Logic

Q1 i) $(p \vee q) \vee \neg(p \wedge q)$

ii) $p \to (q \vee (\neg(\neg r)))$

iii) ~~$((p \vee q) \vee \neg(p \wedge q)) \leftrightarrow r$~~ $p \to q$

iv) $r \to ((p \wedge q) \leftrightarrow ((\neg s) \vee (\neg t)))$

v) $(\neg(p \to q)) \wedge r$

correspodense

Q2 a) i) p: everyone's ordering pizza

q: everyone's having coffee

ii) p: Umberto is here

q: Angela arrived on time

r: Elena decide to come

iii) p: agent knows either optimized or lights on

q: robot swing arm

iv) r: she's the one withdrew them

p: Jacob read The Age...

q: Jacob read Invisible...

s: Jolene return The Age

t: Jolene return Invisible

v) p: you arrive on time

q: Kate arrive on time

r: she's not coming

for any evaluation v,

Q2 $A \models B \to \bot$, so if $h_v(A)$, then $h_v(B \to \bot)$ true

so when $\boxed{h_v(A) = t}$, $h_v(B \to \bot)$ must be t

that is $h_v(B) = f$ or $h_v(\bot) = t$

$\hookrightarrow \boxed{h_v(B) = f}$, ~~so map A to t, B to f~~

~~when $h_v(A) = f$,~~ ⊛

$B \to \bot \models A$, so if $h_v(B \to \bot) = t$, then $h_v(A) = t$

$h_v(B \to \bot) = t$ is $h_v(B) = f$,

so when $\boxed{h_v(B) = f}$ $\boxed{h_v(A)}$ $\boxed{\text{must be } t}$ ⊛

In total we have 4 possible hv, $hv_1 \sim hv_4$

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$hv_1(A) = hv_1(B) = t$, by ⊛, not possible
$hv_2(A) = hv_2(B) = f$, by ⊛⊛ not possible
only hv_3 ~~A~~ : $A \mapsto t, B \mapsto f$,
hv_4 : $A \mapsto f, B \mapsto t$ are possible
thus for any possible V, $h_V(A) \neq h_V(B)$
$\Rightarrow$: so $A \leftrightarrow \neg B$ is ~~always~~ ~~sat~~ evaluate as true
$h_V(\neg(A \leftrightarrow \neg B)) \equiv f$, not satisfiable
$\Leftarrow$: by conter positive at least one of
~~if not $h_V(A) \neq h_V(B)$~~ hv_1, hv_2 above are possible
then when evaluate a V such that $h_V(A) = h_V(B)$
$h_V(\neg(A \leftrightarrow \neg B))$ can be evaluate as true

ii) $h_V(A_1 \vee A_2) \neq h_V(B_1 \vee B_2)$, $\forall V$
$h_V(A_1)$ ~~=t~~ $\vee\ h_V(A_2)$ ~~=t~~ $\neq h_V(B_1)$ ~~=t~~ $\vee\ h_V(B_2)$ ~~=t~~

~~A_1 A_2 B_1 B_2~~
~~V_1 V_1 t t f f possible when A_1 true~~

	A_1	A_2	B_1	B_2	$\neg B_1 \wedge \neg B_2$
V_2 V_2					
V_3 V_3	t		f	f	$t \wedge t = t$
	~~t~~		~~f~~	~~t~~	
	~~t~~			~~t~~ f	

← ~~t~~ by antisatisfied, A_1 true
only ff is possible for B_1, B_2
and this make $A_1 \models \neg B_1 \wedge \neg B_2$ valid

Q3)

	p	q	r	$\neg(((q \to \neg r) \wedge (\neg p \vee q) \wedge \neg(\neg\neg p \wedge (p \to \neg r)))$
$\Rightarrow$	f	f	f	f f t t t t t f t t f f f t t
$\Rightarrow$	f	f	t	f f t f t t t f t t f f f t f
$\Rightarrow$	f	t	f	f t t t t t t t t t f f f t t
	f	t	t	t t f f f t t t t t f f f t f
	t	f	f	t f t t f f f f f f t t t t t
	t	f	t	t f t f f f f f f t t f t f f
	t	t	f	t t t t f f t t f f t t t t t
	t	t	t	t t f f f f t t t t t f t f f

no valid ttf, tft, ftf, fft

Q4 i) a) both b) both c) neither d) neither
e) neither f) CNF g) neither

ii) a) r is pure, so remove {p, q, r}
now q is pure negative, only {{p, s}, {p, ¬s}} left
p is pure {}

b) {r} is unit, so
{{p}, {q, s}, {¬s, ¬p}, {¬p, ¬q}}
now {p} unit, so
{{q, s}, {¬s}, {¬q}}
{¬s} unit, {{q}, {¬q}} ⇒ {{}}

Q5 a) i) $p \vee q$
ii) $\neg q \vee r \vee \neg s$
iii) $(\neg p \wedge s) \vee \neg q \equiv (\neg p \vee \neg q) \wedge (s \vee \neg q)$
iv) $p \vee \neg r$
v) $s \vee p$ vi) $p \wedge s$ negation is $\neg p \vee \neg s$
{{p, q}, {¬q, r, ¬s}, {¬p, ¬q}, {s, ¬q}, {p, ¬r}, {s, p}} {¬p, ¬s}
consider p, ¬p, if p true
{{¬q, r, ¬s}, {¬q}, {s, ¬q} {¬s}}
q is pure so {} valid

Q6 i) Yes ii) Yes
iii) Yes iv) syntax error
v) Yes vi) no , f(X, X) not output true false
vii) Yes

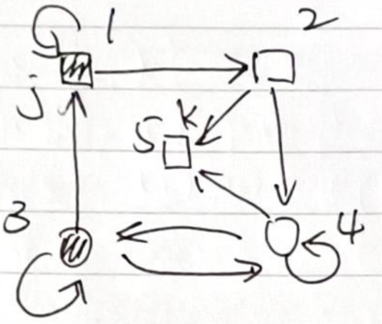
Q7 i) C={} P_1={painter, Paris, London} P_2={color}
$\exists X \forall Y$ painter(X) color(X, Y) Paris(X)
$\rightarrow \neg \exists Z \forall Y$ painter(Z) ~~color~~ London(Z)
painter(X):= X is painter color(Z, Y) color(X, Y) := X use Y

ii) C={Charlie}, P_2={love}
$\forall X \exists Y (\neg X = Charlie) \wedge (\forall Z\, love(Y, Z) \rightarrow love(X, Y))$
$\wedge$ love(Charlie, W) love(X, Y) := X loves Y

iii) C={}, P_1={student, coursework}, P_2={submit, grade}
$\forall X \forall Y$ student(X) coursework(Y)
submit(X, Y) $\vee$ grade(X, Y)
submit(X, Y) := X submit Y on time
grade(X, Y) := X recieve grade for Y

iv) C={Nadia B.} P_2={know, compose, study}
know(X, Y) := X knows Y
compose(X, Y) := X compose Y
study(X, Y) := X study with Y
$\forall X (\forall Y$ study(X, Nadia) $\wedge$ compose(Nadia, Y)
$\rightarrow$ know(X, Y)) $\wedge$
$\forall W \exists Z$ compose(W, Z) $\wedge$ study(W, Nadia)
$\rightarrow$ know(X, Z))

Q8 i)



i) see object 2, 4, 5

for 2, take $Y=5$, $a(2,5)$ ✓

for 4, take $Y=5$, $a(4,5)$ ✓

for 5, no such Y ✗ so false

ii) ~~consider negation~~

~~$\exists X \exists Y (\neg \exists Z (a(X,Z) \wedge a(Y,Z)) \vee (a(X,Y) \vee a(Y,X))))$~~

~~$\equiv \exists X \exists Y (\exists Z (a(X,Z) \wedge a(Y,Z)) \wedge \neg a(X,Y) \wedge \neg a(Y,X))$~~

$Z=1$ $a(1,1) \wedge a(3,1)$ ✓ $X=1, Y=3$

check $a(1,3) \vee a(3,1)$ ✓

$Z=2$

$Z=3$ $a(3,3) \wedge a(4,3)$ ✓ $X=3, Y=4$

check $a(3,4) \vee a(4,3)$ ✓

$Z=4$ $a(3,4) \wedge a(2,4)$ ✓ $X=2, Y=3$

check $a(2,3) \vee a(3,2)$ ✗ False

counter example

iii) only need to check $X=1,3$

for $X=1$, for $\exists Y \exists Z (a(Y,Z) \wedge a(Z,X) \wedge a(Y,k))$

only possible Z are 1, 3 by $a(Z,X)$

~~but no Y s.t. $a(Y,Z)$~~

take $Y=4$, $Z=3$. $a(4,3) \wedge a(3,1) \wedge a(4,k)$

so $\exists Y, Z$, false

iv) ~~possible X = 1,3,4 , by a(X,X)~~.

consider negation:

① $\forall X(\neg a(X,X) \vee \forall Y \neg \exists Z(a(X,Y) \vee a(Z,X) \rightarrow a(Z,k)))$

$\neg a(X,X)$ is true ~~1,3,4~~ 2, 5

so consider $X = 1,3,4$

See if $\forall Y \neg \exists Z(a(X,Y) \vee a(Z,X) \rightarrow a(Z,k))$ ②

~~for X=1 As X≠k~~

consider negation again

③ $\exists Y \exists Z(a(X,Y) \vee a(Z,X) \rightarrow a(Z,k))$ for $X = 1,3,4$

Y can be choose by X to make $a(X,Y)$ false

~~for 1, Y=2~~ so a Z

to make $a(X,Y) \vee a(Z,X)$ false

so the implication is true

for $X=1$, choose $Y=Z=5$

$X=3$, choose $Y=Z=5$

$X=4$, $Y=Z=1$ □

by this we see ③ true

so ② is false for all $X=1,3,4$

so statement ~~false~~ ① is false

so the statement is true