FINAL

IMPERIAL COLLEGE OF SCIENCE, TECHNOLOGY AND MEDICINE

EXAMINATIONS 2018

BEng Honours Degree in Computing Part III
BEng Honours Degree in Electronic and Information Engineering Part III
MEng Honours Degree in Electronic and Information Engineering Part III
MEng Honours Degree in Mathematics and Computer Science Part IV
BEng Honours Degree in Mathematics and Computer Science Part III
MEng Honours Degree in Mathematics and Computer Science Part III
MEng Honours Degrees in Computing Part III
MSc in Computing Science
MSc in Computing Science (Specialist)
for Internal Students of the Imperial College of Science, Technology and Medicine

This paper is also taken for the relevant examinations for the Associateship of the City and Guilds of London Institute

PAPER C317

GRAPHICS

Wednesday 21 March 2018, 10:00
Duration: 120 minutes

Answer THREE questions

Paper contains 4 questions
Calculators not required

1 Transformations, Projections and Shading

a A graphics scene is viewed from position $\mathbf{C} = (10, 10, 0)^T$. The view direction is towards the origin of the world coordinate system.

i) Derive the transformation matrix that transforms the scene to a viewer-centred coordinate system.

ii) Assume $f = 1$ and compute the transformation matrix that achieves perspective projection in the viewer-centred coordinate system.

b The graphics scene defined above contains a triangle with the following vertices defined in world coordinates:

Vertex	x	y	z
$\mathbf{P}_1$	3	1	-2
$\mathbf{P}_2$	1	5	-2
$\mathbf{P}_3$	2	3	2

i) Calculate the Cartesian coordinates of the triangle after projection.

ii) Calculate the normal vector of the triangle that is facing *towards* the viewer.

c The graphics scene above is illuminated by a single point light source and rendered using the Phong illumination model.

i) Using an equation, explain the Phong illumination model. Make sure that you identify the three terms that correspond to the ambient, diffuse and specular components of the illumination model. Explain the geometric meaning of all vectors that appear in the equation.

ii) Assume that the light source is located at the same position as the viewer. Calculate the diffuse illumination at the triangle at the three vertices. You can assume that the diffuse material property of the triangle is $k_d = 1$, that the light source has intensity $\Phi = 1$ and that there is no attenuation between light source and the triangle (*i.e.* perfect vacuum).

The three parts carry, respectively, 30%, 35%, and 35% of the marks.

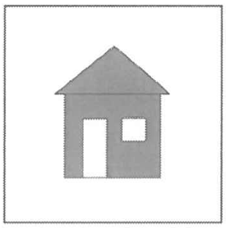
2 Warping

a Assume a Beier-Neely warp that is controlled by one line segment between $\mathbf{Q}$ and $\mathbf{P}$.

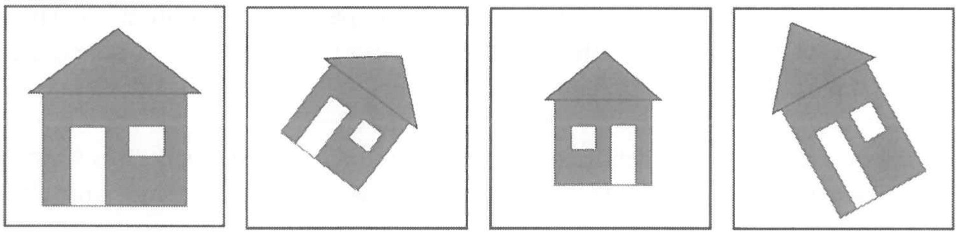
i) Derive the equation that determines the location of point $\mathbf{X}$ after the line segment has moved to $\mathbf{Q}'$ and $\mathbf{P}'$.

ii) Show that any transformation consisting of rotation and translation can be achieved as a Beier-Neely warping with exactly one line segment.

b Consider the image below:



For each of the four images below draw the simplest controls for a Beier-Neely warp that would achieve these transformations.



c A particular Beier-Neely warping is defined by two lines: Line $\mathbf{l}_1$ goes from $(1,1)$ to $(5,1)$ and line $\mathbf{l}_2$ goes from $(1,5)$ to $(5,5)$. After warping line $\mathbf{l}_1$ has not been changed but line $\mathbf{l}_2$ so that it goes from $(1,5)$ to $(5,6)$. Given three points $\mathbf{X}_1 = (1,3)$, $\mathbf{X}_2 = (4,4)$ and $\mathbf{X}_3 = (5,3)$, calculate their new position after warping.

You may use a sketch to determine the quantities u and v relatively to the control lines geometrically. You can further assume that the contribution of each line pair to the warping is equal.

The three parts carry, respectively, 40%, 30%, and 30% of the marks.

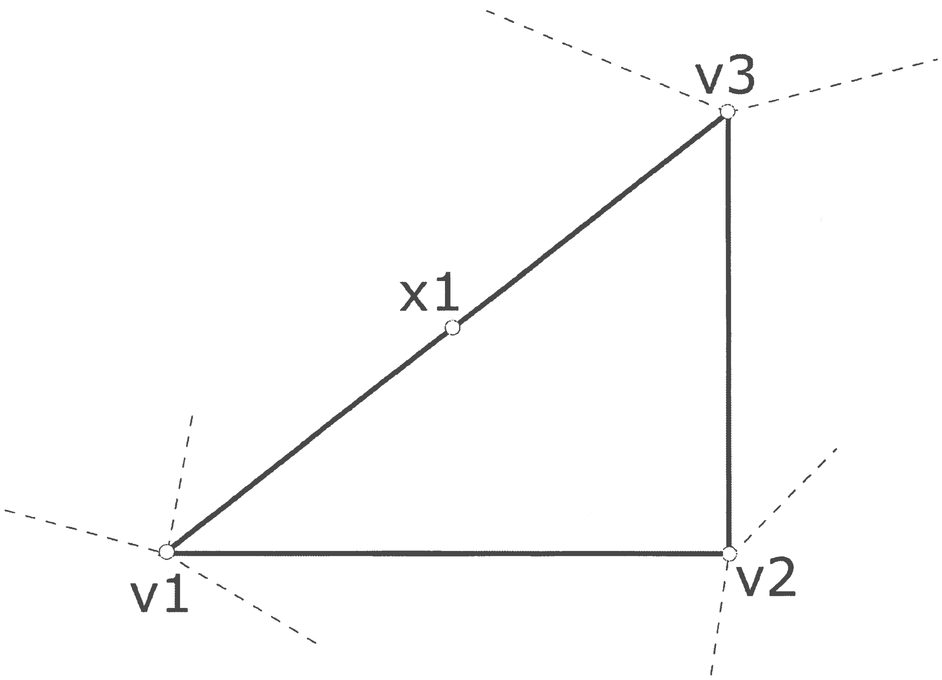


Fig. 1: One highlighted triangle in a triangle mesh.

3 Graphics primitives

Figure 1 shows a 2-simplex (triangle) being part of a polygon mesh. The triangle Δ is defined by three vertices v_1, v_2, v_3. These vertices are connected counter-clockwise with edges.

a For a naïve surface subdivision algorithm it is required to split this triangle into four smaller triangles. For efficiency reasons the smaller triangles have to be arranged using a connected triangle strip.

Draw the triangle from Figure 1 in your answer book and sketch a subdivision strategy that results in an OpenGL triangle strip, *i.e.*, a single connected path, that fully covers the area given by triangle Δ. Clearly enumerate your steps in the sketch, start the triangle strip at v_1 and indicate the direction of your path with arrows. Degenerated triangles are allowed!

b Point coordinates can be defined based on the triangle in Figure 1 in various ways.

i) Name two common strategies to characterise point coordinates relative to a triangle.

ii) What is the difference between these two strategies?

iii) Show a simple way to convert between both.

iv) What is the biggest advantage of point coordinates defined relative to a triangle?

c The coordinates of the vertices v_1, v_2, v_3 in *Cartesian* coordinates are: $v_1 = (0,0,0)$ $v_2 = (2,0,0)$ $v_3 = (2,2,0)$.

Convert v_1, v_2, v_3 to *barycentric* coordinates. Another point is given in *Cartesian* coordinates as $x_1 = (1,1,0)$. Convert this point into *barycentric* coordinates relative to Δ. Show your calculation.

d Is $p_c = (1,2,0)$ (*Cartesian* coordinates) contained inside Δ when orthographically projecting both on the xy-plane? Justify your answer. What is the distance of p to the centroid of Δ in *barycentric* coordinates?

e How can *barycentric* coordinates be used for linear interpolation during triangle rasterization? Show an example interpolating a normal vector for x_1 using *barycentric* coordinates. The normal vectors at the vertices are $v1_n = (-1,0,0)$, $v2_n = (-1,-1,0)$ and $v3_n = (0,1,0)$

f Outline a simple triangle subdivision algorithm that allows splits into arbitrary numbers of sub-triangles in pseudo code. Do not forget to show the necessary steps for correct colour and normal-vector interpolation of newly generated vertices.

The six parts carry, respectively, 20%, 20%, 20%, 10%, 10%, and 20% of the marks.

4 Textures

Raster images can be used to enhance the details of surface meshes. The process of virtual wrapping of images around polyhedral geometry is called *texture mapping*.

a i) Sketch a common graphics pipeline.

ii) Identify the part in the graphics pipeline during which texture mapping is performed.

iii) Which coordinate transformations are required to convert from the identified part in the graphics pipeline to two-dimensional texture space? Name the used coordinate axes (x, y, z), (u, v, w), and (s, t, p) according to the commonly used naming convention for texture mapping.

b Textures are usually sampled from a normalised coordinate space mapped to $(0 <= s <= 1, 0 <= t <= 1)$. Name four common schemes to handle out-of-bounds samples when texture coordinates are sampled $(s < 0 || s > 1, t < 0 || t > 1)$.

c When texture coordinates are added as additional parameters to vertices, these parameters are interpolated during the rasterization stage like any other payload parameter (colour, normal, etc.).

i) Will such an interpolation result in correct texture coordinates for all fragments? Justify your answer.

ii) Name two different and valid schemes to convert screen space fragment coordinates to texture coordinates.

d Two points p' and r' are given that have been projected onto a viewing plane from $p = (x_p, y_p, z_p)$, $r = (x_r, y_r, z_r)$ and a point q' on the projected line between p' and r'. How would you calculate the (s,t) texture coordinates of q' using perspective correct interpolation if $f = 1$ in the projection matrix. You can either derive the process mathematically or describe it in bullet points.

e A point p is defined as $p = (1, 3, 3)$ and a point r as $r = (4, 0, 3)$ **in world coordinates**. p and r are projected to $p' = (1, 3, 3)$ and $r' = (1, 1.5, 3)$ on a viewing plane, which is defined **in world coordinates** by a point $a = (1, 1, 0)$ and its normal vector $n = (1, 0, 0)$. Focal length is $f = 1$ and the position of the eye point is at $c = (0, 2, 3)$ **in world coordinates**.

p carries the texture coordinates $p_{st} = (1, 0)$ and r carries the texture coordinates $r_{st} = (0, 0)$. Calculate the correct texture coordinates for a point q

that is on a line between p and r. The projected point q' yields the texture coordinates $q'_{st} = (0.25, 0)$ after rasterization. What's the perspectively corrected, normalised texture coordinate for q_{st}?

You might find it helpful to draw a sketch in 2D of a projection of the scene.

f Describe *bump mapping* using three bullet points.

i) what is needed?

ii) what is manipulated?

iii) what is the aim of this technique?

The six parts carry, respectively, 20%, 20%, 10%, 20%, 15%, and 15% of the marks.