IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2017

BEng Honours Degree in Computing Part III
BEng Honours Degree in Electronic and Information Engineering Part III
MEng Honours Degree in Electronic and Information Engineering Part III
MEng Honours Degree in Mathematics and Computer Science Part IV
BEng Honours Degree in Mathematics and Computer Science Part III
MEng Honours Degree in Mathematics and Computer Science Part III
MEng Honours Degrees in Computing Part III
MSc in Computing Science
MSc in Computing Science (Specialist)
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER C317

GRAPHICS

Friday 24 March 2017, 10:00
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators required

1 Transformations

a A point $\mathbf{p} = (10, 8, 20)$ is being viewed from the origin with the plane of projection being at $z = 5$.

i) Compute the homogenous coordinates of the point after projection.

ii) Compute the Cartesian coordinates of the point after projection.

b A line is defined by the following equation: $\mathbf{r}(\mu) = \mathbf{r}_0 + \mu\mathbf{d}$

i) Describe the different transformations that are necessary to rotate a point around the line above.

ii) In a concrete example, $\mathbf{r}_0 = (0, 0, 0)$ and $\mathbf{d} = (2, 0, 8)$, determine the homogeneous transformation matrix that will transform this line so that it is aligned with the y axis.

iii) In an animation a graphics scene is rotated 45 degrees around the line defined above (the rotation is to be in the clockwise direction when viewed from the origin). Determine the components of the homogeneous transformation matrix that will achieve this rotation. You do not need to multiply the components together to form one single matrix.

c Give a short geometric explanation of the four column vectors making up the following homogeneous transformation matrix:

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Use this explanation to deduce what the transformation will do to a graphics scene.

The three parts carry, respectively, 20%, 60%, and 20% of the marks.

2 Splines

a Assume that a 2D Bezier curve is defined by the points below:

	x	y
$\mathbf{P}_0$	3	1
$\mathbf{P}_1$	1	5
$\mathbf{P}_2$	3	3
$\mathbf{P}_3$	5	5

Use de Casteljau's construction to identify the location of the curve for $\mu = 0.25$, $\mu = 0.5$ and $\mu = 0.75$ and produce an accurate sketch of the curve.

b Given that a Bezier curve is defined by the equation

$$\mathbf{P}(\mu) = \sum_{i=0}^{N} \mathbf{P_i} W(N, i, \mu)$$

where $W(N, i, \mu)$ is called the Bernstein blending function defined as

$$W(N, I, \mu) = \binom{N}{i} \mu^i (1-\mu)^{N-i}$$

$$\binom{N}{i} = \frac{N!}{(N-i)!i!}$$

Calculate the points on the curve for which $\mu = 1/3$ and $\mu = 2/3$.

c Derive an expression for the gradient of the curve and find its value at the point $\mu = 1/2$.

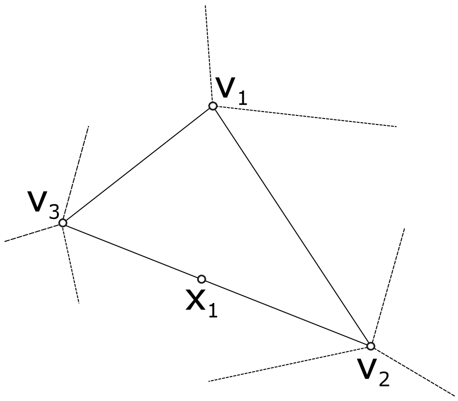
d A cubic spline can be written as $\mathbf{P}(\mu) = \mathbf{a}_3\mu^3 + \mathbf{a}_2\mu^2 + \mathbf{a}_1\mu + \mathbf{a}_0$ with $\mu \in [0, 1]$.

i) Given that the patch is to be drawn between two points $\mathbf{P}_i$ and $\mathbf{P}_{i+1}$ and the gradients at the ends are to be $\mathbf{P}'_i$ and $\mathbf{P}'_{i+1}$ respectively, derive expressions for $\mathbf{a}_3$, $\mathbf{a}_2$, $\mathbf{a}_1$ and $\mathbf{a}_0$ in terms of $\mathbf{P}_i$, $\mathbf{P}_{i+1}$ $\mathbf{P}'_i$ and $\mathbf{P}'_{i+1}$.

ii) Show that the four point Bezier curve above can be expressed as a cubic spline patch that passes through $\mathbf{P}_0$ and $\mathbf{P}_3$.

The four parts carry, respectively, 20%, 20%, 20%, and 40% of the marks.

3 Graphics simplexes and primitive processing



The figure above shows a 2-simplex (triangle) being part of a polygon mesh. The triangle Δ is defined by three vertices v_1, v_2, v_3. These vertices are connected clockwise with edges. Point coordinates can be defined based on this triangle in **Barycentric coordinates**. The coordinates of the vertices v_1, v_2, v_3 in *Cartesian* coordinates are: $v_1 = (1,2,0)$ $v_2 = (2,1,0)$ $v_2 = (1,4,0)$.

a How can the above stated 3D *Cartesian* coordinates be converted to **absolute Barycentric coordinates**?

b What's the difference between Barycentric coordinates and Trilinear coordinates? Show a simple way to convert between these coordinates.

c Convert v_1, v_2, v_3 to Barycentric coordinates. Another point is given in *Cartesian* coordinates as $p = (1,1,0)$. Convert this point into Barycentric coordinates relative to Δ.

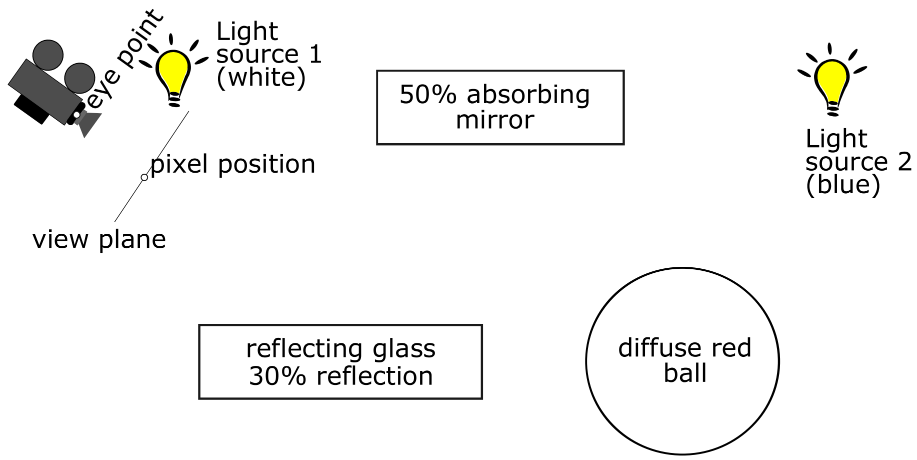
d Is $p_c = (1,1,0)$ (*Cartesian* coordinates) contained inside Δ when orthographically projecting both on the xy-plane? Justify your answer. What's the Distance of p to the Centroid of Δ in Barycentric coordinates?

e What are the Barycentric coordinates of x_1 in the figure above? x_1 is the bisection of the edge between v_2 and v_3.

f Outline a simple triangle subdivision algorithm geometrically in the figure above (a maximum of two subdivision levels!) and describe the steps of the algorithm. Do not forget to show the necessary steps for correct colour and normal-vector interpolation of newly generated vertices.

The six parts carry, respectively, 20%, 20%, 20%, 10%, 10%, and 20% of the marks.

4 Ray tracing



a The scene in the figure above contains a diffuse red ball, which does not reflect or refract any light, a piece of reflecting glass, which reflects 30% of the incoming light, a mirror that absorbs 50% of the light and two light sources.

i) Draw all additional reflection, refraction and shadow rays needed to compute the colour of the marked pixel position on the view plane, starting from the eye point. Label each ray clearly.

ii) How is the initial eye ray direction computed?

iii) Write down a formula which describes the computation of the colour C for the ray. Label your drawing from a part i) or add annotations to your terms in your equation so that it is clear what is being computed.

b Briefly explain which illumination effects can be achieved with ray tracing and how these effects are achieved. What is the key difference between ray tracing and radiosity?

c A simple ray tracing algorithm consists of ray generation, ray processing, intersection evaluation parts and termination criteria.

i) Outline the core ray-tracing loop in pseudo code as you would implement it in a GLSL fragment shader. Also show how to set up rays and how to **terminate** the algorithm.

ii) Which values computed during ray tracing and intersection calculation would be suitable ray termination criteria?

d A ray originates at point $V = (3, 5, 15)$ and has a direction vector $d = (-4, 2, -1)$. The points of a triangle are given as $P_1 = (6, 4, 8)$, $P_2 = (-6, 10, 5)$, and $P_3 = (-6, -10, 7)$. Calculate whether the ray intersects the face defined by the triangle.

The four parts carry, respectively, 35%, 15%, 30%, and 20% of the marks.