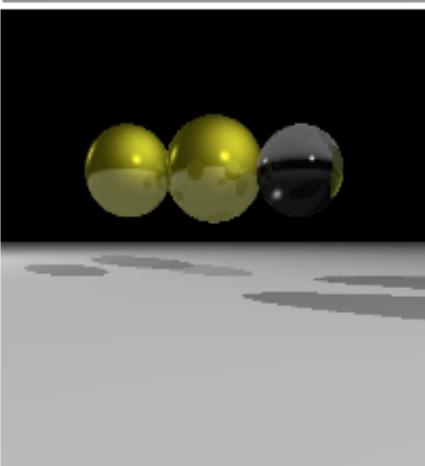
Example



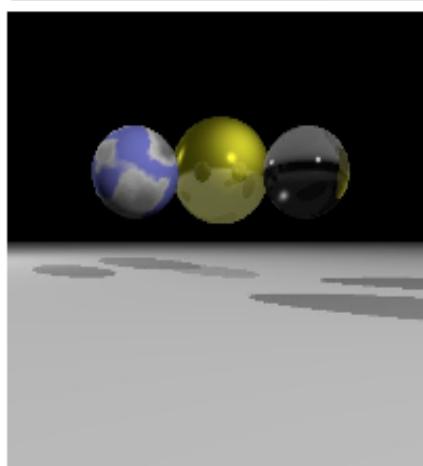
+ Shadows



- $I_{\text{reflected}}$
+ transmitted



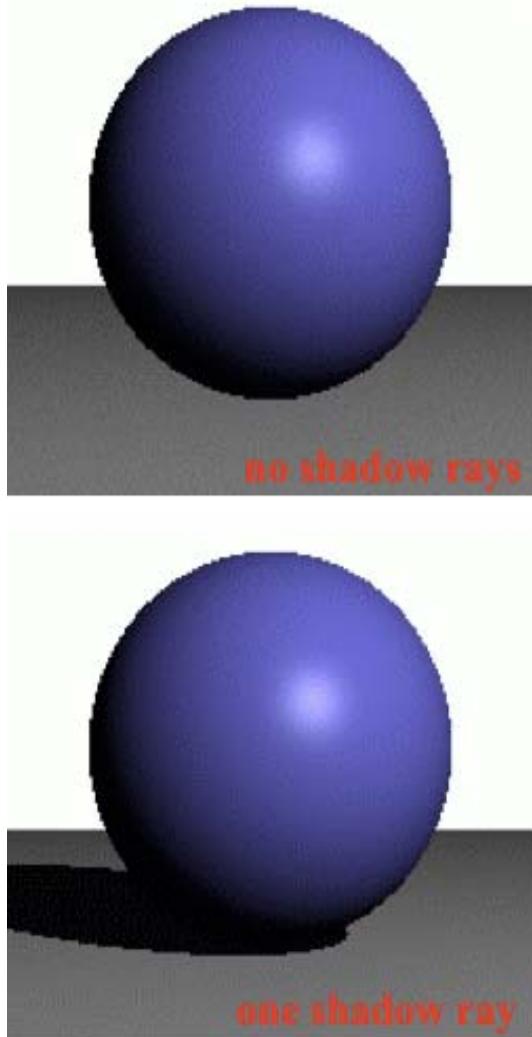
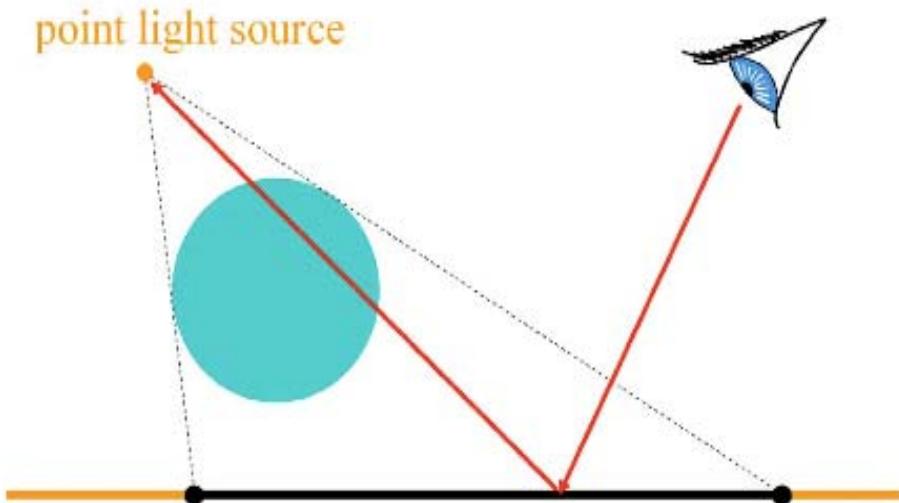
+ $I_{\text{reflected}}$



+ textures

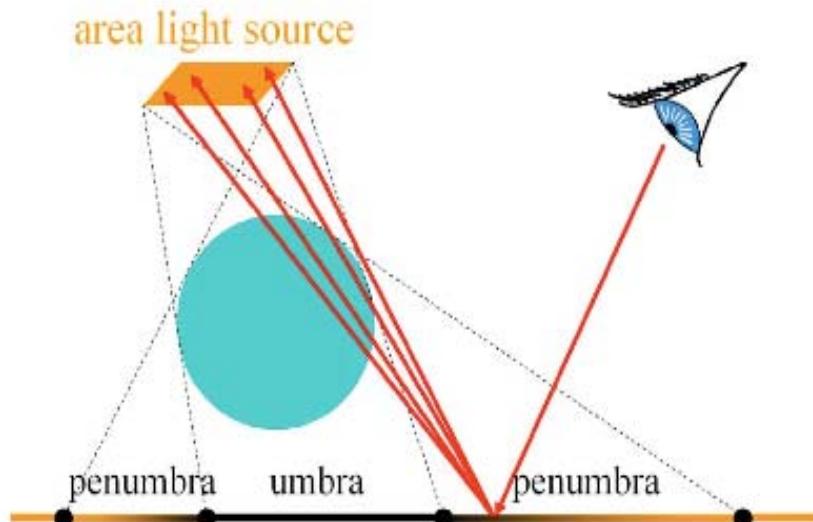
Shadows

- One shadow ray per intersection per point light source

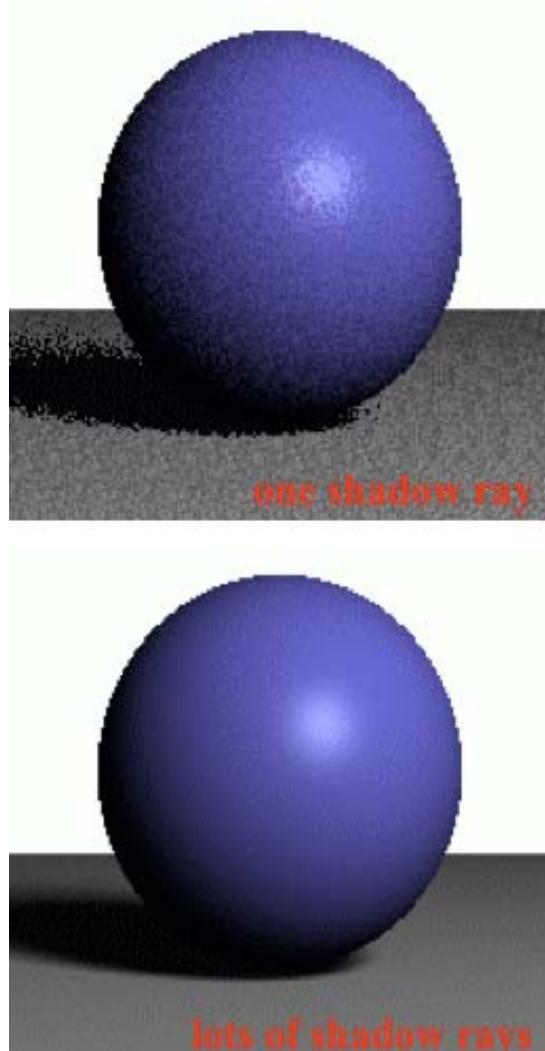


Soft shadows

- Multiple shadow rays to sample area light source

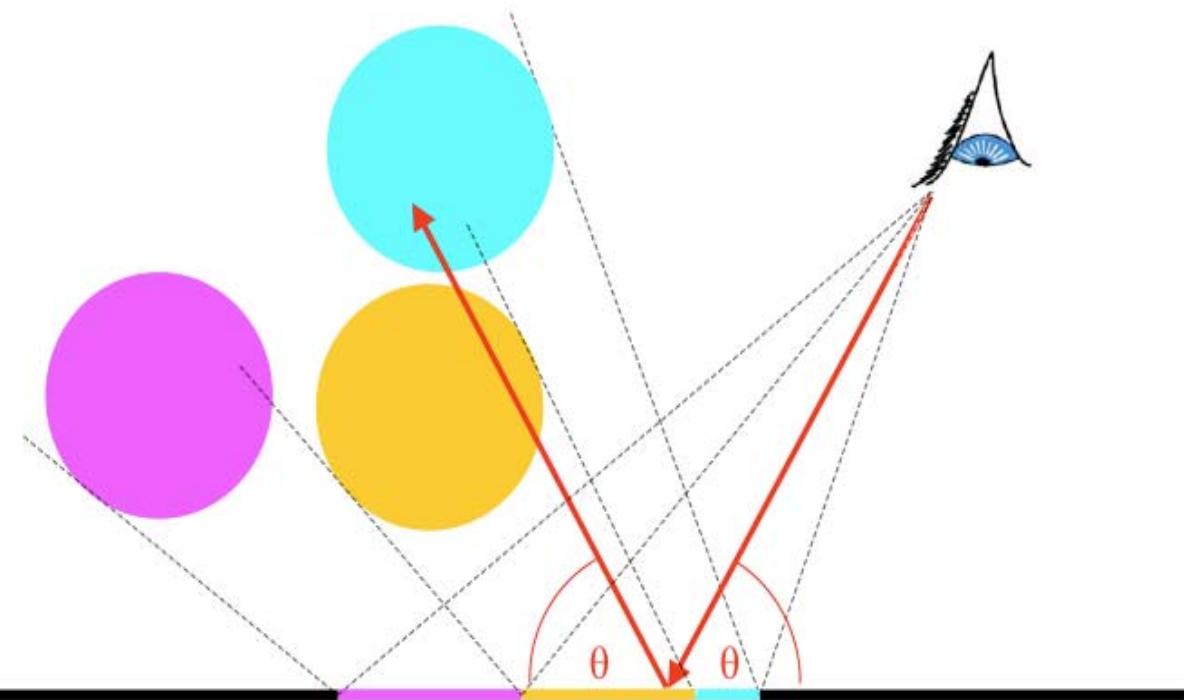


Graphics Lecture 10: Slide 43



Reflection: Conventional ray tracing

- One reflection per intersection



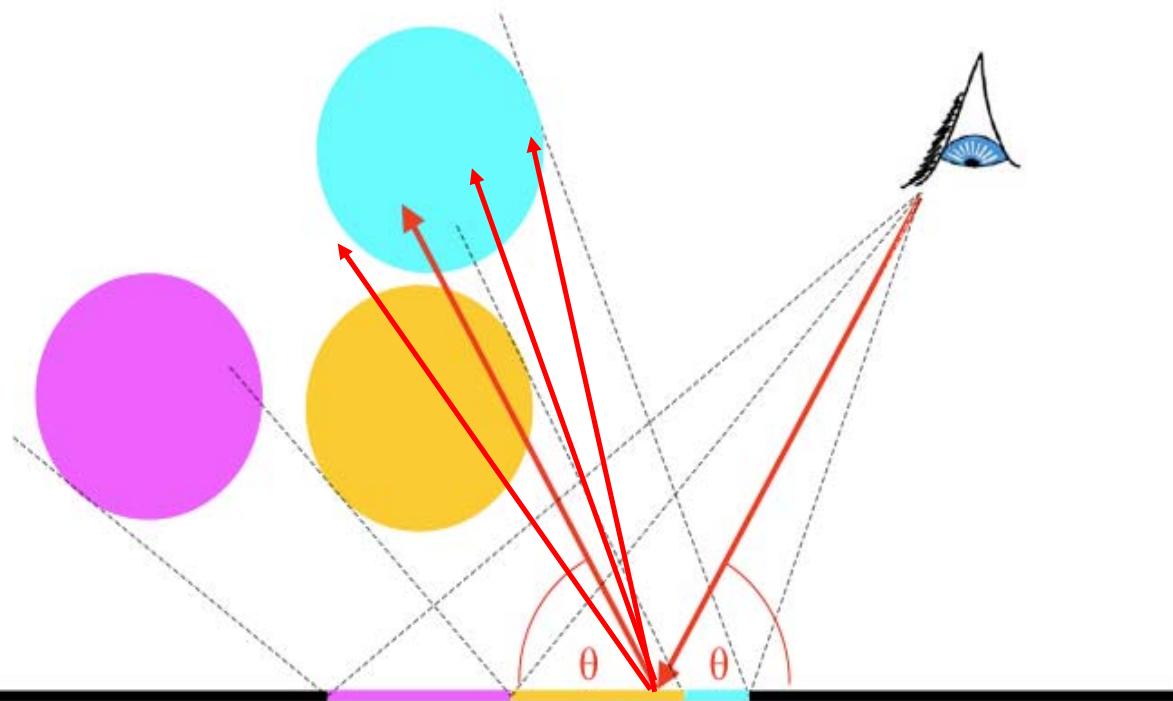
Reflection: Conventional ray tracing

- How can we create effects like this?

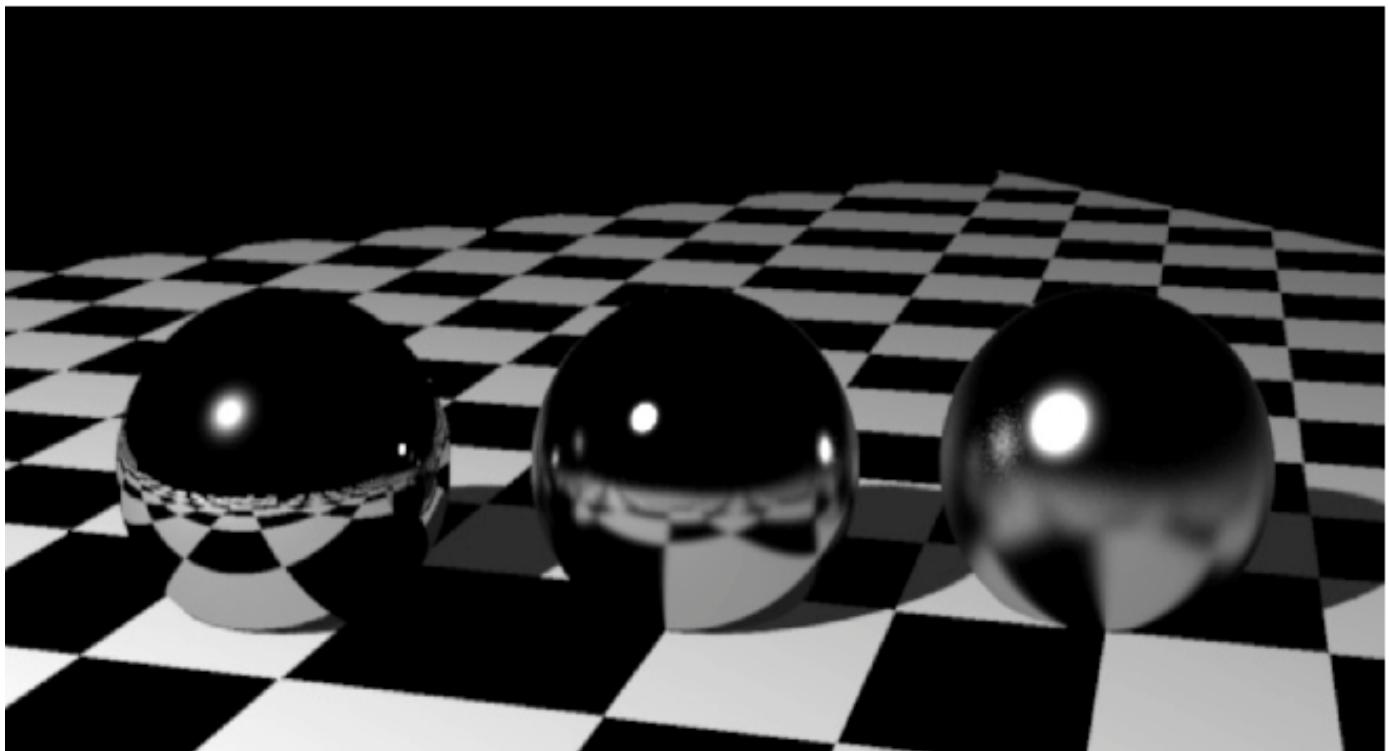


Reflection: Monte Carlo ray tracing

- Random reflection rays around mirror direction

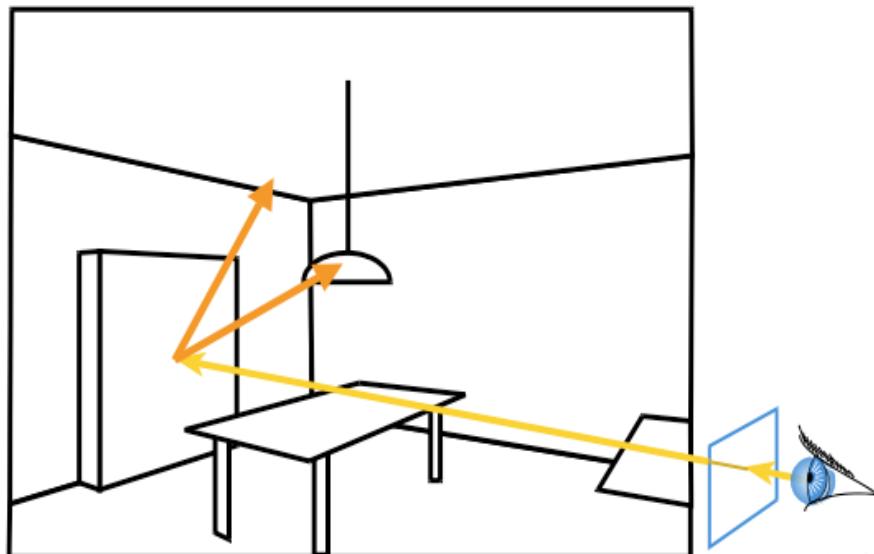


Glossy surfaces



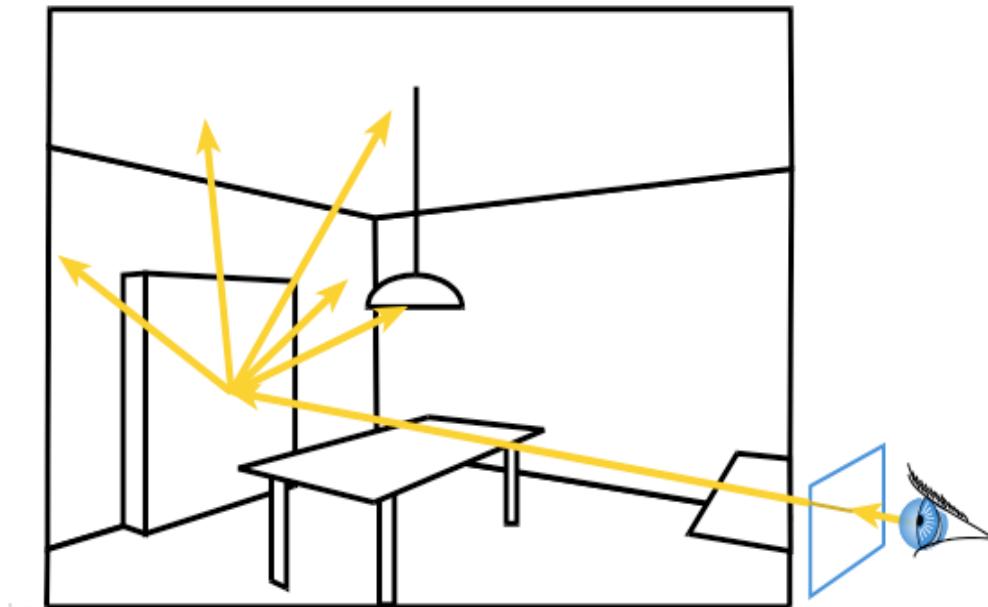
Ray tracing

- Cast a ray from the eye through each pixel
- Trace secondary rays (light, reflection, refraction)



Monte-Carlo Ray Tracing

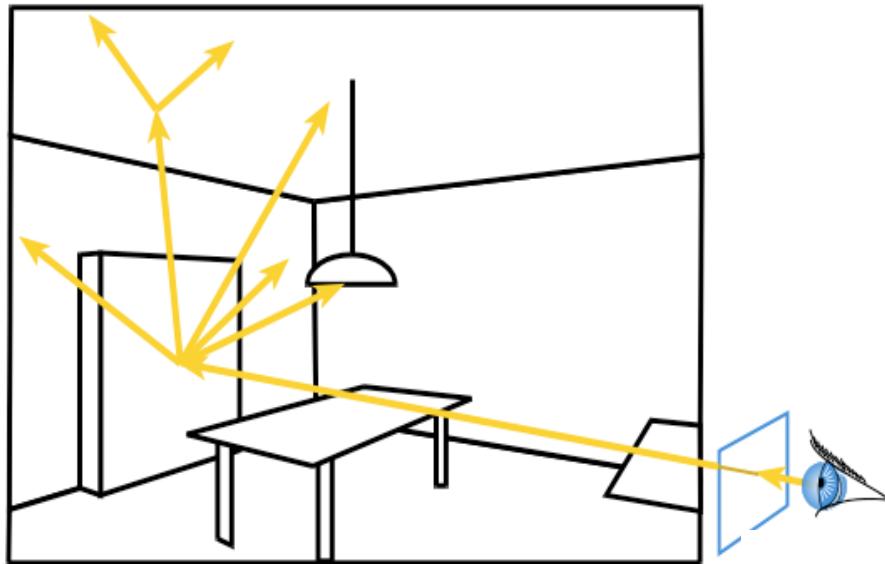
- Cast a ray from the eye through each pixel
- Cast random rays from the visible point
 - Accumulate radiance contribution



Graphics Lecture 10: Slide 49

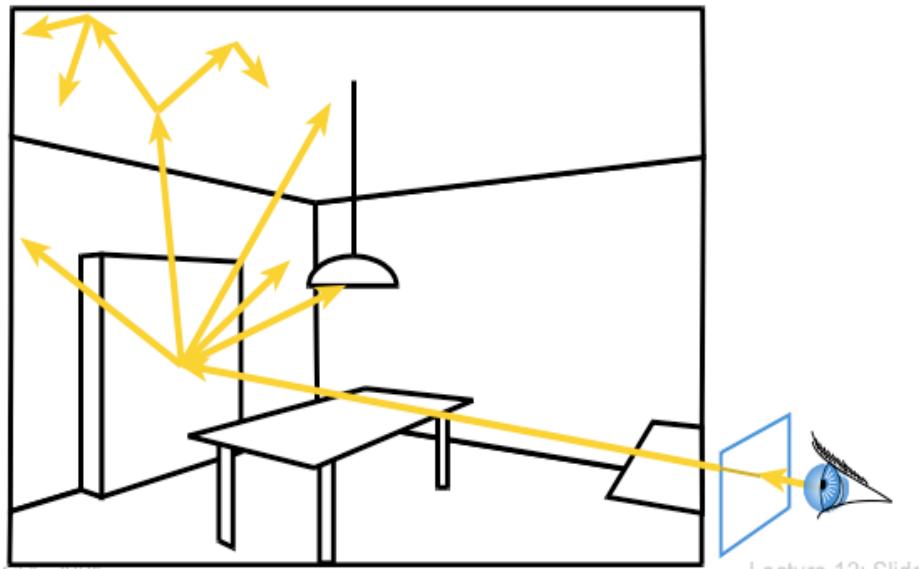
Monte-Carlo Ray Tracing

- Cast a ray from the eye through each pixel
- Cast random rays from the visible point
- Recurse



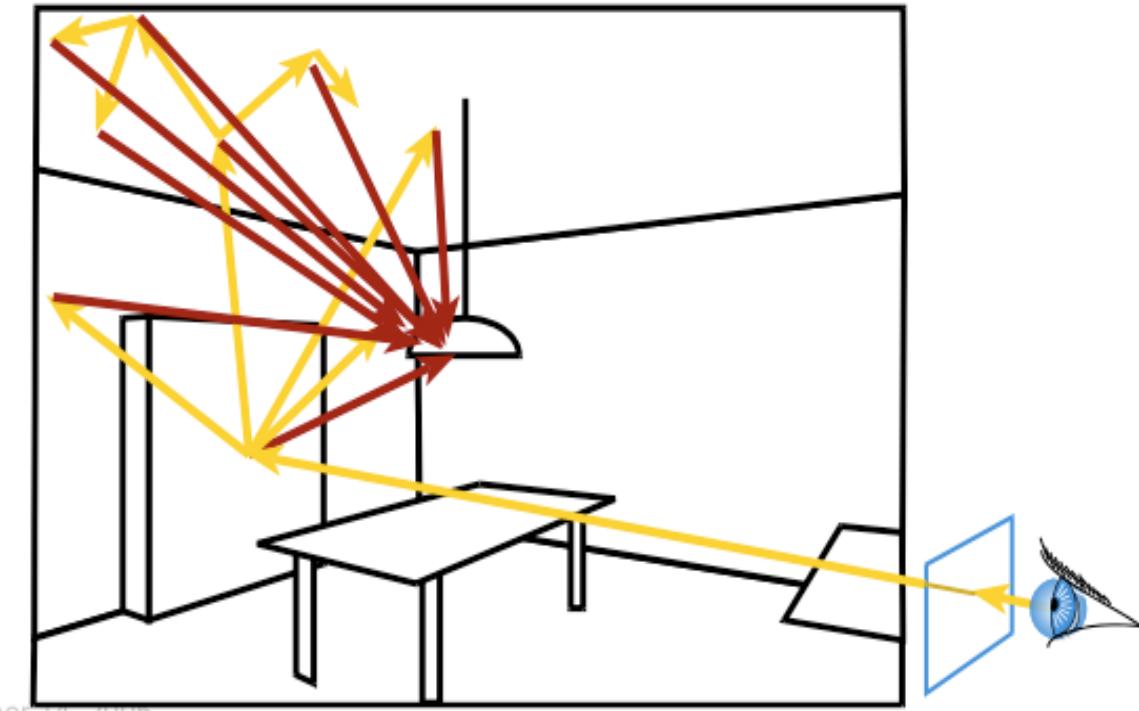
Monte-Carlo Ray Tracing

- Cast a ray from the eye through each pixel
- Cast random rays from the visible point
- Recurse



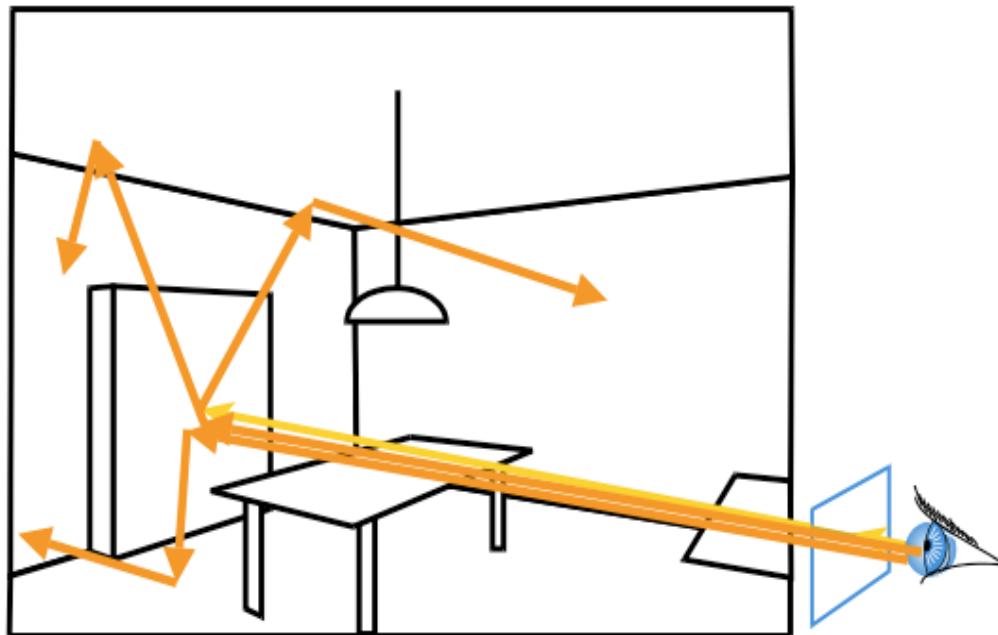
Monte-Carlo Ray Tracing

- Send rays to light



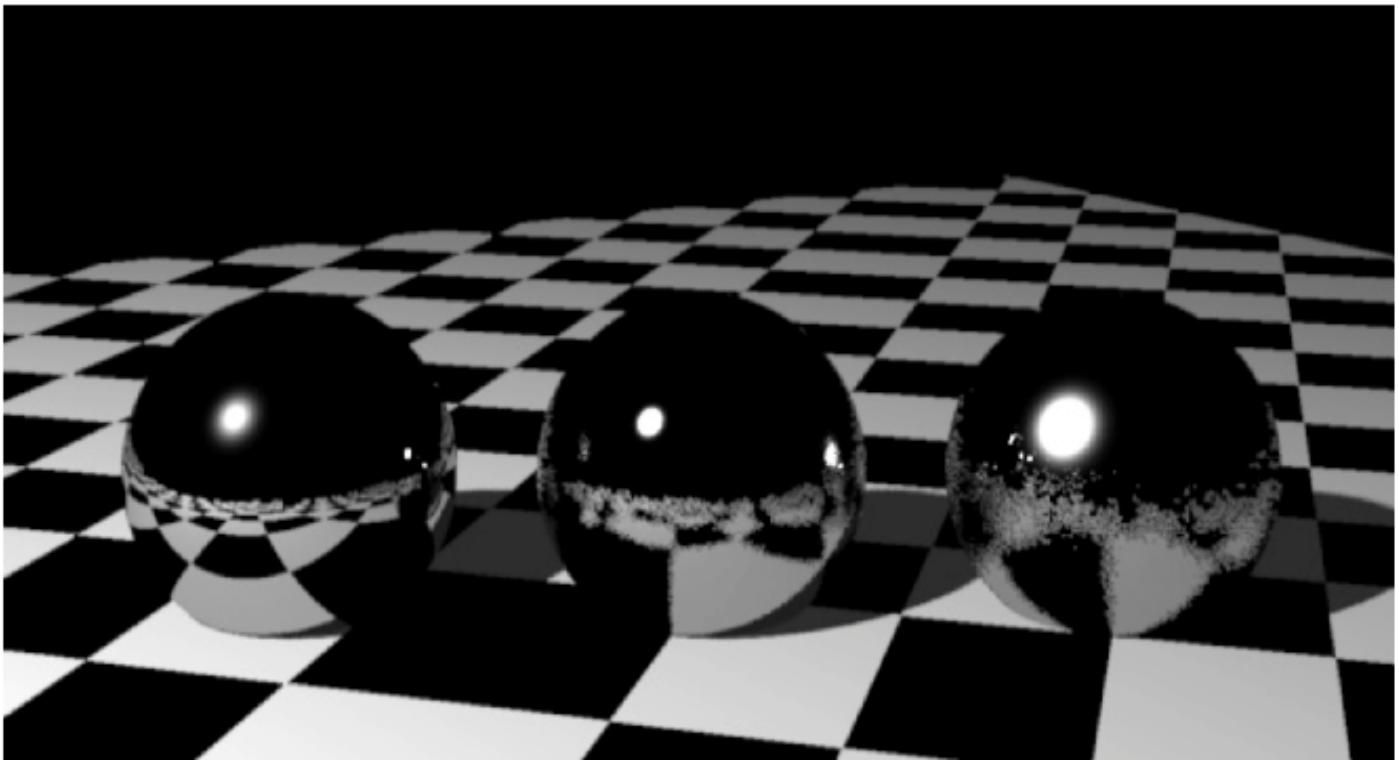
Monte-Carlo Path Tracing

- Trace only one secondary ray per recursion
- But send many primary rays per pixel



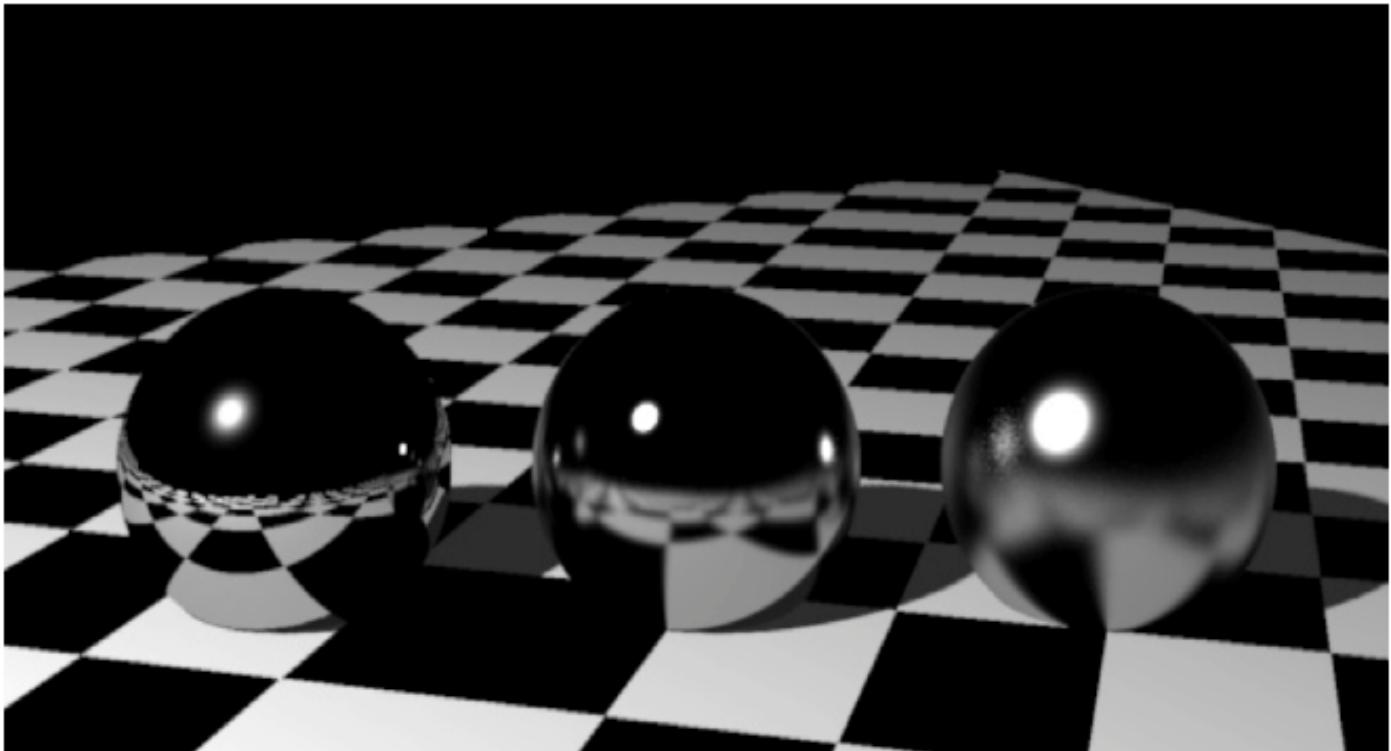
Graphics Lecture 10: Slide 53

Example



1 sample per ray

Example



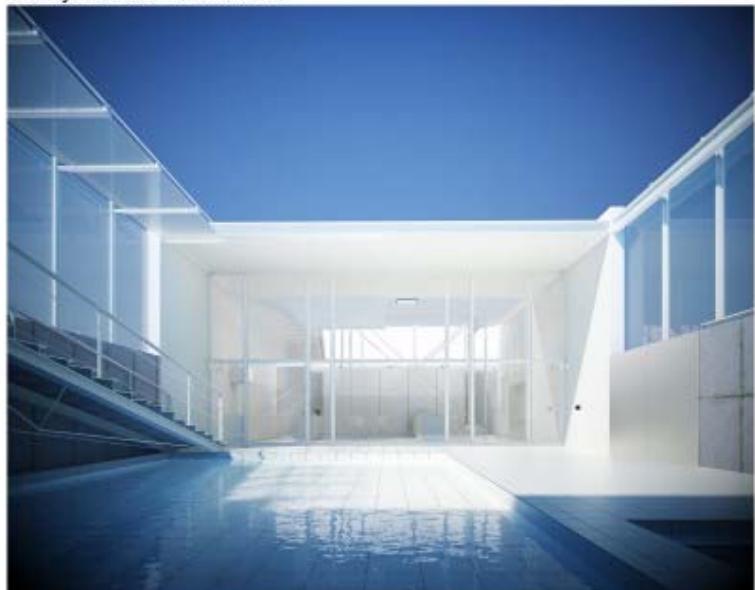
256 samples per ray

Some cool pictures

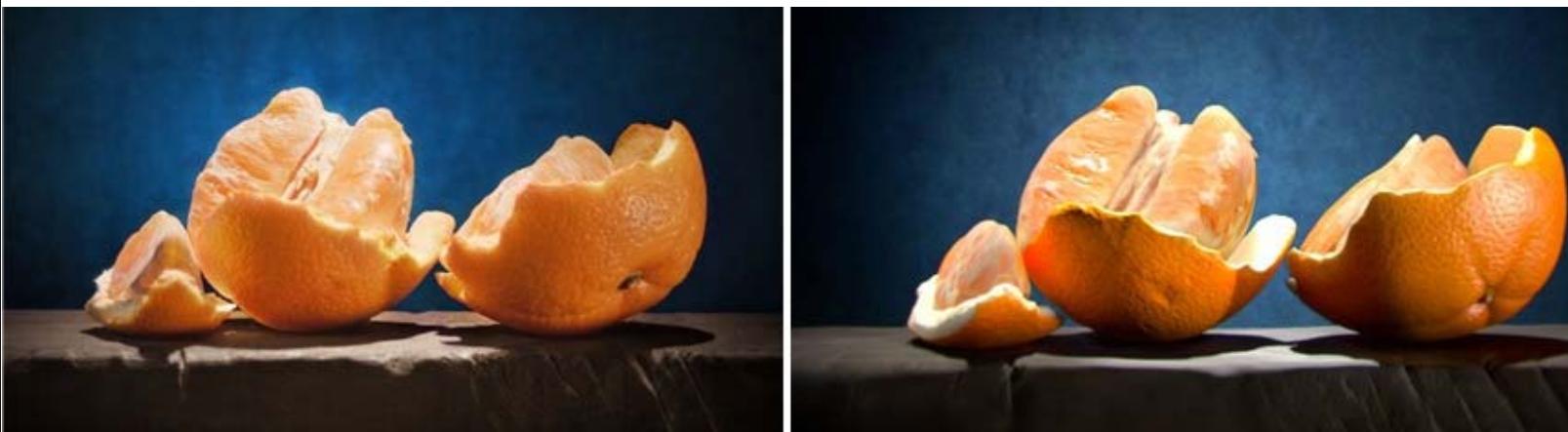


Some cool pictures

"Fukaya House" Waro Kishi



Some cool pictures



Some cool pictures



Some cool pictures



Copyright 2000 Gilles Tran

Some cool pictures

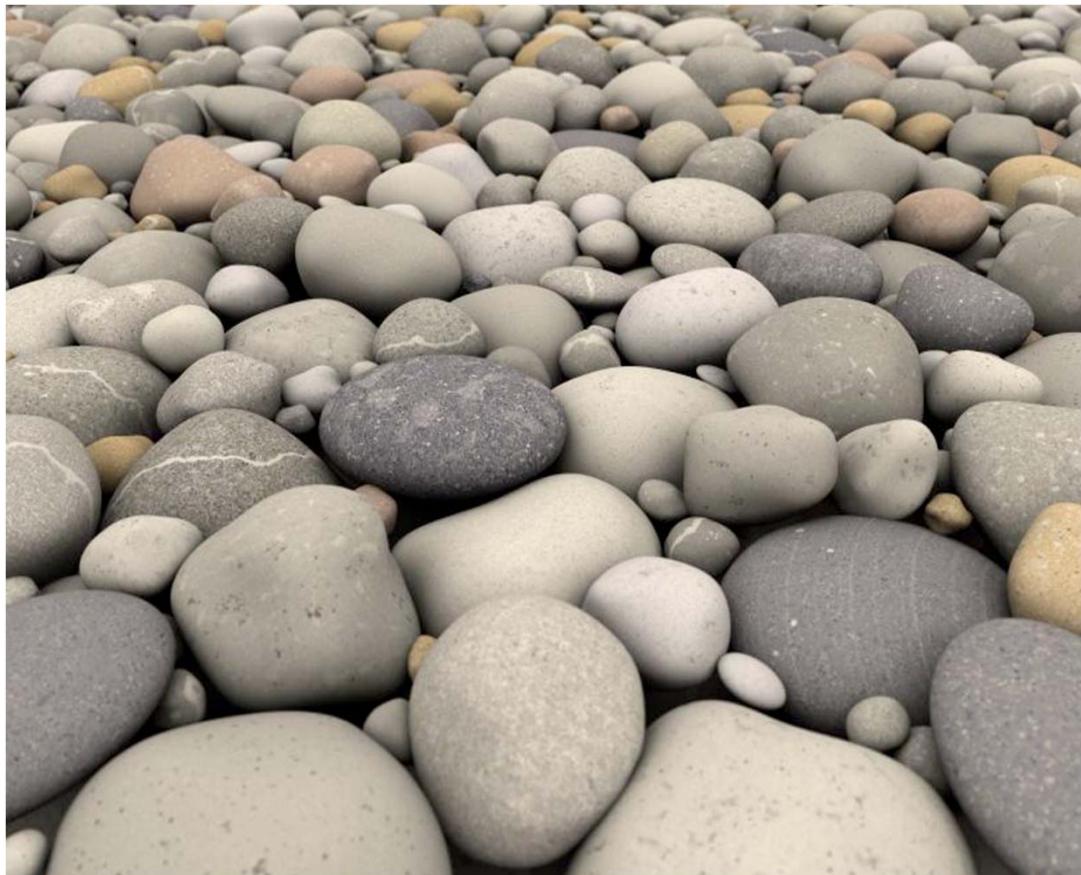


The Office - S01P13 - Inner View Pictures - P27 - May 5, 6

Some cool pictures



Some cool pictures



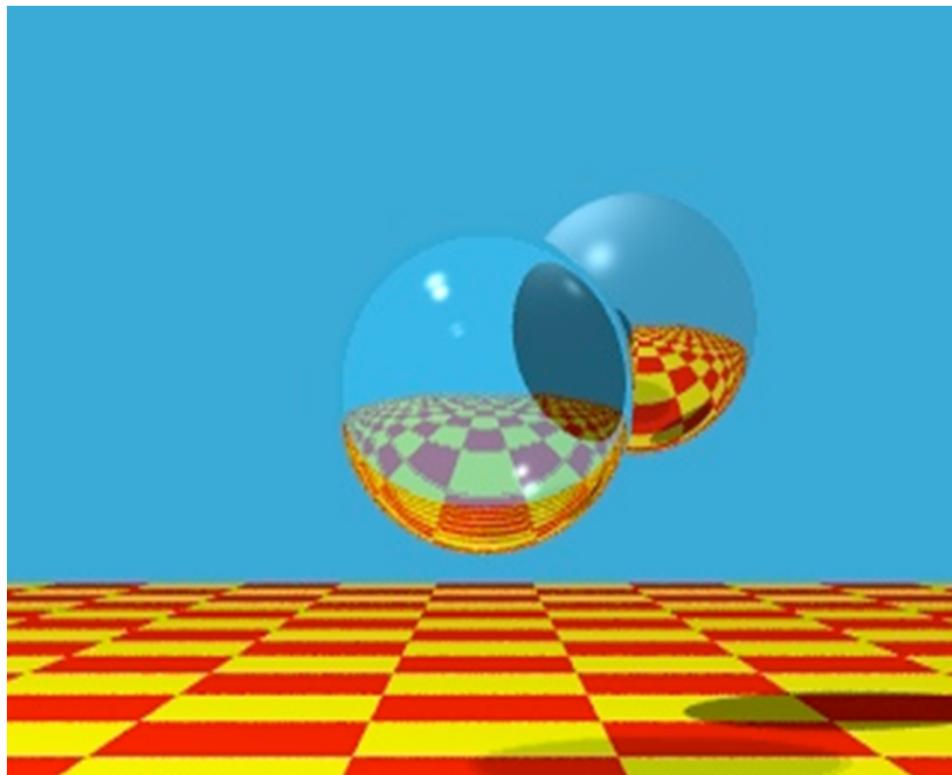
Graphics Lecture 10: Slide 63

took 4.5 days to render! (10 years ago)

Interactive Computer Graphics: Lecture 11

Ray tracing (cont.)

Ray tracing - Summary



Ray tracing - Summary

```
trace ray
```

```
    Intersect all objects
```

```
    color = ambient term
```

```
    For every light
```

```
        cast shadow ray
```

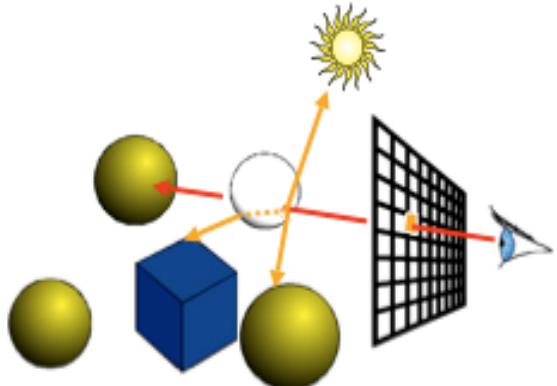
```
        col += local shading term
```

```
    If mirror
```

```
        col += k_refl * trace reflected ray
```

```
    If transparent
```

```
        col += k_trans * trace transmitted ray
```



Ray tracing - Summary

trace ray

Intersect all objects

color = ambient term

For every light

 cast shadow ray

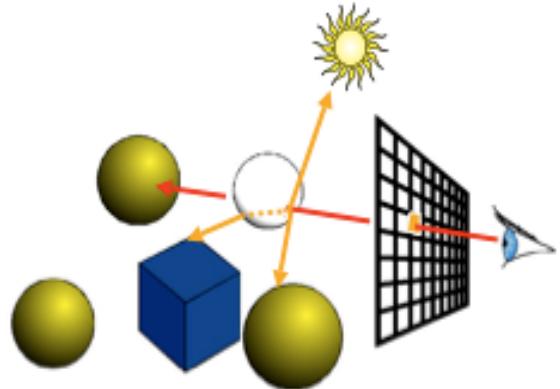
$\text{col} += \text{local shading term}$

If mirror

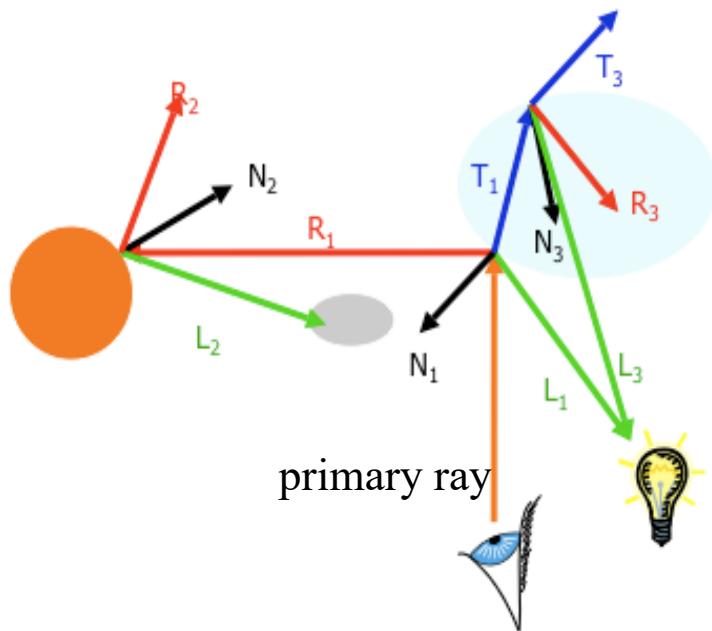
$\text{col} += \text{k_refl} * \text{trace reflected ray}$

If transparent

$\text{col} += \text{k_trans} * \text{trace transmitted ray}$



Ray tracing - Summary

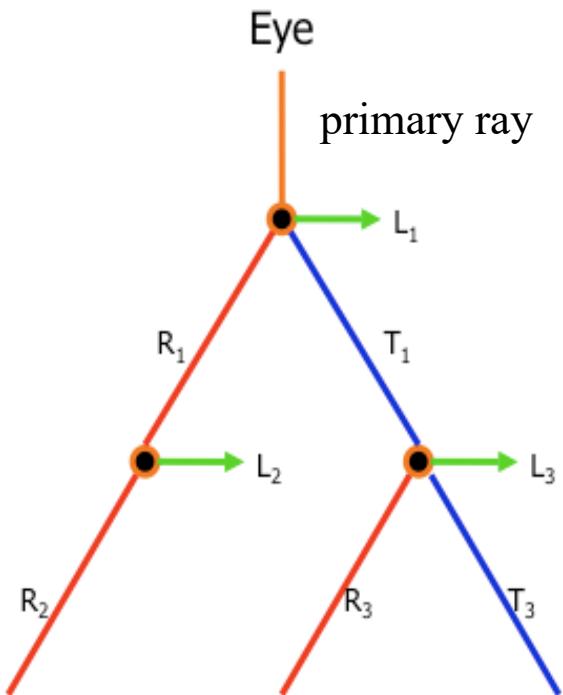


R_i reflected ray

L_i shadow ray

T_i transmitted (refracted) ray

} secondary rays



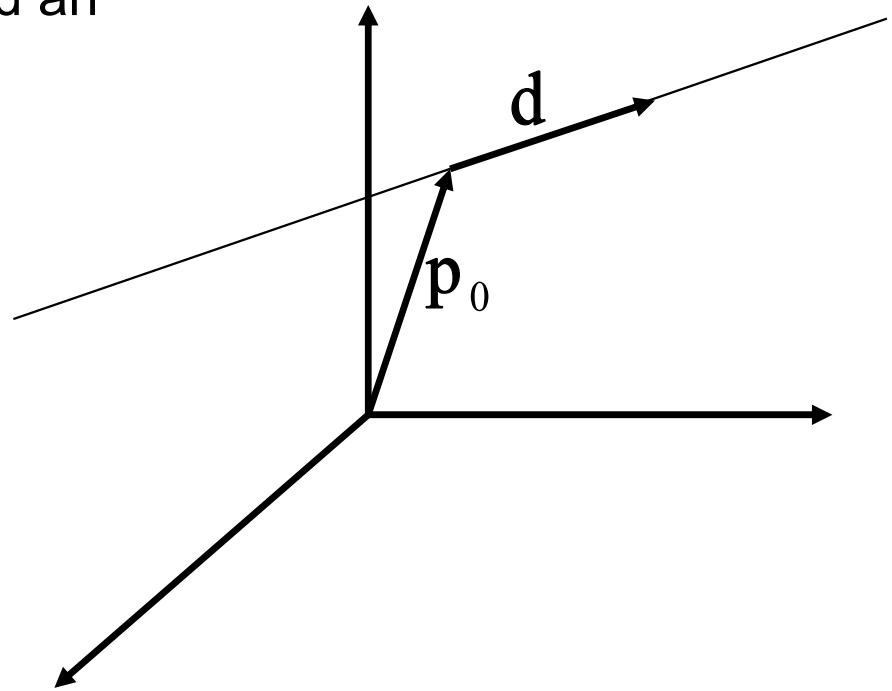
Intersection calculations

- For each ray we must calculate all possible intersections with each object inside the viewing volume
- For each ray we must find the nearest intersection point
- We can define our scene using
 - Solid models
 - sphere
 - cylinder
 - Surface models
 - plane
 - triangle
 - polygon

Rays

- Rays are parametric lines
- Rays can be defined as
 - origin p_0
 - direction d
- Equation of ray:

$$p(\mu) = p_0 + \mu d$$



Ray tracing: Intersection calculations

- The coordinates of any point along each primary ray are given by:

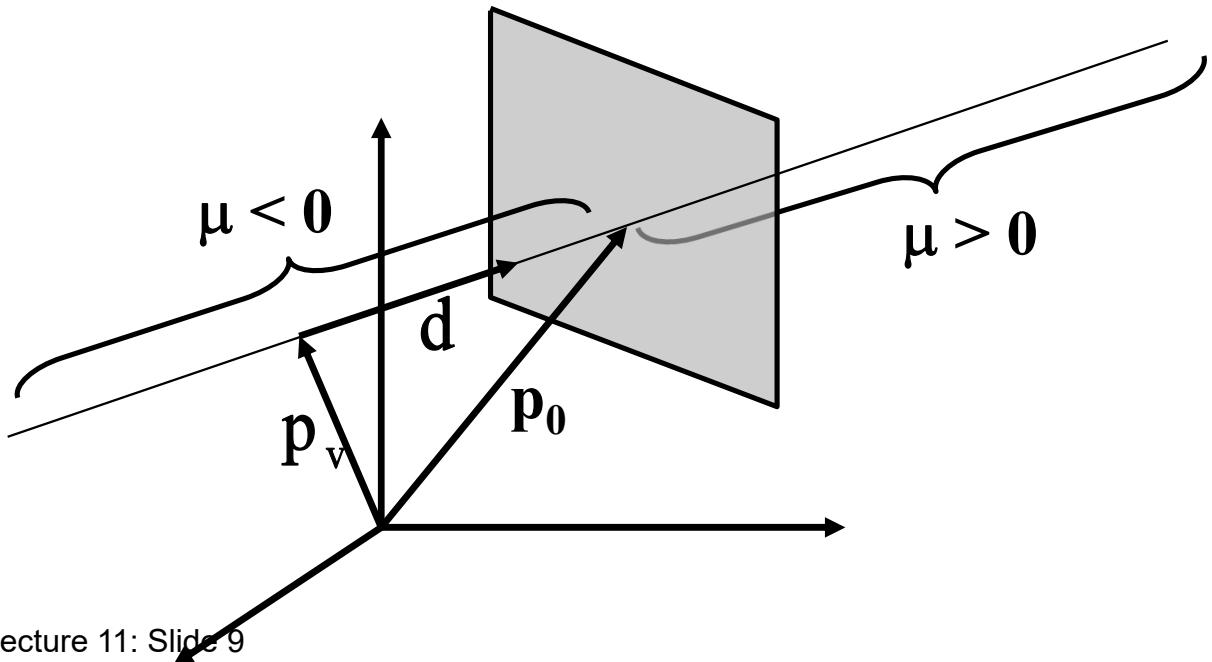
$$\mathbf{p} = \mathbf{p}_0 + \mu \mathbf{d}$$

- \mathbf{p}_0 is the current pixel on the viewing plane.
- \mathbf{d} is the direction vector and can be obtained from the position of the pixel on the viewing plane \mathbf{p}_0 and the viewpoint \mathbf{p}_v :

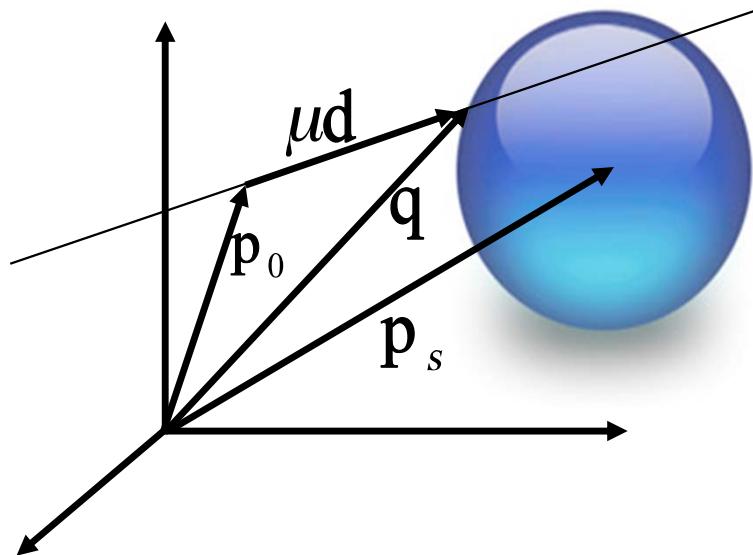
$$\mathbf{d} = \frac{\mathbf{p}_0 - \mathbf{p}_v}{|\mathbf{p}_0 - \mathbf{p}_v|}$$

Ray tracing: Intersection calculations

- The viewing ray can be parameterized by μ :
 - $\mu > 0$ denotes the part of the ray behind the viewing plane
 - $\mu < 0$ denotes the part of the ray in front of the viewing plane
 - For any visible intersection point $\mu > 0$



Intersection calculations: Spheres



- For any point on the surface of the sphere

$$|q - p_s|^2 - r^2 = 0$$

- where r is the radius of the sphere

Intersection calculations: Spheres

- To test whether a ray intersects a surface we can substitute for q using the ray equation:

$$|p_0 + \mu d - p_s|^2 - r^2 = 0$$

- Setting $\Delta p = p_0 - p_s$ and expanding the dot product produces the following quadratic equation:

$$\mu^2 + 2\mu(d \cdot \Delta p) + |\Delta p|^2 - r^2 = 0$$

Intersection calculations: Spheres

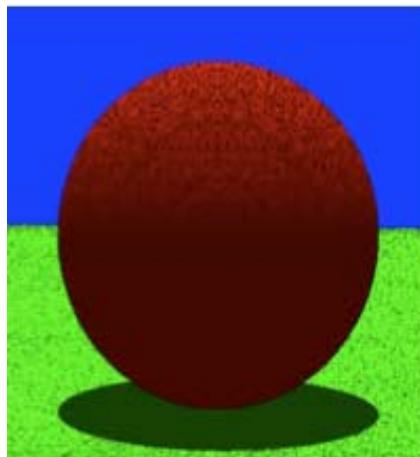
- The quadratic equation has the following solution:

$$\mu = -d \cdot \Delta p \pm \sqrt{(d \cdot \Delta p)^2 - |\Delta p|^2 + r^2}$$

- Solutions:
 - if the quadratic equation has no solution, the ray does not intersect the sphere
 - if the quadratic equation has two solutions ($\mu_1 < \mu_2$):
 - μ_1 corresponds to the point at which the rays enters the sphere
 - μ_2 corresponds to the point at which the rays leaves the sphere

Precision Problems

- In ray tracing, the origin of (secondary) rays is often on the surface of objects
 - Theoretically, $\mu = 0$ for these rays
 - Practically, calculation imprecision creeps in, and the origin of the new ray is slightly beneath the surface
- Result: the surface area is shadowing itself



ε to the rescue ...

- Check if t is within some epsilon tolerance:
 - if $\text{abs}(\mu) < \varepsilon$
 - point is on the sphere
 - else
 - point is inside/outside
 - Choose the ε tolerance empirically
- Move the intersection point by epsilon along the surface normal so it is outside of the object
- Check if point is inside/outside surface by checking the sign of the implicit (sphere etc.) equation

Intersection calculations: Cylinders

- A cylinder can be described by
 - a position vector p_1 describing the first end point of the long axis of the cylinder
 - a position vector p_2 describing the second end point of the long axis of the cylinder
 - a radius r
- The axis of the cylinder can be written as $\Delta p = p_1 - p_2$ and can be parameterized by $0 \leq \alpha \leq 1$

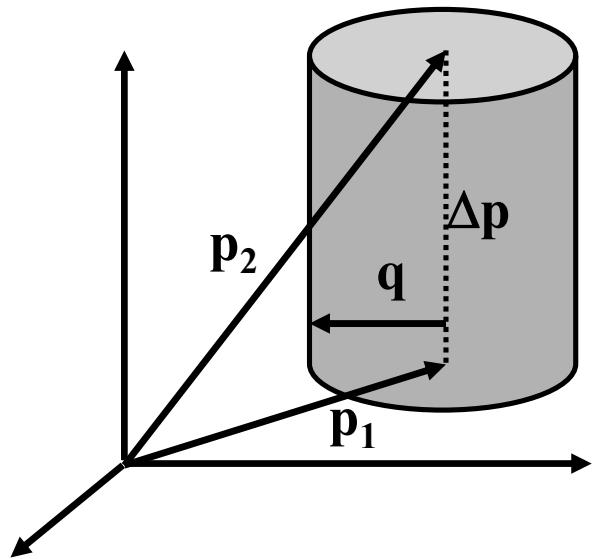
Intersection calculations: Cylinders

- To calculate the intersection of the cylinder with the ray:

$$p_1 + \alpha \Delta p + q = p_0 + \mu d$$

- Since $q \cdot \Delta p = 0$ we can write

$$\alpha(\Delta p \cdot \Delta p) = p_0 \cdot \Delta p + \mu d \cdot \Delta p - p_1 \cdot \Delta p$$



Intersection calculations: Cylinders

- Solving for α yields:

$$\alpha = \frac{\mathbf{p}_0 \cdot \Delta\mathbf{p} + \mu d \cdot \Delta\mathbf{p} - \mathbf{p}_1 \cdot \Delta\mathbf{p}}{\Delta\mathbf{p} \cdot \Delta\mathbf{p}}$$

- Substituting we obtain:

$$\mathbf{q} = \mathbf{p}_0 + \mu \mathbf{d} - \mathbf{p}_1 - \left(\frac{\mathbf{p}_0 \cdot \Delta\mathbf{p} + \mu d \cdot \Delta\mathbf{p} - \mathbf{p}_1 \cdot \Delta\mathbf{p}}{\Delta\mathbf{p} \cdot \Delta\mathbf{p}} \right) \Delta\mathbf{p}$$

Intersection calculations: Cylinders

- Using the fact that $\mathbf{q} \cdot \mathbf{q} = r^2$ we can use the same approach as before to the quadratic equation for μ :

$$r^2 = \left(\mathbf{p}_0 + \mu \mathbf{d} - \mathbf{p}_1 - \left(\frac{\mathbf{p}_0 \cdot \Delta \mathbf{p} + \mu \mathbf{d} \cdot \Delta \mathbf{p} - \mathbf{p}_1 \cdot \Delta \mathbf{p}}{\Delta \mathbf{p} \cdot \Delta \mathbf{p}} \right) \Delta \mathbf{p} \right)^2$$

- If the quadratic equation has no solution:
no intersection
- If the quadratic equation has two solutions:
intersection

Intersection calculations: Cylinders

- Assuming that $\mu_1 \leq \mu_2$ we can determine two solutions:

$$\alpha_1 = \frac{\mathbf{p}_0 \cdot \Delta\mathbf{p} + \mu_1 d \cdot \Delta\mathbf{p} - \mathbf{p}_1 \cdot \Delta\mathbf{p}}{\Delta\mathbf{p} \cdot \Delta\mathbf{p}}$$

$$\alpha_2 = \frac{\mathbf{p}_0 \cdot \Delta\mathbf{p} + \mu_2 d \cdot \Delta\mathbf{p} - \mathbf{p}_1 \cdot \Delta\mathbf{p}}{\Delta\mathbf{p} \cdot \Delta\mathbf{p}}$$

- If the value of α_1 is between 0 and 1 the intersection is on the outside surface of the cylinder
- If the value of α_2 is between 0 and 1 the intersection is on the inside surface of the cylinder

Intersection calculations: Plane

- Objects are often described by geometric primitives such as
 - triangles
 - planar quads
 - planar polygons
- To test intersections of the ray with these primitives we must test whether the ray will intersect the plane defined by the primitive

Intersection calculations: Plane

- The intersection of a ray with a plane is given by

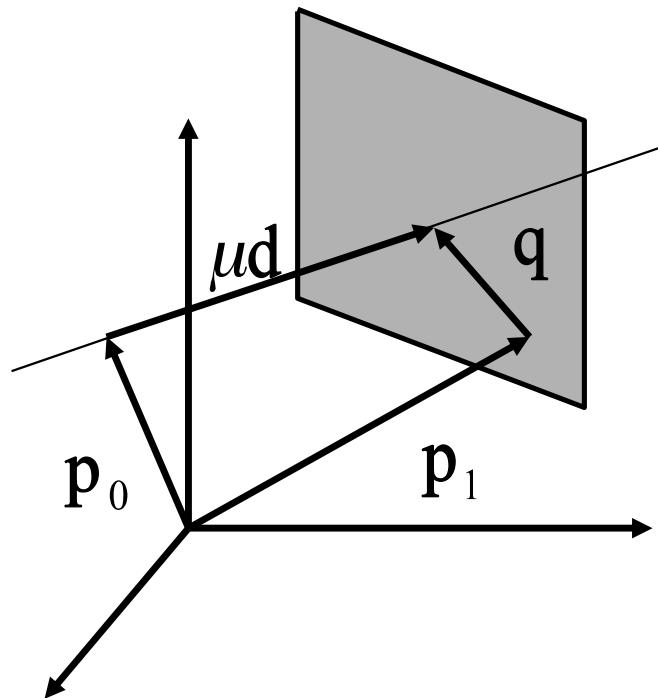
$$\mathbf{p}_1 + \mathbf{q} = \mathbf{p}_0 + \mu \mathbf{d}$$

- where \mathbf{p}_1 is a point in the plane.
Subtracting \mathbf{p}_1 and multiplying with the normal of the plane \mathbf{n} yields:

$$\mathbf{q} \cdot \mathbf{n} = 0 = (\mathbf{p}_0 - \mathbf{p}_1) \cdot \mathbf{n} + \mu d \cdot \mathbf{n}$$

- Solving for μ yields:

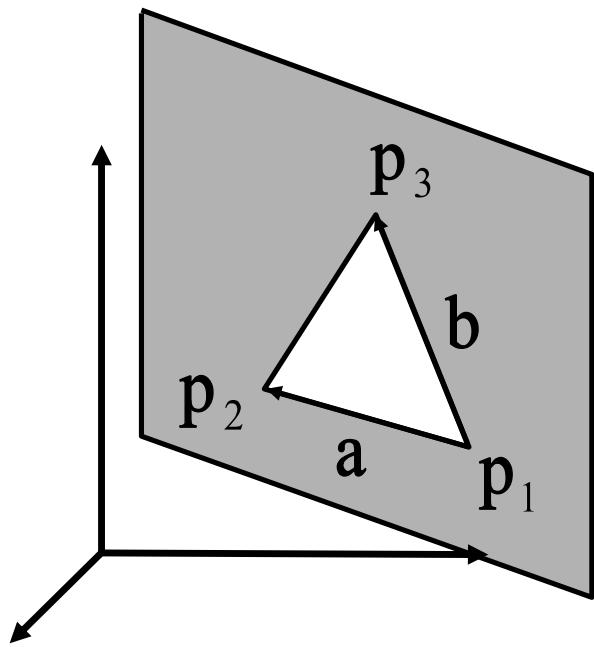
$$\mu = -\frac{(\mathbf{p}_0 - \mathbf{p}_1) \cdot \mathbf{n}}{\mathbf{d} \cdot \mathbf{n}}$$



Intersection calculations: Triangles

- To calculate intersections:
 - test whether triangle is front facing
 - test whether plane of triangle intersects ray
 - test whether intersection point is inside triangle
- If the triangle is front facing:

$$\mathbf{d} \cdot \mathbf{n} < 0$$



Intersection calculations: Triangles

- To test whether plane of triangle intersects ray
 - calculate equation of the plane using

$$\mathbf{p}_2 - \mathbf{p}_1 = \mathbf{a}$$

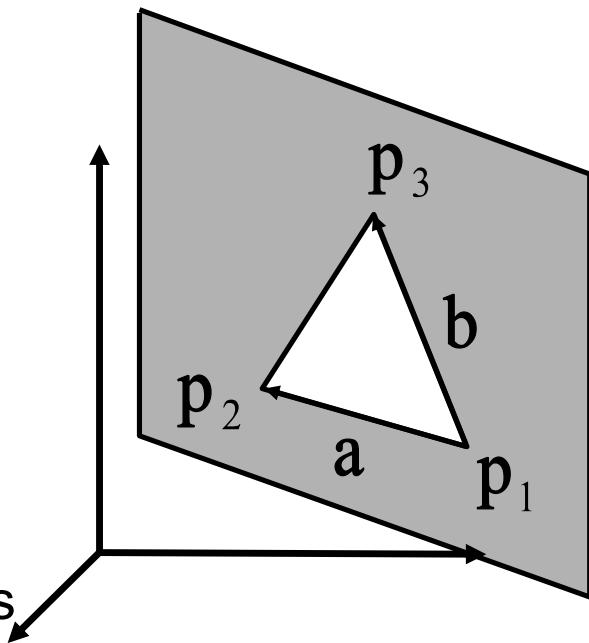
$$\mathbf{p}_3 - \mathbf{p}_1 = \mathbf{b}$$

- calculate intersections with plane as before

$$\mathbf{n} = \mathbf{a} \times \mathbf{b}$$

- To test whether intersection point is inside triangle:

$$\mathbf{q} = \alpha \mathbf{a} + \beta \mathbf{b}$$



Intersection calculations: Triangles

- A point is inside the triangle if

$$0 \leq \alpha \leq 1$$

$$0 \leq \beta \leq 1$$

$$\alpha + \beta \leq 1$$

- Calculate α and β by taking the dot product with a and b:

$$\alpha = \frac{(\mathbf{b} \cdot \mathbf{b})(\mathbf{q} \cdot \mathbf{a}) - (\mathbf{a} \cdot \mathbf{b})(\mathbf{q} \cdot \mathbf{b})}{(\mathbf{a} \cdot \mathbf{a})(\mathbf{b} \cdot \mathbf{b}) - (\mathbf{a} \cdot \mathbf{b})^2}$$

$$\beta = \frac{\mathbf{q} \cdot \mathbf{b} - \alpha(\mathbf{a} \cdot \mathbf{b})}{\mathbf{b} \cdot \mathbf{b}}$$

Intersection calculations: Triangles

- algorithm by **Möller and Trumbore**:
- translate the origin of the ray and then change the base of that vector which yields parameter vector (t, u, v)
- t is the distance to the plane in which the triangle lies
- (u, v) are barycentric coordinates inside the triangle
- plane equation need not be computed on the fly nor be stored

Intersection calculations: Triangles

```
1 bool triangle_intersection( vec3 v1,
2                             vec3 v2,
3                             vec3 v3, // Triangle vertices
4                             vec3 origin, //Ray origin
5                             vec3 ray_dir, //Ray direction
6                             float* out )
7
8 //Find vectors for two edges sharing v1
9 vec3 edge1 = v2-v1;
10 vec3 edge2 = v3-v1;
11
12 //Begin calculating determinant - also used to calculate u parameter
13 vec3 p = cross(ray_dir, edge2)
14
15 //if determinant is near zero, ray lies in plane of triangle or ray
16 //is parallel to plane of triangle
17 float det = dot(edge1, p);
18
19 if(det > -EPSILON && det < EPSILON)
20     return false;
21
22 float inv_det = 1.f / det;
23
24 //calculate distance from v1 to ray origin
25 vec3 t = origin-v1;
```

Intersection calculations: Triangles

```
27 //Calculate u parameter and test bound
28 float u = dot(t, p) * inv_det;
29
30 //The intersection lies outside of the triangle
31 if(u < 0.f || u > 1.f)
32 return false;
33
34 //Prepare to test v parameter
35 vec3 q = cross(t,edge1);
36
37 float v = dot(ray_dir, q) * inv_det;
38 //The intersection lies outside of the triangle
39 if(v < 0.f || u + v > 1.f)
40 return false;
41
42 float mu = dot(edge2, q) * inv_det;
43
44 if(t > EPSILON) { //ray intersection
45     *out = mu;
46     return true;
}
```

Ray tracing: Pros and cons

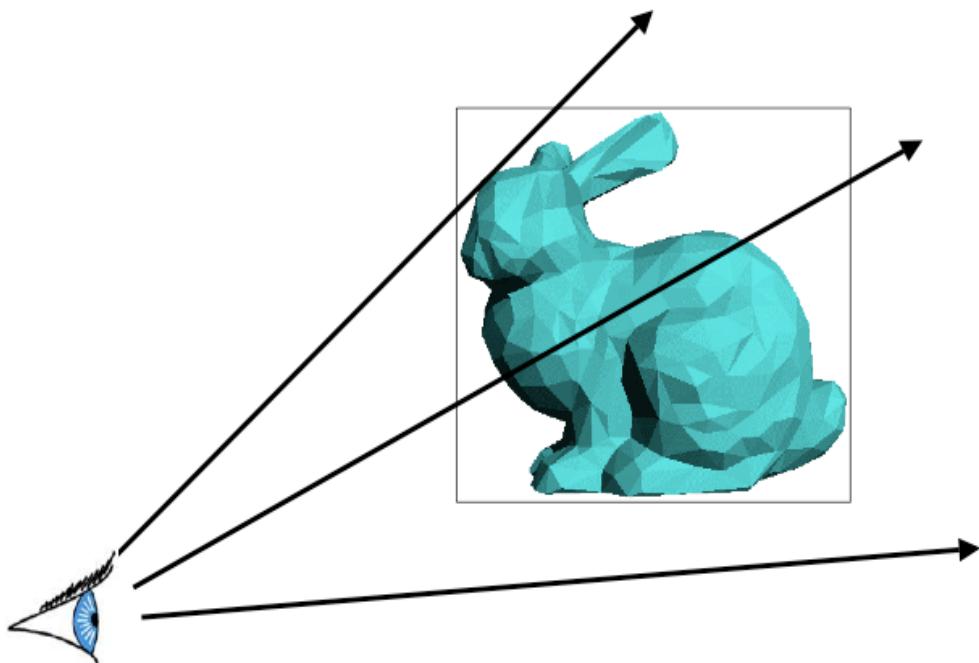
- Pros:
 - Easy to implement
 - Extends well to global illumination
 - shadows
 - reflections / refractions
 - multiple light bounces
 - atmospheric effects
- Cons:
 - Speed! (seconds per frame, not frames per second)

Speedup Techniques

- Why is ray tracing slow? How to improve?
 - Too many objects, too many rays
 - Reduce ray-object intersection tests
 - Many techniques!

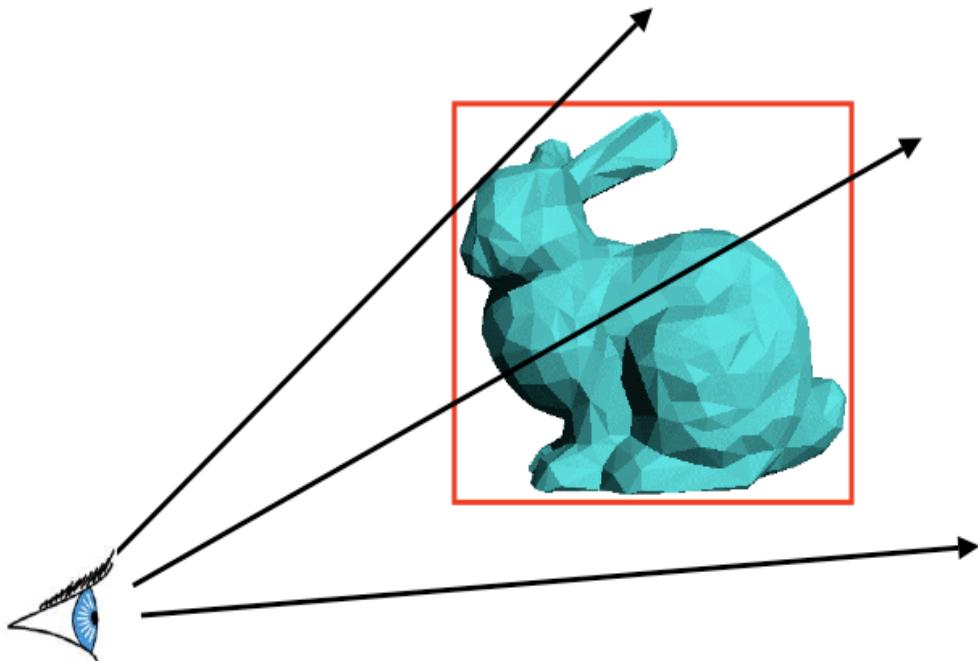
Acceleration of Ray Casting

- Goal: Reduce the number of ray/primitive intersections



Conservative Bounding Region

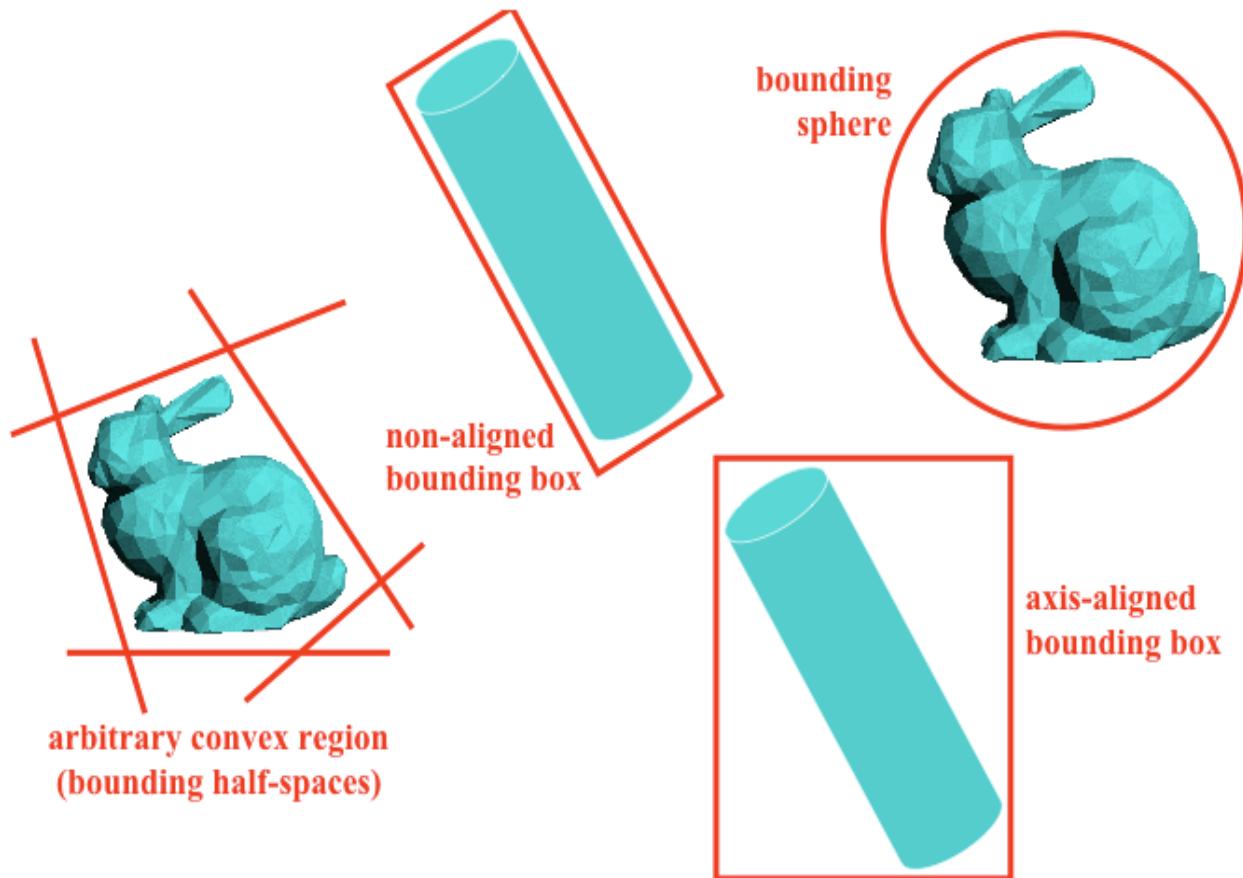
- First check for an intersection with a conservative bounding region
- Early reject



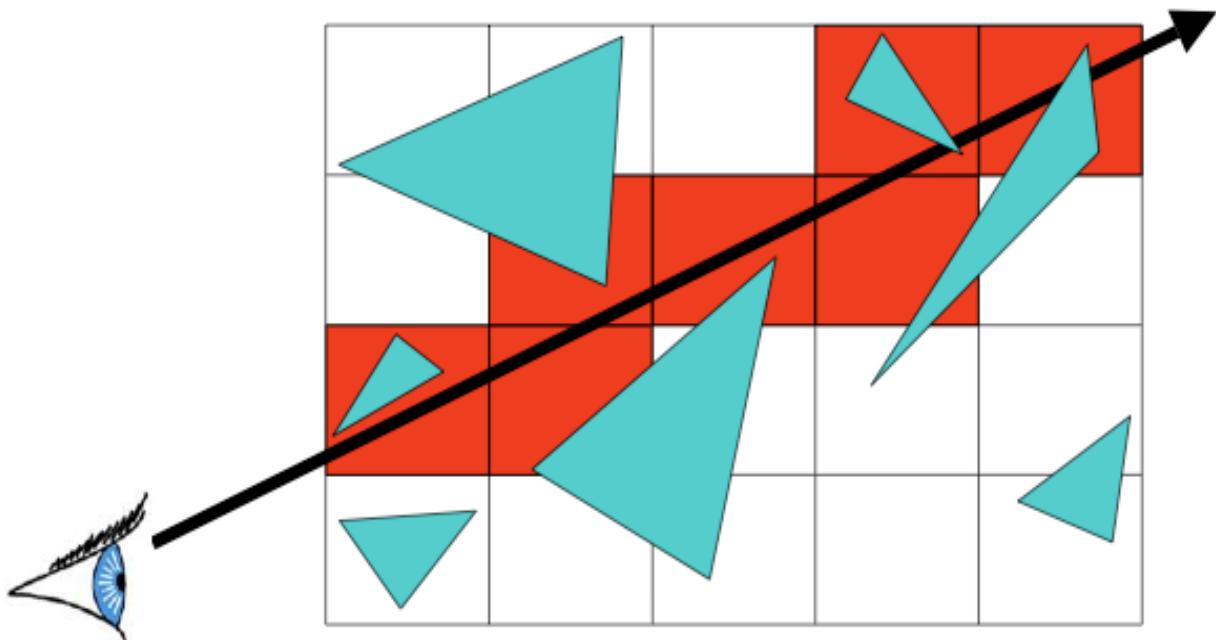
Bounding Regions

- What makes a good bounding region?

Conservative Bounding Regions

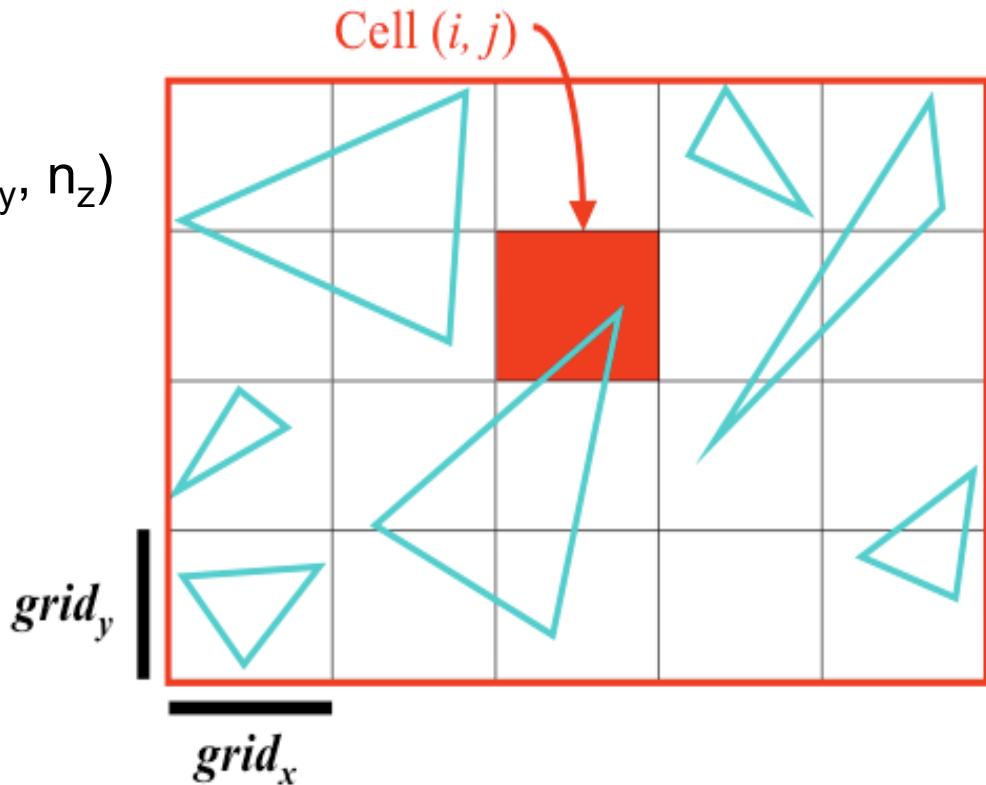


Regular Grid



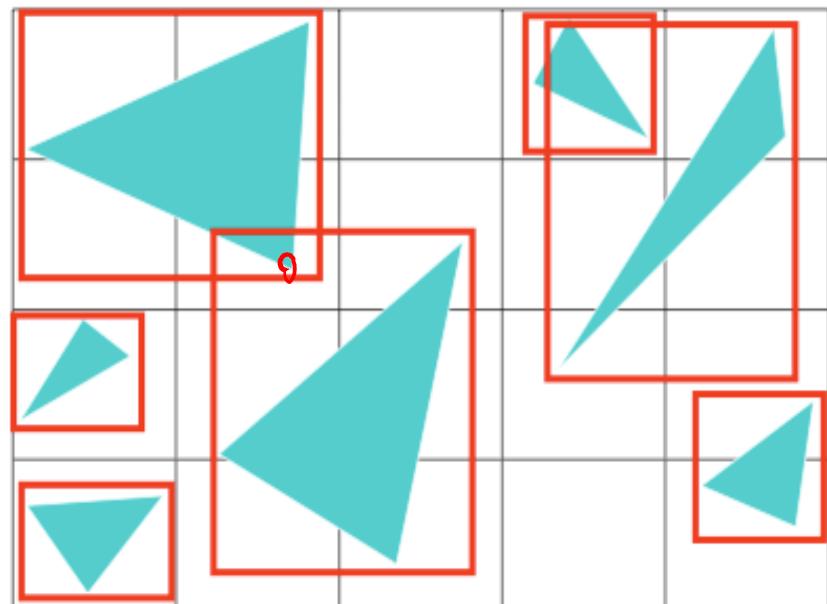
Create Grid

- Find bounding box of scene
- Choose grid resolution (n_x, n_y, n_z)
- $grid_x$ need not = $grid_y$



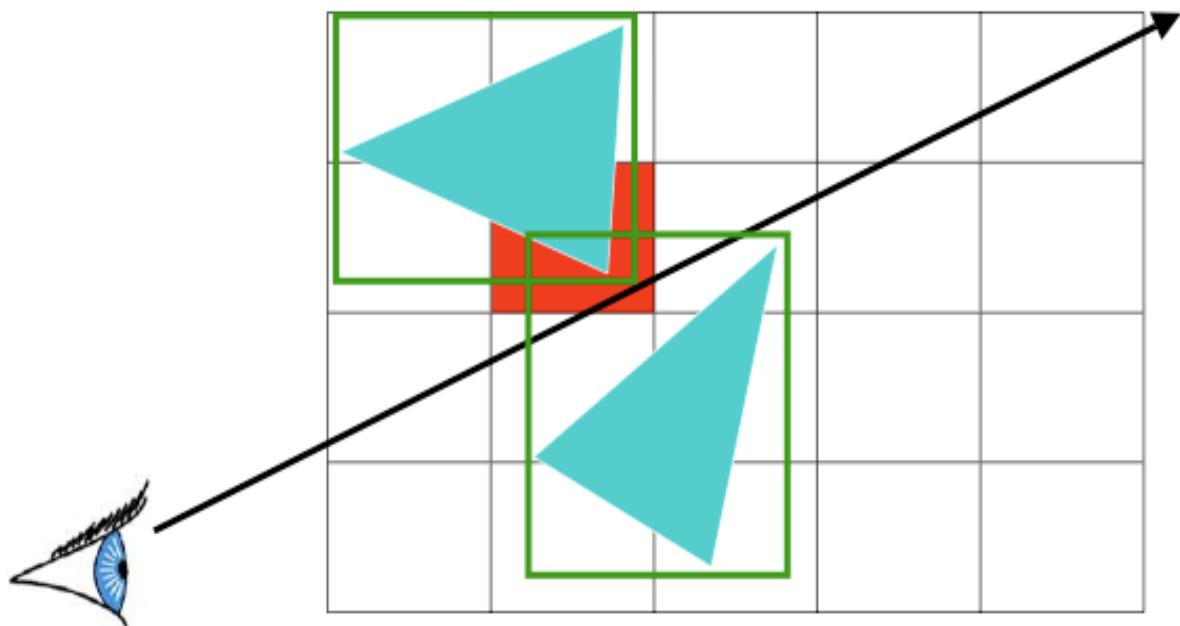
Insert Primitives into Grid

- Primitives that overlap multiple cells?
- Insert into multiple cells (use pointers)



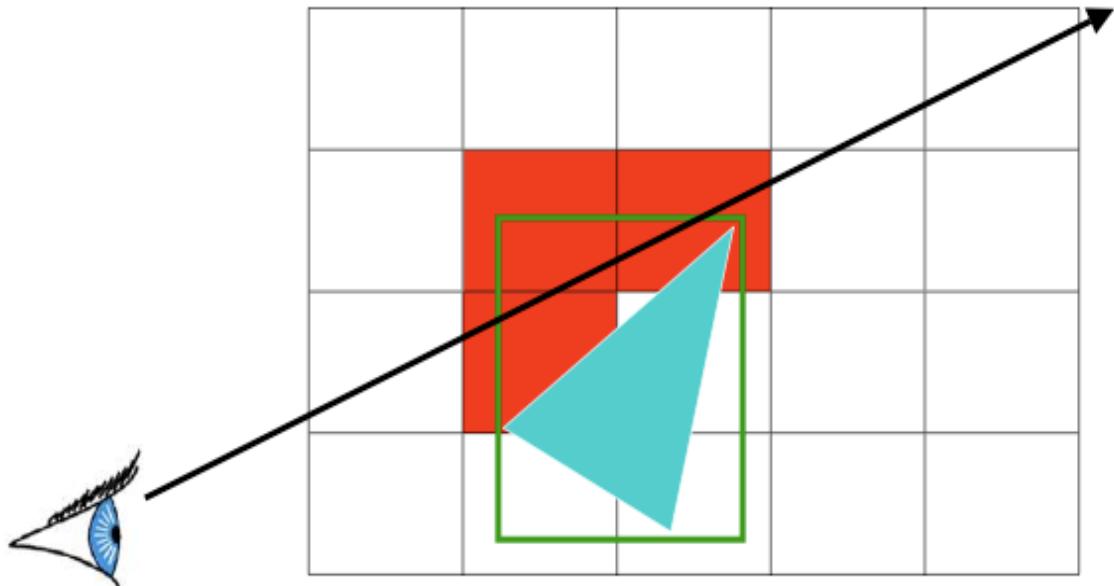
For Each Cell Along a Ray

- Does the cell contain an intersection?
 - Yes: return closest intersection
 - No: continue



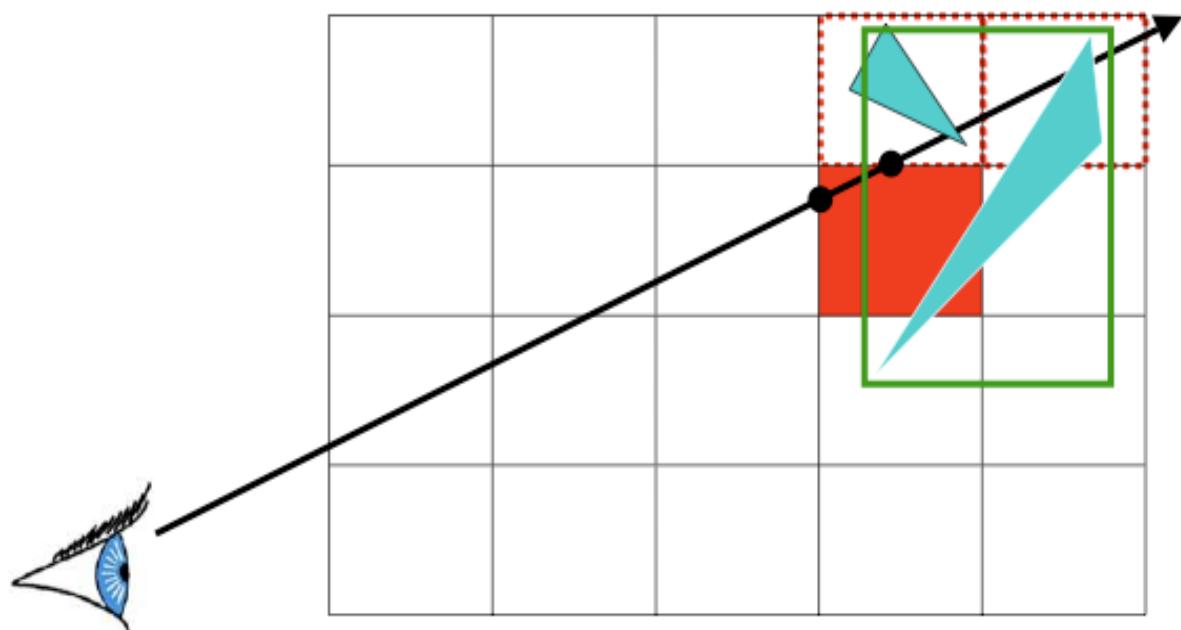
Preventing Repeated Computation

- Perform the computation once, "mark" the object
- Don't re-intersect marked objects



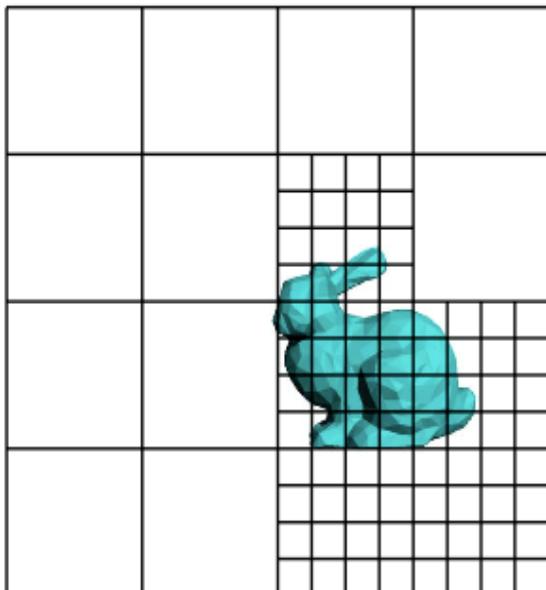
Don't Return Distant Intersections

- If intersection t is not within the cell range, continue (there may be something closer)

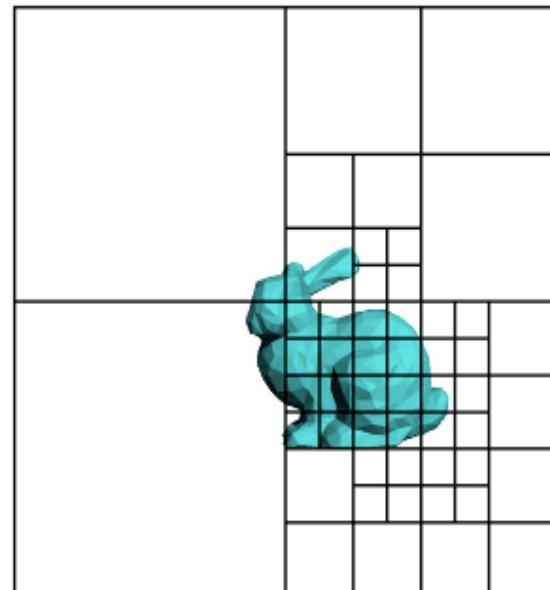


Adaptive Grids

- Subdivide until each cell contains no more than n elements, or maximum depth d is reached



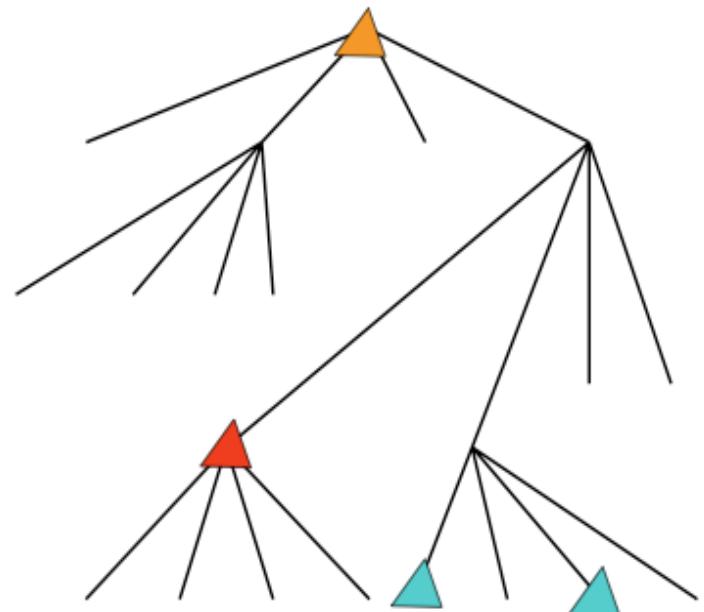
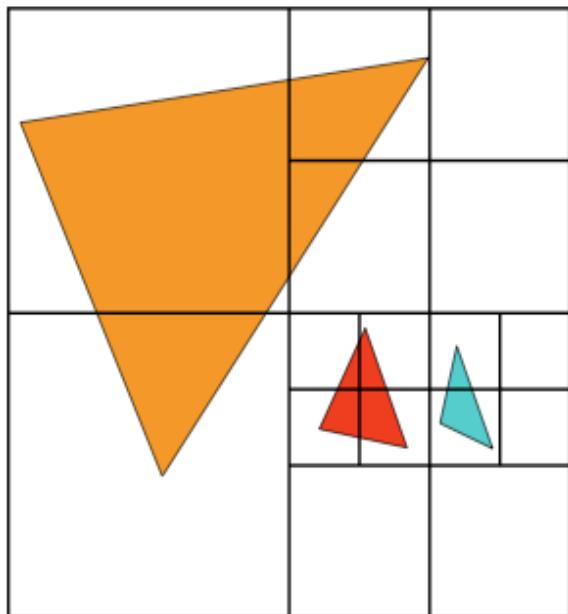
Nested Grids



Octree/(Quadtree)

Primitives in an Adaptive Grid

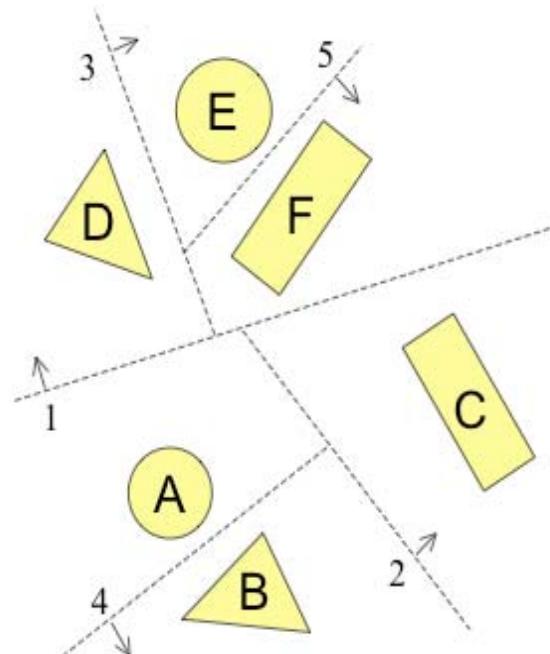
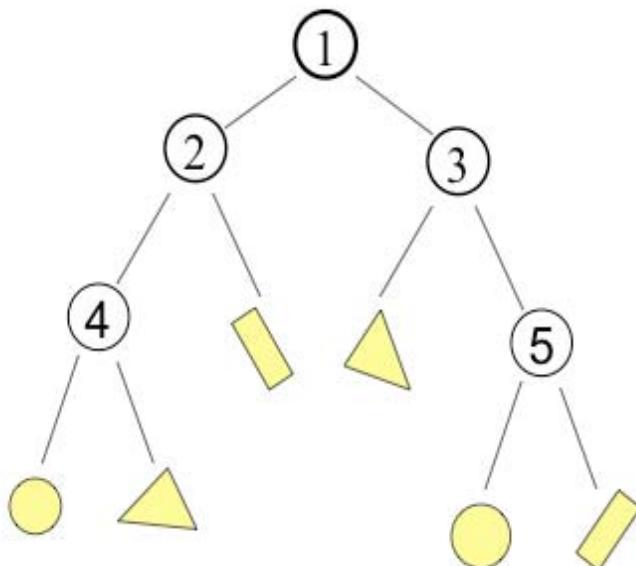
- Can live at intermediate levels, or be pushed to lowest level of grid



Octree/(Quadtree)

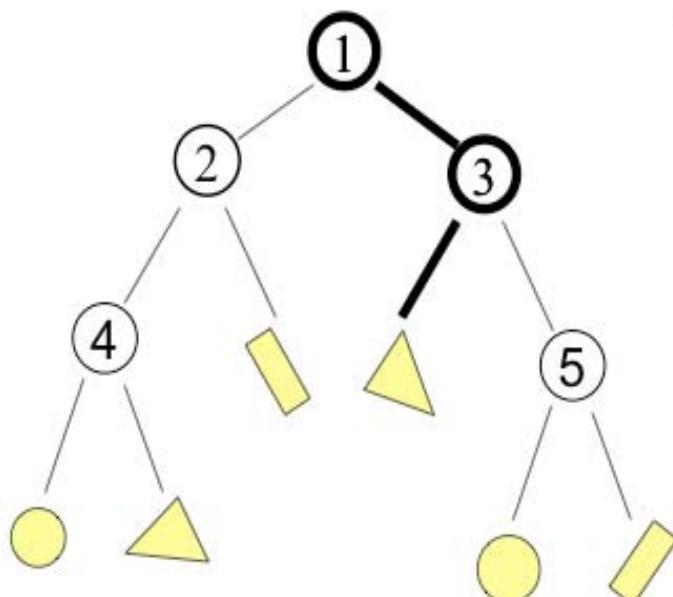
Binary Space Partition (BSP) Tree

- Recursively partition space by planes
- Every cell is a convex polyhedron

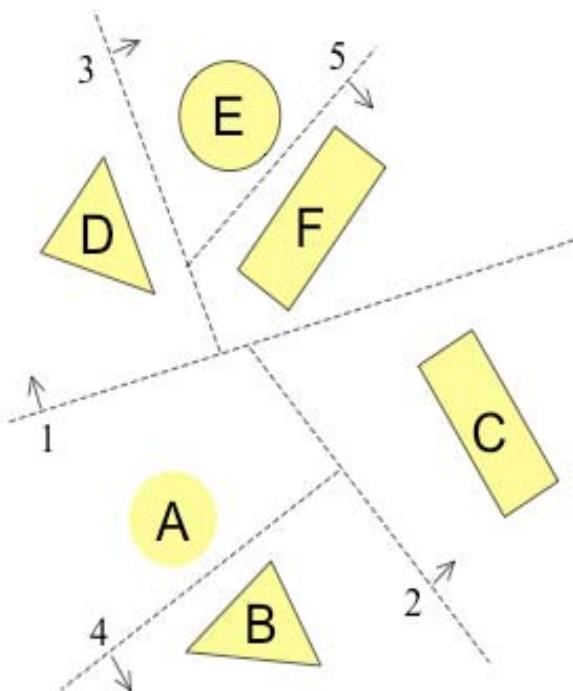


Binary Space Partition (BSP) Tree

- Simple recursive algorithms
- Example: point finding

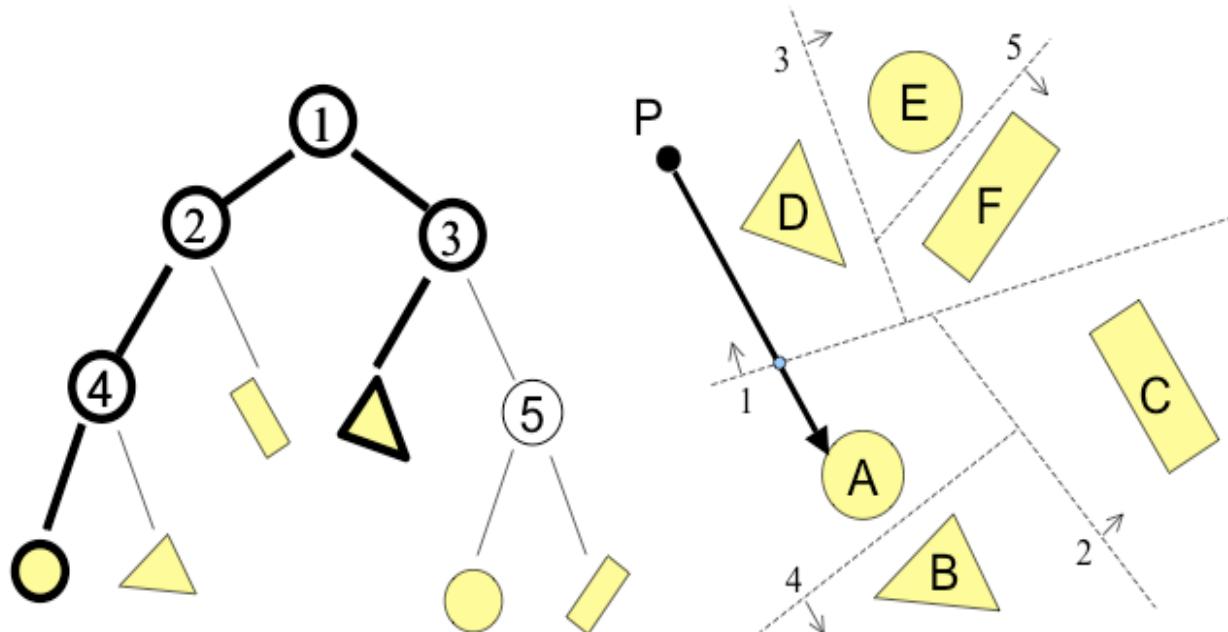


P
●



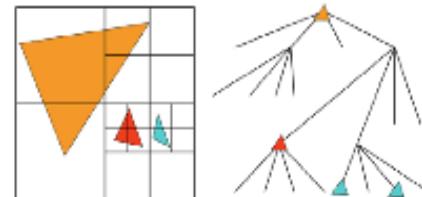
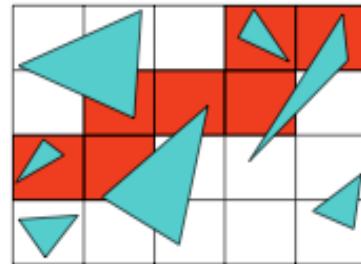
Binary Space Partition (BSP) Tree

- Trace rays by recursion on tree
 - BSP construction enables simple front-to-back traversal



Grid Discussion

- Regular
 - + easy to construct
 - + easy to traverse
 - may be only sparsely filled
 - geometry may still be clumped
- Adaptive
 - + grid complexity matches geometric density
 - more expensive to traverse (especially BSP tree)



Example

- <https://www.youtube.com/watch?v=aKqxonOrI4Q>
- <https://www.youtube.com/watch?v=qlyzo9ll9Vw>
- <https://www.youtube.com/watch?v=AV279wThmVU>
-